# Improving Promotional Effectiveness through Supplier-Retailer Collaboration 

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#### Abstract

In the consumer products industry, retail chains and manufacturers run promotions to maintain consumer and brand loyalty. The two major issues in planning and executing promotions are to accurately forecast demand and to control Out-of-Stock at the shelf. This thesis addresses both these issues. At the strategic level, "Collaborative, Planning, Forecasting and Replenishment" is used to define a process for two companies to collaboratively plan and execute promotions. At an operational level, the single period multi-item newsboy concept with a budget constraint is used to define an optimization model that helps determine the right budget and order quantities for products under a promotion at a targeted service level to improve profit or sales. The concept of Supply Contracts is researched to identify some ways that can be used to optimize the whole supply chain rather than just the retailer's. The value of optimal collaboration was confirmed in the results shown by the model. When optimizing the entire chain, the maximize profit optimization model achieved combined profit improvements of $37 \%$ as compared to an actual promotion. When only the retailer profit was maximized, the optimization model resulted in $5.9 \%$ profit improvements for the retailer and $0.3 \%$ profit improvements for the supplier as compared to an actual promotion. Finally, the revenue maximization model showed that after a certain point, increasing the budget did not result in increased service levels. This research can also be applied to new product launches, seasonality of products as well as daily replenishments.


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## 1 Introduction

This section describes the motivation for research in the area of Supplier-Retailer collaboration to improve promotional effectiveness. It then goes on to provide a brief background about SupplierCo and RetailerCo.

### 1.1 Motivation

Suppliers and retailers are faced with the complexity of managing millions of SKU and Store combinations with the additional complexity of seasonality, new products and promotions. Because of high consumer expectations, positive in-store experience and continuous shelf presence are necessary for suppliers and retailers to win the consumer. Thus an out-of-stock results in lost sales for both the supplier and the retailer. According to a Shoppers Research Study conducted by Gruen, Corsten and Bharadwaj (2002), on average, when retailers face an out-of-stock, they lose the sale $41 \%$ of the time while suppliers lose the sale $28 \%$ of the time. In the case of SupplierCo's top 100 SKUs, $48 \%$ of consumers switch stores when faced with an out-of-stock to buy the same product at another store.

Thus, for heavily promoted products, suppliers and retailers find it difficult to determine the right budget and product quantities to minimize Out-of-Stocks for a given promotion, due to conflicting objectives and limited collaboration. Promotional discounts are used to increase sales volumes of certain products or to increase traffic of consumers in the store so that they will purchase other high margin products.

The scope of our thesis is to determine how a supplier and retailer can better collaborate on the promotion planning process as well as optimize inventory and service levels for a category of

SKUs under promotion to improve sales and profits. It is important to note that this research can also be applied to new product launches, seasonality of products, as well as daily replenishment.

The thesis is based on a case study of SupplierCo, a leading consumer product goods manufacturer and RetailerCo, a leading pharmacy retailer. Currently, SupplierCo and RetailerCo conduct more than $\$ 1$ billion worth of business with each other, which is approximately $2 \%$ of SupplierCo revenue and 3\% of RetailerCo revenue respectively. However, joint value creation opportunities resulting in supply chain efficiencies will enable SupplierCo and RetailerCo to expand their relationship and maximize profitability as well as revenue.

### 1.2 SupplierCo Overview

SupplierCo is a world class leader in the consumer product goods industry with a global consumer base. SupplierCo has a product portfolio in the household care, beauty and healthcare market segments. SupplierCo products are primarily sold through mass merchandisers, grocery stores, membership club stores and drug stores.

SupplierCo's core competency lies in understanding the consumer, innovation, branding, go-to-market capability, and scale. On a daily basis, SupplierCo faces two opportunities to win the consumer; one when the consumer is at the store shelf and the other when consumers use the product. The SupplierCo supply chain plays an integral part in achieving the first opportunity by making sure that the product is available to the consumer.

### 1.3 RetailerCo Overview

RetailerCo is one of the nation's largest drug retailers. RetailerCo's product portfolio includes prescription drugs as well as general merchandise, including, over-the-counter drugs, beauty products and cosmetics, film and photofinishing services, seasonal merchandise, greeting cards, household care products and convenience foods.

RetailerCo's strategy is to provide high levels of customer service and value to its customers by meeting their healthcare needs and making their shopping experience as easy as possible. RetailerCo's Supply Chain helps drive this strategy by working with its suppliers to ensure high levels of in-store availability for its diverse product portfolio.

### 1.4 Thesis Roadmap

This section outlines the chapter by chapter roadmap for the entire thesis. Chapter 2 of the thesis discusses a literature review of the key concepts that could be applied to improve promotional collaboration between a retailer and a supplier. Chapter 3 then goes onto discuss the as-is promotions planning and execution process for SupplierCo and RetailerCo. The next step is to describe the methodology behind the formulation of the promotional profit maximization and revenue maximization models in Chapter 4. Chapter 5 provides an analysis of the demand pattern data as well as the model results at the national and region distribution center level respectively. Finally, Chapter 6 goes on to provide final recommendations and potential future research opportunities in the area of promotions collaboration.

## 2 Literature Review

In order to understand how SupplierCo and RetailerCo could improve promotions collaboration, we looked at three areas in our literature review, which include Collaborative Planning, Forecasting and Replenishment (CPFR), Single Period Mult-item Newsboy problem and Supply Contracts.

The concept of Collaborative Planning, Forecasting and Replenishment (CPFR) was researched to develop an understanding of the process and organizational changes needed for effective collaboration between SupplierCo and RetailerCo for promotional events. The MultiProduct Newsboy Model was then researched to develop the promotions budget optimal allocation model to maximize profit and revenue. Lastly, the concept of overall supply chain optimization was researched in the literature termed Supply Contracts.

### 2.1 Collaborative Planning Forecasting and Replenishment

Collaborative Planning, Forecasting and Replenishment (CPFR) was pioneered by the Voluntary Interindustry Commerce Standards Association (VICS), which was founded in 1986. VICS is a non-profit organization which is geared towards improving trading partner relationships in the end-to-end retail supply chain with the goal of improving product availability to the consumer. The VICS committee has over 190 sponsoring member companies.

After extensively researching literature in the area of Collaborative Planning, Forecasting and Replenishment (CPFR), we were able to find literature on the description, standards and technology solutions for CPFR, as well as four CPFR case studies published by the VICS committee and one case study published by Supply Chain Management Review magazine. This
section will cover the relevant literature for the scope of the thesis, which includes an overview of the CPFR process and tasks as well as the details of the Retail Event Collaboration standard which is relevant to the promotion planning process. Finally, this section will cover learnings from two case studies, Procter \& Gamble's pilot CPFR project with multiple retailers and how West Marine improved communication with its suppliers.

### 2.1.1 CPFR Framework

VICS defines CPFR as, "A business practice that combines the intelligence of multiple trading partners in the planning and fulfillment of customer demand." The primary objective of CPFR is to increase product availability while reducing inventory, transportation and logistics costs. As per the VICS committee overview, CPFR has brought about improvements in in-stock availability of products from $2-8 \%$ and inventory reductions of $10-40 \%$ in the supply chain.

The VICS committee has defined a standard framework for the CPFR process. This framework has the consumer at the center surrounded by concentric circles, which consist of the activities which need to be performed individually by the retailer, collaboratively between the retailer and the manufacturer, and individually by the manufacturer, respectively. The framework is further divided into four major quadrants with major activities which include Strategy and Planning, Demand and Supply Management, Execution and Analysis.


Figure 1 VICS CPFR Model (Source: VICS Committee)

## Collaboration Tasks:

CPFR takes place at multiple levels of collaboration right from strategy to execution. Below are the tasks which are executed at each level of collaboration as defined in the CPFR Overview published by the VICS Committee.

Strategy and Planning:

- Collaborative Arrangement: Define business goals and the scope of collaboration, including roles and responsibilities, checkpoints and escalation procedures.
- Joint Business Plan: Identify the events that impact supply and demand planning. For example, promotions, inventory policy changes etc.

Demand and Supply Management:

- Sales Forecasting: Project the future demand based on previous trends
- Order Planning / Forecasting: Determine future product ordering and delivery requirements.
- Replenishment planning and modeling

Execution:

- Order generation: Convert forecasts into demand
- Order fulfillment: Produce, ship, deliver and stock products
- Store compliance and execution

Analysis:

- Exception Management: Monitor planning and operations for exceptions.
- Performance Assessment: Calculate key metrics to evaluate achievement of business goals and uncover trends
- Conduct root cause analysis and implement continuous improvement initiatives


### 2.1.2 Retail Event Collaboration

One area where most retailers and suppliers struggle to balance demand and supply is promotions. In a GMA / FMI Retail Out-of-Stocks study (Gruen, Corsten and Bharadwaj, 2002), $75 \%$ of these Out-of-Stocks come from poor planning and communication. This is also a challenge faced by retailers and suppliers while executing promotions that involve continuous changes in details such as ad placement and prices impacting production volumes.

The VICS committee has defined a standard business model and implementation guidelines for collaboration on retail events such as promotions, etc. Retail event collaboration can be implemented in the event synchronization or event collaboration phases. Event synchronization is focused more on the communication aspects of collaboration, to ensure that the two trading partners are up to date with any changes or updates. Event collaboration, on the
other hand, includes both the communication aspects of event synchronization and collaborative execution and decision making. The table below compares the activities which are conducted during the different phases of collaboration for Event collaboration and Event Synchronization.

|  | Retail Event Synchronization | Retail Event Collaboration |
| :---: | :---: | :---: |
| Strategy and Planning | Define scope and process of information sharing |  |
|  | Individual Supplier and Retailer process | Assign exception handling roles and procedures |
|  | Communicate event details. Send updates as details change |  |
| Demand and Supply Management | Individual Supplier and Retailer process | Develop and share event sales forecast estimates. |
|  | Individual Supplier and Retailer process | Develop and share event order plan estimates. |
| Execution | Place promotional order or release promotional quantity in continuous replenishment system. |  |
|  | Deliver promotional quantities to third parties, retailer DCs or stores |  |
|  | Individual Supplier and Retailer process | Monitor store inventory and sales performance during the event. |
| Analysis | Individual Supplier and Retailer process | Trigger, communicate and resolve exceptions. |
|  | Communicate event performance results |  |

## Table 1: Comparing Retail Event Synchronization and Retail Event Collaboration Processes

(Source: VICS - Retail Event Collaboration Business Process Guide)
One example of the benefits of retail event collaboration can be seen in the $P \& G^{\circledR}$ Case Study published on the VICS committee website. $\mathrm{P} \& \mathrm{G}^{\circledR}$ conducted a CPFR pilot with Tesco ${ }^{\circledR}$ and Sainsbury ${ }^{\circledR}$ in Europe to collaborate on demand and supply management. By using an online interactive tool, $\mathrm{P} \& \mathrm{G}^{\circledR}$ had access to the retailers forecast as well as the actual POS data during the promotion. In the pilot study, the online view of the promotion status helped $P \& G^{\circledR}$ react to a decrease in in-store availability and save three to four days of Out-of-Stocks. The online promotions management process also helped $P \& G^{\circledR}$ improve their forecast accuracy by $20 \%$.

The VICS committee has also published a case study on a CPFR pilot in which HewlettPackard ${ }^{\circledR}$ developed a CPFR web tool to collaborate with its distributors on forecasts for its short lifecycle products which have made its demand and supply management processes more efficient.

### 2.1.3 Communication

For successful implementation of CPFR, it is very important for different functions in the retailer and supplier organizations to collaborate at regular intervals. West Marine, a boat retailer, set up regular meetings with their key suppliers to successfully identify and manage collaborative initiatives as well as build their relationship. This cross-functional approach has helped West Marine and its suppliers overcome discrepancies in their goals and metrics at the same time developing accountability.

| Frequency | Objective | Resources |
| :---: | :--- | :--- |
| Quarterly | Identify supply chain improvement initiatives and <br> opportunities as well as define timelines and <br> resources to execute on initiatives. | Stakeholders from sales, <br> marketing and merchandising. <br> Resources responsible for <br> managing buying, forecasting, <br> inventory control, production <br> planning, distribution and <br> transportation. |
| Monthly | Review performance metrics, discuss current or new <br> initiatives and assign deliverables to owners, resolve <br> supply chain constraints | Relevant supply chain <br> stakeholders |
| Bi-annual | Supply chain summit: Quarterly meetings with <br> suppliers, Team Building and cultural change <br> activities | Suppliers Senior Sponsors and <br> Sales and Marketing, Retailer <br> Collaborative team members |

Table 2: Frequency, Objective and Resources of Meetings between West Marine and its Suppliers (Source: West Marine Case Study)

### 2.2 Single Period Multi-product Newsboy Problem with Budget Constraint

In the classic newsboy problem, a newsboy needs to decide the order quantity of each day's newspaper. The actual demand for the newspaper faced by the newsboy is a random variable. If the order quantity exceeds actual demand, the newsboy incurs a cost for the leftover newspapers. If actual demand exceeds the order quantity, the newsboy incurs a penalty for any lost sales, which includes the opportunity cost of lost profit and the loss of customer goodwill.

The problem faced by newsboy is to determine the optimal order quantity of newspapers to maximize profit.

The classic newsboy problem considers a single product, with no budget constraint. However, in reality, many businesses not only need to order multiple products in a single period but also face several resource constraints. For example, the manager of a sea food restaurant needs to decide the order quantity of different sea foods, such as lobsters, shrimps and crabs, with the constraint of budget and storage space. A garment manufacturer needs to decide what quantity of each style good should be produced prior to the short selling season under the constraint of production capacity.

Although many papers have appeared in the past two decades to extend the newsboy problem, an overwhelming majority of them consider only a single-product scenario and are inapplicable to our problem. To find the solution of multi-product newsboy problem with budget constraint, we reviewed research from Hadley and Whitin (1963); Silver, Pyke and Peterson (1998); Nahmias and Schmidt (1984); Lau and Lau (1988, 1994, 1995); and Abde-Malek (2004). The literature provided us with a formulation and derivation for the single period multi-product news boy problem with the budget constraint, which has been further explained in Chapter 4.

### 2.3 Supply Contracts

A literature review of Supply Contracts (Cachon, 2002, Supply Chain Coordination with Contracts) was performed to introduce some methods to achieve overall supply chain optimization. This would provide SupplierCo and RetailerCo with an assessment of the gains that could be achieved by collaboration leading to total supply chain optimization.

### 2.3.1 Introduction

Retailers and Suppliers are primarily concerned with maximizing profitability by optimizing their, own part of the chain instead of looking at how to jointly maximize profitability for both. This results in sub optimal performance. The Supply Contracts literature looks at the methods retailers and suppliers can use to optimize the entire chain and improve profits of each party thus creating a win-win situation.

In most literature on supply contracts, the single period newsboy problem is used to research ways that can help both the retailer and supplier achieve optimal performance. In the single period newsboy problem, the retailer faces one selling season with stochastic demand. The retailer must order the product from the manufacturer before the selling season begins, and the retailer doesn't have the opportunity to replenish the inventory during the selling season. In the example below (Sheffi, lecture notes, ESD260, MIT, 2006) the retailer would order 800 units from the supplier to optimize its own profit. This results in a combined retailer and supplier expected profit of $\$ 110,120$. However, if the retailer and supplier collaborated to optimize the entire supply chain, the combined retailer and supplier expected profit would increase to \$113,150.
Stochastic Demand

| Demand | Probabiltiy |
| ---: | ---: |
| 400 | $0 \%$ |
| 500 | $4 \%$ |
| 600 | $10 \%$ |
| 700 | $20 \%$ |
| 800 | $29 \%$ |
| 900 | $19 \%$ |
| 1,000 | $10 \%$ |
| 1,100 | $6 \%$ |
| 1,200 | $2 \%$ |
| 1,300 | $0 \%$ |

Price and Cost

| Retailer Sale Price | $\$$ | 200 |
| :--- | :--- | ---: |
| Supplier Wholesale Price | $\$$ | 135 |
| Supplier Product Cost per Unit | $\$$ | 50 |
| Salvage Value of Product | $\$$ | 10 |
|  |  |  |

Table 3: Product demand, price and cost


Figure 2 Whole Channel Optimization
(source: Yossi Sheffi, lecture notes, ESD.260, MIT, 2006)

|  | Order Q | Retailer Profit |  | Supplier Profit | Chain Profit |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Sub-Optimal | 800 | $\$$ | 42,120 | $\$$ | 68,000 | $\$$ |
| Chain Optimal | 900 | $\$$ | 36,650 | $\$$ | 76,500 | $\$$ |

Table 4 Optimal Order Q

The single period newsboy problem is simple, but it can provide sufficient information to study three important questions in supply chain coordination. First, which collaboration approaches coordinate the supply chain? Second, which collaboration approaches have sufficient flexibility to allow for any division of the supply chain's profit among the firm? Third, which collaboration approach is worth adopting? The literature describes the different collaborative approaches as Supply Contracts or agreements (Cachon, 2002).

### 2.3.2 Types of Supply Contracts

This section briefly describes the different types of Supply Contracts. Currently, the Sales Rebate contract is used at SupplierCo and RetailerCo.

## Sales Rebate Contract

This contract was studied by Krishan, Kapuscinski and Butz (2001). With a sales rebate contract, the supplier charges W per unit purchased but then gives the retailer a rebate R per unit sold above a threshold or per unit sold during the promotion period as an incentive to increase sales.

## Buy Back Contract

In a buy back contract, the supplier charges the retailer $W$ per unit purchased, but pays the retailer B per unit remaining at the end of the season. Pasternack (1985) did a detailed analysis of buy back contracts in the context of the newsboy problem. Through a buy back contract, the supplier shares the risk of demand variability and induces the retailer to order more products to cover the demand.

## Revenue Sharing Contract

With a revenue sharing contract the supplier charges the retailer the Wholesale Price per unit purchased and the retailer gives the supplier a percentage of its revenue. Cachon and Lariviere (2000) provide an analysis of revenue sharing. Similar to the buy back contract, the revenue sharing contract achieves the optimal supply chain by risk sharing. The main limitations of revenue sharing include high administrative costs, and a negative impact on sales effort.

## Option / Quantity Flexibility Contract

With a quantity flexibility contract, the supplier charges the retailer a wholesale price per unit purchased but then compensates the retailer for losses on unsold units. The supplier provides a full refund for returned items as long as the number of returns is no larger than a certain quantity. Eppen and Iyer (1997) have performed a detailed study on quantity flexibility contract.

## Quantity Discount Contract

There are mainly two types of quantity-discount contract. Incremental discount contract and all unit quantity discount contract. Tomlin (2000) discusses the quantity-discount contract. The quantity-discount contract induces the retailer to order more by shifting the retailer's marginal cost curve so that the supplier earns progressively less on each unit.

### 2.3.3 Summary

The various collaboration approaches in the literature discuss a variety of agreements that can be used to move closer to total supply chain optimization. This represents a subset of the approaches that can be used by SupplierCo and RetailerCo in the collaboration relationship, and is provided as an area for future research. The thesis is concerned with assessing the opportunities that exist in collaboration leading to total supply chain optimization.

## 3 RetailerCo and SupplierCo Promotions As-is Process

This section covers the promotional planning as-is processes for SupplierCo and RetailerCo. The Logistics Manager, Account Executive, and DC Analyst from SupplierCo as well as the DC Planner and Store Planner from RetailerCo were interviewed to develop an understanding of the activities and collaboration between SupplierCo and RetailerCo for Brand A and Brand B product promotions.

### 3.1 SupplierCo Promotions Key Account Planning / S\&OP Process

Twelve months prior to the start of the fiscal year, SupplierCo's headquarter provides a list of priorities for the coming year. The Account Executive teams then develop a category level monthly forecast for each customer for the upcoming year. The key drivers for the forecast include Ad Frequency, New Product Introduction and Inventory Turns. This forecast is updated by the Account teams on a monthly basis and sent to the headquarters for review.

SupplierCo's Account Executive for RetailerCo develops forecasts for the Brand A and Brand B product categories. The forecast accuracy is measured to determine if there is any bias in the forecast. Three or more consecutive forecasts which are consistently above or below the actual demand indicate a bias and call for a correction in future forecasts.

Six months prior to the promotion, SupplierCo's Account Executive and RetailerCo's Category Manager plan the aggregate dollar amount for the promotion and the list of SupplierCo's products which will be promoted. The aggregate dollar amount for the promotion is then confirmed four and a half months prior to the promotion. Currently, there is limited
communication between SupplierCo's Key Account Executive and RetailerCo's Category Manager around changes in the ad price or aggregate dollar amount for the promotion beyond this period.

### 3.2 SupplierCo and RetailerCo Promotions As-is Forecasting Process

This subsection covers the promotions forecasting and planning process for Brand A and
Brand B products from 16-18 weeks prior to the promotion to 1 week after the promotion. Each step in the process, per the numbering in Figure 3 is described below.


Figure 3 As-is Promotions Planning and Execution Process

## 16-18 weeks prior to Promotion:

1.1> RetailerCo's Category Manager works with SupplierCo's Key Account Executive to determine the aggregate dollar amount chain wide for the upcoming promotion. This information is conveyed from RetailerCo's Category Manager to the Store Planner for the category. The

Store Planner determines the feasibility of the aggregate dollar amount for the promotion based on previous promotions data. The Store Planner and Category Manager work together to agree on the aggregate dollar amount for the promotion. This new aggregate dollar amount for the promotion is not communicated to SupplierCo.
1.2> The Store Planner then identifies 2 previous promotions with similar product, price point, advertisement, location of ad in catalog and seasonality (if applicable). This data is loaded into the Promotions Tool. The Store Planner takes into account the sell through of the previous promotion and lost sales to make an adjustment to the aggregate dollar amount based on judgment. This is usually a factor of 1.8 .
1.3> The promotions data is then loaded from the Promotions Tool to the ASR tool, which then allocates data from the chain level to the store level.

## 10 weeks prior to Promotion:

2.1> Each RetailerCo's Store Manager reviews the store level forecast and can accept or adjust the forecast based on the store's specific experience.
2.2> The promotions data is then aggregated to the DC level in the ASR system. This is reviewed by RetailerCo's Store Planner who may make any further revisions to the aggregate promotions forecast. The RetailerCo's Store Planner reviews the DC Purchasing System to ensure that the orders for the promotion have been accurately placed from RetailerCo's store to the DC. The RetailerCo's Store Planner also continues to keep track of any communications around promotional changes regarding prices or advertisements.

## 8 weeks - 6 weeks prior to Promotion:

3.1> The DC Planner reviews the DC level forecast for the promotions and determines the product quantities which need to be ordered from SupplierCo. The DC planner takes into account
the existing on hand inventory at the DC for the regular sales period and reserves any excess inventory for the promotional sales period. The DC Planner reviews the 4 week forecast as well the days of supply for the regular sales period and as a rule of thumb keeps double the amount of required inventory for the regular sales period and anything above that as excess inventory which could be used for the promotion. No formal safety stock policy is implemented for reserving regular period inventory at the DC , which could lead to holding too much or too little inventory by the DC planner. The reserved inventory for the promotion is not used if the inventory for the regular period is depleted at the DC. Additionally, no safety stock is kept at the DC for the promotion. The DC Planner function is performed by SupplierCo's DC Analyst who works closely with RetailerCo's DC Planner.
$3.2>$ The Ad goes to print. If the Ad price changes 10 to 6 weeks prior to the promotion, RetailerCo's Store Planner works with the DC planner to place an order for additional quantities to SupplierCo.

## 4 weeks prior to Promotion:

4.1> SupplierCo delivers the order to the RetailerCo Distribution Center.

## 2 weeks prior to Promotion:

5. $1>$ The Inventory Management System(IMS) at each store makes an adjustment to the promotions order, which was placed to the DC, based on how much existing inventory of the product is at the store. RetailerCo's DC then ships only the adjusted amount to the store and keeps the balance as inventory in the DC. The primary metric used to measure the success of the promotion is sell through.

Post Draw: If RetailerCo's stores sell more than expected they will reorder products from RetailerCo's DC. The DCs will then react by placing an order to SupplierCo to manage the post
draw situation. It is important to note that even though stores reorder during the promotion, the product is received after the promotion.

## Promotion:

During the promotion, no replenishments are made from RetailerCo's DC to the Stores if there is an Out-of-Stock. This could lead to lost sales and impact customer service.

## Post Promotion Event Analysis:

Upon completion of the promotion, the RetailerCo team conducts a post promotion analysis internally. This takes place weekly. However, there is no formal post promotion analysis and communication which takes place between SupplierCo and RetailerCo.

## 4 Methodology

The main features of the multi-product newsboy problem with budget constraint are (Silver, Pyke and Peterson, 1998):

- There is a relative short selling season with a well-defined beginning and end.
- There is more than one product to be sold during the short selling season.
- The decision maker faces a budget constraint.
- Buyers (at the stocking point) or producers have to commit themselves to a large extent, in terms of how much of each stock-keeping unit to order or produce, prior to the start of the selling season.
- There may be one or more opportunities for replenishment after the initial order is placed. Such replenishment actions may be taken prior to the selling season (if the forecast of demand has risen appreciably) or during the early part of the selling season itself (if actual demand to date indicates that the original forecast was considerably low).
- When the demand in the season exceeds the stock made available, there are associated underage costs, or lost sales.
- When the total demand in the season turns out to be less than the stock made available, overage costs result.

Hadley and Whitin (1963) originally implemented the Lagrangian approach to solve the multi-product newsboy problem with a budget constraint. The problem with this approach is that it does not consider the lower bound of order quantities. Lau and Lau (1996) indicate that order quantities in stochastic environments can be infeasible (negative order quantities) if the lower bounds of item order quantities are relaxed. Therefore, to obtain the optimum order quantities
for each of the considered items, their lower bounds should be imposed (non-negativity constraints) and the Kuhn-Tucker conditions must be observed (Abdel-Malek and Montanari, 2004).

In Silver, Pyke and Peterson's book, Inventory Management and Production Planning and Scheduling, an Excel model is introduced to solve the multi-product newsboy problem with budget constraints. To solve the problem in Excel, Excel's Goal Seek function is used to search the optimal Lagrangian multiplier.

The following two sections discuss the derivations for the profit maximization model and revenue maximization models. Since we will modify this approach to solve the maximize promotion revenue problem, we will list out the details of the derivation.

### 4.1 Profit Maximization Model Formulation

The objective of the profit maximization model (Silver, Pyke and Peterson, 1998) is to maximize total profit of SKUs under promotion with a budget constraint. The total profit takes into account the revenue, cost, lost sales and salvage value of the SKU. The budget constraint is a product of the quantity ordered and RetailerCo's cost for the product.

Traffic Builder SKUs such as Brand A and Brand B are used by RetailerCo to increase consumer traffic in the store which leads to an increase in profitability from the sales of other products. A 'basket' comprises of the traffic builder SKU and the other products which are bought by a consumer when they enter the store to buy a traffic builder SKU. The objective of the promotion is to therefore maximize the entire basket under a budget constraint for the promotion. Below are the variable notations and the formulation of the profit maximization model.

For SKU i, the following variable notations are used
$Q_{i}{ }^{*}$ : optimal order quantity under
$R_{i}$ : the retailer sale price
$C_{i}$ : the supplier's wholesale price
$S_{i}$ : Salvage value
$L_{i}$ : the penalty of loss sales
$x_{i}$ : the demand of product
$f_{i}(x)$ : probability density function of demand for SKU i
$F_{i}(x)$ : cumulative density function of demand for SKU i
$F_{i}(x)$ is differentiable, strictly increasing and $0 \leq \mathrm{F}(0) \leq 1$

## Objective function:

$\operatorname{Max} \sum_{i}^{N} \operatorname{profit}\left(Q_{i}\right)=$
$\sum_{i}^{N}\left[R_{i} \int_{0}^{Q_{i}} x_{i} f_{i}(x) d x_{i}+R_{i} Q_{i} \int_{Q i}^{\infty} f_{i}(x) d x_{i}+S_{i} \int_{0}^{Q_{i}}\left(Q_{i}-x_{i}\right) f_{i}(x) d x_{i}-C_{i} Q_{i}-L_{i} \int_{Q i}^{\infty}\left(x_{i}-Q_{i}\right) f_{i}(x) d x_{i}\right]$

## Subject to

$\sum_{i}^{N} C_{i} Q_{i} \leq$ Budget
Applying the Lagrange multiplier M ,
$W=\sum_{i}^{N} \operatorname{profit}\left(Q_{i}\right)-M\left(\sum_{i}^{N} C_{i} Q_{i}-B u d g e t\right)$
To find the optimal $M$, apply the partial differentiation and find the solution of $\frac{\partial W}{\partial Q_{i}}=0$
The optimal order quantity of $S K U_{i}$ is as below:
$Q i^{*}=F_{i}^{-1}\left[\frac{R_{i}+L_{i}-(M+1) C_{i}}{R_{i}-S_{i}+L_{i}}\right]$
For the detail of derivation, please see Appendix A.

### 4.2 Revenue Maximization Model Formulation

The objective of the revenue maximization model is to maximize the total revenue for SKUs under promotion, subject to a budget constraint. The total revenue depends on RetailerCo's promotion price, SupplierCo's wholesale price and the demand of each SKU. The budget constraint is a product of the quantity ordered and RetailerCo's cost for the product. Below is the variable notation and formulation of the revenue maximization model.

For SKU i, the following notations are used
$Q_{i}$ : order quantity
$Q_{i}{ }^{*}$. optimal order quantity under
$R_{i}$ : the retailer sale price
$C_{i}$ : the supplier's wholesale price
$x_{i}$ : the demand of product
$f_{i}(x)$ : probability density function of demand for SKU i
$F_{i}(x)$ : cumulative density function of demand for SKU i
$F_{i}(x)$ is differentiable, strictly increasing and $0 \leq \mathrm{F}(0) \leq 1$
Objective function
$\operatorname{MAX} \sum_{i}^{N} \operatorname{Re}$ venue $\left(Q_{i}\right)=\sum_{i}^{N}\left[R_{i} \int_{0}^{Q_{i}} x_{i} f_{i}(x) d x_{i}+R_{i} Q_{i} \int_{Q_{i}}^{\infty} f_{i}(x) d x_{i}\right]$

Subject to
$\sum_{i}^{N} C_{i} Q_{i} \leq$ Budget

## Apply the Lagrange multiplier M

$W=\sum_{i}^{N} \operatorname{revenue}\left(Q_{i}\right)-M\left(\sum_{i}^{N} C_{i} Q_{i}-\right.$ Budget $)$
To find the optimal $\mathbf{M}$, apply the partial differentiation and find the solution of $\frac{\partial W}{\partial Q_{i}}=0$
The optimal order quantity of $S K U_{i}$ is as below:

$$
Q_{i}^{*}=F_{i}^{-1}\left(\frac{R_{i}-M C_{i}}{R_{i}}\right)
$$

For the detailed derivation, please see Appendix $\mathbf{A}$.

## 5 Data Analysis

This section discusses an analysis of the promotions demand pattern, out-of-stocks and promotions optimization model results. The promotions demand pattern analysis focuses on 13 SKUs under RetailerCo's distribution center A. Then, a method is introduced to identify out-ofstocks through daily POS data. Finally, the results from the promotions profit maximization and revenue maximization models at the national and distribution center level are discussed.

### 5.1 Promotions Demand Pattern Analysis

This subsection covers the data scope and analysis of the promotions demand pattern. The data scope includes the rationale for selection of distribution center A as the representative distribution center and 13 SKUs as the representative products. These 13 SKUs cover two product categories, Brand A and Brand B products. The promotions demand pattern analysis at the national and distribution center level includes segmentation of SKUs by coefficient of variation and price sensitivity.

### 5.1.1 Data Scope

RetailerCo's distribution network comprises of more than 10 distribution centers and more than thousand stores nationwide. The data under distribution center A was chosen for the scope of promotions data analysis. This is because the number of stores in the distribution center A has remained stable over the past five years. For instance, the store count increased from 400 stores in the year 2000 to 450 stores in the year 2006. Brand A and Brand B products promotions sales for distribution center A comprise of $20 \%$ of the national promotions sales for these categories.

For the scope of data analysis, Brand A and Brand B products were chosen due to the maturity as well as the heavy promotional activity of these products at RetailerCo. Brand A and Brand B products are traffic builder SKUs and are promoted in RetailerCo for one week of every month to attract customers to the store so that they buy other higher margin products.

### 5.1.2 Brand A SKU Demand Pattern Analysis

Promotions account for more than $80 \%$ of total unit sales and $70 \%$ of total revenue nationwide for Brand A and Brand B products. Since, each SKU is promoted for one week each month, the weekly demand in promotional weeks is around 15 times the weekly demand in regular weeks for units sold and 10 times the weekly demand in regular weeks for revenue, respectively (See Figure 4 and 5). This makes effective promotions planning and execution even more critical to reducing out-of-stocks. One important aspect of promotions planning is SKU segmentation of demand by volume, coefficient of variation and demand pattern. The tables below show the demand volume in units sold and revenue for distribution center $A$.

2006 Sales in Unit (in thousand), Distribution Center A

| SKU | Total | Promotion | Regular | Promotion in \% | Weekly demand, Prom. <br> week $/$ Regular week |
| :---: | ---: | ---: | ---: | ---: | :---: |
| SKU-1 | 2,246 | 1,991 | 255 | $89 \%$ | 23 |
| SKU-2 | 1,461 | 1,312 | 148 | $90 \%$ | 27 |
| SKU-3 | 493 | 399 | 95 | $81 \%$ | 13 |
| SKU-4 | 484 | 368 | 116 | $76 \%$ | 10 |
| SKU-5 | 453 | 355 | 98 | $78 \%$ | 11 |
| SKU-6 | 499 | 387 | 112 | $78 \%$ | 10 |
| SKU-7 | 441 | 345 | 96 | $78 \%$ | 11 |
| SKU-8 | 290 | 233 | 57 | $80 \%$ | 12 |
| SKU-9 | 434 | 324 | 110 | $75 \%$ | 9 |
| SKU-10 | 61 | 39 | 22 | $64 \%$ | 5 |
| SKU-11 | 837 | 692 | 145 | $83 \%$ | 14 |
| SKU-12 | 920 | 765 | 156 | $83 \%$ | 15 |
| SKU-13 | 159 | 128 | 31 | $80 \%$ | 12 |
| Total | 8,780 | 7,338 | 1,442 | $84 \%$ | 15 |

Figure 4 Promotional Sales vs. Regular Sales in Units, DC A

2006 Sales in \$K, Distribution Center A

| SKU | Total |  | Promotion |  | Regular |  | $\begin{aligned} & \text { Promotion } \\ & \text { in } \% \\ & \hline \end{aligned}$ | Weekly revenue, prom. week / Regular week |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SKU-1 | \$ | 14,270 | \$ | 11,878 | \$ | 2,392 | 83\% | 14.9 |
| SKU-2 | \$ | 7,236 | \$ | 6,273 | \$ | 964 | 87\% | 19.5 |
| SKU-3 | \$ | 2,905 | \$ | 2,146 | \$ | 759 | 74\% | 8.5 |
| SKU-4 | \$ | 2,913 | \$ | 1,983 | \$ | 929 | 68\% | 6.4 |
| SKU-5 | \$ | 2,698 | \$ | 1,911 | \$ | 787 | 71\% | 7.3 |
| SKU-6 | \$ | 2,981 | \$ | 2,084 | \$ | 896 | 70\% | 7.0 |
| SKU-7 | \$ | 2,620 | \$ | 1,857 | \$ | 763 | 71\% | 7.3 |
| SKU-8 | \$ | 1,709 | \$ | 1,252 | \$ | 457 | 73\% | 8.2 |
| SKU-9 | \$ | 2,621 | \$ | 1,743 | \$ | 879 | 66\% | 6.0 |
| SKU-10 | \$ | 386 | \$ | 212 | \$ | 174 | 55\% | 3.6 |
| SKU-11 | \$ | 4,882 | \$ | 3,722 | \$ | 1,160 | 76\% | 9.6 |
| SKU-12 | \$ | 5,356 | \$ | 4,111 | \$ | 1,245 | 77\% | 9.9 |
| SKU-13 | \$ | 999 | \$ | 693 | \$ | 306 | 69\% | 6.8 |
| Total | \$ | 51,575 | \$ | 39,865 |  | 11,711 | 77\% | 10.2 |

Figure 5 Promotional Sales vs. Regular Sales in \$, DC A

For the years 2005 and 2006, an analysis of demand pattern for 11 Brand A SKUs was performed. Under a budget constraint, a SKU with a higher coefficient of variation will be less sensitive to an increase in the budget and a higher budget would be needed to decrease the likelihood of lost sales. Figure 6 and Table 5 show the 2005 and 2006 on-promotion unit sales for each of these SKUs.


Figure 6 Promotional Weekly Sales in Thousand Units

Based on demand volatility, the demand pattern for the SKUs can be categorized as high with a Coefficient of Variation (CV) of greater than $20 \%$, and low with a Coefficient of Variation of less than $20 \%$.

## Brand A SKU Promotion Sales (in Thousand Units) at Distribution Center A, 2006

|  | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-7 | SKU-8 | SKU-9 | SKU-10 | SKU-11 | SKU-12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | SKU-13 9 (183

Table 5 Promotional Demand variability

SKU-11 and SKU-12 follow the same demand pattern and can be aggregated for the purpose of forecasting. SKU-13 demand is very stable and is almost flat. SKU-8 and SKU-9 were new products introduced in week 35 year 2005 and the demand stabilized in one year. This should be taken into account while forecasting these products.

### 5.1.3 Brand B SKU Demand Pattern Analysis

A high level of sensitivity between demand and price for promotions impacts the ability to forecast accurately. This should be taken in to account when determining which SKUs should be promoted to minimize out-of-stocks and lost sales. An analysis of the demand pattern with different promotion prices was performed for Brand B SKU-1and SKU-2. As per Figure 7 below, the demand pattern of SKU-1 is very sensitive to the promotion price. This makes it more challenging to forecast products and manage demand. Also, there is a strong negative correlation between demand and promotion price.


Figure 7 Brand B SKU-1

On the other hand, SKU-2 is very insensitive to price as can be seen in Figure 8, which makes it easier to forecast and manage demand.


Figure 8 Brand B SKU-2

Below is a summary of Brand A and Brand B characteristics. Similar profiles should be created for other SKUs.

| Characteristic | Brand A | Brand B |
| :--- | :--- | :--- |
| Price Sensitive | Medium | High |
| Profitability | Negative during promotion | Negative during promotion |
| Promotional Volume | High | High |
| Promotion demand predictable | Yes | No sure |
| In-store availability | Medium during promotion | Low during promotion |
| Inventory cost | Low | Low |
| Demand on shelf space | Medium or high | High |
| Challenge to supply chain | Difficult during promotion | Difficult during promotion |
| Transportation cost | High | High |
| Retailer business need | Traffic builder | Traffic builder |
| Benefit | Bring customer to the store | Bring customer to the store |

Table 6 Product Profile

### 5.2 Promotions Out-of-Stocks Analysis

Maximizing on shelf-availability of supplier's products at retailer's stores leads to collaborative success. Owning a fully stocked retail shelf improves consumer value, builds consumer loyalty to the brand, shopper loyalty to the store, increases sales, and - most importantly - boosts category profitability. In today's competitive consumer products and retail industries, no one can afford to ignore out-of-stocks.

According to a GMA study on Retail Out-of-Stocks(Gruen, Corsten and Bharadwaj, 2002), ineffective Store Forecasting, Store Stocking and Store Ordering policies and processes represent over $70 \%$ of the causes for Out-of-Stocks in the industry.


Figure 9: Root Causes for shelf Out-of-Stocks
Source: Retail Out-of-Stocks (Gruen, Corsten and Bharadwaj, 2002)

Manufacturers and retailers can use point-of-sale (POS) data to determine how much product has been sold through the retail chain, and estimate out-of-stock levels. Currently, there are multiple ways to solve the out-of-stock problem, ranging from reliance on safety stocks to physical audits. Holding safety stock leads to higher costs, while it may not be feasible to conduct physical audits every day to provide a complete picture of stocking conditions. The following two subsections illustrate how to estimate lost sales at the distribution center and store respectively. At the DC level, out of stock estimations are used as an input to the baseline scenario when comparing with the result from the optimization model. Store level analysis can be used to make replenishment decisions during a promotion.

### 5.2.1 DC Level

Since RetailerCo's out-of-stocks data is not available, one way to estimate out-of-stocks at DC level is to analyze rain-checks ${ }^{1}$. According to RetailerCo's policy, a customer can ask for a rain check if the item is Out-of-Stock during the promotional week and come back to pick up the product at the promotional price, when it is available. In non-promotional weeks, the customer

[^0]will go back, redeem the rain-check and receive the product at the previous promotion price. An important factor to take into account is that not every customer asks for a rain check when an item is Out-of-Stock and not every customer who takes a rain check ends up claiming it. Assuming that only $70 \%$ of customers who ask for rain checks, and only $70 \%$ of those customers really redeem their rain check, the estimated lost sales in unit can be estimated in the table below.

According the table below, the estimated out-of-stocks for Brand B, SKU-1 and SKU-2, is higher than Brand A SKUs.

Estimated Lost Sales (in Thousand Units), 2006

|  | SKU-1 | SKU-2 | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-7 | SKU-8 | SKU-9 | SKU-10 | SKU-11 | SKU-12 | SKU-13 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Total promotional sales in unit | 1,305 | 1,980 | 398 | 367 | 354 | 386 | 344 | 233 | 323 | 39 | 690 | 763 | 127 |
| Sales in prom. Week in unit | 1,226 | 1,733 | 385 | 354 | 340 | 372 | 331 | 224 | 310 | 37 | 665 | 735 | 122 |
| Redeemed rain check in unit | 79 | 248 | 12 | 13 | 14 | 14 | 13 | 9 | 13 | 2 | 25 | 28 | 6 |
| Redeemed rain check in \% | $6.1 \%$ | $12.5 \%$ | $3.1 \%$ | $3.5 \%$ | $3.9 \%$ | $3.7 \%$ | $3.7 \%$ | $3.8 \%$ | $3.9 \%$ | $5.1 \%$ | $3.7 \%$ | $3.7 \%$ | $4.5 \%$ |
| Factor 1, asking for RC | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ |
| Factor 2, redeeming RC | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ | $70 \%$ |
| Estimated out-of-stocks | $12.4 \%$ | $25.5 \%$ | $6.3 \%$ | $7.2 \%$ | $7.9 \%$ | $7.6 \%$ | $7.6 \%$ | $7.7 \%$ | $8.0 \%$ | $10.4 \%$ | $7.5 \%$ | $7.5 \%$ | $9.2 \%$ |
| Estimated Lost Sales after RC | $6.3 \%$ | $13.0 \%$ | $3.2 \%$ | $3.7 \%$ | $4.0 \%$ | $3.9 \%$ | $3.9 \%$ | $3.9 \%$ | $4.1 \%$ | $5.3 \%$ | $3.8 \%$ | $3.8 \%$ | $4.7 \%$ |
| Estimated lost Sales in unit | 82 | 258 | 13 | 14 | 14 | 15 | 13 | 9 | 13 | 2 | 26 | 29 | 6 |

Figure 10 Lost Sales Estimation by SKU, DC A

The chart below (Figure 11) is an example of promotional sales during the promotional week and redeemed rain checks after promotional week.


Figure 11 Actual Rain Check Amount vs. Promotional Week Sales, SKU-11

### 5.2.2 Store Analysis

During a weekly promotion, if the store runs out of inventory in the middle of the week there is a trade off between losing the sale versus incurring transportation costs to replenish the store with more inventories. Point of sale data is used to understand the demand during the promotion week and assess the potential lost sales value. This can be then compared with the transportations costs of an emergency shipment to replenish the store. A method to estimate the potential lost sale is described in the next paragraph.

RetailerCo's promotional period is for one week starting from Sunday to Saturday. On average, the sales volume in the first day of the promotion, Sunday, is about $32 \%$ of total sales during the promotional period. Intuitionally, the sales volume will go down during weekdays and increase again the coming Saturday if the store doesn't run out of any inventory. To test this assumption, four stores which don't run out of inventory for SKU-11 during the promotional period January 14 to January 20, 2007, were selected. Figure 12 below shows the U shape trend of sales in the promotional week. From the figure, we can see that the last day of the promotional period has the second largest sale volume in the four stores, which were analyzed.


Figure 12 Daily Demand Pattern, SKU-11

If a store runs out of inventory during the promotional period, the lost sales can be estimated by taking a sum of historical averages for the remaining days of the promotion. This is illustrated in Figure 13 below.


Figure 13 Estimate Lost Sales from Out-of-Stocks by Daily POS

### 5.3 Model Data Analysis

The Profit Maximization and Revenue Maximization models can be applied to the SupplierCo and RetailerCo promotions budget and inventory allocation decision at the National Chain, Distribution Center and Store levels. Based on the available data, this section covers an analysis of the model results at the national chain and distribution center level. It confirms that the model provides similar results \& insights at both these levels.

### 5.3.1 National Chain Level Optimization

This section discusses data pre-processing, data analysis and compares the results between actual promotions data, profit maximization and revenue maximization models at the national chain level.

### 5.3.1.1 Data Pre - Processing

Data Pre-Processing was performed to overcome the challenges of obtaining historical promotions forecast data and actual demand data. At the national chain level, the analysis focuses on seven traffic builders, SKU-3, 4,5,6,11, 12 and 13, at 1,000 stores because of the availability of consistent forecast data at the same promotional price for these SKUs.

The actual promotion result in week 31 year 2006 was used to compare the expected result based on the model. A forecast of the promotion demand was generated for each SKU based on historical demand data from 4 previous promotions at the same promotional price, which is listed in Figure 14 below.
Actual Promotional Sales in Unit

| Week Nbr | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-11 | SKU-12 | SKU-13 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2006-14$ | 32,160 | 37,592 | 17,180 | 29,992 | 42,444 | 42,052 | 21,688 |
| $2006-18$ | 41,428 | 35,088 | 23,100 | 28,888 | 53,224 | 50,480 | 16,172 |
| $2006-23$ | 44,064 | 38,776 | 25,716 | 31,676 | 58,732 | 57,104 | 18,672 |
| $2006-27$ | 40,204 | 35,884 | 25,216 | 31,836 | 52,344 | 49,612 | 17,916 |

Figure 14 SKU level Actual Sales, National Chain Level

The naïve forecast for each SKU was developed by calculating the mean and standard deviation of demand from four previous promotions. The naïve forecast for each SKU is listed in the Figure 15 below.

|  | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-11 | SKU-12 | SKU-13 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Mean | 39,464 | 36,835 | 22,803 | 30,598 | 51,686 | 49,812 | 18,612 |
| Std Dev. | 5,129 | 1,663 | 3,916 | 1,413 | 6,779 | 6,161 | 2,302 |

Figure 15 SKU level Forecasted Demand

### 5.3.1.2 Profit Maximization Model

This subsection covers data analysis for the actual promotion data as well as the profit maximization model for traffic builder SKUs. Firstly, it shows that maximizing RetailerCo profit results in a profit improvement for both SupplierCo and RetailerCo. Next, it shows that the most optimal scenario is to optimize the entire chain (both SupplierCo and RetailerCo). In both scenarios, the profit generated from the 'basket' of SKUs is maximized. The 'basket' of SKUs includes the traffic builder SKU under promotion as well as the additional profit generated from the sales of higher margin SKUs as an effect of the traffic builder SKU. In addition, the analysis also shows the profit improvement by comparing baseline profit with expected optimal profit. Finally, sensitivity analysis of budget versus expected optimal profit and budget versus service level shows the tradeoffs between a budget which is too low or too high. The following assumptions were made in the input data for the model:

1. RetailerCo Traffic Builder Benefit: the profit of whole basket which is trigged by traffic builder SKU. This value is estimated to be $\$ 2$ per unit of Brand A and $\$ 3$ per unit of Brand B , since real data was not available.
2. RetailerCo customer Goodwill: the value of damage on RetailerCo brand once the traffic builder is Out-of-Stocks. This value is also estimated to be $\$ 2$ per unit.
3. SupplierCo rebate to RetailerCo: SupplierCo will give RetailerCo 25 cents for each unit of Brand A SKU sold and 50 cents for each unit of Brand B SKU sold during the promotional period.

## Baseline: Actual Promotion

For the promotion in week 31, 2006, the budget constraint was $\$ 1,707,411$. The actual RetailerCo profit was $\$ 226,792$. This was calculated by using the following formula:

Actual Profit $=$ actual unit sales * (prom. price - cost + rebate + traffic builder benefit $)$

- (lost sales units * lost sales penalty) + (order quantity - demand)*Salvage

The RetailerCo Profitability and service level for the SKUs under consideration are shown in the table below.

| SKU | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-11 | SKU-12 | SKU-13 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Actual Order Quantity (in Thansand) | 40 | 33 | 25 | 31 | 56 | 50 | 18 |  |
| RetailerCo Fill Rate | $95 \%$ | $100 \%$ | $95 \%$ | $95 \%$ | $96 \%$ | $96 \%$ | $95 \%$ |  |
| RetailerCo Actual Profit | $\$ 22,879$ | $\$ 52,722$ | $\$ 33,634$ | $\$ 43,649$ | $\$ 33,508$ | $\$ 30,133$ | $\$ 10,270$ | $\$ 226,793$ |
| SupplierCo Actual Profit | $\$ 46,587$ | $\$ 32,716$ | $\$ 24,281$ | $\$ 30,576$ | $\$ 64,528$ | $\$ 57,939$ | $\$ 21,381$ | $\$ 278,008$ |
| