Electromagnetic Scattering by Open-Ended Cavities: An Analysis Using Precorrected-FFT Approach

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***Abstract*--In this paper, the precorrected-FFT method is used to solve the electromagnetic scattering from two-dimensional cavities of arbitrary shape. The integral equation is discretized by the method of moments and the resultant matrix equation is solved iteratively by the generalized conjugate residual method. Instead of directly computing the matrix-vector multiplication, which requires N^2 operations, this approach reduces the computation complexity to $O(N \log N)$ as well as avoids the storage of large matrices. At the same time, a technique known as the complexifying k is applied to accelerate the convergence of the iterative method in solving this resonance problem. Some examples are considered and excellent agreements of radar cross sections between these computed using the present method and those from the direct solution are observed, demonstrating the feasibility and efficiency of the present method.** [1]

***Indexs*--precorrected-FFT method, method-of-moments, electrical-field integral equation, electromagnetic scattering, cavity**

I. INTRODUCTION

The analysis of electromagnetic scattering by open cavities has been extensively investigated because of its importance in radar cross section (RCS) problems. Different approaches have been used, depending on the frequency and the geometry of problems. Among these methods, model analysis is only applicable when the geometry permits analytical expressions of the modes to be obtained [1]. Some high frequency techniques, such as shooting and bouncing rays approach (SBR), generalized ray expansion (GRE), and Iterative Physical Optical (IPO) Approach, can be used to model the scattering properties of electrically large cavities of arbitrary shape. However they have been shown to work only for cavities with very simple terminations [2-4]. Low frequency or fullwave numerical techniques such as the method of moments (MoM), the finite element method and the finite-difference time-domain method can accurately model the complex terminations as well as other parts of cavities. Unfortunately, the computation costs and memory requirement restrict the capability of these methods. In recent years, hybrid methods that combine high and low frequency methods are proposed to overcome the shortcomings of either method [5]. As an alternative of the prospective hybrid method, an approach combining the precorrected-FFT method in conjunction with the traditional MoM is developed in this paper. This approach can reduce the memory requirement and computation complexity significantly and enable large-scale problems to be solved using a small computer.

The precorrected-FFT method was originally proposed by Philips and White [6,7] to solve electrostatic integral equation associated with capacitance extraction problems. The key idea of the algorithm is to represent the long-range part of the field by current distributions lying on a uniform grid. This grid representation allows the Fast Fourier Transform (FFT) to be used to efficiently perform field computations. In this paper, we make appropriate modifications and extend the precorrected-FFT method to analyze electromagnetic scattering by two-dimensional cavities of arbitrary shape. The integral equation is first discretized to form a matrix equation by the method of moments (MoM), and then the resultant matrix equation is solved by the generalized conjugate residual method (GCR). The precorrected-FFT technique is introduced to avoid the filling of the moment matrix and accelerate the matrix-vector multiplication in the iteration procedure. It can be demonstrated that

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the computation complexity is thus reduced to $O(N \log N)$ and memory requirement to, $O(N)$ respectively. For some wavenumber k of an incident wave, the integral equation of EM scattering problems can be extremely ill conditioned. In this paper, the complexifying k-technique is used to solve the resonance problem. This technique reduces the condition number of the matrix dramatically, accelerating the convergence of the iteration process. Numerical results will be presented to validate the algorithm that we developed.

II. PROBLEM FORMULATION

Consider an E_z-polarized plane wave incident on a two-dimensional conducting cavity in free space, as shown in Fig. 1. According to the equivalent theorem, the external scattered field from the open-ended cavity can be regarded as the radiation of the equivalent electric and magnetic currents on the aperture of the cavity, which can be obtained by the induced current on the inner surface of the cavity. The induced current satisfies the following integral equation

$$i\omega\mu_0 \oint_C d\, l' g_0(\rho - \rho') J_z(\rho') = -E_z^{inc}(\rho), \ \rho \in C \quad (1)$$

where $J_z(\rho')$ denotes the *z*-directional induced current, $E_z^{inc}(\rho)$ is the *z*-component of incident field on C, and $g_0(\rho - \rho')$ represents the two-dimensional scalar Green's function in free space. The integral equation in (1) can be discretized by the method of moments to yield

