On Queue Audience:

Calculating Reach and Frequency for Supermarket Television

by

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A.B. in Mathematics, Bowdoin College
(1988)

Submitted to the Sloan School of Management
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Abstract

This thesis is intended to define a new means of calculating the marketer's audience measures of reach, frequency, and gross rating points (GRP) from a new advertising medium. It is intended to, for perhaps the first time, blend marketing science and queueing theory.

Consider televisions mounted at the checkout lanes of a supermarket which broadcast specialized programming. Customers waiting in queue constitute a captive audience for advertisers. The crucial input to determining audience size is the time the customers spend waiting in the queue: the longer they wait the more likely they are to see an advertisement.

Three different models are considered to illustrate how the supermarket queue behavior can be studied and the waiting times determined: (1) M/M/1 queue model, (2) M/M/k queue model, and (3) a derived model which combines elements of the M/M/1 and the M/M/k models. Realistic data, based on an actual test of this technology, show the most optimistic values of reach, frequency, and GRP result from the M/M/1 model.

Thesis Advisor: Richard C. Larson
Professor of Electrical Engineering and Computer Science

Acknowledgments

This thesis would never have happened if Dick Larson hadn't given me the opportunity to be a part of the Checkout Channel Project and free rein to write this paper. And while I may never forgive him for making it impossible for me to go to the grocery store without feeling like I am collecting scientific observations, I can't thank him enough for the chance to present the work in Norwalk. For almost a year now I have quite literally eaten, drunk, and slept grocery stores. It has been a pleasure; I am glad to have had the chance.

Though they weren't directly involved in this thesis, there are people in the OR Center who made my life bearable for the last two years. Thanks go to Karla for never letting me give up on myself and for letting me cry and celebrate in her office; to Tom, Mitchell, and Mark for keeping me on my toes; to Kim, Kerry, Martha, and Suguna for those women's bonds; to Rob for relieving the extra 6.041 stress this spring; to Joe for helping me understand it all. Thanks go to Michelle and Laura who got me through the paper work and the long summer. Thanks to Al Drake, especially, for believing in teaching and me.

Outside the OR Center are some very special people who experienced MIT inadvertently. Thanks, Jane, for doing my chores when I needed it most and taking me shopping when we could afford it least. Thanks to Nina, Becky, Charlie, and Clare for being able to live with me. And even thanks to Nate and Jack, my surrogate children.

Last but far from least: thanks, David, for absolutely everything: from the tough decisions to the Computer Museum. Thanks most for lightening and varying my merry but logical load.

Contents

Chapter 1

Introduction and Background

Section 1.1: Introduction

In the 1990's, American businesses trying to reach the buying public are faced with two increasingly important marketing issues. While neither of these issues is new, new difficulties and emphases are arising. Innovative solutions are required. These two growing challenges are the ability to reach the consumer with advertisements and the rising demand for customer service.

The need for new and creative means of advertising is expanding. Network television was once considered the surest way of reaching the American buying public, but recently the overall level of ratings as reported by Nielsen Media Service has fallen. This drop in the ratings of network television has sparked concern over both Nielsen's ability to accurately measure the audience size [Schlossberg, 1990b] and the American people's switch to cable and VCR's for their entertainment [Henke and Donohue, 1989]. In addition to network spending, advertisers are now having to spend for cable ads as well [Katz and Lancaster, 1989]. The combination of remote controls and VCR's means ads during regular network programs may be 'zapped' (the rapid changing of channels during an ad or the muting of the sound) and during the watching of taped programs, commercials may be 'zipped' (fast-forwarding the tape through the commercials) [Stout and Burda,1989; Kaatz, 1987; Marketing News, May 9, 1990], thus reducing the effectiveness of the ad. Further there is the issue of using VCR's to watch rented 'theater' movies, which displace television watching and have no advertisements [Sims, 1989].

No one with any connection to television or advertising is happy with the current state of affairs. The networks returned almost $200 million dollars to advertisers from their 1989 - 1990 spending for refunds for advertisements during programs when the viewership fell below the guaranteed level [Advertising Age, March 11, 1991]. There is increasing hesitation on the part of advertisers to commit to network television ads [Advertising Age, Feb 11, 1991]. Clients of advertising agencies are demanding the more creative use of their advertising budget [Advertising Age, March 4, 1991].

More creative advertising ideas have begun to be visible. The home-shopping channels are like the point of sale items at the grocery checkout, playing up to the impulse buy. Print ads on grocery carts act like little billboards, trying to reach the subconscious or trigger the forgotten desire to purchase the item [Schuman, *et al*, 1991]. The scratch-'n-sniff cards or the scented cards in magazines are really free samples, an old marketing technique [Smith, 1989]. Thus old ideas are being turned around with new technology to provide new ways of reaching the consumers. The mass media, mass marketing approach is no longer the norm, but rather more specific targeting is on the rise [Light, 1990].

Advertisers need more original ways to reach consumers. Almost equally important is the need that these new ways be measurable. Advertisers want to know who they are reaching with their messages and how often.

On top of these challenges, businesses are also being faced with greater demands for customer service. The buying public is no longer concerned with just the lowest price, but rather is demanding increased levels of attention from sellers. Japanese competition comes not just from products, but from the services they offer [Schlössberg, 1990a]. New ways to satisfy customers in this country are being implemented [Michaelson, 1990; Miller, 1990]. The installation of Automatic Teller Machines to increase the banking hours and decrease the lines for live tellers are a prime example. The increasing number of money-back guarantees and the posted store policies of providing more cashiers if the lines reach a certain threshold length are the beginnings of attempts to satisfy customers in non-price related ways. Whereas the 800 number for questions and comments used to be a rarity, it is now the norm for many consumer product companies [Major, 1990]. Articles in the professional journals about service and the relationship of service to continued business by a consumer are more visible in recent editions; for example, two articles on service appeared in the 1990 Journal of Marketing, while there are none in 1989 or 1988 [Bitner, Booms, Tetreault, 1990; Bitner, 1990]. An entire issue of Marketing News was devoted to articles on service

[Marketing News, May 28, 1990]. The articles document attempts to improve service in everything from car sales to fast food sales.

It is against the backdrop of these new business challenges that the concept of placing televisions at supermarkets was born. If customers waiting in line to pay for their groceries could be exposed to advertisements which they could not turn off or mute, and if at the same time they could be distracted by specialized programming so that they felt they waited less time in line, then both the buyers, the sellers, and the store managers could be satisfied.

Groceries stores would provide a sizable audience for the advertiser. The average grocery store completes 12,000 transactions a week in this country. The primary shoppers are still women, aged 24-52, who are married and have children [Pagano, 1990]. Ninety percent advertisements during prime time network television programs are targeted at men and women between the ages of 18 and 54 [Advertising Age, April 22, 1991]. Thus almost half of the prime time audience purchasers are also at the grocery store. Moreover, studies have shown that 65% to 81% of buying decisions are actually made in the supermarket [Schuman, *et al*, 1990]. Thus for the advertiser there is an audience in the process of trying to decide what to buy at the moment the advertisement is running.

The idea of placing television and advertisements where people wait is not entirely new. The United Airlines "Air Report" which is shown on board flights is really the same idea. Students in Canada have also brainstormed the idea of televisions in supermarkets. * Turner broadcasting is also starting a television service in airports to appeal to customers waiting to check in at the ticket counters .

There is at least one precedent which leads one to believe these types of distractions can work. In a study completed in the Bank of Boston, customers waiting on line for teller service were exposed to "silent radio", a LCD rolling screen of news and information. It was found that customers who were videotaped and surveyed greatly reduced their perceived waiting time while the silent radio was in operation [Katz, Larson, and Larson, 1990]. Thus there is a reason to believe this new technology can not only succeed for advertisers, but for consumers as well.

In this paper, I shall concentrate on the potential of this new medium for the advertiser. Just how much audience exposure could an advertiser expect? How many people will see the advertiser's ad in a week or in six months? How many people will see it twice or three times in a week or a month? These issues are important to the advertiser for determining how much she is willing to pay for the ad. These issues also come into play in determining the relationship between the level of exposure and increased sales. Thus the flow of traffic through the supermarket and the time the customer spends waiting in line (when the customer is available to watch the television) are crucial.

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* The student authors of a paper regarding their ideas for this concept sent a copy of their report to Richard Larson, who in turn shared it with the author of this paper.

The work here is an attempt to blend marketing science and queueing theory to develop new formulations of old definitions. Marketers measure audiences with the concepts of reach, frequency, and gross rating points. The goal of this paper is to present some illustrative ways of approaching new definitions of these concepts through the use of queueing theory. The project is motivated by a real life test of the Checkout Channel, completed in the fall of 1990.

Section 1.2 of this chapter defines and discusses how reach and frequency have been calculated in the past. Section 1.3 describes the Pilot Test of the Checkout Channel and the differences between the Checkout Channel advertising situation and advertising through other mediums. Section 1.4 of this chapter will discuss the data used in this paper. Chapter Two will discuss the formulation of each model, as well as its strengths and weaknesses and some general concerns regarding modelling the supermarket checkout system. The chapter will also derive the reach and frequency equations in the general situation and then the specific formulations under the models. Chapter Three contains the empirical results of the three models and a discussion of the findings, as well as some consideration of how the results would change with changes in the parameters. Chapter Four gives an overview of the work, discusses conclusions, details other places this type of technology could be applied, and makes suggestions for further research.

Section 1.2: Reach, Frequency, and Gross Rating Points

Reach, frequency, and gross rating points (GRP) are the marketer's measures and evaluators of the audience of an ad. Those who sell advertising space in magazines, on television or on radio base their prices on the values of the reach, frequency, or GRP numbers [Katz and Lancaster, 1989]. While the concepts of what each number is trying to measure are fairly well understood, there is little agreement on the methods of calculation.

Section 1.2.1: Definitions

Consider a particular ad which is placed in an advertising schedule; for example, six issues of a monthly magazine or once during each evening's episode of 'Jeopardy.' Let *x* be a discrete, non-negative variable which represents the number of times the ad is seen by an individual or a household in a particular time period. Let *f(x)* be the number of individuals or households who see the ad *x* times in the time period in question. The function, *f(x)*, is defined to be the frequency distribution of the ad. When the total size of the target population of the ad, call it *N*, is known, the frequency distribution is sometimes written as $\frac{f(x)}{N}$ for each *x* and thus resembles a probability mass function.

When marketers use the term frequency, they are usually referring the average frequency of individuals or households who saw the ad. This quantity answers the question, Of people who saw the ad, what is the average number of times any one of them saw the ad? The average frequency is equal to

$$\frac{\sum_{x=1}^{\infty} f(x) \cdot x}{N - f(0)}$$

When $\frac{f(x)}{N}$ is thought of as a probability mass function, average frequency is equal to $E(x \mid x > 0)$.

Reach is defined to be the total number of individuals or households who saw the ad at least once in a given time period. Using the notation already defined, reach is equal to

$$\sum_{x=1}^{\infty} f(x) \qquad \text{or} \qquad N - f(0)\,.$$

Often reach is expressed as a percentage of the total audience, or $\dfrac{\sum_{x=1}^{\infty} f(x)}{N}$.

Occasionally marketers are interested in the number of people who saw the ad at least twice or at least thrice. Effective reach is the term used in these cases. Effective reach at level *i* is equal to

$\sum_{x=i}^{\infty} f(x)$ or, in terms of percentages, is equal to $\dfrac{\sum_{x=i}^{\infty} f(x)}{N}$.

The level of the effect is the minimum number of times the ad must be seen to be counted in the 'reached' group.

The third measure of interest is the gross rating points (GRP). GRP is often used to price advertisements. GRP is the total number of exposures to the advertisement in the time period of interest. Therefore, GRP is equal to

$$\sum_{x=1}^{\infty} f(x) \bullet x\,.$$

While reach is concerned with the number of people exposed, GRP is concerned with the number of exposures, regardless of how many people make up the exposed group. Frequency is the merger of the two concepts. A good summary of these measures can be found in Dickson, 1991.

Section 1.2.2: Methods of Calculation

In the past, reach and frequency have been calculated in a number of ways. The expected GRP of an advertising schedule can be obtained from media rating services, like Nielsen for television, and thus if the reach figure is known, a simple division of GRP by reach gives the average frequency:

$$\frac{\text{GRP}}{\text{Reach}} = \frac{\sum_{x=1}^{\infty} f(x) \bullet x}{\sum_{x=1}^{\infty} f(x)} = \frac{\sum_{x=1}^{\infty} f(x) \bullet x}{N - f(0)} = \text{Ave. Frequency}$$

Thus many authors have worked on estimating reach accurately [Agostini, 1961; Caffyn and Sagovsky, 1963; Metheringham, 1964; Young, 1972; Friedman, 1971; Cannon, 1983]. Yet it has been pointed out by other authors that this type of analysis does does not provide the full frequency distribution. Clearly there are benefits from the additional information that the full distribution provides, as opposed to the information the mean alone provides. For example, it may be important to a company to know how many people saw the ad for a product at least three times, for three exposures may be the level considered necessary for inducing the customer to try the product. Thus while knowing the gross rating points and the reach are important, they are not sufficient information, and more calculation is necessary to fully understand frequency.

A variety of methods have been used to evaluate the full frequency distribution. Some methods for determining frequency in magazine advertising have used survey data to estimate the probability that an individual saw the ad in question [Greene, 1970]. Then the distribution of exposures is determined by a binomial distribution or Monte Carlo simulation. For television advertisements, the negative binomial and the beta-binomial have been used to model the exposure of households to the advertisements. In the latter model, the beta distribution gives the probability that any portion of the audience is exposed to the ad and the binomial distribution gives the percentage which is exposed once, twice, etc. Other models begin by estimating the probability of an individual seeing the ad and extend from there [Greene, 1970]. Other specialized techniques like the Modal-2 , the Metheringham method,the Kwerel-Geometric, and the Hofmans-Geometric [Headen, Klompmaker, and Teel, 1976; Leckenby and Kishi, 1982a] have been studied and improved upon over the years. A good review of the various models and the way they are viewed by media directors can be found in Leckenby and Kishi, 1982b and Leckenby and Boyd, 1984.

Section 1.3: The Checkout Channel Pilot Test

The Checkout Channel Pilot Test was completed in the Fall of 1990 as a joint venture between Turner Broadcasting and Actmedia, Inc., a major, national supermarket chain marketer. For six weeks, a satellite feed of news, weather, sports, feature pieces and advertisements was supplied by Turner Broadcasting , to twelve supermarkets around the U.S. Television monitors were placed at the checkout lanes so as to be seen by the people standing in line to have their groceries totaled. A small speaker was placed just above the conveyor belt, with two sound levels (loud and soft), but no switch which would allow it to be shut off.

Section 1.3.1: Monitor and Speaker Placement

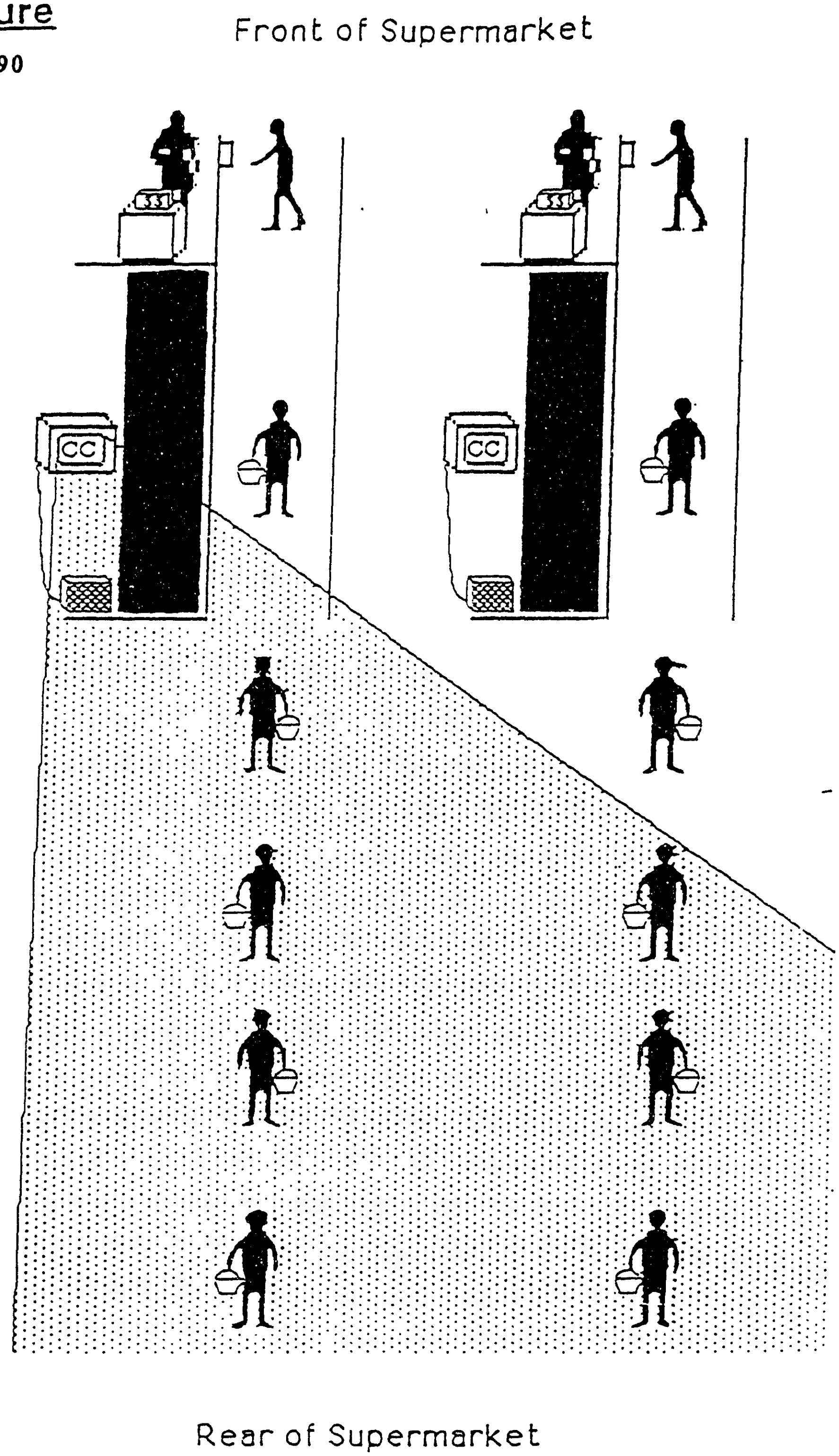
Clearly the placement of the monitors at the lanes will have a profound impact on the effectiveness of this medium. The monitors were placed about six feet high on the left side of each lane above the magazines and point of sale items. They were angled so as to be particularly visible to the second person in queue and the people behind him, as is shown in Figure 1.1. Once in service, a customer would have a difficult time actually seeing the monitor, without backing up into the lane. The angle of the monitors also allows a customer in queue to see the monitor in the next lane to his left. Observation of customers in the test stores showed that for the third or fourth customer in queue, especially taller customers, it was actually preferable to watch the monitor one lane to the left. The angle was a bit less acute and appeared to be more comfortable.

For customers still shopping in the store, when moving down the main horizontal aisle (the one which runs perpendicular to the checkout lanes and forms the queueing area), the televisions were quite visible when queueing was low. I observed one older gentleman leaning against a display case at the end of an aisle watching the Channel and guarding the shopping cart while his wife moved about the store gathering the groceries. Thus customers may be exposed to the Channel for more time than simply their queue time.

Figure 1.1:

Video Exposure

Hass-Hill, 1990



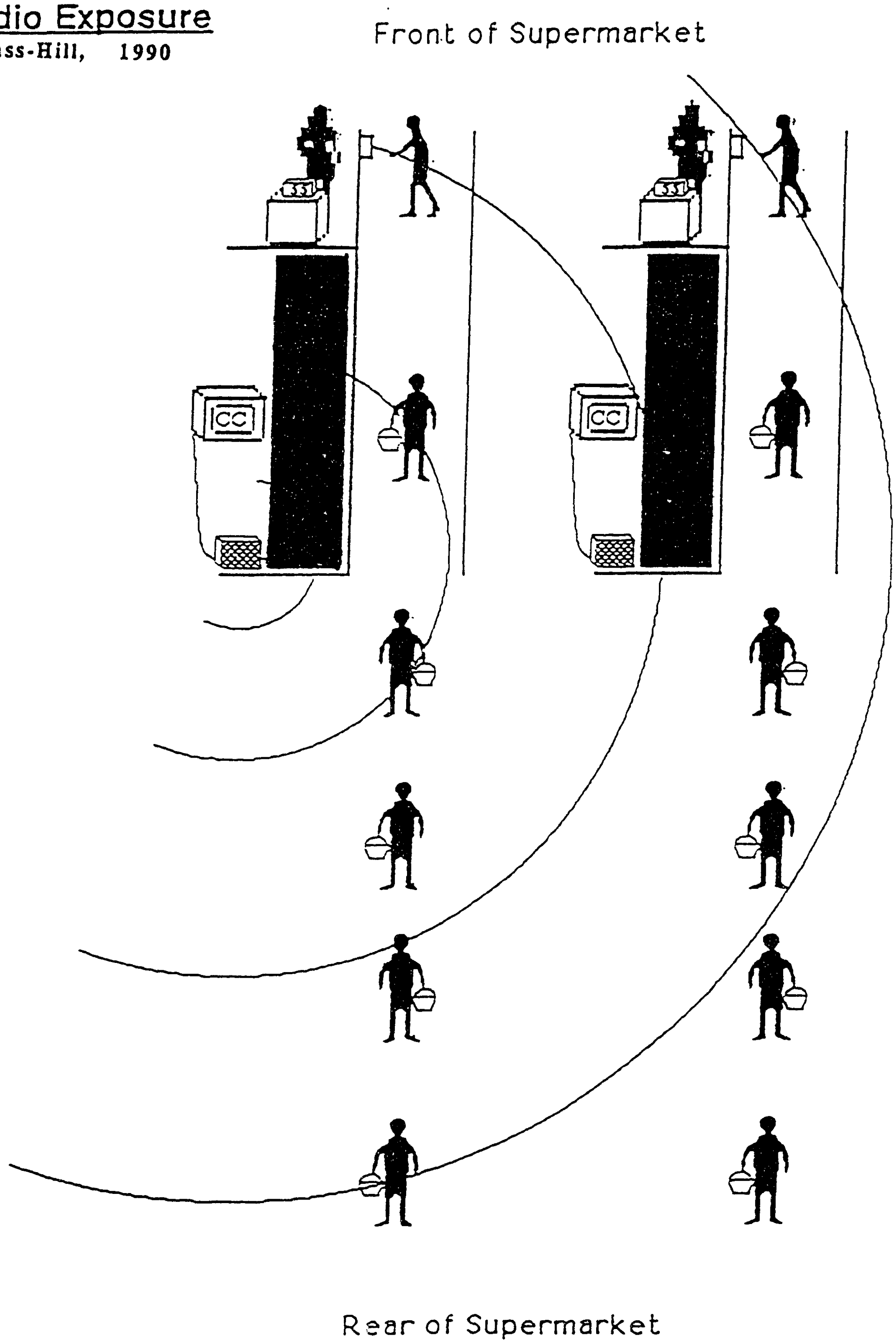
The speaker was also angled to most effectively reach the customers in queue. The intent was to avoid distracting the cashiers any more than absolutely necessary, and thus the single speaker for each lane was placed at the end of the conveyor belt facing the queueing customers, as is shown in Figure 1.2. The design was to have the sound only be audible in a four to six foot radius from the speaker [Schlossberg, 1990d]. The first and second customers in queue thus received the greatest impact of the speaker in their lane, but the overlap of the speakers allowed customers further along in the queue to also hear the audio. The first or second customer in queue could reach the small button on the bottom of the speaker to adjust the sound level to high or low.

The speakers also had sensors. If no customer arrived to the queue to trigger the sensor, the volume dropped to virtually inaudible. When a customer arrived to the lane, the sensor triggered the sound to return to the lower of the two settings. In this way, when a lane was closed, its speaker was not adding to the din of the store or overwhelming the cashiers who were busy. The more lanes idle, the lower the overall sound. The more lanes busy, the greater the overall level of sound which helped compensate for the noise of the additional customers.

Section 1.3.2: Programming

The format of the programming was quite similar to that of Turner's CNN/Headline News program, with some additional consumer information particularly aimed at grocery shoppers. Because the programming was supplied by satellite and not a taped version, it could be updated frequently or pre-empted to cover news of immediate importance. The programming was

Figure 1.2:
Audio Exposure
Hass-Hill, 1990



updated hourly for the test. The programming changed slightly over the course of the day to reflect the differences in the shopping population at the various times of day: programming aimed at parents during the day, programming aimed at single adults or adults with no children in the evening, family programming on the weekends. The feed was available twenty-four hours a day, seven days a week for the length of the test.

Section 1.3.3: Data Gathered in the Pilot Test

Data gathered included videotape from seven test stores (with programming) and two control stores (without programming). Survey data were also gathered from each of four waves of surveys, which occurred before, during (2 waves) and after the test, at a minimum of sixty surveys completed per store per wave. Most of the stores' management also provided the number of transactions completed each hour of the day for a week during the test. These data were used to examine the length of queueing in the stores, understand the overall level of business and customer activity in the stores, and determine if customers who watched the televisions had a perceived waiting time lower than those who did not watch the televisions.

This situation provides some unique opportunities for study, as well as unique opportunities for the advertiser. For the advertiser, there is a captive audience which cannot turn off the monitor, cannot mute the sound and cannot fast-forward through the ad. There is clearly an audience with money to spend in grocery stores and thus is an audience advertisers would like to reach. Moreover, beyond just the shopper, there is the potential of the additional companion to the shopper: a spouse, friend or child over the age of sixteen, who though while not purchasing at that moment, is a valuable target for the advertiser. This issue will be discussed further in the following chapters. Thus we have a large, desirable, captive audience for the advertiser.

The supermarket environment also provides the researcher with a unique opportunity. Many of the weaknesses of the previously mentioned models for calculating reach and frequency have come from their inability to predict audience size. Often there is a reliance on outside sources, like the Nielsen ratings service or magazine readership studies, to determine the potential audience size. Thus there is the possibility of error from two sources: the model itself and the data fed into it. However, the grocery store world is different. Whereas magazine readership and television audience size may need estimation, we at least have the potential audience size from the number of transactions the store completes. The only estimation we may need is the estimation of the contributing effect of the shoppers' companions. Thus we need not worry about inaccurate data for our model, at least in terms of potential audience size component.

Section 1.4: Data for this Work

The data used in this paper are not the actual data which resulted from the Checkout Channel Pilot Test. Difficulties in getting permission to use the data, due to its proprietary nature, have prevented its actual use. However, using some of its patterns and some known industry standards, I have created examples for this work which are realistic and illustrative, but are not the results from any particular store in the study. The following guidelines were used:

1. The following are industry standards, as told to me by Actmedia researchers:

a) The average supermarket in the U.S. completes 12,000 transactions per week.

b) The average customer shops 2.3 times per week.

2. I have maintained the pattern found in many of the stores that the number of transactions per day has a small peak in the noon - 1PM hour and then a larger peak in the 5PM - 6PM hour, as shown in Figure 1.3 at the end of the section.

3. I have only considered stores to be open from 8AM to midnight. The late night hours are not considered in this study, because of the extremely small number of transactions which take place during those hours.

The primary data of importance for this work are the average number of transactions per hour, the number of lanes open per hour, and the average service time. These values are shown in Table 1.1. These averages total to 11,970 transactions per week, just under the industry average. I also take the average service time to be 90 seconds.

Once more before closing this section I stress that these are not the actual data, but are reasonable values considering the actual averages and the ranges of the values which resulted from the study.

Figure 1.3:
Pattern of Transactional Data

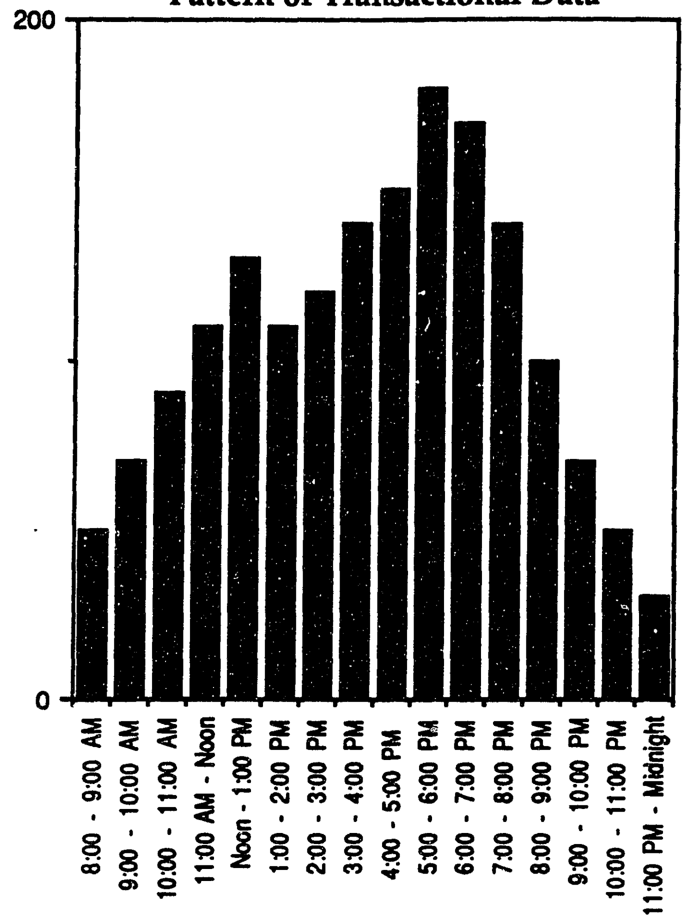


Table 1.1: Transactional Data and Number of Lanes Open by Hour of the Day

Time of Day	Average Number of Transactions	Number of Lanes Open
8:00 - 9:00 AM	50.00	3.00
9:00 - 10:00 AM	70.00	4.00
10:00 - 11:00 AM	90.00	5.00
11:00 AM - Noon	110.00	5.00
Noon - 1:00 PM	130.00	6.00
1:00 - 2:00 PM	110.00	5.00
2:00 - 3:00 PM	120.00	5.00
3:00 - 4:00 PM	140.00	6.00
4:00 - 5:00 PM	150.00	6.00
5:00 - 6:00 PM	180.00	7.00
6:00 - 7:00 PM	170.00	7.00
7:00 - 8:00 PM	140.00	6.00
8:00 - 9:00 PM	100.00	5.00
9:00 - 10:00 PM	70.00	4.00
10:00 - 11:00 PM	50.00	3.00
11:00 PM - Midnight	30.00	2.00

Chapter 2

Three Models of Supermarket Queueing

In this chapter three different models for a grocery store checkout system will be discussed and considered. The techniques for calculating reach, frequency, and GRP under the models will be outlined. As the key to these calculations is the distribution of waiting times of the customers, the equations will be derived in terms of a general waiting time distribution in Section 2.2, and then the specific cases under the different modelling assumptions will be demonstrated in Section 2.3. In section 2.1 I shall present the three models and their viability.

Section 2.1: The Three Models in Question and Their Viabilities

To characterize the checkout system at a supermarket, three different models were considered. The first two are somewhat standard models: the M/M/1 and the M/M/k queueing models. The third model arose from the inability of these two models to fully capture the idiosyncrasies of the queueing at a supermarket, and is an attempt at combining the two.

Section 2.1.1: The M/M/1 Model

Suppose we consider each checkout lane at a supermarket to be an M/M/1 queue. The number of customers who arrive each hour at the checkout lane to have their groceries totalled is a Poisson random variable with parameter λ_h , where *h* designates the hour of the day. There is one server or cashier at each lane whose service time for a customer is an exponential random variable, with a mean of 90 seconds. We shall assume that the distribution of service times does not vary with the hour of the day.

The M/M/1 model divides the traffic equally among the available lanes. Quite literally, the model requires dividing the total arrival rate by the number of open lanes and therefore assumes that the traffic is evenly distributed over the lanes. This model allows us to use the well-known results of the M/M/1 queue and allows the lanes to function independently, which is somewhat like how they actually behave in the supermarket.

While this division of the arrival rate may seem reasonable decision, it is not exactly how supermarkets work. Arriving shoppers are not assigned a checkout lane number at the entrance of the store which dictates which lane they must check out through, but the model assumes this sort of structure. It is possible that the the 'forcing' of customers into a line, as this formulation does, may in fact increase waiting times by not allowing faster services to be taken advantage of by waiting shoppers. If one queue empties, the shoppers in the other lines are not allowed to switch under this structure, and thus the model may over-represent the actual waiting times. Thus we may expect slightly higher waiting times, which will result in higher probabilities of seeing advertisements and thus overly-optimistic reach, frequency, and GRP numbers.

Section 2.1.2: The M/M/k Model

For the second model, consider all the checkout lanes together as one system, instead of considering each lane individually. Consider customer arrivals to the system of lanes to be Poisson, with rate λ_h which changes for each hour of the day and h designates the hour. In this model there are now k cashiers or servers available to all the customers. Service times are exponential random variables with a mean of 90 seconds. As with the M/M/1 case, we shall assume that service times are not dependent on the hour of the day.

This model considers the entire system of checkout lanes as a whole and thus allows for the possibility that customers may do better in terms of waiting than the 'assigned lanes' approach of the M/M/1 model. Thus in a sense it allows customers to continually change lanes to find the shortest waiting time. However, this model actually assumes that customers form one line to wait for the next available cashier. Supermarkets do not use these 'serpentine' queues, but rather shoppers choose their own lanes upon arriving to the checkout area. By assuming that customers form one line the model assumes the most efficient allocation of cashiers. Shoppers may not actually choose their lines this efficiently and thus this model may under-represent waiting times. If waiting times are underestimated then the reach, frequency, and GRP figures will also be underestimated.

Section 2.1.3: The Derived Model

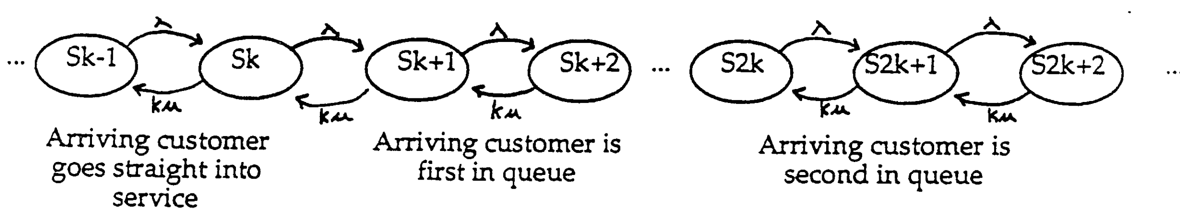
Because of the specific concerns of overestimating and underestimating waiting times by the M/M/1 and M/M/k models respectively, a third model was considered. If the M/M/1 and M/M/k model formed bounds for the true behavior of the supermarket queueing, perhaps a model which predicted waiting times somewhere in between could be developed. The following model was proposed.

Suppose customers are allowed to choose their own lane upon arriving at the checkout area. Customers must choose the lane with the least people in it and once chosen, the lane may not be changed. This procedure is fairly close to how customers actually choose their lanes at the grocery store. Further, assume that there are k servers available, or k checkout lanes open. To simplify calculation, assume that when there are k customers in the system, they are distributed evenly over the k available lanes. Thus we cannot have a situation in which there are more than k customers in the system and there is an empty checkout lane.

Consider the state transition diagram, Figure 2.1. Under this model, when the system is in state S_{k+3} then the assumption is that all k servers are busy and three of them have one person in queue. For example, when there are k+3 customers in the system, the situation of k-2 servers busy (two servers are idle) and five of them with queues cannot occur. These assumptions also imply that if there is a transition from S_{k+3} to S_{k+2} then a customer must have left one of the lanes with a queue. This assumption is not wholly unreasonable. If a customer has a choice of a number of lanes which have the same length queue, then the choice of which queue to join is arbitrary.

Similarly, when a service finishes, there is no reason to assume it is not a service by one of the servers with the maximum queue. There may be a combinatorial term for the number of servers which could be the appropriate one to finish, but I have assumed this to be a second order effect.

Figure 2.1: State Transition Diagram for the New Model



In an entirely parallel way, if the system is in state S_{2k+4} then all k servers have one person in service and at least one more person in queue. When a service is finished, we assume that is by one of the servers with a maximum queue.

In order to determine the waiting time distribution under this model, I assume that the probability a customer arrives and joins a line of length L (remembering that customers must join the shortest possible line) is the probability that there are more than (L-1)k customers in the system but less than Lk customers in the system. If a customer arrives to find less than k people in the system, then the customer goes straight into service without waiting in queue. I have assumed that the probability the customer arrives to find between (L-1)k and Lk customers in the system is

$$\sum_{i=(L-1)k}^{Lk-1} P(i)$$

where P(i) is the steady probability of *i* customers being in the system under the M/M/k queueing model.

Once a customer is in a line of length *L*, the time the customer waits until his service begins is an Erlang random variable, order *L*, and the parameter of this density function is equal to the parameter of the exponential service time distribution, in this case 90 seconds. As in the other models, random variable *w* is the time a customer spends waiting in queue. The total time the customer spends in the system is also an Erlang random variable with order *L+1*.

By combining these results, the waiting time distribution for this model can be derived. I have the conditional density function for how long a customer waits, conditional on *L*:

$$f_{w|L}(w_o \mid L_o) = \frac{\mu^{L_o}\, w_o^{L_o-1}\, e^{-\mu w_o}}{(L_o-1)!}.$$

I have the probability law for the customer joining a line of length *L*:

$$p_L(L_o) = \sum_{i=(L-1)k}^{Lk-1} P(i)$$

where P(i) is the probability of i people in the system under the M/M/k model. By multiplying these two probability laws I obtain the joint probability law for joining a line of length *L* and waiting time *w*. By summing over all the possible values of *L*, I obtain the marginal probability law for w, which is what I need for the reach, frequency, and GRP calculations:

$$\sum_{L_o=0}^{\infty} p_L(L_o)\, f_{W|L}(w_o \mid L_o) = \sum_{L_o=0}^{\infty} \left(\sum_{i=(L-1)k}^{Lk-1} P(i) \right) \frac{\mu^{L_o} w_o^{L_o-1} e^{-\mu w_o}}{(L_o-1)!}$$

This model allows for some of the idiosyncrasies of the supermarket checkout. It allows customers to choose their own lanes reasonably, but also allows the system to function as a whole. By not forcing customers into specific lanes as the M/M/1 did, the model may compensate for the overestimated waiting times. By not allowing the customers to switch lines, the model may allow for the possibility of not having the most efficient allocation of cashiers. However, this model still does not accurately describe the checkout system.

By using the probabilities from the M/M/k system, this model may underestimate the probability of finding i people in the system. The use of the M/M/k probabilities assumes that the cashiers are being allocated most efficiently, and thus may in effect assume that people are leaving sooner than they are. Thus the number of people in line may be underestimated. Numerical results will show that this seems to be the case.

Section 2.1.4: General Modelling Concerns

While each of these models raises its own questions as to its viability, there are some general concerns about modelling this situation which apply to all three models.

The first concern is the use of steady state probabilities to obtain the waiting time distributions. Each of the models implicitly assumes that the probability of being in state *i* is not dependent on what state the system was in when the process started. This would imply that the number of people in the system at the beginning of the hour is independent of the number of people a customer will find in the system at any point in the hour. Clearly this is a broad assumption. The probability that a customer who arrives only moments after the beginning of an hour is not independent of the number of customers in the system at the beginning of the hour, but rather is dependent on this number of customers, which is the number of customers left behind by the previous hour.

The steady state probabilities smooth out the variation within the hour, but they also smooth out the carry-over effects between hours. When there are a number of high traffic (high arrival rate) hours in a row, a time when there is apt to be large carry-over, the steady state probabilities may be underestimating the number of people in queue. Similarly, during low traffic hours (low arrival rates)when there are high probabilities of no one in queue, the steady state probabilities may be overestimating the number of people in queue. For this work, the underestimating during high traffic hours is of particular concern. This concern will be discussed further in Chapter 4.

Another concern regarding modelling supermarket queueing is the difference between express and regular lanes. Express lanes, where customers usually have only a few items, may have a very short average service times and the regular lanes in which customers may have large carts full of groceries may have very long average service times. Express lanes may behave very differently, for the lines can get very long but can also be dispersed very quickly. It may be that while the probability of seeing the ad is very small in express lanes, it is quite large in regular lanes. Thus the issue would be how much of the audience goes through the regular lanes and thus has a real chance of seeing the ad. The two lane types may require two separate models, which must somehow be weighted by the amount of business which is completed through the two lane types. It may also be possible to model the express and regular lanes as a queueing system in which some customers have priorities and a different service time distribution. These ideas will be discussed again in Chapter Four.

Though less pressing than the previous issues, another area for exploration is the question of companions. It may be that a shopper is more likely to have a companion along when he is doing a large shopping trip or planning to purchase many items. Shoppers in express lines stopping in for the proverbial quart of milk may be less likely to have someone with them. Thus there is a chance that the people who stand in line longest also increase the audience size more often with their companions, or in other words, companions are not distributed over the frequency distribution categories in proportion to the shoppers, but are more concentrated in the higher categories. Depending on this concentration and the overall number of companions, this increase could greatly increase the number of gross rating points. Some sample calculations are done in Chapter Three.

A final issue to be explored in terms of modelling is the service time distribution. We assumed here that service times are independent of the hour of the day, but this may not be the case. It is possible that the average service time decreases as lines get longer. Cashiers under pressure may scan groceries with more vigor. Thus there may be a correlation between the length of the line and the speed of service. Such a correlation, if found to exist, would have to be included in an accurate model.

This work does not address all these concerns individually, but it is illustrative of the ways in which this question can be approached. Alternative models and ideas are discussed in Chapter Four.

Section 2.2: Developing the Equations for Reach, Frequency, and GRP

As previously mentioned, the key to developing the new equations for reach, frequency, and GRP for this new medium of advertising is the waiting time distribution. Clearly the longer a customer waits in line at the store, the greater the probability of seeing a particular advertisement.

Section 2.2.1: The Probability of Seeing an Ad on a Single Shopping Trip

Consider a programming loop which is t minutes long. The news, weather, feature and sports segments as well as the advertisements repeat every t minutes, with slight variations in the reported stories. The importance of the length of the programing loop is that in each repetition of the loop, an advertiser's ad is shown once. A firm is only interested in the number of people who see <u>its</u> ad; the total number of ads seen by the customer which belong to other firms is irrelevant. A customer arriving to the queue is equally likely to arrive during any moment of the programming. Thus if the customer waits w minutes then the probability that the customer sees the ad in question is the probability that the ad falls somewhere in the w minutes that the customer waits, with possible end effects. Because the loop repeats and the ad shown in each repetition of the loop, if a customer waits an amount of time longer than the loop of programming, then the customer must be exposed to the ad at least once. It is possible to formalize this argument into a probability statement.

Let r be the random variable for the number of times a customer is exposed to an ad. If a customer waits a time w which is less than the t then the probability a particular firm's ad falls entirely in the section of tape covered by the time the customer waits is $\frac{w}{t}$. However, the customer could arrive during the ad, or begin service during the ad, thus creating the end effects.

Define a to be the length of each advertisement in the loop of programming. Suppose a customer is considered to have seen the ad if he sees at least half of the ad, or $a/2$. Consider again that w is less than t. As Figure 2.2a shows, the probability the customer sees the ad once is now, given that w is less than t:

$$\frac{w + 2(\frac{a}{2})}{t}$$

Suppose w is greater than t. Then the probability the customer is exposed to the ad at least once is 1, because the customer has been exposed to the entire length of the programming loop. Thus it is possible to define:

$$p_{r\geq 1w}(1 \mid w) = \begin{cases} 0 & w < 0 \\ \frac{w+a}{t} & 0 \leq w \leq t - a \\ 1 & w > t - a \end{cases}$$

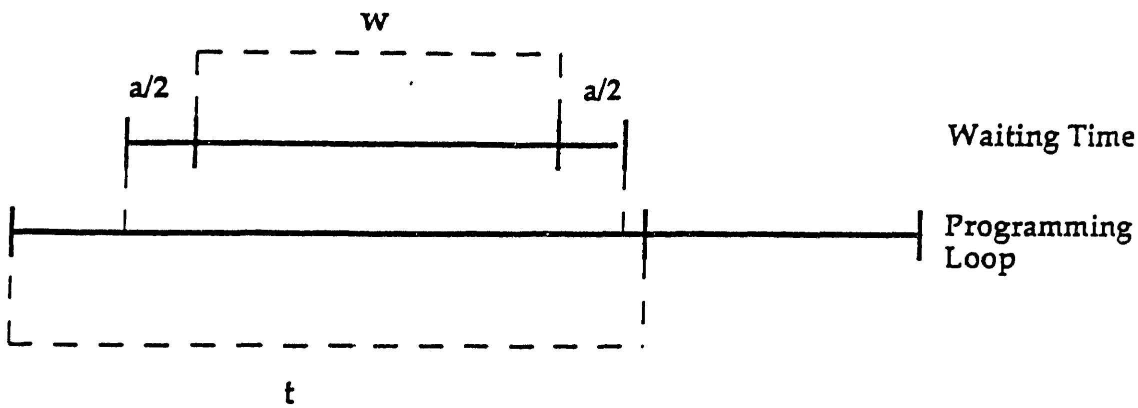


Figure 2.2a
Determining the probability of seeing an ad when w < t.

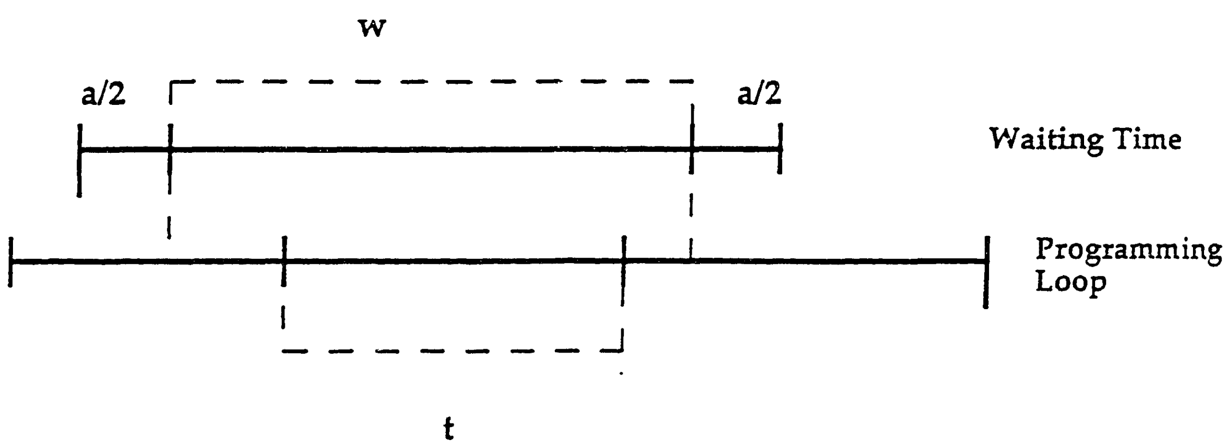


Figure 2.2b
Determining the probability of seeing an ad when w > t.

The probability that the customer sees the ad twice is, as shown in Figure 2.2b:

$$\frac{w - t + 2(\frac{a}{2})}{t}$$

With a parallel argument to the case with w less than t, it is possible to define:

$$p_{r\geq l w}(2 \mid w) = \begin{cases} 0 & w < t - a \\ \dfrac{w - t + a}{t} & t - a \leq w \leq 2t - a \\ 1 & w > 2t - a \end{cases}$$

Generalizing to seeing the ad at least $r = h$ times the following probability law conditional on w, is obtained:

$$p_{r\geq l w}(h \mid w) = \begin{cases} 0 & w < (h-1)t - a \\ \dfrac{w - (h-1)t + a}{t} & (h-1)t - a \leq w \leq ht - a \\ 1 & w > ht - a \end{cases} \qquad h_o = 1,2,\ldots$$

Having derived a probability law for the number of times the ad is seen conditional on the time the customer waits in queue, it is possible to obtain a joint probability law for the time waited and the number of ads seen by multiplying the two probability laws:

$$f_{r,w}(r_o, w_o) = p_{r \mid w_o}(r_o \mid w_o)\, f_w(w_o)$$

By integrating over all possible values of w, the marginal probability law for the number of times the ad is seen on a trip to the store is achieved:

$$p_r(r_o)= \int_{w_o} f_{r,w}(r_o,w_o)\, dw_o = \int_{w_o} p_{r|w_o}(r_o \mid w_o)\, f_w(w_o)\; dw_o$$

For each model, there is a different probability law for w, and thus a different probability law for the number of times the ad is seen will result.

