Micro-Electromagnetic Formation Flight of Satellite Systems

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Abstract

Electromagnetic formation flight (EMFF) investigates the concept of using electromagnets to provide the forces to maintain a satellite's relative position in a formation. Thus far, high temperature superconducting (HTS) wire has been considered the enabling technology and the concept has been sized for aggressive maneuvering over large distances with concepts such as terrestrial planet finder in mind. A nominal mode of operation, of simply keeping a fleet of satellites within a volume, calls for a simpler system. Micro-EMFF (μ EMFF) investigates the use of conventional conductors, capacitors and solar cells for use on formations at small separation distances and requiring small forces. Simple one-dimensional models investigate this concept and the optimal mass implementations are compared to traditional propulsion systems as well as HTS EMFF and shown to be advantageous in close proximity formations. Because the forces involved with μ EMFF are so small, a mobile-like device was built to validate the simple models and to allow for the further investigation of control algorithms. Overall, this thesis proves the viability of the μ EMFF concept in close proximity, small force requiring formations.

Thesis supervisor: Raymond J. Sedwick

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Chapter 1 Introduction

1.1 Motivation

1.1.1 Motivation for Formation Flight

Multiple spacecraft formation flight provides an alternative to traditional monolithic satellites for future space missions. As missions come with increasingly stringent requirements, formations of small satellites can provide many advantages. For example, with current launch capabilities, the Hubble is a good representation of the largest satellites we can launch. However, using the concept of formation flight, multiple smaller spacecraft can be flown for a space-based interferometer to allow for a larger diameter aperture, and thus a finer resolution telescope. There is also the added benefit that a configuration can be repaired or upgraded by replacing only a part of the formation. Furthermore, small spacecraft can be cheaper to manufacture and launch, and thus may allow for a faster response, more flexible system.

However, formation flight comes with several challenges. The most researched, and the one emphasized in the following work, is disturbance rejection. Disturbance forces, such as the effects of the J_2 potential or drag, can be counteracted using traditional propulsion technologies. However, the propellant required to thwart these disturbances over the lifetime of the mission may be significant.

1.1.2 Motivation for Electromagnetic Flight

Thus, the main advantage of electromagnetic formation flight (EMFF) highlighted for this thesis is that it removes a non-replenishable resource. EMFF uses solar panels to provide the power needed; thus the fuel does not limit the lifetime of the mission. In

addition, thruster plumes, which cause damage and disturbance to neighboring spacecraft, can be avoided.

1.2 Introduction of Electromagnetic Formation Flight

There are two ways for using electromagnets for actuation of spacecraft. The first involves the use of the Earth's magnetic field; for example, the existing technology of magnetic torque rods. The rods interact with the Earth's magnetic field to control the orientation of the spacecraft. The second, EMFF, entails the use of electromagnets for each spacecraft to create their own field for another spacecraft to react against. This is achieved by creating a steerable magnetic dipole by driving current through three orthogonal electromagnetic coils. With the addition of reaction wheels, the EMFF system can provide all of the necessary forces and torques needed to maintain a satellite's relative position and attitude in a formation of satellites.

1.3 Previous work at Space Systems Laboratory

Many aspects of the EMFF concept have been investigated by the Massachusetts Institute of Technology Space Systems Laboratory (MIT SSL). Kong conducted initial analysis of mass fractions, power demands, and volume requirements, and found the EMFF design to be comparable to current propulsion based systems [1]. He also investigated initial controllability issues. Elias developed a linearized model and optimal controller design for a two-vehicle array [2]. Models for the forces and torques and the equations of motion were developed and described by Schweighart [3]. The benefits of EMFF were analyzed on simple satellite formations by Kwon [4]. The effects of varying coil mass, coil mass distribution, number of spacecraft, and other parameters were characterized. The subsystem design and technology levels for the electromagnet coils, thermal systems, structure and power were presented and the results of the trade studies were applied to the Terrestrial Planet Finder (TPF) mission. Furthermore, Ahsun investigated optimal time path planning for reconfiguration and showed that an n-satellite formation can be stabilized [5].

A two-dimensional testbed has been developed to test control algorithms using electromagnetic forces and reaction wheels. Two vehicles have been built containing two orthogonal high temperature superconducting (HTS) wire coils each. They are supported on air carriages to simulate a frictionless environment. A thorough description of this system is given by Neave [6]. The testbed uses HTS wire which requires a thermal system to cool the electromagnet. Presently, liquid nitrogen is poured into a reservoir and allowed to boil off. However, with this method, liquid nitrogen merely replaces propellant as the non-replenishable resource. Thus a thermal subsystem development for a flight system is being performed by Kwon.

In addition, the MIT SSL has developed a formation flight testbed called Synchronized Position Hold Engage and Reorient Experimental Satellites (SPHERES). This system uses bowling ball sized vehicles to demonstrate and test formation flight maneuvers. The current EMFF testbed borrows the avionics developed for these vehicles.

1.4 Thesis overview

The overall objective of this thesis is to analyze the viability of the Micro-EMFF (μ EMFF) approach. In order to achieve this, the following steps were outlined. First, develop models of the μ EMFF system to investigate the control authority achievable. Then, compare the system to traditional propulsion techniques. Last, build and operate a testbed to validate the models.

The remainder of this thesis is divided into five main chapters and a conclusion. The first main chapter, Chapter two, illustrates the basics of EMFF. The simple, far field model description of the force is given as well as the explanation of how μ EMFF is different from what has been previously studied. After familiarizing with the general concept, the model for the μ EMFF system is given in Chapter three. The necessary equations are described as well as the different parts required for the system. The Simulink model is described here and some sample inputs and outputs are shown. Chapter four explains the

large design space available and the optimization of the system over that space for minimum mass. Using the optimized systems, a few missions are analyzed in Chapter five and μ EMFF is compared to traditional propulsion techniques. In addition, the hybrid concept of using both HTS EMFF and μ EMFF together to achieve better performance is investigated. Chapter six discusses the need for a new testbed, the design, and the test results compared to the simulated values. The last chapter, Chapter seven, concludes with the thesis summary and contributions.

Chapter 2 EMFF Overview

2.1 EMFF basics

The magnetic fields for EMFF are generated by sending current through coils of wire. Solar arrays provide the power required, and reaction wheels are used to maintain the attitude. Thus far, the concept developed features three orthogonal coils to create a steerable dipole. The magnetic dipoles created are most easily understood with a far field approximation where the separation distance between two vehicles is large compared to the physical size of the coils. Imagine the two magnetic dipoles as conventional bar magnets.

Figure 2.1 Forces and torques on two dipoles in far field

It is easy to see that when two dipoles are placed aligned with their axes, they can attract or repel depending on the direction of the dipoles. What may not be obvious are the shear forces that can be produced when the dipoles are placed perpendicular to each other. In the right figure, the left dipole feels an upward force while the right dipole feels a downward force. These forces are half that of the forces in the coaxial case. When the satellites begin to move in this formation, angular momentum must be conserved; thus the individual satellites begin to rotate as shown. Combined with reaction wheels, any desired maneuver can be performed as long as the formation's center of mass is not required to change. The detailed derivation of the exact, far field, and mid field models

are described in Schweighart [3]. For the purposes of this thesis, the main equation to note is the force between two dipoles. The two dimensional equation for two dipoles restricted to a plane is the most intuitive for use in the following analysis.

Figure 2.2 Two dipoles in far field

For two dipoles as in the figure above, the force acting on vehicle A is [3]

$$\vec{F}_A = \frac{3}{4\pi} \frac{\mu_0 \mu_A \mu_B}{d^4} \left[(2 \cos \alpha \cos \beta - \sin \alpha \sin \beta) \hat{i} - (2 \cos \alpha \cos \beta - \sin \alpha \sin \beta) \hat{j} \right] \quad (2.1)$$

For the two aligned dipoles attracting case,

$$\alpha = 0^\circ$$

$$\beta = 0^\circ$$

d = distance apart

μ_0 = permeability of free space

and for each vehicle A and B,

$$\mu_i = N I \pi R^2 \quad (2.2)$$

where

N = number of turns of wire

I = current through wire

R = loop radius

Thus the equation for force is simply

$$\vec{F}_A = \frac{3}{2\pi} \frac{\mu_0 \mu_A \mu_B}{d^4} \hat{\mathbf{i}} \quad (2.3)$$

An equal and opposite force acts on vehicle B. One dipole would need to be flipped in order to achieve repulsion; as when both are flipped together, they will still attract.

Looking at equations 2.3 and 2.2, we can see that the force increases with increasing number of turns of wire, current in the wire, and the size of the coil. In fact, the force increases proportionally to the radius of a coil to the fourth power. The force quickly decreases, however, as the distance between the coils increases.