$$\sum_{i=1}^{N} P_{ji} x_i = f_j, \qquad j = 1,2,\cdots N \quad (2)$$

where

$$P_{ji} = \begin{cases} \dfrac{\omega\mu_0}{4}\left[1 - \dfrac{2i}{\pi}\ln\left(\dfrac{\gamma}{4e} k\Delta_i\right)\right]\Delta_i & i = j \\ \dfrac{\omega\mu_0}{4}\Delta_i H_0^{(2)}(k\rho_{ji}) & i \neq j \end{cases} \quad (3)$$

$$f_j = E_z^{inc}(\rho_j), \qquad x_i = J_z(\rho_i) \quad (4)$$

with $\rho_{ji} = |\rho_j - \rho_i|$, $\gamma/4e = 0.163805$, $H_0^{(2)}$ being the zeroth-order Hankel function of the second kind, and k as the wave number in free space.

For a solution of the system the generalized conjugate residual (GCR) method [8] is used, which relies on an efficient evaluation of the matrix-vector products associated with Eqn. (2). The computational complexity in executing the matrix-vector products is $O(N^2)$ and storage requirements are of the same order, where N denotes the number of unknowns. For a more efficient implementation of the method, it is therefore crucial to reduce the complexity of the matrix-vector product. We propose to accomplish this by using the precorrected–FFT algorithm, which eliminates the requirement of filling the moment method matrix and produces an approximation to the matrix-vector product in the order of $N \log N$ operations.

3. THE PRECORRECTED-FFT ALGORITHM

To implement the precorrected–FFT algorithm, we enclose the entire object in a quadrangle after it has been discretized into segments. The quadrangle is then subdivided into a $k \times l$ array of small squares, each containing only a few segments. We refer to these small squares as cells. Fig. 1 shows a discretized arbitrary cavity, with the associated space subdivided into a 3×3 array of cells. Based on the fact that fields at evaluation points distant from a cell can be accurately computed by representing the given cell's current distribution using a small number of weighted point currents, the matrix–vector product can be approximated in four steps: (1) to project the segment currents onto a uniform grid of point currents, (2) to compute the fields at the grid points due to grid currents using the FFT, (3) to interpolate the grid fields onto the segments, and (4) directly compute nearby interactions. This process is summarized in Fig. 2. Note that the number of grid points is required to be a factor of two in order to perform the FFT.

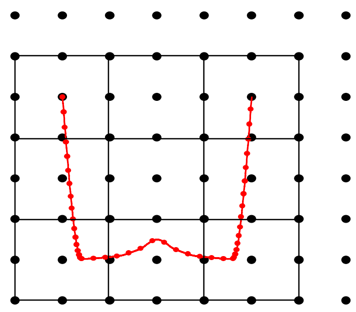

Figure 1 The superimposed grid currents with $p = 3$

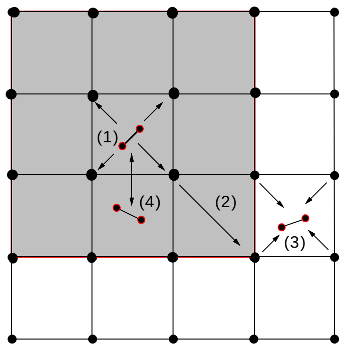

Figure 2 Four steps of the precorrected-FFT algorithm with $p = 2$

3.1 Projecting segment currents onto a uniform grid of point currents

Firstly, we describe the construction of the grid projection operator W. Assume that a $p \times p$ array of grid currents is used to represent the current in a cell and N_c test points are selected on the surface of a circle of radius r_c whose center is coincident with the center of the given cell k. We then enforce electric field due to the total p^2 grid currents to match that due to the cell's actual current distributions at the test points and thus yield

$$\boldsymbol{P}^{gt}\hat{\boldsymbol{J}}(k) = \boldsymbol{P}^{qt}\boldsymbol{J}(k) \tag{5}$$

where $\boldsymbol{J}(k) \in R^{n(k)}$ and $\hat{\boldsymbol{J}}(k) \in R^{p^2}$ are the vectors of segment currents and grid currents, respectively. $n(k)$ denotes the number of segments in cell k. $\boldsymbol{P}^{gt} \in R^{N_c \times p^2}$ represents a mapping between grid currents and test point fields, given by

$$\boldsymbol{P}^{gt} = \frac{\omega\mu_0}{4} H_0^{(2)}\left(k\left|\rho_i^t - \hat{\rho}_j\right|\right) \tag{6}$$

The relative positions of the grid currents and the test points can be constructed to be identical for each cell, therefore $\boldsymbol{P}^{gt}$ is the same for each cell and needs to be computed only once. $\boldsymbol{P}^{qt} \in R^{N_c \times m}$ identifies a mapping between segment currents and test point fields. m stands for the number of segments in the cell. Since the collocation equation (5) is linear in the segment and grid current distributions, the contribution of the j th segment in the cell k to $\hat{\boldsymbol{J}}(k)$ can be represented by a column matrix $\boldsymbol{W}(k, j)$,

$$\boldsymbol{W}(k, j) = \left[\boldsymbol{P}^{gt}\right]^{+} \boldsymbol{p}^{qt,j} \tag{7}$$

where $\boldsymbol{P}^{qt,j}$ denotes the j th column of $\boldsymbol{P}^{qt}$ and $\left[\boldsymbol{P}^{gt}\right]^{+}$ denotes the generalized inverse of $\boldsymbol{P}^{gt}$. Since the matrix size is small and the same for each cell, the relative computational cost of computing $\left[\boldsymbol{P}^{gt}\right]^{+}$ is insignificant. For any segment current j in cell k, this projection generates a subset of grid currents $\hat{\boldsymbol{J}}(k)$. The total $\hat{\boldsymbol{J}}(k)$ can be obtained by summing up all the contributions of the currents in cell k.

3.2 Computing grid fields due to grid currents using the FFT

Once the current has been projected to a grid, the electric field at the grid points due to the grid currents can be computed by a two-dimensional convolution

$$\hat{E}(i, j) = H\hat{J} = \sum_{i', j'} h(i - i', j - j')\hat{J}(i', j') \tag{8}$$

where i, j and i', j' specify the grid points and $h(i - i', j - j')$ is the mapping between grid currents and grid fields. $h(0,0)$ can be arbitrarily defined, usually set to zero. The convolution in Eqn. (8) can be rapidly computed by using the Fast Fourier Transform (FFT) [10], *i.e.*,

$$\hat{E} = F^{-1}\left(\tilde{H} \cdot \tilde{J}\right) \tag{9}$$

where F^{-1} denotes the inverse FFT while $\tilde{H}$ and $\tilde{J}$ are the FFT of $h(i, j)$ and $\hat{J}(i, j)$ respectively. Note that $\tilde{H}$ needs to be computed only once.

3.3 Interpolating grid fields onto segments

The interpolating process is essentially the same as the projecting process. It has been proved that the projection and interpolation operators have comparable accuracies [6].

Assume that $[V(k, j)]^T$ denotes an operator that interpolates fields at grid points onto segment coordinates. Thus, projection, followed by convolution and interpolation, gives the grid-current approximation E_G to the electric field that can be represented as

$$E_G = V^T HWJ\ . \tag{10}$$

3.4 Computing near-zone interactions directly

Since the error due to the grid-current approximation is inversely proportional to the interaction distance [6,7], the interactions between nearby segments have been poorly approximated in the projection or interpolation process. In order to get correct results, it is necessary to compute the near-zone interactions more accurately and remove the inaccurate contribution due to the grid-currents from Eqn. (10) as well. This process is referred to as "precorrection".

Define a "precorrected" direct interaction operator as follows:

$$\widetilde{P}(k,l) = P(k,l) - V(k)^T H(k,l) W(l) \qquad (11)$$

The exact field $E(k)$ for each cell k can be obtained by

$$E(k) = E_G(k) + \sum_{l \in N(k)} \widetilde{P}(k.l) J_l \qquad (12)$$

where $E_G(k)$ is the grid-approximation to $E(k)$, including the inaccurate near zone portions. $N(k)$ denotes the indices of the set of cells which are "close to" cell k. The second term in (12) represents the near-zone interactions that can be computed directly. Because for each k, $N(k)$ is a small set, each matrix $\widetilde{P}(k,l)$ is also small.

Combining the above steps leads to the precorrected-FFT algorithm. The effort of this algorithm is so made as to replace the dense matrix-vector product ***PJ*** with the sparse operation $[\widetilde{P} + V^T HW]J$. The cost of the P-FFT consists three components. The cost of the direct interactions is $O(klN_c^2) = O(N)$, the cost of grid projection and interpolation is $O(np^2) = O(N)$, and the cost of the FFT is $O(p^2 kl \log p^2 kl) = O(N \log N)$. So the total cost of the algorithm is $O(N \log N)$.

3.5 Complexifying k-technique

At some certain values of the wavenumber, the integral equation of scattering problems can be extremely ill conditioned, leading the condition number of the discretized matrix to be very large and the iterative technique to fail to converge. This problem occurred frequently in resonance problems. A combined field integral equation (CFIE) method has been presented to overcome resonance problem. CFIE cannot be used for an open conductor, but for a closed one only. The electric field integral equation (EFIF) is frequently used to solve electromagnetic scattering from a conductive cavity. However, it needs very high iteration number to converge. In this paper, the complexifying k-technique [10] is used in EFIE to accelerate the convergence of the iteration in resonance problems. In this technique, a small imaginary part of the wavenumber k was introduced, and extrapolated back to the real axis. Two-point linear extrapolation or parabolic extrapolation is used to attain the results corresponding to the wavenumber with a zero imaginary part.

This method reduces the condition number of the matrix, and needs considerably fewer iteration than the general case(no complexifying k).

III. NUMERICAL RESULTS

In this section several examples are considered to verify correctness and efficiency of the algorithm. Fig. 3 shows a 3-D offset bend cavity. The RCS in the principal plane of the 3-D cavity can be obtained from that of the corresponding 2-D model that extends infinitely in the z-direction [5]. The problem domain is divided into an array of 127×63 cells for $p = 3$ and an array of 254×126 cells for $p = 2$. The grid spacing is about 0.12λ in both cases. The backscattering patterns obtained at 10 GHz from the Gaussian elimination method and the precorrected-FFT method with $p = 2$ and $p = 3$, respectively. The RCSs are plotted in Fig. 4. It is observed that all the three sets of results agree very well near the normal incidence. But the results for $p = 2$ deviate somewhat from the other two sets of results near the graze incidence. In order to get more accurate results, we therefore select $p = 3$ in the following examples.

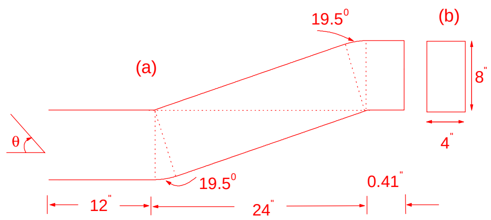

Figure 3 Geometry of a 3D offset bend cavity (a) Side view (b) Cross-sectional view

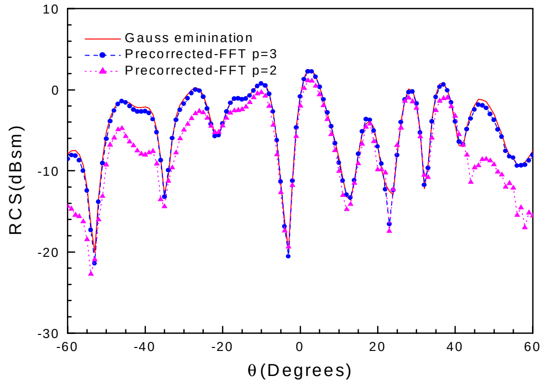

Figure 4 Comparison of backscattering RCSs of a 3D offset bend cavity obtained from the precorrected-FFT method and the Gauss elimination method

Fig. 5 shows a tapered cavity model given in [1]. The backscattering RCSs obtained from the direct solution and by the precorrected-FFT method with different grid constructions are compared in Fig. 6. It can be seen that the results from P-FFT method for 256×64 grids (with a grid spacing of about 0.124λ) agree very well with the direct solutions. But when using 128×32 grids (with a grid spacing of about 0.248λ), the dispersions are unacceptable. More numerical examples demonstrate that a grid spacing of 0.15λ and a near-zone radius of 0.5λ are sufficient to maintain a good accuracy desired.

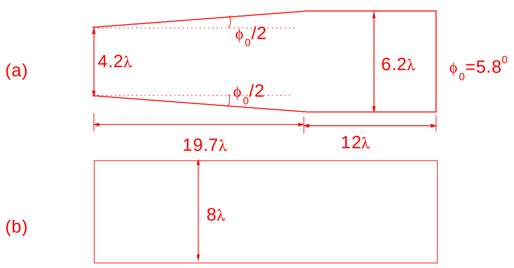

Figure 5 Geometry of a 3D tapered cavity (a) Side view (b) Top view

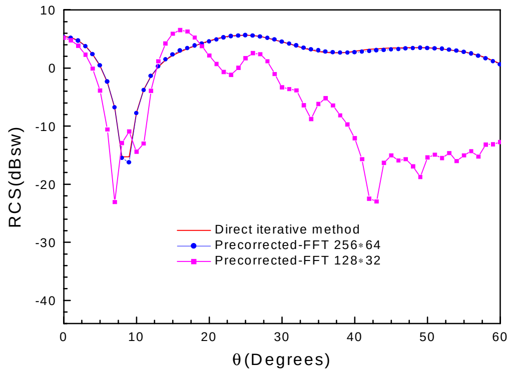

Figure 6 Backscattering RCSs of the tapered cavity

A long 3D bend cavity shown in Fig. 7 is also calculated for comparison between the present method and the standard GCR method. A good agreement is also observed. To accelerate the convergence, we use the current solution from the previous angle with phase correction as the initial guess for the next angle. In these examples, the cavity is discretized into about 1200 segments and the problem domain is divided into an array of 96×96 grids. So the number of the grid currents is about seven times that of the segments, but the precorrected-FFT algorithm performs the matrix-vector multiplication 3 to 4 times faster than the direct computation. In terms of memory requirement, the present method only consumes about 8% that of the direct iterative method.

To prove that complexifying k-technique can reduce the number of iterations, the bistatic RCS of the long bend cavity for incident wave at 0° is calculated by use of the present method and PFFT with real k. In this example, two-point extrapolation ($k=k_0+i0.1$, $k=k_0+i0.2$) is used. It is observed that PFFT with real k needs 170 iteration for relative residual error $1.0\ 10^{-4}$, but PFFT with the complexifying k-technique only needs 46 and 36 iterations for $k_0+i0.1$ and $k_0+i0.2$, respectively.

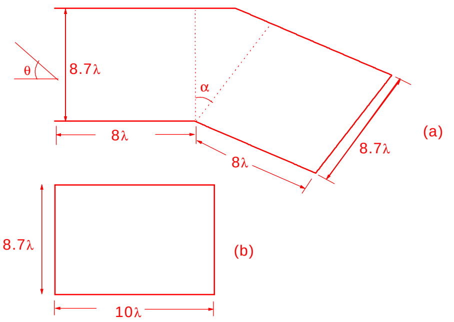

Figure 7 Geometry of a 3D long bend cavity endplate (a) Side view (b) Cross-sectional view

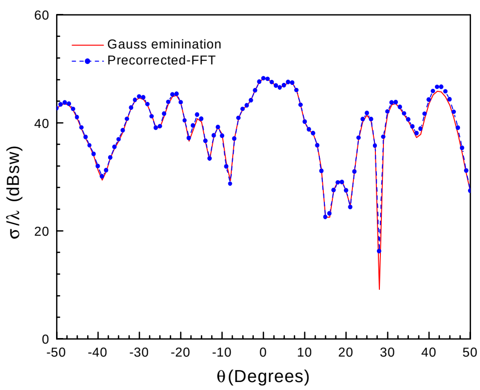

Figure 8 Backscattering RCSs of the 3D long bend cavity

Fig. 9 shows the geometry of a waveguide cavity with a semicircle hub or a flat endplate at the termination. The RCSs plotted in Fig. 10 clearly indicate that the termination geometry has significant effects on the RCS level of the cavity near normal incidence. So in order to get accurate results, the termination should be discretized in a finer manner compared to that of the front section of the cavity. In this example, the cavity with a flat endplate at the termination is discretized into 1100 segments and the cavity with a semicircle hub into 1600 segments. The problem domain is divided into an array of 192×48 grids for both cases. In the direct solution, each of iterations for the latter cavity costs about twice of the time for the former one. But in the present method, the computational time consumed for each of iterations for the latter cavity increases no more than 5% in contrast to the former cavity. The memory requirements of the P-FFT keep about the same for the two cases because the grid numbers are the same. On the other hand, the memory requirements of the direct iterative method increase significantly because they are in order of N^2. In other words, the advantage of the present method is more pronounced when the cavities have complex interior structures or terminations.

The last example considered here is an S-shaped engine inlet, as shown in Fig. 11. The numbers of grid currents and segments are assumed to be 128×128 and 2400, respectively. Different from the above examples, this example has two open ends and the scattered field is radiated on both apertures. It can be seen from Fig. 12 that the backscattering RCS is about 10 dB lower when the incident angles are larger

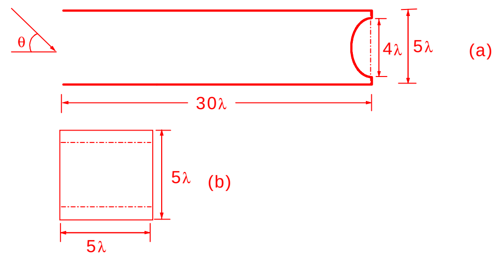

Figure 9 Geometry of a waveguide with semicircle hub or a flat endplate (a) Side view (b) Cross-sectional view

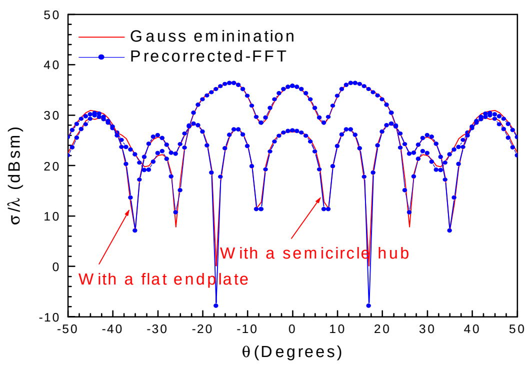

Figure 10 Backscattering RCSs of the waveguide with different terminations

than zero. This is because more energy is transformed through the inlet and radiated from the lower end of the inlet, which generates a weaker field in the back direction. Again, a good agreement between the P-FFT solution and direct iterative solution is observed. It is also found that at incident angles of up to 100 degrees, the P-FFT method costs only about 1/2 time that of the Gauss elimination and 1/4 that of the direct iterative solution. The memory requirement of the P-FFT method is about 7.8% those of other two methods. So in terms of a memory-speed product, the precorrected-FFT algorithm is still far more superior to the Gauss elimination procedure and the direct iterative method even for monostatic RCS computations involving many incident angles. In the convergence check of all the above examples, relative errors are chosen to be within 10^{-3}.

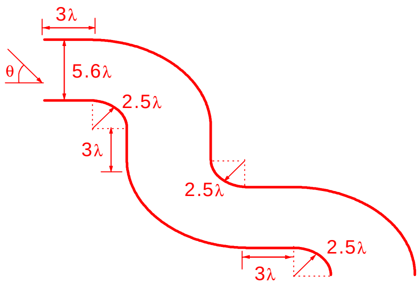

Figure 11 Geometry of a S-shaped inlet

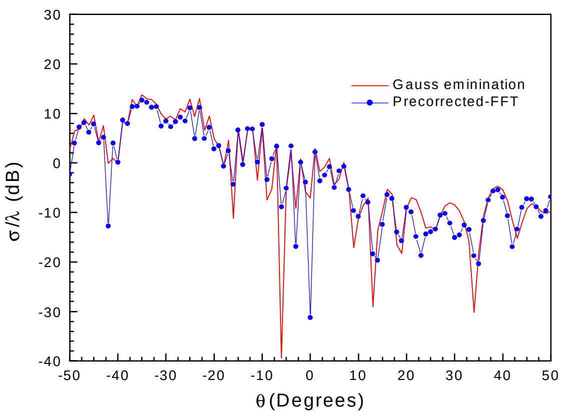

Figure 12 Backscattering of the S-shaped inlet

CONCLUSIONS

In this paper, the precorrected-FFT algorithm that used to be utilized in the solution to electrostatic integral equations is successfully extended to analyze electromagnetic scattering from two-dimensional conductive open-ended cavities. Several examples are specifically considered, namely, a 3-D offset bend cavity, a tapered cavity, a 3-D long bend cavity, a waveguide cavity with different terminations, and a S-shaped inlet. Numerical results of radar cross sections of these objects are obtained using both the direct iterative method and the precorrected-FFT approach. Good agreements of RCSs between the two methods are demonstrated for all the objects. However, the precorrected-FFT method reduces the memory requirements and computational complexity from $O(N^2)$ for direct computation to $O(N)$ and $O(N \log N)$, respectively, where N denotes the number of the grids. This significantly improves the capability of the conventional method of moments for analyzing electrically large objects.

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