Section 2.2.2: Extending to a Week

Consider random variable y to be the number times in a week that a customer sees a particular ad. Then y is equal to the number of times the ad is seen on a single trip summed over the number of trips per week, which is also a random variable which we shall call s. Thus, y is a random sum of random variables, or

$$y = r_1 + r_2 + \dots + r_s.$$

It is possible to obtain from survey data something which looks like the probability mass function for the number times per week a customer shops. However, if the survey is taken at a supermarket, the attained percentages cannot be used for the density function of s, because an interviewer at a supermarket is more likely to question the customers who go to the supermarket more often, for the simple reason the frequent shoppers are more likely to be there on the given day when questions are asked. Thus the resulting density is skewed towards the customers who shop more often. This problem can be fixed.

Suppose in a survey we obtain the following percentages: *i%* of the questioned customers shop once a week, *j%* shop twice a week, *k%* shop three times a week, and *l%* of the customers shop four (or more) times per week. If we take these percentages as probabilities and define random variable *d* to be the number of shopping trips per week as described by the survey data, then the 'random incidence' relationship gives the result that

$$p_d(d_o) = \frac{d_o p_s(s_o)}{E(s)} .$$

This equation can be used to solve for $p_s(s_o)$. The following equations can be set up from the data:

$E(s) = p_s(1) + 2p_s(2) + 3p_s(3) + 4p_s(4)$

(a) $\frac{i}{100} = \frac{p_s(1)}{E(s)}$ (b) $\frac{j}{100} = \frac{2p_s(2)}{E(s)}$

(c) $\frac{k}{100} = \frac{3p_s(3)}{E(s)}$ (d) $\frac{l}{100} = \frac{4p_s(4)}{E(s)}$

(e) $p_s(1) + p_s(2) + p_s(3) + p_s(4) = 1$

By substituting E(S) into equations (a), (c), and (d), and using equations (a), (c), (d), and (e) we have system of four equations in four unknowns and we can solve for the values of *s* that are needed.

We now have a probability mass function for *s*, and one for *r*, though it is tedious, we can determine a density function for *y* by noting that since $y = r_1 + r_2 + \dots + r_s$, the z-transform for *y* is just the z-transform for *s* evaluated at the z-transform for *r*, or

$$p^T{}_y(z) = p^T{}_s(h) \Big|_{h = p^T{}_r(z)}$$

However, for our calculations here, we have different probability mass functions for *r* for each hour of the day. Without additional information, we assume that the probability of shopping during a particular hour of the day is equal to the average number of shoppers during that hour divided by the total number of shoppers in the day, or the proportion of shoppers in that hour of the day. Thus the random variable we need in the transform equation is really the resulting value from the random choice of the random variables for each hour of the day. Thus $p^T{}_r(z)$ as noted above is equal to

$$p^T{}_r(z) = p(8\text{ AM})p^T{}_{r(8\text{ AM})}(z) + p(9\text{ AM})p^T{}_{r(9\text{ AM})}(z) + \dots + p(11\text{ PM})p^T{}_{r(11\text{ PM})}(z)$$

We can substitute this piece into the above equation and determine the probability mass function for *y*. The final result is notationally cumbersome, but once the data are applied, it simplifies a great deal. The results under each model will be shown in Chapter 3.

Section 2.2.3: Reach, Frequency, and GRP

We can now use the probability mass function for *y* to determine how many people see the particular ad once, twice, etc, within a week. We cannot however, simply multiply $p_y(1)$ times the number of transactions in a week, or $p_y(2)$ times the number of transactions in a week to get the number of people who see the ad once or twice respectively. The number of transactions per week counts twice each person who shopped twice, counts three times the number of people who shopped three times. etc. Thus to perform these multiplications would be to treat each transaction like it represents a separate customer, which is <u>not</u> the case. We must determine the number of individuals who constitute this number of transactions.

Define T to be the total number of transactions completed in a week. Previously it was shown how use the survey data to solve for the true percentage of customers who shop *s* times per week (*s* = 1,2,3, or 4). Suppose s_1% of the customers shop once a week, s_2% shopped twice, s_3% shopped three times, and s_4% shopped four or more times. Thus $.s_1T$ represents the number of individuals who shopped once. Similarly, $\frac{s_2T}{2 \times 100}$ represents the number of people who shopped twice; $\frac{s_3T}{3 \times 100}$ represents the number who shopped three times; and $\frac{s_4T}{4 \times 100}$ represents the number who shopped four times. If we define P to be the total number of individuals who shopped in the week, then

$$P = \sum_{i=1}^{4} \frac{s_iT}{i \times 100} .$$

The frequency distributions determined by calculating $P \times p_y(y_o)$ for each value of y_o. If n_o is defined to be the number of people out of the total available audience who saw the ad zero times, then $n_o = P \times p_y(0)$.

Similarly, if n_1 is defined to be the number of people who saw the ad once, then $n_1 = P \times p_y(1)$. The same calculations can be made for each possible value of y.

Summing the $P \times p_y(y_o)$ over all values of y_o not equal to zero gives the gross rating points, or

$$GRP = \sum_{i=1}^{\infty} P \times p_y(i) .$$

Having the frequency distribution makes the calculation of reach rather straightforward. If reach is defined to be the total number of people who see the ad at least once, then we can obtain the needed values from the frequency distribution. We simply add up the all the values in the frequency distribution which are defined for values greater than one, or $\sum_{i=1}^{\infty} p_y(i)$. In practical terms it would make more sense to take the total population P and subtract out the number of people who never saw the ad or $P - P \times p_y(0)$.

I note however, that if one wishes to take a broader view of the definition of reach, it could be defined as the total available audience. If this is the case, the the total population P is the actual reach figure. Clearly this number will be higher than the that of the first definition and is perhaps overly optimistic, unless a specific example presents probabilities of viewing the ad which are close to one during any given hour.

Section 2.2.4: Summary of Algorithm to Determine Reach, Frequency, and GRP

The purpose of this section is to review the algorithm described for the new calculations of reach and frequency. The algorithm can be summarized as follows:

<u>Step 1:</u>

Determine the probability mass function for r_h, the number of times an individual sees the ad on a trip to the supermarket, given the customer shopped in hour h.

$$p_{r\geq|w}(r_o|w) = \begin{cases} 0 & w < (r_o - 1)t - a \\ \dfrac{w - (r_o - 1)t + a}{t} & (r_o - 1)t - a \leq w \leq r_o t - a \\ 1 & w > r_o t - a \end{cases} \qquad r_o = 1,2,\ldots$$

<u>Step 2:</u>

Using the probability density function for the time a customer spends waiting in queue from the model of choice, determine the joint density function for the time the customer waits and the number of ads he sees, given the hour he shopped in. Integrate over all values of w to obtain the unconditional, marginal probability mass function for the number of ads a customer sees in a single shopping trip for that hour:

$$p_{r_h}(r_o) = \int_{w_o} f_{r_h,w}(r_o, w_o)\, dw_o = \int_{w_o} p_{r_h|w_o}(r_o|w_o)\, f_w(w_o)\, dw_o$$

Step 3:

Determine the mass function for r for the whole day by weighting the individual mass functions for r for each hour of the day by the appropriate weights, here taken to be the proportion of customers in the hour, and summing. This may be most easily done via the sum of the weighted z-transforms of r_h :

$$p^T_r(z) = p(8\ AM)p^T_{r(8\ AM)}(z) + p(9\ AM)p^T_{r(9\ AM)}(z) + \ldots + p(11\ PM)p^T_{r(11\ PM)}(z)$$

Step 4:

Using random variable s, the number of times a customer shops in a week (adjusted from survey data if necessary), determine the probability mass function for y, the total number of times an individual sees an ad in a week by summing the random variable r, s times:

$$y = r_1 + r_2 + \ldots + r_s$$

This step may also be most easily accomplished by using the z-transforms, by recalling

$$p^T_y(z) = p^T_s(m) \Big|_{m = p^T_r(z)}.$$

Step 5:

Using random variable s, determine the actual number of customers, P, to complete the week's number of transactions, T.

Step 6:

Determine the frequency distribution by calculating

$$n_i = P \times p_y(i).$$

Determine reach by calculating

$$\sum_{i=1}^{\infty} p_y(i) \qquad \text{or} \qquad P - P \times p_y(0) .$$

Determine GRP by calculating

$$\sum_{i=1}^{\infty} P \times p_y(i) .$$

Section 2.3: Waiting Time Distributions under Each Model

Each of the following subsections details the waiting time distribution under one of the models and gives the derived probability law for the number of times a particular ad is seen on a trip to the store.

Section 2.3.1: M/M/1 Model

The waiting time distribution under the M/M/1 model is well known and thus is stated here, not derived in detail. The probability law for the time a customer spends waiting in queue for his service to begin, *w*, is:

$$f_{w_q}(w_o) = \begin{cases} 1-\rho & w_o = 0 \\ \rho\mu(1-\rho)\exp(-\mu(1-\rho)w_o) & w_o > 0 \end{cases} .$$

The impulse at zero results from the probability that the customer enters service immediately and thus waits no time at all in queue. The total probability of the impulse is equal to the probability that there are no customers in this system. Using the procedure outlined in the previous section, the obtained probability mass function for *r* is:

$$p_r(1) = \frac{\rho}{\mu(1-\rho)t_o}[1-\exp(-\mu(1-\rho)t_o)] ,$$

$$p_r(2) = \frac{-\rho}{\mu(1-\rho)t_o}[\exp(-\mu(1-\rho)2t_o)] + \frac{\rho}{\mu(1-\rho)t_o}[\exp(-\mu(1-\rho)t_o)] ,$$

or in general

$$p_r(i) = \frac{-\rho}{\mu(1-\rho)t_o}[\exp(-\mu(1-\rho)it_o)] + \frac{\rho}{\mu(1-\rho)t_o}[\exp(-\mu(1-\rho)(i-1)t_o)] .$$

Section 2.3.2: M/M/k Model

The resulting probability density function for the time ,*w*, that a customer spends in queue waiting for service in the M/M/k model is also fairly well documented [Saaty, 1961]. If k is the number of servers, then the density for w is the following:

$$f_{w_q}(w_o) = P(>0)k\mu(1-\rho)\exp(-k\mu(1-\rho)w_o) \qquad w_o \geq 0$$

where

$$P(>0) = \frac{k\rho^k}{k!\,(1-\rho)} P_o$$

$$P_o = \left[\sum_{n=0}^{k-1} \frac{(k\rho)^n}{n!} + \frac{(k\rho)^k}{k!\,(1-\rho)}\right]^{-1}$$

and

$$\rho = \frac{\lambda}{k\mu} .$$

The derived distribution for the number of times the ad is seen on a trip to the store, r, is then:

$$p_r(1) = \frac{-P(>0)}{k\mu(1-\rho)t_o}\exp(-k\mu(1-\rho)t_o) + \frac{P(>0)}{k\mu(1-\rho)t_o} ,$$

$$p_r(2) = \frac{-P(>0)}{k\mu(1-\rho)t_o}\exp(-k\mu(1-\rho)2t_o) + \frac{P(>0)}{k\mu(1-\rho)t_o}\exp(-k\mu(1-\rho)t_o) .,$$

or in general

$$p_r(i) = \frac{-P(>0)}{k\mu(1-\rho)t_o}\exp(-k\mu(1-\rho)it_o) + \frac{P(>0)}{k\mu(1-\rho)t_o}\exp(-k\mu(1-\rho)(i-1)t_o)$$

Section 2.3.3: The Derived Model

As mentioned in section 2.1.3, the waiting time distribution under the new model must be derived by using the conditional density for the time waited given the length of the line joined and the probability of joining a line of that length. Multiplying these two and summing over all possible line lengths gives the density for the time spent waiting in queue. However, this infinite sum does not have closed form. The sum is:

$$f_{w_q}(w) = P_L(0)\times(0) + P_L(1)\mu e^{-\mu w} + P_L(2)\mu^2 w e^{-\mu w} + \ldots$$

where

$$P_L(0) = \sum_{i=0}^{k-1} \frac{\left(\frac{\lambda}{\mu}\right)^i}{i!} P_o$$

and

$$P_o = \left[\sum_{n=0}^{k-1} \frac{(k\rho)^n}{n!} + \frac{(k\rho)^k}{k!\,(1-\rho)}\right]^{-1}$$

$$P_L(n) = \sum_{i=(n-1)k}^{nk-1} \frac{\left(\frac{\lambda}{\mu}\right)^i}{k^{i-k}\,k!} P_o$$

Because no closed form for the infinite sum, in this paper, the summation was expanded until the coefficient of the Erlang term was not significant to five decimal places. The resulting probabilities for *r* are found by integrating the expansion. The integration requires many repeated integrations by parts and contains over twenty-five terms. The form is not particularly useful, and the expanded version is not included here, but may be found in the appendix. The numerical results are presented in Chapter 3.

Chapter 3

Empirical Results and Discussion

In this chapter, the data described in Chapter One are put into each of the theoretical models and the numerical results are calculated. Some parametric analysis is completed for the M/M/1 and M/M/k models. The values and the models are compared, and the implications for the Checkout Channel are discussed.

For each of the models, the mean number of arrivals per hour and the number of open lanes is important to the formulation. For the specific values being used, the reader is referred back to Table 1.1. For all the calculations, I shall assume the length of the programming loop, t_0, is 8.5 minutes.

Section 3.1: Numerical Results

Random variable *s*, described in Chapter Two, is defined to be the number of times per week a customer shops. For each of the models, the probability law for *s* is the same.

A survey was conducted during the Pilot Test at one of the test stores. Suppose the survey found that 15% of the queried customers shop once per week; 22% shop twice per week; 27% shop three times per week; and 36% shop four times (or more) per week. Using the equations set out in Section 2.2.2, it is possible to solve for the probability law for *s*. In this case, these are not the actual survey values, and thus the values of *s* are not those which were obtained in this study. However, after solving I obtained the values

$$p_s(1) = .35$$
$$p_s(2) = .25$$
$$p_s(3) = .20$$
$$p_s(4) = .20 \quad ,$$

which results in an expected value of s is 2.25, just slightly lower than the industry standard of 2.3. This probability law for *s* will be used in all three models.

Section 3.1.1: Results for the M/M/1 Model

From the data described in Chapter One, the model describes a grocery store with checkout lanes with different arrival rates per hour. Table 3.1 shows the number of lanes open for each hour and the arrival rate per hour for each open lane.

Table 3.1: Number of Transactions and Arrival Rate

Time of Day	Number of Lanes Open	Ave. Arrival Rate per Hour per Lane
8:00 - 9:00 AM	3.00	0.417
9:00 - 10:00 AM	4.00	0.389
10:00 - 11:00 AM	5.00	0.375
11:00 AM - Noon	5.00	0.458
Noon - 1:00 PM	6.00	0.433
1:00 - 2:00 PM	5.00	0.458
2:00 - 3:00 PM	5.00	0.5
3:00 - 4:00 PM	6.00	0.467
4:00 - 5:00 PM	6.00	0.5
5:00 - 6:00 PM	7.00	0.5
6:00 - 7:00 PM	7.00	0.472
7:00 - 8:00 PM	6.00	0.467
8:00 - 9:00 PM	5.00	0.417
9:00 - 10:00 PM	4.00	0.389
10:00 - 11:00 PM	3.00	0.417
11:00 PM - Midnight	2.00	0.25

Using the equations defined in 2.1.1 above for the probability mass function for r, the values in Table 3.2 were obtained. I note that for most hours of the day there is between a 0.2 and 0.4 chance of seeing the ad once. Only during the most congested hours does the probability of seeing the ad twice get near 0.10. Thus the probabilities of seeing the ad at least twice per trip are extremely small. There are no values in the table for the probability of seeing the ad three or more times in a single trip, as they were equal to zero when rounded to five decimal places. Thus the probability that the customer did not see the ad, $p(r=0)$, is taken to be $1 - p(r=1) - p(r=2)$. While this means customers will not be bored by repetition, it does not bode well for a great many exposures in a week.

Table 3.2: Probabilities of Seeing An Ad *r* Number of Times Under M/M/1

Time of Day	P(r=0)	P(r=1)	P(r=2)
8:00 - 9:00 AM	0.741	0.228	0.031
9:00 - 10:00 AM	0.776	0.205	0.019
10:00 - 11:00 AM	0.792	0.192	0.016
11:00 AM - Noon	0.678	0.272	0.05
Noon - 1:00 PM	0.717	0.247	0.036
1:00 - 2:00 PM	0.678	0.272	0.05
2:00 - 3:00 PM	0.599	0.314	0.087
3:00 - 4:00 PM	0.664	0.28	0.056
4:00 - 5:00 PM	0.599	0.314	0.087
5:00 - 6:00 PM	0.599	0.314	0.087
6:00 - 7:00 PM	0.654	0.286	0.06
7:00 - 8:00 PM	0.664	0.28	0.056
8:00 - 9:00 PM	0.741	0.231	0.028
9:00 - 10:00 PM	0.776	0.205	0.019
10:00 - 11:00 PM	0.741	0.231	0.028
11:00 PM - Midnight	0.897	0.1	0.003

The probability of seeing the ad at least once or at least twice changes as $\frac{\lambda}{v}$ changes. Figure 3.1 shows this relationship. The pattern of the change, which is exponential, is similar to the behavior of the expected waiting time in the M/M/1 queue. This similarity of pattern is reasonable, because of the relationship between the waiting times and the probability of seeing the ad. Note that the probability of seeing the ad three or more times will be come less negligible and indeed significant as $\frac{\lambda}{v}$ increases.

Figure 3.1: Changes in Probabilties of Seeing an Ad Versus Lambda/Mu Under M/M/1 Model

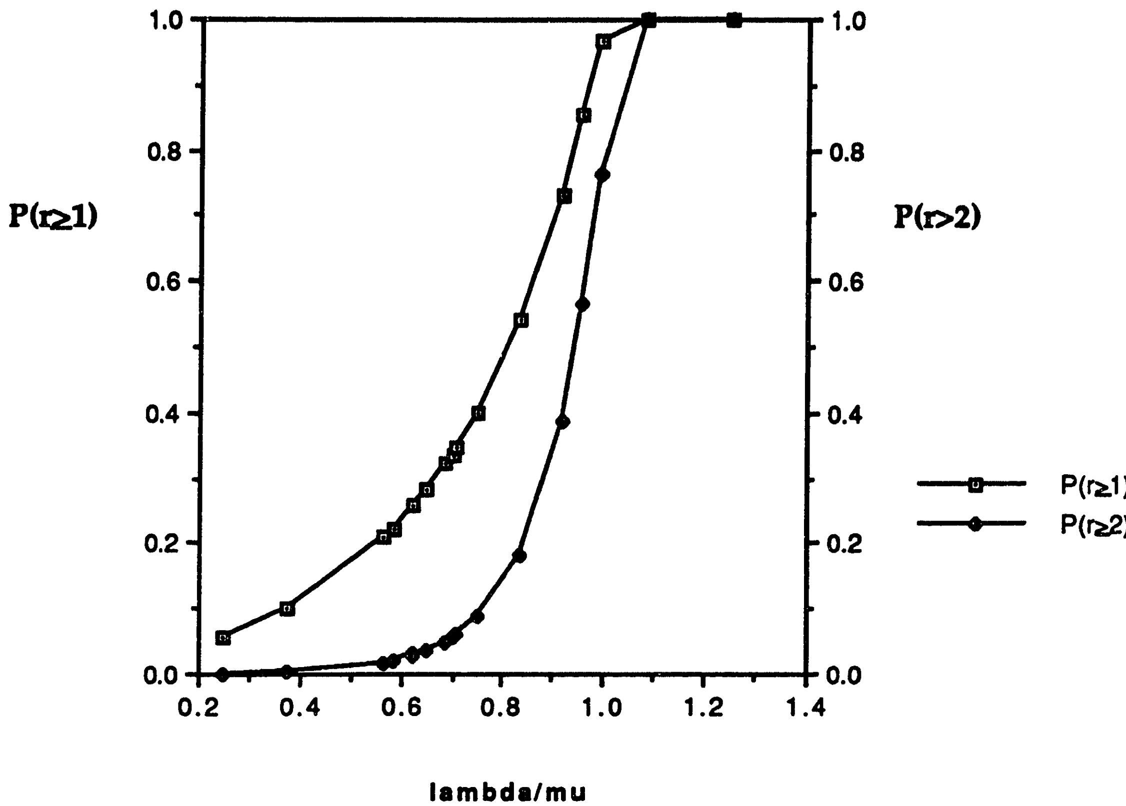


Table 3.3 presents the probabilities of shopping during the various hours of the day, based on the proportion of shoppers whose transactions were during the hour. Table 3.4 presents a representative probability mass function for the random variables r_h where h represents the hour of the day. Combining the weighted mass functions for r_h results in the probability mass function for r. The probability mass function for s, the number of shopping trips per week is also presented, as is the resulting probability mass function for *y*, the number times the ad is seen per week. Note that while it should be possible to see the ad eight times in a week - twice per trip and four trips in the week - the probability of this occurring does not appear in the probability mass function of *y*. The probability of seeing the ad seven or eight

times in a week was zero to four decimal place accuracy. This result is not surprising, as the probability of seeing the ad twice on any trip is so small.

Table 3.3: Probability of Shopping in each Hour of the Day

Time of Day	P(shop in this hour)
8:00 - 9:00 AM	0.029
9:00 - 10:00 AM	0.041
10:00 - 11:00 AM	0.052
11:00 AM - Noon	0.064
Noon - 1:00 PM	0.076
1:00 - 2:00 PM	0.064
2:00 - 3:00 PM	0.072
3:00 - 4:00 PM	0.082
4:00 - 5:00 PM	0.088
5:00 - 6:00 PM	0.105
6:00 - 7:00 PM	0.099
7:00 - 8:00 PM	0.082
8:00 - 9:00 PM	0.058
9:00 - 10:00 PM	0.041
10:00 - 11:00 PM	0.029
11:00 PM - Midnight	0.018

Table 3.4: Probability Mass Functions Under M/M/1 Model

Example of hourly probability mass function for r:

$$p_{r(8AM)}(r_o) = \begin{cases} .741 & r_o = 0 \\ .228 & r_o = 1 \\ .031 & r_o = 2 \end{cases}$$

Probability mass function for r:

$$p_r(r_o) = \begin{cases} .681 & r_o = 0 \\ .266 & r_o = 1 \\ .053 & r_o = 2 \end{cases}$$

where *r* is the number of times the ad is seen on a single trip to the supermarket.

Probability mass function for s:

$$p_s(s_o) = \begin{cases} .35 & s_o = 1 \\ .25 & s_o = 2 \\ .20 & s_o = 3 \\ .20 & s_o = 4 \end{cases}$$

where *s* is the number of trips to the supermarket per week.

Probability mass function for y:

$$p_y(y_o) = \begin{cases} .460 & y_o = 0 \\ .325 & y_o = 1 \\ .151 & y_o = 2 \\ .048 & y_o = 3 \\ .013 & y_o = 4 \\ .002 & y_o = 5 \\ .0003 & y_o = 6 \end{cases}$$

where *y* is the total number of times the ad is seen in a week.

Table 3.5 presents the breakdown of the 11,970 (= 1710 x 7) transactions which occurred during the sample week. The resulting figure is that 7074 individuals completed these 11,970 transactions.

Table 3.5: Proportion of Transactions Completed by Distinct Individuals

T = 11,970	
.35T	4190
.25T/2	1496
.20T/3	798
.20T/4	590
	Total = P = 7074

Table 3.6 presents the frequency distribution under this formulation. Dividing through by the total potential audience size, I can express these values as percentages of the total audience, and thus extrapolate to other stores if I believe they behave similarly. Note also that Table 3.6 has the total reach figure as well as the corresponding gross rating points figure.

Table 3.6: Frequency Distribution Under M/M/1 Model

Total Number Available to See Ad: 7074

Number of Times Exposed to Ad in a Week	Number of Shoppers
0	3258
1	2298
2	1066
3	342
4	91
5	17
6	2
7	0
8	0

Total Reach: 3816 or 53.9% of the total number of shoppers

Gross Rating Points: 5917

I noted in Chapter One that there is the possibility of customers shopping with companions, which are defined to be adults or teen-age children over 16. Though not counted in the number of transactions, these supermarket-goers are very much a part of the audience. If I assume that approximately 20% of the customers have a companion with them, then the total audience size increases by 20% as well. It is difficult to speculate on the frequency distribution for companions, as I do not know how often a companion accompanies the shopper, even if I know how often the shopper visits the store. I can distribute the 20% more people over the categories of the frequency distribution in proportion to the distribution of shoppers, but this may not truly be reflective of the distribution of companions. If companions were distributed in the manner just described, the resulting reach and frequency values can be found in Table 3.7. I note the corresponding 20% increase in GRP. While percent reach remains the same, total number of individuals reached also increases by 20%.

Table 3.7: Frequency Distribution Under M/M/1 with Companions

Total Number Available to See Ad: 8488

Exposed to Ad in a Week	Number of Shoppers
0	3909
1	2757
2	1279
3	410
4	109
5	20
6	2
7	0
8	0

Total Reach: 4579 or 53.9% of the total number of shoppers

Gross Rating Points: 7093

If the service time could be considered as time available for exposure to advertisements, then it is possible to obtain new values of reach, frequency, and GRP. Table 3.8 shows the new values under this change in the model.

Table 3.8: Frequency Distribution Under M/M/1 Including Service Time

Total Number Available to See Ad: 7074

Number of Times Exposed to Ad in a Week	Number of Shoppers
0	2157
1	2040
2	1641
3	755
4	360
5	127
6	32
7	6
8	1

Total Reach: 4962 or 69.7% of the total number of shoppers

Gross Rating Points: 9862

Section 3.1.2: Results for M/M/k Model

The results for the M/M/k model parallel the results for the M/M/1 model. Table 3.9 presents the calculated probabilities of seeing the ad once or twice in a shopping trip. There is almost no chance of seeing an ad twice, even during the most congested hours of the day. This also implies the chance of seeing the ad three or more times is negligible. Thus $p(r=0) = 1 - p(r=2) - p(r=1)$.

Table 3.9: Probabilities of Seeing an Ad *r* times Under M/M/k

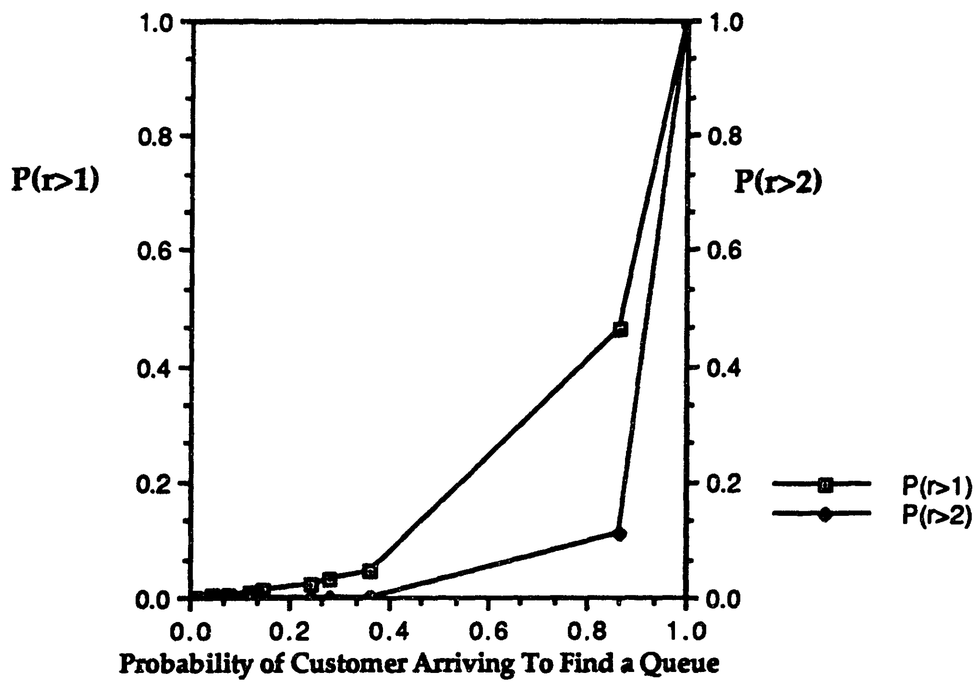
Time of Day	P(r=0)	P(r=1)	P(r=2)
8:00 - 9:00 AM	0.887	0.111	0.002
9:00 - 10:00 AM	0.953	0.047	0
10:00 - 11:00 AM	0.976	0.024	0
11:00 AM - Noon	0.942	0.058	0
Noon - 1:00 PM	0.969	0.031	0
1:00 - 2:00 PM	0.942	0.058	0
2:00 - 3:00 PM	0.909	0.09	0.001
3:00 - 4:00 PM	0.956	0.044	0
4:00 - 5:00 PM	0.935	0.065	0
5:00 - 6:00 PM	0.95	0.05	0
6:00 - 7:00 PM	0.65	0.35	0
7:00 - 8:00 PM	0.956	0.044	0
8:00 - 9:00 PM	0.962	0.038	0
9:00 - 10:00 PM	0.953	0.047	0
10:00 - 11:00 PM	0.888	0.111	0.001
11:00 PM - Midnight	0.97	0.03	0

In Table 3.10, the probability that an arriving customer will wait more than 0 seconds for each hour of the day are shown. Only one of these probabilities is greater than 0.5, thus demonstrating that many customers are able to go straight into service in this queueing situation. Figure 3.2 shows the relationship between the probability of seeing the ad at least once on a trip and the probability that a customer arrives to find he must wait, i.e., not go into service immediately because a queue exists. As the figure shows, the greater the probability of a queue existing, the greater the probability of seeing the ad at least once.

Table 3.10: Probability of Queueing Under M/M/k Model

Time of Day	P(wait more than 0)
8:00 - 9:00 AM	0.481
9:00 - 10:00 AM	0.334
10:00 - 11:00 AM	0.241
11:00 AM - Noon	0.409
Noon - 1:00 PM	0.302
1:00 - 2:00 PM	0.409
2:00 - 3:00 PM	0.509
3:00 - 4:00 PM	0.378
4:00 - 5:00 PM	0.462
5:00 - 6:00 PM	0.422
6:00 - 7:00 PM	0.349
7:00 - 8:00 PM	0.378
8:00 - 9:00 PM	0.32
9:00 - 10:00 PM	0.334
10:00 - 11:00 PM	0.481
11:00 PM - Midnight	0.205

Figure 3.2: Probability of Customer Seeing at Least One Ad Versus Probability of Customer Arriving to Find a Queue Under M/M/k Model



Referring to Table 3.3 for the probabilities of shopping in the particular hours of the day, the probability mass function for *r* is obtained from the weighted probability mass functions for $r(h)$. Table 3.11 shows the resulting probability mass function for *r*, repeats the mass function for *s* (the number of shopping trips per week), which was found in the above section, and shows the resulting probability mass function for *y*, the number of times the ad is seen per week by a customer. Since there was virtually no chance of seeing the ad two or more times in a single visit to the store, it makes sense that the highest number of times the ad could be seen in a week is 4, however the probability of seeing the ad 4 times was 0, rounded to five decimal places.

Referring to Table 3.5 we have the total number of shoppers who completed the 11,970 transactions in our week, 7074. Using this figure and the density function for y, we obtain the frequency distribution in Table 3.12. Table 3.12 also shows the total reach and gross rating points.

Table 3.11: Probability Mass Functions Under M/M/k Model

<u>Example of hourly probability mass function for r:</u>

$$p_{r(8AM)}(r_o) = \begin{cases} .887 & r_o = 0 \\ .111 & r_o = 1 \\ .002 & r_o = 2 \end{cases}$$

<u>Probability mass function for r:</u>

$$p_r(r_o) = \begin{cases} .916 & r_o = 0 \\ .083 & r_o = 1 \end{cases}$$

where *r* is the number of times the ad is seen on a single trip to the supermarket

<u>Probability mass function for s:</u>

$$p_s(s_o) = \begin{cases} .35 & s_o = 1 \\ .25 & s_o = 2 \\ .20 & s_o = 3 \\ .20 & s_o = 4 \end{cases}$$

where *s* is the number of trips per week

<u>Probability mass function for y:</u>

$$p_y(y_o) = \begin{cases} .824 & y_o = 1 \\ .160 & y_o = 2 \\ .012 & y_o = 3 \\ .004 & y_o = 4 \end{cases}$$

where *y* is the total number of times the ad is seen in a week.

Table 3.12: Frequency Distribution Under M/M/k

Total Number Available to See Ad: 7074

Number of Times Exposed to Ad in a Week	Number of Shoppers
0	5832
1	1131
2	88
3	30
4	0

Total Reach: 1249 or 17.6% of the total number of shoppers

Gross Rating Points: 1397

Once again the issue of companions needs consideration. If 20% of the shoppers are accompanied by a companion, and if that 20% is distributed over the frequency categories proportional to the percentage of shoppers in that category,the new reach and frequency values are obtained. The results are in Table 3.13. The table shows a 20% increase in GRP and number of people reached.

Table 3.13: Frequency Distribution Under M/M/k with Companions

Total Number Available to See Ad: 8488

Number of Times Exposed to Ad in a Week	Number of Shoppers and Companions
0	6998
1	1357
2	105
3	36
4	0

Total Reach: 1498 or 17.6% of the total number of shoppers

Gross Rating Points: 1675

Section 3.1.3: Results for the Derived Model

The results for the derived model were calculated as they were for the other two models. Table 3.14 shows the calculated probabilities of seeing an ad once, twice, or not at all on a trip. The probabilities of seeing the ad once are lower than those under the M/M/1 model as expected, but are also actually lower in this model than in the M/M/k model, which is something of a surprise. Again the probability of seeing the ad three or more times is negligible, and p(r=0) = 1 - p(r=1) - p(r=2).

Table 3.14: Probability of Seeing an Ad *r* Times Under Derived Model

Time of Day	P(r=0)	P(r=1)	P(r=2)
8:00 - 9:00 AM	0.933	0.055	0.012
9:00 - 10:00 AM	0.98	0.016	0.004
10:00 - 11:00 AM	0.995	0.003	0.002
11:00 AM - Noon	0.973	0.022	0.005
Noon - 1:00 PM	0.993	0.005	0.002
1:00 - 2:00 PM	0.973	0.022	0.005
2:00 - 3:00 PM	0.946	0.044	0.01
3:00 - 4:00 PM	0.984	0.012	0.004
4:00 - 5:00 PM	0.967	0.027	0.006
5:00 - 6:00 PM	0.981	0.015	0.004
6:00 - 7:00 PM	0.991	0.006	0.003
7:00 - 8:00 PM	0.984	0.012	0.004
8:00 - 9:00 PM	0.987	0.01	0.003
9:00 - 10:00 PM	0.98	0.016	0.004
10:00 - 11:00 PM	0.933	0.055	0.012
11:00 PM - Midnight	0.992	0.006	0.002

Table 3.15 shows the resulting probability mass function for *r*, after weighting by the values in Table 3.3, and the probability mass function for *y*, the total number of times the ad is seen in a week. Since the probabilities of seeing the ad twice on a trip are so small, it again makes sense that the probability of seeing an ad more than four times is negligible.

Table 3.15: Probability Mass Functions Under Derived Model

Example of hourly probability mass function for r:

$$p_{r(8AM)}(r_o) = \begin{cases} .933 & r_o = 0 \\ .055 & r_o = 1 \\ .012 & r_o = 2 \end{cases}$$

Probability mass function for r:

$$p_r(r_o) = \begin{cases} .977 & r_o = 0 \\ .017 & r_o = 1 \\ .001 & r_o = 2 \end{cases}$$

where *r* is the number of times the ad is seen on a single trip to the supermarket.

Probability mass function for s:

$$p_s(s_o) = \begin{cases} .35 & s_o = 1 \\ .25 & s_o = 2 \\ .20 & s_o = 3 \\ .20 & s_o = 4 \end{cases}$$

where *s* is the number of trips to the supermarket per week.

Probability mass function for y:

$$p_y(y_o) = \begin{cases} .824 & y_o = 1 \\ .160 & y_o = 2 \\ .012 & y_o = 3 \\ .004 & y_o = 4 \end{cases}$$

where *y* is the total number of times the ad is seen in a week.

Table 3.16 shows the frequency distribution as well as the reach and GRP achieved under this model. As the results are dependent on the probabilities for he number of times the ad is seen on a trip, the results are consistent with the previous findings: the overall reach and GRP figures are lower than those of the M/M/k queue. Table 3.16 shows the frequency distribution for the derived model if companions are also counted. Again note the 20% increase in GRP and number of people reached.

Table 3.16: Frequency Distribution Under Derived Model

Total Number Available to See Ad: 7074

Number of Times Exposed to Ad in a Week	Number of Shoppers
0	6715
1	260
2	96
3	3
4	1
5	0
6	0
7	0
8	0

Total Reach: 360 or 5.1% of the total number of shoppers

Gross Rating Points: 465

Table 3.17: Frequency Distribution Under Derived Model With Companions

Total Number Available to See Ad: 8488

Number of Times Exposed to Ad in a Week	Number of Shoppers
0	8058
1	312
2	115
3	4
4	1
5	0
6	0
7	0
8	0

Total Reach: 432 or 5.1% of the total number of shoppers

Gross Rating Points: 558

It is also possible with this model to consider the time in service as time when the customer could be exposed to the advertisements. With this change to the model included, the new results for reach, frequency and GRP are shown in Table 3.18.

Table 3.18: Frequency Distribution Under Derived Model Including Service Time

Total Number Available to See Ad: 7074

Number of Times Exposed to Ad in a Week	Number of Shoppers
0	5657
1	1337
2	324
3	46
4	6
5	1
6	0
7	0
8	0

Total Reach: 1714 or 24.3% of the total number of shoppers

Gross Rating Points: 2152

Section 3.2: Discussion

The empirical results are both surprising and not surprising. It was hypothesized that the M/M/1 model might overestimate waiting times and thus give generous figures for reach, frequency and GRP. When compared with the results for the M/M/k model, which was thought to underestimate waiting times, it would appear that the hypotheses were correct. However, the derived model was intended to produce results within the bounds set by the two other models. Indeed, the derived model produced results even more pessimistic to the advertiser than the M/M/k model.

One interesting note is that while the the derived model estimates a lower probability of seeing an ad once than the M/M/k model, it estimates a higher probability of seeing an ad twice. Thus while it seems to indicate that the overall probability of seeing the ad is lower than it is in the M/M/k model, once the ad has been seen, it is more likely to be seen a second time under this model.

The results from the derived model may have resulted for a combination of reasons. First, the use of the M/M/k steady state probabilities for the number of people in the system may be underestimating the number of people in the system so severely as to cause these results. The description of the model does not allow customers to change lanes, but the steady state probabilities are in effect letting them. Because these probabilities implicitly assume the most efficient use of cashiers, they are assuming customers are leaving faster than they are in the rest of the model - and perhaps in real life. Thus this double discrepancy, both within the model and in interacting with the real data, may be the cause of the very low probabilities of exposure.

The fact that the derived model produces higher probabilities of seeing the ad twice (higher than the M/M/k model), there is some indication that it is the steady state probabilities which are causing the discrepancy. The higher probability of seeing the ad more than twice indicates that once there is waiting it is longer than in the M/M/k model, which is part of what the model was intended to achieve. However the lower probabilities for seeing the ad once indicate an overall lower level of queueing, which could arise from the underestimation by the M/M/k steady state probabilities of the people in the system.

An additional concern is that the models are do not account for the possible carry-over effects between hours. The use of the steady state probabilities in each of the models ignores the the dependence of the number of people in the system in an hour on the number of people who were in the system at the end of the last hour. Of particular concern is the underestimating during successive high traffic hours. These hours are of great interest because the most business is done during these hours. Thus to underestimate the waiting times in these hours decreases the probabilities of seeing the advertisements for a disproportionately large part of the audience.

The main implication of these results is that the true behavior of the supermarket queueing must be determined. It may be that the assumptions involved in the M/M/k and derived model are not necessary and that the M/M/1 model is actually the best fit to real world behavior.

Chapter 4

Conclusions and Further Ideas

In this chapter, I review the work done in this paper and suggest areas for further research. Section 4.1 contains an overview of the models and results. In Section 4.2, ideas for other locations where the concept could be effective are proposed. Section 4.3 outlines possible extensions of this work.

Section 4.1: Overview

This paper was an attempt to define new measurements for reach, frequency, and GRP in a very specific advertising situation. This new situation is the installation of televisions with specialized programming in grocery stores. The goal was to try to merge the definitions of marketing science with the theory of queueing.

Traditional methods of reach and frequency calculations have required the estimation of the audience size, but in this particular situation, the audience size is known almost exactly. In this new situation, the key to accurate measurement is the waiting time distribution of customers waiting to have their purchases totalled. With this in mind, three different models for queueing in a supermarket were proposed: the standard M/M/1 and M/M/k models, and a derived model which was an attempt to combine the two models. It was believed that the M/M/1 model would overestimate the waiting times and the M/M/k model would underestimate the waiting times, thus the third model was an effort to find something in between.

The third model was a combination of the M/M/1 and M/M/k, an attempt to capture more of the idiosyncrasies of supermarket queueing. By assuming that customers arrive to the system in a Poisson manner and then choose the lane with the fewest available people in it, I can determine that the time they wait for service to begin is an Erlang random variable, with order equal to the number of people in line ahead of them, including the customer in service. I assumed that the customers in the system are evenly distributed over the lanes, i.e., if there are k customers in the system, then all k servers are busy. The model assumed there was not a server with a queue if another server is idle. Thus I used the total number of people in the system to determine what length line the customer will join. I estimated the probabilities for the number of people in the system with the steady state probabilities of the number of people in the system under the M/M/k model. Thus the model contains a component of the M/M/1 model, the conditional density of the waiting times, conditional on the number of people n queue, and a component of the M/M/k model, the steady state probabilities for the number of people in queue.

The M/M/1 model did produce the highest probabilities of seeing a particular advertisement. The M/M/k model produced much lower probabilities of seeing an ad. However, the third model did not produce probabilities which fell in the range between the M/M/1 and the M/M/k. The probability for seeing an ad at least once was actually lower than under the M/M/k queue. The probability for seeing the ad at least twice was higher, thus suggesting that it is the steady state probabilities which are underestimating the number of people in the system, for once an ad is seen it is more likely to be seen a second time under this model. The conclusion was that it must be determined what model best fits the true queueing situation at a supermarket. Ideas as to how to determine the best model are presented in Section 4.3.

Section 4.2: Other Locations

While this paper has focussed on the televisions being placed at the checkout lines of supermarkets, there are other locations where this type of technology could be implemented, both within the supermarket and outside the supermarket realm.

Section 4.2.1: Other Locations within the Grocery Store

Even if the waiting times are not as long as suggested by the M/M/1 model, thus producing the high probabilities of seeing the ad, there may be other ways to make this concept a success in the supermarket. Replacing the monitors is one of the easier ways to improve the audience size.

One reconfiguration of the monitors could involve a monitor at the end of the checkout lane, so that customers in service can continue to watch the programming. In this way, the time waited during service would not be lost. If the service time is included in the M/M/1 case, using the distribution for the total system time in this model, the reach figure climbed to 69.7% and the gross rating points to 9862. (The frequency distribution is shown in Table 3.8.) Including a way for customers to be exposed to the monitors during service increases the GRP by 67.5% and the reach increases by 1,146 people or 30.0%. There is a significant advantage under the M/M/1 model to reposition the monitors.

The similar calculations were done for the derived model. The results were shown in Table 3.18. The reach figure increases by 376% and the GRP increases by 362%. As is evident, there is much to be gained by allowing the monitors to be visible by those in service.

Another possible placement of the monitors in the supermarket would be at the deli/bakery/seafood counters. At these counters where customers usually take a number and wait for service, during peak shopping hours long lines may grow. Many customers may wander a little ways away to gather other groceries, but few stray far for fear of missing their turn. Monitors placed at these counters could increase the probability of seeing the ad greatly.

As more and more states institute bottle bills offering the return of deposits on glass and plastic bottles and aluminum cans, more stores will have to have return centers. These locations, like the bakery/deli/seafood counters can develop lines at the peak shopping hours. Thus monitors at these locations could also increase the probability of seeing the ads.

Section 4.2.2: Other Locations Outside Supermarkets

While grocery stores are the focus of this work, they are not the only locations where this concept may succeed. Any place people wait and are impatient is a valid location. Other types of stores and transportation centers are the two most obvious possibilities. As mentioned in Chapter One, Turner Broadcasting is already beginning the same concept in airports.

If airports can be considered, then perhaps train stations should be as well. Busy train stations where interstate as well as instate commuter trains arrive and depart could be a perfect location for this medium. During rush hours and holiday times of year, these locations could provide particularly sizable audiences.

Other types of stores have adopted payment procedures similar to those in the supermarkets. The discount department stores like Zayre/Ames, K-Mart, and Bradlee's have begun to install scanners like those in the supermarkets, and their lane configurations are quite similar. Many of these stores do not have the extreme number of checkout lanes that the new, large supermarkets do and thus may incur more substantial lines. Monitors placed where impatient shoppers could see them in these stores could achieve positive results. The audience size is actually quite sufficient in these stores to warrant exploration. A recent survey conducted in Boston asked residents to name a department or other major store where they shopped in the last 90 days. Table 4.1 shows the percentages who mentioned specific stores [Advertising Age, April 22, 1991]. Though the major department stores with checkout islands and not checkout lanes may not be good locations, at least five of the mentioned stores could be potential locations and would appear to have sufficient potential.

Table 4.1: Large Boston-area Retail Stores and the Percentage of Survey Respondents who Shopped There in Last 90 Days

Store Name	Percentage who Shopped there in last 90 days
Bradlees	49.9
Sears	49.9
Jordan Marsh	42.7
Filene's	40.5
Zayre/Ames	32.1
Lechmere	31.3
K-Mart	27
Filene's Basement	23.6
JC Penney	18

As mentioned in Chapter One, a similar type of experiment has already been tried in banks. There is potential for this medium in banks also, as well as post offices, doctors' and dentists' offices and maybe even hair salons. Any place where people wait and get impatient is a possible location. While some may prove logistically difficult or difficult to target, they should not be ruled out too quickly.

One advantage of almost all of these locations is the possibility of measuring the audience. Unlike television which must rely on projections from the Nielsen sample, many of these audiences can be determined fairly accurately. In the age of growing dependence on quantitative measures, this advantage may outweigh some of the logistical problems.

Section 4.3: Further Research

Clearly the most important piece of research which needs to be completed is a real life analysis of how grocery store queues behave. The wide variation of results under the different models would suggest very different courses of action for those attempting to launch the Checkout Channel nationwide.

It is quite possible that to accurately model supermarket queueing, a very complex model is needed. The number of changing elements of supermarket queueing is quite large. The model may need to require a priority system to account for express lane customers versus regular lane customers. The service times for these two types of customers may be different. Moreover, the model must capture the different arrival rates over the hours of the day, which may be correlated. The model may also need to take into account the fact that additional checkout lanes are often opened when lines reach a certain length. Thus the number of lanes open is dependent on the number of customers in line, which is dependent on the number of servers available. Additionally, servers may work faster when queues are long, thus service time could be dependent on queue length which is dependent on service time. These circular, time-dependent characteristics may require a very complex model to determine an accurate waiting time distribution. Monte Carlo simulation may be required to capture all the various elements.

It may well be that the best to determine the waiting time distribution is a non-parametric model. The Queue Inference Engine (QIE) [Larson, 1990] may in fact be an excellent means of determining the waiting times of supermarket customers. The QIE uses only transactional data, which are the time a service begins and the time the service ends for each customer, and assumes Poisson arrivals. For busy periods, which are clearly the times advertisers are most interested in, the QIE can determine the mean waiting time in queue as well as the probability law for the number in queue. One of the most useful aspects of the QIE is that the results do not depend on λ, the arrival rate of the Poisson process of arrivals. Thus the results could be used in the grocery store with its changing arrival rate, as long as the arrival rate does change significantly during the busy period. The QIE then removes the need for a complicated model and parameter estimation. Moreover, it removes the assumption that the system is in steady state, one of the key modelling concerns. With the advancements in checkout register technology, it should be feasible to obtain the necessary transactional data and implement the QIE.

The second important are of study is customer reaction to the Checkout Channel. If the Channel is well liked in many regions of the U.S. then it has promise as a new advertising medium, as well as a way to distract customers in line. However, if it is perceived as a gimmick, the results could be quite negative for both for the supermarket ad the advertiser who would be associated with the Channel. Customer reaction to placing television programming like that of the Channel in other locations should be determined before large sums of money are spent on hardware. The buying public may like the idea of the Channel in some locations where they wait, but in others they may find it intrusive.

Further work should also be done to clarify the issue of perceived versus actual waiting time. If distractions like the Channel do lower the perceived waiting time of customers in line, then they could be a much more viable option.

Clearly this technology has a great deal to offer both consumers and advertisers for its potential to increase customer satisfaction and provide a new means to advertise in a wide variety of locations and situations. It is equally clear that accurate measurement of the audience size will continue to be a crucial element to its success.

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