2.2 μ EMFF concept description

The EMFF concept thus far has been sized for precise and aggressive maneuvering over fairly large distances with concepts such as interferometry and telescope assembly in mind. HTS wire has been considered the enabling technology; however, the use of HTS wire requires a thermal control system, as they need to be cooled to around 100 K to achieve superconducting properties. This adds significant complexity and mass. A nominal mode of operation, such as simply keeping a fleet of microsatellites within a volume, calls for a smaller, simpler system. μ EMFF investigates the use of conventional conductors, capacitors, and solar cells for applications requiring less authority and accuracy than the traditional HTS EMFF concept. The currents through the coils for μ EMFF must be of short duration or of low enough amperage such that the system does not require an elaborate thermal control system. The pulsing operation does not allow for the precise control of the formations; however, upcoming space missions may only

require that the satellite modules stay within a certain range for communications and to avoid collisions.

In addition, the traditional HTS EMFF investigated by MIT-SSL has included three orthogonal coils and reaction wheels for each vehicle to create a steerable dipole. To reduce the mass for the μ EMFF system, it may be advantageous to have only one coil which can be reoriented by changing the spacecraft attitude. If the attitude of the vehicle does not need to be controlled, a reaction wheel is also not necessary.

Overall, μ EMFF is the EMFF concept implemented on a smaller force scale. Mass and complexity due to the thermal control system required by the HTS are saved, but a great deal of control authority is lost. However, the “ μ ” does not necessarily restrict its use solely on micro-satellites. The restrictions arise from the size of the coil, the amount of force required, and the average separation distance between the vehicles.

Chapter 3 One Dimensional Simulation

For initial investigations, two satellites are considered to be free to move in one dimension, drifting apart due to J_2 perturbations. At a specified separation distance, the μ EMFF systems on both satellites are pulsed to create an attractive force for a short duration. The satellites move in towards each other then start to drift apart again.

3.1 Necessary Equations

3.1.1 Coil properties

To model the circuit, the resistance and inductance of the coil must first be calculated. The resistance is easily found using

$$R = \frac{L\rho}{A} \quad (3.1)$$

where

L = length of wire

ρ = resistivity of material

A = cross sectional area of wire

However, there is no closed form solution for the inductance of a circular coil; thus, an approximation is used. The equation shown below [7] is used in the Simulink model.

$$L = N^2 R \mu_0 \left[\ln \left(\frac{8R}{a} \right) - \frac{7}{2} \right] \quad (3.2)$$

where

N = number of turns

R = radius of wire loop

μ_0 = permeability of free space

a = radius of the wire

This equation was verified by discretely calculating the inductance of the loop divided into smaller pieces. First, because the inductance of a straight piece of round wire or rod is given by [8]

$$L_{segment} \approx \frac{\mu_0 l}{2\pi} \left[\ln\left(\frac{2l}{a}\right) - \frac{3}{4} \right] \quad (3.3)$$

where

$L_{segment}$ = inductance of segment

l = length of segment

if l is much larger than a . Dividing the circle into n pieces, and using equation 3.3 to calculate the inductance of each piece and taking the sum, gives a similar inductance as equation 3.2 if n is specified such that l is much larger than a . However, equation 3.3 does not hold as n is increased. Thus another approximation method was used. The MATLAB code for a sample calculation can be found in Appendix A. Because the inductance can be defined as the magnetic flux over current

$$L = \frac{\Phi}{i} \quad (3.4)$$

where

L = inductance

i = current

Φ = magnetic flux

The loop was first approximated as an octagonal shape. This shape was then divided into

slices, which were then cut further as shown in the figure.

Figure 3.1 Octagonal approximation of circular loop

The magnetic field at the center of each piece was calculated assuming a 1 amp current. Then the flux was approximated as the product of the field at the center of the piece and the area of the piece. Because the current was set at 1 amp, the flux equals the inductance from equation 3.4. The inductance calculated using this method was very similar to the inductance calculated by equation 3.2 for a variety of loop sizes. Thus, equation 3.2 is used in the simulation.

3.1.2 Near field forces

The electromagnetic force equation from Chapter 2 is only valid in the far field, thus needs to be modified for the close proximity formations considered for μEMFF . The exact model derived by Schweighart [3] is computationally expensive as it requires a double integration calculation. However, the difference is a function of the ratio between the loop diameter and the separation distance. Several points were calculated for a variety of ratios and this information was fitted to a curve.

To calculate the exact force between two coils, the magnetic field at each location along

the coils must first be calculated. Using the Biot-Savart law the first integration is carried out around one current carrying loop.

$$\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{i \, d\vec{l} \times \hat{r}}{|\vec{r}|^2} \quad (3.5)$$

where

$\vec{B}$ = magnetic field at point

$\vec{r}$ = vector from current element to point

$d\vec{l}$ = segment of current carrying element

i = current

μ_0 = permeability of free space

Figure 3.2 Two current loops for exact force

So for each element $d\vec{l}_2$, equation 3.5 must be integrated around loop 1. Then the force on each element due to the magnetic field on the other coil can be calculated. The force on a small element $d\vec{l}_2$ is given as

$$d\vec{F}_{d\vec{l}_2} = i_2 \, d\vec{l}_2 \times \vec{B} \quad (3.6)$$

Thus, to find the force on the whole loop, this must be integrated around loop 2.

The MATLAB code used to calculate the exact and far field forces is found in Appendix B. The difference between the far field model and the exact model only becomes apparent when the separation to diameter ratio falls below a value of two as seen in Figure 3.3.

Figure 3.3 Exact and far field forces

Thus, the ratios of exact to far-field forces were calculated, as shown in the plot below, and placed in a lookup table used by the simulation when the vehicles travel into the close proximity region.

Figure 3.4 Force correction factor in near field

For example, a coil at a separation distance equal to the diameter of the coil, with a constant current pulse may have a far field force profile of the top half of Figure 3.5, but the exact force used for the dynamics is the bottom half.

Figure 3.5 Far field force corrected to actual

3.1.3 J_2 perturbation

J_2 perturbations are the result of the oblateness of the Earth around the equator. In orbits that are at and below GEO, the J_2 perturbations dominate over the Sun and the moon's disturbance. The J_2 perturbation acceleration is calculated from the equation below [9].

$$a = \frac{F}{m} = 3 J_2 \left(\frac{R_{eq}}{R_{orbit}} \right)^2 n^2 R_{separation} \quad (3.7)$$

Where

a = acceleration

$$J_2 = 1.083 \times 10^{-3}$$

R_{eq} = equator radius

R_{orbit} = orbit radius

$n = .001$ rad/s

$R_{separation}$ = separation distance between vehicles

At a separation of two meters the acceleration is around $3.25 \times 10^{-9} m/s^2$.

3.2 Simulation components

3.2.1 Ultra-capacitor

The Maxwell Technologies BOOSTCAP Ultracapacitors are used as the baseline for the capacitor in the simulation [10]. These capacitors are only slightly larger than a D-cell battery and have a range of capacitance from 120 to 350 farads.

Figure 3.6 Ultra-capacitor

They would be charged by the energy from the solar panels and were chosen instead of batteries for their low internal resistance. The capacitance is an input to the simulation and the mass to capacitance ratio of the BCAP0350 E250 is used to calculate the mass of the capacitor. The typical values used range from 100 to 1000 farads.

3.2.2 Aluminum or copper

Aluminum is used as the conductor with a density of 2750 kg/m^3 and resistivity of

$2.82 \times 10^{-8} \Omega \cdot m$. Copper was initially considered for its low resistivity. The resistivity of copper is typically around $1.72 \times 10^{-8} \Omega \cdot m$ but the density is around 8920 kg/m^3 .

$$\begin{aligned} 2750 \text{ kg/m}^3 \times (2.82 \times 10^{-8} \Omega \cdot m) &= 7.76 \times 10^{-5} \text{ kg } \Omega / m^2 \\ 8920 \text{ kg/m}^3 \times (1.72 \times 10^{-8} \Omega \cdot m) &= 1.53 \times 10^{-4} \text{ kg } \Omega / m^2 \end{aligned} \quad (3.8)$$

The smaller value for the product of density and resistivity achieved by aluminum proved it to be the more efficient choice.

3.2.3 Solar panels

The solar panel's specific power is assumed to be 25 W/kg, a conservative value for silicon solar panels. Larger specific power was also tested as many triple junction solar panels possess specific power specifications well above 75 W/kg. However, the solar panel mass was always insignificant compared to the coil and capacitor mass, thus the conservative value was left in the simulation.

3.4 Simulation description

A setup file, which initiates the model inputs, is run first in the MATLAB environment. This is where the mass of the satellite is calculated from the satellite bus mass, wire mass, capacitor mass, and solar panel mass. The dynamics of the system coded in Simulink includes the coil and capacitance circuit as well as the dynamics of the total mass of the satellite. The forces acting on each satellite are simply the J_2 and electromagnetic forces along the center axis. The circuit is modeled as a resistor, inductor, and capacitor (RLC), as shown below.

Figure 3.7 RLC circuit simulation

The current output from this section is put into equations 2.2 and 2.3 to calculate the electromagnetic force and multiplied by the near force correction factor if necessary. The above circuit and the calculation of the force are triggered when the satellites drift apart to a specified separation distance.

The dynamics for one satellite are modeled as shown below. The position output is fed back to the trigger for the RLC circuit as well as the calculation of the force.

Figure 3.8 Dynamics simulation

3.5 Simulation inputs and outputs

The inputs into the simulation are shown along with typical values. These properties are initialized in the setup file written in MATLAB.

Table 3-1 Typical inputs into simulation

Wire loop radius	0.25 m
Wire material	Aluminum
Number of turns or wire	25
Total wire mass	2 kg
Capacitance	500 F
Maximum separation	2 m

The number of turns of wire was set at 25 for the particular case using the data from the graph below.

Figure 3.9 Change in velocity due to number of turns of wire

We know from equation 2.2 that increasing the number of coil turns, N , results in increasing force. However, not only does the resistance increase due to increasing length of wire, when the total mass of the coil is limited, in order to increase the number of turns, the radius of the wire must decrease. The wire radius is calculated from the wire

loop radius, wire mass, and number of turns. Thus when the number of coils is increased, the resistance is increased two-fold, and with a higher resistance, the force dissipates at a faster rate in the circuit. However, when the number of turns is decreased, the inductance decreases, resulting in smaller force amplitude. Thus, for this case, the number of turns of wire is set to 25, where the two opposing effects seem to find a good balance.

The operation of the initial model is set such that at a specified separation distance, the EMFF system pulses for a short duration creating an attractive force. The satellites move in towards each other until the J_2 forces overcome the pulse acceleration and push the satellites apart again. The J_2 forces are calculated using the specified pulse distance and assumed to be constant over the maneuver. Because the satellites move in beyond and not out past this distance, this makes the simulation more conservative.

A typical force profile is shown.

Figure 3.10 Typical force profile from simulation

The overdamped resistance, inductance and capacitance circuit has a fairly long time constant. For efficient operation, the circuit is broken after a short duration as when the current drops, the force is not as high. The system performs better when the pulse is cut off and the capacitor is allowed to charge back to its original voltage often instead of letting the voltage drop continue.

The plot below shows the position, velocity, and acceleration of one of the satellites in the simulation. Note the position is measured from the center of mass of the two satellites, thus the actual maximum separation distance is set at two meters.

Figure 3.11 Position, velocity, and acceleration output from simulation

The satellite has an assumed bus mass of 10 kg, and an added mass of 2.12 kg: 2 kg wire

and 0.12 kg capacitor. Some parameters of interest are given in the table below.

Table 3-2 Sample model parameters

Force period	3.6 hours
Radius of wire	2.5 mm
Peak current	32.5 Amps
Peak magnetic field	20.5 G

The system is forced every 3.6 hrs with a pulse duration of around 0.2 sec. To charge the capacitors for this case, the total energy stored is 2200 J, thus to charge in the 3.6 hours between pulses, 0.17 W is needed. Assuming silicon solar cells, this requires only 7 g or 8.4 cm² of solar array. This again is a conservative estimate as the total energy is not dissipated in each pulse.

Using less capacitance, the force period becomes shorter.

Figure 3.12 Position, velocity, and acceleration output from simulation, shorter period

The case graphed above results in the following parameters.

Table 3-3 Sample model parameters, frequent pulses

Force period	1.3 hours
Total capacitance	260 F
Peak current	25 Amps
Total mass	12.45 kg

In this case, the charge time is reduced to 1.3 hours. However, the energy stored is also reduced to 820 J, thus the Watts required and amount of solar array required are the same as above. Less mass is required while keeping the same separation distance by having more frequent pulses.

Thus, using the same model, a different mode of operation was investigated. Powering the coils constantly with small oscillating currents provides an oscillating attractive force between the satellites, which may be a more effective approach. In addition, the oscillating magnetic field would cancel out the effect of the Earth's magnetic field.

The torque produced due to the Earth's magnetic field is the cross product of the magnetic dipole of the coil and the local geomagnetic field vector, $\mathbf{B}$. The geomagnetic field vector is calculated as

$$\begin{bmatrix} B_r \\ B_n \\ B_x \end{bmatrix} = \begin{bmatrix} \frac{-2M_e}{r^3 \cos n} \\ \frac{-M_e}{r^3 \sin n} \\ 0 \end{bmatrix} \quad (3.9)$$

The angle n is measured from the dipole axis. M_e represents the strength of the Earth's magnetic dipole, and has a value of 8.1×10^{15} Wb.m [V.s.m]. The worst case angle, 90 degrees, and a satellite shape of a sphere for the moment of inertia calculations are assumed.

The first plot below shows the position, velocity, and acceleration of a satellite in such a configuration. Although at first glance the satellite seems to be moving significantly back and forth, upon noting the scale of the plots, it can be seen that the satellites are scarcely moving. The position scale is always 1.9191 meters and both the velocity and acceleration plots have a scale of 10^{-9} .

Figure 3.13 Position, velocity, and acceleration output from simulation: oscillating dipole

Figure 3.14 Angular position output from simulation, oscillating dipole

The oscillating dipole of the system cancels out the effect of the Earth's magnetic field over time and thus results in angular position as shown in Figure 3.14. The satellite oscillates slightly as the dipole oscillates.

Although this mode of operation seems attractive, the heat produced by having constant current running through the coils may require a thermal control system.

3.6 Temperature of Coil

Because the major advantage μ EMFF has over HTS EMFF is the simplification of the thermal control system, the temperature of the coils should be estimated. If the coil becomes too hot, a thermal system will still be required. However, if the temperature is within a reasonable range, the cooling system may be as simple as adding a different coating with higher emissivity. A simple radiation model is used to approximate the temperature.

Assuming the total energy into the coil is the total energy stored in the capacitor

$$U = \frac{CV^2}{2} \quad (3.10)$$

where

U = energy in Joules

C = capacitance

V = voltage across capacitor

This is an over estimate of the energy, as it assumes that all the energy stored in the capacitor goes into the coil each pulse. This is not the case as the pulse duration is cut off. However, this is just an approximation of the mean temperature.

Divide the above by the time between pulses, t , to get $\dot{E}_{in}$ in Watts. The rate of energy dissipated by radiation is

$$W = \sigma A k_B (T^4 - T_s^4) \quad (3.11)$$

where

W = energy rate in Watts

σ = emissivity

A = surface area

k_B = Boltzmann constant

T = temperature of material

T_s = surrounding temperature

The surface area is assumed to be a torus, with the cross sectional area equal to the bundle of wire in the coil. Thus the surface area, A , is calculated as

$$A = 4\pi^2 R r \quad (3.12)$$

where

R = coil radius

$$r = \sqrt{N r_{wire}^2}$$

N = number of turns

r_{wire} = wire radius

Setting the energy in equal to the energy out, the temperature of the material at steady state, T , can be found.

$$T = \sqrt[4]{\frac{CV^2}{2t\sigma A k_B} + T_s^4} \quad (3.13)$$

For example, for a coil with the following properties

$$r_{\text{wire}} = 0.0006 \text{ m}$$

$$C = 250 \text{ F}$$

$$N = 250$$

$$R = 0.5 \text{ m}$$

$$t = 15 \text{ s}$$

The steady state temperature assuming that the surrounding temperature is absolute zero gives $T = 296 \text{ K}$. This however is a very conservative estimate as stated before, it assumes that all the energy stored in the capacitor goes into the coil each pulse. Taking into account that the pulse is cut off, if the voltage drop for each pulse is only 0.2 V , the steady state temperature of the above case becomes $T = 185 \text{ K}$.

Chapter 4 Optimization of model

As was seen with the number of turns of wire in the sample case described in Chapter 3, the system can be optimized for minimum mass.

4.1 Design space

The system allows for a large design space, with many parameters that can be varied. There are six parameters input into the simulation, which can be varied to output a different mass and different average acceleration rejection. In addition, there are seven constants that are required for calculating the acceleration and thus are all used in the dimensional analysis. Simply optimizing using the six parameters which are varied seemed to be a large trade space. Hence the Buckingham pi theorem was employed to try to reduce the space.

Table 4-1 Varied and constant parameters

Wire radius	r	Resistivity of wire	ρ_{res}
Number of loops	N	Density fo wire	ρ_{mass}
Time between pulses	t_{charge}	Mass density of solar power	α_{solar}
Radius of loop	R	Permeability of free space	μ_0
Impulse	Im	Distance between satellites	d
Capacitance	Cap	Mass density of capacitance	β_{cap}
		Voltage across Capacitor	Volts

The theorem loosely states that if there is a physically meaningful equation involving a

certain number of parameters, N , and these variables are expressible in terms of M independent fundamental dimensions; then the original expression is equivalent to an equation involving a set of $N-M$ dimensionless variables, called π 's, constructed from the original parameters.

$$N = \text{number of parameters (including min mass)} = 14$$

$$M = \text{number of fundamental dimensions used} = 4$$

Therefore, there should be ten independent dimensionless variables. If the seven parameters kept constant in the optimization can form more than four independent dimensionless variables, then the six varied parameters can combine to create a dimensionless variable. Thus, when the combined parameters are varied such that the dimensionless π 's do not change, the value of the expression does not change.

For example, if wire radius and radius of the loop could be combined to form a dimensionless π , and not used in any other π

$$\frac{r \left[\frac{m}{m} \right]}{R \left[\frac{m}{m} \right]} = \pi_1 \quad (4.1)$$

the final expression, in this case the mass, would not change if we increased the wire radius by a factor of two and also increased the loop radius by a factor of two.

To find the dimensionless variables you can write the parameters' dimensions' powers in a matrix. For example, in the column for density, $\rho_{\text{mass}} \left[\frac{kg}{m^3} \right]$, -3 goes in the meter row

for the $\left[\frac{1}{m^3} \right]$, and 1 in the kilogram row for $[kg]$.

Table 4-2 Units dimensions of parameters

		ρ_{res}	ρ_{mass}	α_{solar}	μ_0	d	β_{cap}	Volts	r	N	t_{charge}	R	Im	Cap	Min Mass
Units/Dims	m	3	-3	-2	1	1	2	2	1	0	0	1	1	-2	0
	kg	1	1	0	1	0	2	1	0	0	0	0	1	-1	1
	s	-3	0	3	-2	0	-4	-3	0	0	1	0	-1	4	0
	A	1	0	0	-3	0	-2	-1	0	0	0	0	0	2	0

For a matrix T, the null space of T gives the set of linearly independent vectors X such that $TX = 0$. X is a linear combination of the parameters' dimensions which gives a zero vector. This combination can be interpreted as the product of the parameters which give a dimensionless quantity.

The null space of the above matrix shown below gives the independent dimensionless combinations.

Table 4-3 Null space of units dimensions of parameters

	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}
ρ_{res}	-9	-32	-19	-9	0	-6	-9	-30	5	-27
ρ_{mass}	12	-16	-4	12	0	8	12	-4	8	-8
α_{solar}	-11	0	11	-11	0	-22	-11	-22	-33	-33
μ_0	-3	-40	-21	-3	0	-2	-3	-10	31	-9
d	44	0	0	0	0	0	0	0	0	0
β_{cap}	0	44	0	0	0	0	0	0	0	0
Volts	0	0	44	0	0	0	0	0	0	0
r	0	0	0	44	0	0	0	0	0	0
N	0	0	0	0	44	0	0	0	0	0
t_{charge}	0	0	0	0	0	44	0	0	0	0
R	0	0	0	0	0	0	44	0	0	0
Im	0	0	0	0	0	0	0	44	0	0
Cap	0	0	0	0	0	0	0	0	44	0
Min Mass	0	0	0	0	0	0	0	0	0	44

There is no way to manipulate this matrix such that the constant variables, the seven from the top, combine to make more than three independent dimensionless π 's, thus the number of varied parameters cannot be reduced. The design space of the system cannot be reduced by combining any of the varied parameters; however, each varied parameter can have its own π . Varying these dimensionless π 's is the same as varying the parameters, as all other variables in each π is constant.

This procedure was also repeated with the addition of the percentage of time constant. However, because that parameter is already non-dimensional, the results were the same.

4.2 Matlab optimization and results

To optimize the system for minimum mass while still meeting the acceleration constraint, the *fmincon* function of MATLAB was utilized. A sample of the code used can be found in Appendix C. The function finds a constrained minimum of a function of several parameters starting at an initial estimate varying the parameters within set ranges. The *fmincon* function takes in the mass calculation as the function to minimize. The constraint is that the impulse achieved by a single pulse divided by the product of the mass and the time between pulses, the acceleration, must be at least some prescribed value. Thus the acceleration is found from

$$a = \int_0^t \frac{F}{m(T)} \quad (4.2)$$

where

$$t = P\tau$$

P = percentage of pulse

τ = circuit time constant

a = acceleration

F = force

m = mass of satellite

T = time between pulses

The fmincon function calls the minimizing and constraint functions for every combination of parameters it tries; making the integral in the above calculation time consuming. So we use equation 2.3 for force from Chapter 2 and plug in equation 2.2 assuming $\mu_A = \mu_B$ to get

$$\bar{F}_A = \frac{3}{2} \frac{\mu_0 N^2 I^2 \pi R^4}{d^4} \hat{\mathbf{i}} \quad (4.3)$$

Then, if we assume that the duration of the pulse is short enough that d^4 , the fourth power of the distance between the vehicles, does not change significantly during the pulse, the only time dependant variable is I , the current. Using the fact that the system is, except in extreme cases, always overdamped:

$$R^2 > \frac{4L}{C} \quad (4.4)$$

where

R = resistance

L = inductance

C = capacitance

We can use the following formula for current

$$I(t) = \frac{V_0}{\tilde{\omega}_0 L} e^{\frac{-R}{2L}t} \sinh(\tilde{\omega}_0 t) \quad (4.5)$$

where

$$\tilde{\omega}_0 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad (4.6)$$

V_0 = initial voltage on capacitor

t = time

MATLAB can symbolically integrate the square of the above function from t_1 to t_2

$$\int_{t_1}^{t_2} (I(t))^2 \quad (4.7)$$

and this can be multiplied by the rest of the variables in equation 4.3 and divided by the mass to find the average acceleration the satellite can reject.

For a particular distance apart, and thus a particular J_2 acceleration that needs to be combated, the system can be optimized for minimum mass by varying the capacitance, the radius of wire, the radius of the loop, the number of loops, and the time between pulses. The output of the code can be set to simply give the results of this optimization. However, to analyze the behavior and to get a feel for how the mass varies; one input was varied systematically while the others were allowed to reach their optimum. The minimum mass was calculated for a system at several different separation distances.

For example, the plots below show the results from varying the time between pulses from 1 to 200 seconds, but allowing the rest of the parameters to reach their optimal values within set constraints. The *fmincon* function does not always settle on the absolute minimum thus there may be a few outliers to the plots.

Figure 4.1 Optimization output, varying time between pulses

There are several things to note from the above plots. First the minimum mass systems for these particular cases, with the satellites positioned at 20, 30, and 40 meters can be found in the top left plot. The shortest time between pulses provides the minimum mass system for these cases. The rest of the plots show the value of the particular parameter

that f_{mincon} found to produce the minimum mass for that T . For the above results, the maximum allowed radius of the loop, R , is 0.75m, and the minimum mass system always reaches this maximum. A similar observation can be made for the number of loops, N . The pink points of the plots are cases such that the constraint was not satisfied or the bounds of a parameter were not met. In the above case, a 40 meter apart system which will combat J_2 forces cannot be achieved at low pulse rates without increasing the loop radius to above the set bound.

Figure 4.2 Optimization output, varying number of turns

The above plots are for similar systems as before, however, the number of loops, N , was varied from 1 to 250. The minimum mass is once again found in the top left plot. The minimum mass system for each separation distance for the above system is reached at the maximum allowed number of loops. The systems always utilize the maximum allowed

loop radius as well as the minimum allowed time between pulses in the above plots.

Although the code was initially written to analyze J_2 combating behavior, the acceleration required can be manipulated to provide the minimum mass system for a variety of missions. The system behavior changes significantly depending on the separation distance, satellite bus mass, and acceleration. The only consistent result seemed to be that for minimum mass, the radius of the loop was always the maximum allowed. The other parameters all have different characteristics depending on the operating range.

Several sample systems found using the optimization code were verified using the Simulink model. The minimum mass system's parameters were input into the simulation and it was verified that the expected acceleration was achieved.

Chapter 5 Mission Analysis

Using the minimum mass systems found with the optimization code, the use of μ EMFF was analyzed for several different mission scenarios. These scenarios were motivated by the DARPA F6 project, for which fractionated modules in close proximity would need to avoid collision and stay within cross-link supportable range [11].

Because the mass of the EMFF system changes significantly with varying satellite bus mass, separation distance, and ΔV required, a plot of mass fraction of EMFF is shown below for the different cases.

Figure 5.1 Mass fraction of μ EMFF for 5 yr missions

The upper shaded area is for satellites with a separation distance of 10 meters, while the lower is for 5 meters. The heavier the satellite bus mass, the lower the mass fraction.

5.1 J_2 perturbation

The J_2 perturbation scenario was the initial mission used in the simulation and optimization code and is explained in Chapter 3. The ΔV required for this mission scenario is fairly small, even for mission lifetime of 5 to 10 years. The small forces required are ideal for the μ EMFF system. To get a general idea of how much a μ EMFF system for J_2 perturbation combat would weigh, below is a plot of the μ EMFF system for a satellite bus mass of 50 kg, for a variety of average separation distances. Note that mission lifetime is not a factor in the mass of the system, as no consumables are being expended.

Figure 5.2 Mass of μ EMFF for J_2 perturbations, bus mass 50 kg

J_2 accelerations scale with separation distance, and for a 10 year mission, separation distance in meters effectively equals ΔV in meters per second. To compare the μ EMFF system to traditional propulsion systems, the masses of PPT, Cold Gas, and FEEP systems for the same mission were calculated. The detailed description of how to estimate these masses is outlined by Reichbach [13].

Figure 5.3 Mass of μ EMFF and traditional propulsion technologies for small delta V

For the 10 year mission life scenario, PPT and FEEP propulsion system masses in this regime are dominated by solar array mass and thus do not change significantly over the small ΔV range. μ EMFF is most favorable for small ΔV , under 10 m/s; and at less than 5 m/s, the μ EMFF system is less than half the mass of the cold gas system.

5.2 Drag makeup

Atmospheric drag in LEO results in ΔV required to stay in orbit. With traditional thruster technology alone, each fractionated module would require a traditional propulsion system. Using EMFF, one module could have a traditional propulsion system while all modules would have the EMFF system.

Approximating the mass of a satellite as

$$M_{SAT} = M_0 + M_{PS} + M_k + M_p \quad (5.1)$$

where

M_0 = dry mass of satellite

M_p = mass of propellant

M_k = mass of tank

M_{PS} = mass of propulsion system

M_{EMFF} = mass of EMFF

N = number of satellites

Using the rocket equation,

$$\frac{M_p}{M_{FINAL}} = e^{\frac{\Delta V}{I_{sp}g}} - 1 \quad (5.2)$$

and

$$\alpha = \frac{M_p}{M_k} \quad (5.3)$$

For the case of having traditional propulsion systems on all the modules the total mass is,

$$|M_{total}|_{ALL} = N(M_0 + M_{PS}) \left[1 + \frac{(1 + \alpha) \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)}{1 - \alpha \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)} \right] \quad (5.4)$$

For the case of having EMFF on all the modules and a traditional propulsion system on just one,

$$|M_{total}|_{EMFF} = (NM_0 + M_{PS}) \left[1 + \frac{(1 + \alpha) \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)}{1 - \alpha \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)} \right] + N(M_{EMFF}) \quad (5.5)$$

By taking the difference between the two cases, we see that we need

$$M_{EMFF} < \frac{(N-1)}{N} \left[1 + \frac{(1 + \alpha) \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)}{1 - \alpha \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)} \right] M_{PS} \quad (5.6)$$

for the EMFF system to be more mass efficient than the traditional case.

From the $\frac{(N-1)}{N}$ term we see that as the number of modules increases, the more massive the EMFF system may be. Assuming that N is large enough that this term is close to unity, we plot

$$\frac{M_{EMFF}}{M_{PS}} \approx \left[1 + \frac{(1 + \alpha) \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)}{1 - \alpha \left(e^{\frac{\Delta V}{I_{sp}g}} - 1 \right)} \right] \quad (5.7)$$

and use

$$\beta = \frac{\Delta V}{I_{sp}g} \quad (5.8)$$

Figure 5.4 Mass of EMFF over mass of traditional propulsion system using α and β

We see from this plot that as α and β increase, the more massive the EMFF system may be.

We can calculate a typical value for α

$$\alpha = \frac{M_p}{M_k} = \frac{2}{3} \frac{\sigma}{\rho_T RT} \quad (5.9)$$

where

σ = ultimate stress of tank material

ρ_T = tank material density

R = gas constant of fuel

T = temperature of fuel

For cold gas with Freon

$$\alpha \approx 6.5$$

Taking the cross-section of the above plot at $\alpha = 6.5$ and bringing back the $\frac{(N-1)}{N}$ term,

Figure 5.5 Mass of EMFF over mass of propulsion system for $\alpha=6.5$

The shaded area has no physical meaning as this is when the tank mass equals the total dry mass.

To use the above plot, we need to estimate a ΔV required. For drag makeup, ΔV depends on ballistic coefficient, altitude, and mission life [12].

Figure 5.6 Delta V required per year at different altitudes

For example, if the Ballistic coefficient is 10 kg/m^2 at an altitude of 550 km, $\Delta V / \text{yr} = 100 \text{ m/s/yr}$. If the mission life is 5 yrs, $\Delta V = 500 \text{ m/s}$. Going back to Figure 5.5 for a $\Delta V = 500 \text{ m/s}$, and assuming $I_{sp} g = 1000 \text{ m/s}$, the EMFF system may be almost as twice as massive as the traditional system to have the overall system mass still be the same.

The above analysis simply shows that if the EMFF system can be made comparable in

mass to other propulsion systems, it may be beneficial to use for drag-makeup in the above mentioned manner. It would at least be comparable in total system mass, and may add the advantage only needing to replace a single thruster module when refueling, and easier thrust plume management.

5.4 Hybrid analysis

Because the μ EMFF system mass increases significantly as the distance between the vehicles increases, the idea of having a “mother ship”, employing HTS EMFF, was investigated. One could imagine a cluster in which the HTS EMFF satellite would sweep its dipole around to the μ EMFF satellites which pulse when aligned to pull itself in to the cluster. Although the HTS EMFF system will add the complexity and mass of a thermal control system, it will also greatly increase the range with just the addition of one HTS vehicle. To analyze this concept, the optimization code was edited to include an HTS EMFF model, and the following configurations were modeled.

Figure 5.7 Modeled hybrid configurations and their operations

The μ EMFF model as before includes the wire, capacitor, and solar array. The HTS EMFF model includes the wire, thermal control system (TCS), and solar array. The TCS consists of a copper jacket, multi-layer insulation, and a cryogenic cooler, but assumes that the multi-layer insulation is negligible compared to the copper jacket. The solar array provides enough power for the cryogenic cooler and the coil, and assumes that both are powered continuously. Some sample masses of the systems are given below.

Table 5-1 Hybrid analysis masses

Separation [m]	Acceleration [m/s]	Micro Total [kg]	Hybrid Total [kg]	hybrid- micro [kg]	hybrid- HTS [kg]	HTS Total [kg]	Minimum Mass system
5	1.63E-08	3.23	6.1	0.7	5.4	10.794	micro
10	3.25E-08	7.99	7.75	1.65	6.1	11.425	hybrid
20	6.50E-08	35.1	14.61	4.19	10.4	12.559	HTS
30	9.75E-08	96.8	50.2	34.7	15.5	14.7	HTS
5	1.63E-07	4.77	6.96	1.06	5.9	11.4	micro
10	3.25E-07	19.84	10.9	3.17	7.73	12.6	hybrid
20	6.50E-07	111.1	88.1	77.4	10.7	15.3	HTS
30	9.75E-07	309.7	148	88.1	59.9	26.2	HTS

A range of separation distances, as well as a range of acceleration requirements were analyzed. The last column gives which system, micro, hybrid, or HTS, supplied the required acceleration with the least mass.

The plot below shows at what separation distance and acceleration required the various configurations are the most advantages. The J_2 perturbation acceleration is labeled and is used as the minimum that the satellites must perform. Using a ballistic coefficient of 100 kg/m², the drag at different LEO altitudes is also labeled.

Figure 5.8 Hybrid analysis, 2 vehicles

The masses given in table 5-1 are for two satellites. In the hybrid case, the μ satellite is much lighter than the HTS satellite. In the μ and HTS cases, the two satellites are the same mass. Thus, adding more satellites is advantageous for the hybrid case. For example, if there are 3 vehicles,

$$\mu \text{ mass } 3 \text{ vehicles} = 1.5(\mu \text{ mass } 2 \text{ vehicles})$$

$$\text{HTS mass } 3 \text{ vehicles} = 1.5(\text{HTS mass } 2 \text{ vehicles})$$

$$\text{Hybrid mass } 3 \text{ vehicles} = 2(\text{hybrid-}\mu \text{ mass } 2 \text{ vehicles}) + (\text{hybrid-HTS mass } 2 \text{ vehicles})$$

Assuming the two μ satellites pulse at different times, the area in which the hybrid case is the least massive increases significantly.

Figure 5.9 Hybrid analysis, 3 vehicles

5.5 Summary

The analysis in this chapter shows that the μ EMFF masses are comparable to traditional propulsion technologies, especially at small separation distances and small required acceleration as in the J_2 perturbation operation. Thus, the drag makeup analysis showed that if the μ EMFF mass is comparable to traditional technologies, there could be mass savings involved in using μ EMFF for drag make up operations. In addition, if a mother ship concept is used such that one vehicle employs HTS EMFF, the operating range is greatly expanded without a significant mass increase.

Chapter 6 Testbed Design and Tests

6.1 The need for a new testbed

Because the μ EMFF concept is for such small forces, a system with small disturbances was needed for validation of the simulation. The use of a system similar to the existing EMFF testbed, which floats on air carriages on an optics table, was investigated.

First, several accelerations for reasonably sized systems were found using the simulation described in Chapter 3.

Table 6-1 Simulated accelerations for reasonably sized test systems

separation distance (m)	force (N)	# of turns	mass of wire (kg)	radius (m)	radius of wire(m)	acceleration (m/s ²)
1	0.017	30	2	0.25	0.00220	0.0028
1	0.025	30	2.5	0.25	0.00250	0.0033
1	0.01	30	1.5	0.25	0.00200	0.0022
1	0.004	30	1	0.25	0.00160	0.0013
1	0.0038	40	1.25	0.25	0.00150	0.001
1	0.0025	60	1.5	0.25	0.00140	0.0006
1	0.0025	40	1	0.25	0.00140	0.0008
0.75	0.011	30	1	0.25	0.00160	0.0037
0.75	0.01	40	1.25	0.25	0.00150	0.0027
0.75	0.0064	60	1.5	0.25	0.00140	0.0014
0.75	0.0063	40	1	0.25	0.00140	0.0021

A range of .0006 to .0037 m/s² was identified as the acceleration a practically sized coil

could produce.

The disturbances were then measured on a system similar to the existing testbed, by simply measuring how long it took a module to drift a certain distance when let go without any initial velocity.

Table 6-2 Air carriage disturbance data

Distance (cm)	12	22
Time (s)	11.34	17.74
	12.11	20.28
	11.55	24.99
	12.59	24.46
	11.99	20.4
	10.66	21.31
	10.7	21.45
	11.38	22.63
	13.41	20.88
Average time (s)	11.74778	21.57111
Average acceleration (m/s²)	0.00087	0.000473

The disturbances on the table fall within the range of reasonable values of acceleration the μ EMFF system could produce with a practical sized coil. Thus, it would be hard to differentiate what motion is intentional and what is disturbance with the existing testbed.

6.2 Testbed description

The new system requires a low friction, low disturbance environment such that the small forces produced by the μ EMFF system can be measured. Thus, a device like the one in figure 6.1 was built. There is an air bearing at the joint of the T structure which allows for low friction rotation of vehicle A and a counter mass. In addition, each vehicle

rotates around its center pole, controlling their attitude, such that the two vehicles may align their dipoles at different distances apart.

Figure 6.1 Testbed description

A carbon fiber tubing is used as the top bar of the T. Plastic caps were fitted to the ends to which aluminum rods were affixed to hold the vehicle and counter mass.

The air-bearing requires a 60-80 psi air source and floats a stainless steel part which holds the carbon fiber tube. The stainless steel part has a tight flatness tolerance that must be met on the face that encounters the air-bearing in order to ensure the low friction characteristics. A level is attached at the top of the steel part to ensure that the structure is level.

Figure 6.2 Air bearing and steel fitting

The electronics for the testbed are being developed in two phases. Currently, at the end of phase 1, the testbed is radio controlled (RC). The attitude of each vehicle is controlled by servos and a dipole pulse is also activated by the RC transmitter. This system is used to validate the simulations and to get a feel for the operating range of the system. In phase 2, a SPHERES module, developed by the MIT-SSL, will be incorporated into each vehicle. The SPHERES vehicles will provide a programmable platform with which different operations and controls schemes can be tested. This will also progress the technology towards a more flight-like state.

A block diagram of the phase 1 electronics is given below.

Figure 6.3 Phase 1 electronics

The only difference between the two vehicles is the second signal out of the receiver. The first signal is the same, Ch1, so that the two vehicles pulse their dipoles at the same time. The second signal, Ch2 and Ch3, controls the servo, thus must be different on each vehicle to control the attitudes separately.

6.2.1 D cell batteries

The coil is powered by the same D-cell nickel metal hydride (NiMH) batteries as the HTS EMFF testbed. Thus far, the system has been modeled to have ultra-capacitors charged by solar panels. However, because we do not have solar panels and a source of sunlight in the lab, and any umbilicals on the vehicle would affect the dynamics, batteries are required for power. Although the NiMH batteries provide the power we require, the

internal resistances are slightly higher than the ultra-capacitors. However, because of the mass savings of having just the batteries and the complexity of the charging electronics of the ultra-capacitors, the decision was made to power simply using the batteries. In addition, the added resistance of the batteries is negligible compared to the resistance of the connections in the system.

6.2.2 OSMC

The Open Source Motor Controller (OSMC) is used to switch power to the coils on and off. The board is simply an H-bridge designed to withstand high currents. This same board is used on the HTS EMFF testbed. The OSMC comes in a kit form. The only modification to the kit made was the replacement of the 16 MOSFETs with a lower resistance model, IRF2903Z, which kept the same footprint.

Figure 6.4 Open source motor controller board

6.2.3 DALF

The Dual Advanced Closed-Loop Motor control (DALF) is a microcontroller based interface to the OSMC. The DALF generates the PWM signals required by the OSMC, and with the addition of a SPHERES vehicle, may not be needed in phase 2. For the tests

described below, the DALF is programmed to accept RC receiver signals and activate a pulse in the OSMC.

Figure 6.5 Dalf board

The power to both the OSMC and Dalf board are provided by ten NiMH AA batteries.

6.2.4 RC electronics

The Airtronics VG 400, 4-Channel FM transmitter was chosen for controlling the phase 1 system. Channel 1, the right stick is devoted to the activation of a pulse, while channels 3 and 4, the left stick, control the attitudes of the two vehicles.

Figure 6.6 RC transmitter

To interface with the transmitter, the Airtronics DC FM Micro Receiver is placed in each vehicle, along with the Airtronics 94102Z Heavy Duty Servo to control the attitude. The power is provided to both by a 4.8 volt nickel-cadmium battery.

Figure 6.7 RC receiver, servo, and battery

In an attempt to shave the mass of the system down, at first the RC receiver, servo, OSMC, and Dalf boards were all powered by the same batteries. However, this caused unexpected movements in the servo when the pulse was activated and thus the original battery from the RC kit was added.

6.2.5 Coil

The coil is made out of a seven strand, five gauge, industrial power transmission cable. The wire was hammered into shape using a wheel like template and ratchet system and held together using heavy duty zip ties. The template was a two sided wheel with pegs sandwiched in between to form the correct diameter. The zip ties were placed in the structure before the wire was wound as loosening the sides of the template to fit them in after the fact caused the coil to fall apart.

Figure 6.8 Coil

When the system was first powered, the current through the coil was much smaller than expected, thus the resistance of the coils was measured to investigate this problem. The resistance of the coil was calculated assuming a single strand wire of equal diameter and using the standard resistivity of aluminum, $2.82 \text{ E-}8 \text{ } \Omega \text{ m}$.

$$R = \frac{L\rho}{A} = \frac{(47.12 \text{ m})(2.82 \text{ E-}8 \text{ } \Omega \text{ m})}{\pi(2.3 \text{ mm})^2} = .0800 \text{ } \Omega \quad (6.1)$$

When the resistance was calculated by applying a known current through the coil while measuring the voltage, it was found to be slightly larger.

$$R = \frac{V}{I} = \frac{3.5 \text{ V}}{40 \text{ amps}} = .0875 \text{ } \Omega \quad (6.2)$$

However, the difference is small, and can be attributed to the inaccuracy in the current measurement, the resistance in the connections in the test system, or the resistivity of the

particular aluminum.

The highest resistance was found in the battery holder connections when an end-to-end type battery holder was in use. The end connections in the holder were found to have a resistance of almost $0.05\ \Omega$ at each end. Thus, they were replaced with side-by-side aluminum battery holders which were enhanced by soldering higher gauge wires to the connections between the batteries as well as the end connections. The connections resistances were then found to be less than $.01\ \Omega$.

6.2.6 Completed Phase 1 System

The electronics are housed in two polycarbonate boxes. One holds the core EMFF equipment which will be needed in phase 2, while the other holds the phase 1, RC equipment.

Figure 6.9 Completed phase 1 vehicles

One vehicle is mounted onto the carbon fiber rod, and the other is mounted upside-down onto a base plate.

Figure 6.10 Complete testbed

6.3 Tests and Results

In order to validate the simulation, first the testbed must be characterized. To make sure that the dipoles are not interacting with any other magnetic field, the mobile vehicle was powered alone to show that there was no significant movement. There is a little movement in the mobile due to the air circulation around the room, but this is only noticeable when the mobile is left for around a minute or there is other movement around the vehicles. Next, the friction and any disturbances in the system need to be modeled. Thus, without the coils powered, the vehicle was given a small push and the deceleration was measured. Initially, there was a higher friction force than was expected from the air-bearing; giving a coefficient of friction of almost 0.001 while the expected value was two orders of magnitude less. However, this was found to be due to the stainless steel fitting not being made to specifications. Thus, the part was remade.

With the new part in place, friction measurements were taken.

Figure 6.11 Friction and disturbance tests

Tests 1 and 2 show that at high velocity the friction is small enough to ignore; however, test 3 shows a significant friction at slow velocities. Thus, after taking more samples at the slow velocities and fitting a curve to the data, the acceleration due to friction was found to be between -0.00001 and -0.00002 m/s^2 . These values are negligible when performing the following validation tests; however, they are included in the simulation as a constant deceleration.

6.3.1 Repulsion Tests

To compare the simulation and testbed, repulsion maneuvers were performed from a certain distance apart. A laser pointer was placed on the end of the carbon fiber tube and equal angle separation markings were made on the walls. The times at which the light moved over marked positions were recorded and the distance around the circle was calculated. This was compared to the position data from the simulation.

The simulation was altered to have a short constant voltage across the coils; the duration of each pulse is 0.92 seconds. Because the voltage of the battery drops after a few pulses, a test current is noted before and after a set of maneuvers. These are used for the sim and sim end values.

Figure 6.12 Half meter repulsion test and simulation

At a starting distance of a half a meter, the test results mostly lie within the simulated

positions. The differences can be attributed to such things as the air circulating in the room and misalignment of the coils.

Figure 6.13 Quarter meter repulsion test and simulation

The differences are more pronounced in the quarter meter separation tests. However, because small misalignments can cause a fairly large difference in the force, these results are still reasonable.

6.3.2 Sources of Errors

Because the alignment is performed by eye, the two coils may not be perfectly parallel to each other. In addition, the stationary coil may not be perfectly upright as this is also set by eye. The figure below depicts dipoles created by misaligned coils.

Figure 6.14 Misaligned dipoles

As given in Chapter 2, when aligned the force on vehicle A is

$$\vec{F}_A = \frac{3}{2\pi} \frac{\mu_0 \mu_A \mu_B}{d^4} \hat{i} \quad (6.3)$$

With a 2.5 degree tilt in each direction,

$$\alpha = \beta = 2.5^\circ$$

$$\vec{F}_A \Big|_{2.5^\circ \text{ off axis}} = 0.95 (\vec{F}_A) \hat{i}$$

With a 5 degree tilt in each direction,

$$\alpha = \beta = 5.0^\circ$$

$$\vec{F}_A \Big|_{5^\circ \text{ off axis}} = 0.90 (\vec{F}_A) \hat{i}$$

There is a 10% difference in the force in the $\hat{i}$ direction.

In addition, the stationary vehicle is not exactly at the same height as the mobile vehicle.

Assuming that the dipoles are parallel, but 2 cm misaligned, the d between the two vehicles becomes slightly larger.

For the half meter case,

$$d = \sqrt{.02^2 + .50^2} = 0.5004$$

$$\vec{F}_A \Big|_{2cm.5apart} = 0.997(\vec{F}_A) \hat{i}$$

There is less than a 1% difference. However, for the quarter meter case,

$$d = \sqrt{.02^2 + .25^2} = 0.2508$$

$$\vec{F}_A \Big|_{2cm.25apart} = 0.987(\vec{F}_A) \hat{i}$$

Combined with the 5 degree tilt in each direction, this could account for up to 11.3%.

The actual difference between the simulation and the tests fell within this error.

The impulses from the simulations are calculated by integrating the force curve, and the impulses for the testbed are calculated by fitting a curve to the position data to in the change in velocity, and multiplying by the mass of the mobile. The impulses with 5% and 10% error bars are shown.

Figure 6.15 Impulses from testbed and simulation

Thus, the test results lie within the expected errors of the simulated values.

6.3.3 Maneuvers

Several different maneuvers were also demonstrated using the testbed. The shear forces can be seen by placing the vehicles in the orientation shown below.

Figure 6.16 Top view of coils

A collision avoidance maneuver is demonstrated by giving the mobile vehicle a small force towards the stationary vehicle. When the two vehicles are close enough, the repelling pulse is activated, thus avoiding collision.

Furthermore, a dead band maneuver can be performed by pulsing the coils to repel, rotating one coil a full 180 degrees, pulsing the coils to stop the movement, then pulsing again to attract the coils. One coil needed to be rotated to reverse the direction of one dipole. Because the same signal was used to activate the pulse on both vehicles, the direction of the dipoles could not be controlled separately on the two vehicles. In order to achieve the full 180 degree turn, the servo was modified by taking out the potentiometer feedback.

6.4 Summary

Because the existing testbed would not allow for the validation of the small forces of the μ EMFF system, a new testbed was built to provide a low friction, low disturbance system with which to validate the simulation used throughout the prior analysis. The mobile structure is enabled by the low friction air bearing, and a modified μ EMFF was built, which could be effectively operated in the laboratory environment. This phase 1 testbed system, controlled by RC, was used to successfully confirm the simulation results. The phase two of this testbed will include a SPHERES vehicle on each vehicle to allow for more accurate relative position knowledge and control.

Chapter 7 Thesis summary and Conclusions

7.1 Thesis summary

The overall goal of this thesis was to show the viability of the μ EMFF approach. This was achieved using the following steps. First the system was modeled to investigate the authority of the system. Then the system was optimized for least mass. A range of optimized systems were compared to traditional propulsion techniques and analyzed for several different mission scenarios to show the advantages of the system. Finally, a testbed was designed and built to validate the model, and to allow for future testing of operational and control schemes.

Chapter one illustrated the motivation for formation flight by presenting advantages such as higher resolution space telescopes, flexibility, lower cost, etc. Then the motivation for electromagnetic flight was given and a small introduction to the EMFF concept was presented. Previous and current work at the MIT SSL was described and an overview of the thesis was given.

Chapter two described in more detail the EMFF basics and the forces and torques that it could provide. The two dimensional force equation was presented and reduced to the one dimensional case used in the following chapters for simulation. The basic variables affecting the force EMFF could provide were listed here. This chapter also described how the μ EMFF concept differs from the HTS EMFF that has been investigated up to now and presented the advantages and disadvantages of this difference.

The following chapter, Chapter three, described the one dimensional simulation of the

μ EMFF system. The equations to calculate the coil's properties were given and the verification of the inductance equation was shown. Next, because the force equation given in chapter two is only valid in the far field, the exact force calculations were shown for the close proximity operations. A force correction factor was calculated and incorporated into the simulation to save computation time. The J_2 perturbation control, the initial mission scenario, was also presented. The ultracapacitor, wire metal, and solar panel properties included in the simulation were described before the simulation in Simulink was given. Some typical inputs and output from the simulation were presented and the coil temperature estimation calculation was shown to ensure that μ EMFF would not require the complicated thermal system as in the HTS EMFF.

Chapter four started with the description of the vast design space for the concept. In an attempt to reduce the number of variable parameters, the Buckingham Pi theorem was employed. However, the variable space could not be reduced, and the optimization that followed varied each parameter independently. The MATLAB optimization code was described with a particular emphasis on the constraint equation of minimum acceleration. Some results of the optimization were shown and the optimized system was verified to work in the simulation from Chapter three.

The mass optimized designs from Chapter four were then used for mission analysis in Chapter five. The J_2 perturbation mission was presented again with the minimum mass μ EMFF for different separation distances. This was compared to the masses of other traditional propulsion technologies and found to be better at small distances and comparable overall. Then, the drag make up scenario was investigated. The results show that a system using one module with a traditional propulsion subsystem and μ EMFF on all the modules would be more mass efficient than a system with traditional propulsion subsystems on all modules if the individual μ EMFF subsystem was designed to be of comparable mass to the individual traditional propulsion subsystem. Furthermore, the hybrid scenario was also discussed in this chapter. The use of both HTS EMFF and μ EMFF extends the operation range of the system significantly for a given mass especially when more than two vehicles are involved.

Chapter six described the testbed required in order to validate the models described in the former chapters. The reasoning for a new EMFF testbed, the significant disturbance, was quantified, and the individual components were described. The completed phase 1 system was shown. Then the repulsion test results were presented with the simulated values and the sources of errors between the two were discussed. The results fall within the expected error ranges. Finally a description of other maneuvers that can be performed was given.

7.2 Contributions

This thesis contributed to the EMFF development by performing the initial investigation into the capability of μ EMFF. The initial simulation of the concept was developed and the models were optimized and validated. The advantages of the μ EMFF concept were highlighted, and the favorable range of operation for the system was located. A testbed, which furthered the development of the subsystems, was designed and built and now is available for further testing of the operation and control schemes.

7.3 Recommendations for future work

In the whole of the EMFF project, there are still many topics to be researched. Specifically concerning μ EMFF, the control scheme of the system will need to be well examined. With multiple satellites in a pulsing mode there are many operational choices that need to be made. Pairs of satellites may pulse at once or the whole cluster could pulse together. In addition, to avoid collision as well as stay within range of each other many different formations could be made; this however, may be dictated by the mission. Investigations should be made into these different modes to find the optimum approach.

In addition, the hybrid concept should be validated. This could be done in the short term by combining the two testbeds. The authority gained by including just one HTS vehicle in the cluster is significant and this concept should be further investigated.

Appendix A

The MATLAB code for validating inductance equation.

inductance.m

```
% Inductance calculations
clc
% octagon
r=.25; %[m]
alpha=(45/2)*pi/180; %[rad]

l_side=sqrt(r^2+r^2-2*r*r*cos(alpha*2)); %length of side
rc=sqrt(r^2+(l_side/2)^2-2*r*l_side/2*cos(pi/2-alpha)); %r length out
to a side
rpt1_l=3/4*rc; % r length out to point 1
rpt2_l=1/4*rc;
%vectors out to center of each segment
rseg=zeros(2,8);
rseg(:,1)=rc*[cos(alpha);sin(alpha)];
rseg(:,5)=-rseg(:,1);
rseg(:,8)=rc*[cos(alpha);-sin(alpha)];
rseg(:,4)=-rseg(:,8);
rseg(:,2)=rc*[cos(3*alpha);sin(3*alpha)];
rseg(:,6)=-rseg(:,2);
rseg(:,7)=rc*[cos(3*alpha);-sin(3*alpha)];
rseg(:,3)=-rseg(:,7);

% vector to point1
rpt1=rpt1_l*[cos(alpha);sin(alpha)];
bfield1=0;
% vector from segment1 to point 1
for i=1:8
rb=rpt1-rseg(:,i);
rsq=rb'*rb;
cos_of=rb'*-rseg(:,i)/(sqrt(rb'*rb)*sqrt(-rseg(:,i)'*-rseg(:,i)));
angle=(90*pi/180-real(acos(cos_of)));
bfield1=bfield1+l_side*sin(angle)/rsq;
end

% vector to point2
rpt2=rpt2_l*[cos(alpha);sin(alpha)];
bfield2=0;
for i=1:8
rb=rpt2-rseg(:,i);
rsq=rb'*rb;
cos_of=rb'*-rseg(:,i)/(sqrt(rb'*rb)*sqrt(-rseg(:,i)'*-rseg(:,i)));
angle=(90*pi/180-real(acos(cos_of)));
bfield2=bfield2+l_side*sin(angle)/rsq;
end
```

```

l_side_2=sqrt((r/2)^2+(r/2)^2-2*r/2*r/2*cos(alpha*2));
a_1=((l_side+l_side_2)/2)*rc/2;
a_2=l_side_2*rc/4;
L_cal=8*10^(-7)*(bfield1*a_1+bfield2*a_2)
% formula from web assume wire of .001m
L_form=.25*4*pi*10^-7*(log(8*.25/.001)-7/2)

```

Appendix B

The MATLAB code for the calculation of the exact forces.

exact_model.m

```
% exact model
% calculates force using inner.m and outer.m quadv numerically
integrates
mu_0=4*pi*10^-7;
i=1; %current
N=1; % number of turns
count=1; % counter

% loop for distances between the vehicles .5m to 1.5
% can take out for loop and simply run for a specific rc vector
for dis=.25:.01:.5
rc=[0 0 dis]'; % vector from center of loop1 to center of loop2
R=.25; % radius of loop
n=[0 0 1]'; % normal vector from center of loop2 (determines attitude
of loop2 wrt loop1)

force_ex=(mu_0*(i*N)^2/(4*pi))*quadv(@(theta)outer(theta, rc, R, n), 0,
2*pi, 1.e-9);
force(count,1)=dis;
force(count,2)=-force_ex(3);

% compare to far field model
A=pi*R^2;
mu=N*i*A;
force(count,3)=(3*mu_0/(2*pi))*mu^2/(norm(rc)^4);
count=count+1;
end

% plot far field and near field
plot(force(:,1), force(:,2), force(:,1), force(:,3))
save('forces','force') % save force matrix
```

inner.m

```
function y1=inner(rc, R, n, phi, theta)

loop_2=[R*cos(theta); R*sin(theta); 0]; %radial vector in second loop

rot_1=cross([1; 0; 0], n);
rot_2=cross(n, rot_1);
ROT_inv=[rot_1, rot_2, n];
```

```

ROT=inv(ROT_inv); % rotation matrix from coord1 to coord2

loop_1=[R*cos(phi); R*sin(phi); 0]; % radial vector in first loop

r_1=rc+ROT*loop_2-loop_1; % vector from loop1 to loop2
r_1n=r_1/norm(r_1); % normalized vector

dl_1=[-R*sin(phi); R*cos(phi); 0];

y1=cross(r_1n, dl_1)/(norm(r_1))^2;

```

outer.m

```

function y2=outer(theta, rc, R, n)

v=quadv(@(phi)inner(rc,R,n,phi, theta), 0, 2*pi, 1.e-9);

rot_1=cross([1; 0; 0], n);
rot_2=cross(n, rot_1);
ROT_inv=[rot_1, rot_2, n];
ROT=inv(ROT_inv); % rotation matrix from coord1 to coord2

dl_2=[-R*sin(theta); R*cos(theta); 0];
dl_1=ROT*dl_2;
y2=cross(v, dl_1);

```

Appendix C

Sample MATLAB code used for finding the minimum mass system.

minimizing_functions.m

```
% minimizing using fmincon

clear all
close all

format compact
count=1;
for var=50:10:250
    den = 2750; %kg/m3
    V_c = 2.5; %V of one capacitor
    res = 2.82*10^-8;
    cap_den = 350/.60; %C/kg
    sol_den = 1/25; % kg/W
    s_apart= 5; %m
    sat_mass=60; %kg

    % % Variables
    % R_l  loop radius
    % N    number of turns
    % r    wire radius
    % C    capacitance
    % t    time between pulses

    x0 = [.75, var, .001, 500, 9]; % starting guess
    lb = [.1, var, .0005, 10, 5];
    ub = [1, var, .005, 1000, 1000];
    options = optimset('LargeScale','off', 'TolFun', 1.e-6,
        'MaxFunEvals',3000);

    [x, fval] = fmincon(@myfun,x0,[],[],[],[],lb,ub,@confun, options);
    wire_mass=den*x(1)*2*pi*x(2)*pi*x(3)^2;
    cap_mass=x(4)/cap_den;
    V=V_c*x(4)/350;
    solar_mass=.5*x(4)*V^2/x(5)*sol_den;

    R=x(2)*2*x(1)*res/(x(3)^2);

    answer(count,:)=[x fval, wire_mass,cap_mass,solar_mass];
    count=count+1;
end
figure
plot(answer(:,1),answer(:,6))
title('Loop Radius')
ylabel('min mass')
```

```

xlabel('Loop radius')
figure
plot(answer(:,2),answer(:,6))
title('Number of loops')
ylabel('min mass')
xlabel('Number of loops')
figure
plot(answer(:,3),answer(:,6))
title('Wire Radius')
ylabel('min mass')
xlabel('Wire Radius')
figure
plot(answer(:,5),answer(:,6))
title('Time')
ylabel('min mass')
xlabel('Time')
figure

plot(answer(:,4),answer(:,6))
title('Capacitance')
ylabel('min mass')
xlabel('Capacitance')

```

myfun.m

```

function f=myfun(x)

% mass function to minimize
%%%%%%%%%%%%
% Constants
den = 2750;
V_c = 2.5;
res = 2.82*10^-8;
cap_den = 350/.60;
sol_den = 1/25;

V=V_c*x(4)/350;
% mass of emff system
f=den*x(1)*2*pi*x(2)*pi*x(3)^2+x(4)/cap_den+.5*x(4)*V^2/x(5)*sol_den;

```

confun.m

```

function [c, ceq] = confun(x)
% Nonlinear inequality constraints
%%%%%%%%%%%%
c = [];

% Nonlinear equality constraints
% Constants
den = 2750;
V_c = 2.5;
res = 2.82*10^-8;

```

```

cap_den = 350/.60;
sol_den = 1/25;
s_apart= 5;
sat_mass = 60;

V = V_c*x(4)/350; % voltage
% force integral

R=x(2)*2*x(1)*res/(x(3)^2);
L=(x(2))^2*x(1)*4*pi*10^-7*[log(8*x(1)/x(3))-7/2];
beta=sqrt(1/(L*x(4))-R^2/(4*L^2));
alpha=R/(2*L);

constant=(1/(x(5)*sat_mass))*(3/2)*4*pi*10^-
7*(x(2))^2*(x(1))^4*V^2/(s_apart^4*beta^2*L^2)*(1/(4*alpha)-
alpha/(4*alpha^2+4*beta^2));

ceq = [3.25*10^-9*s_apart-constant];
%ceq = [constant];

```


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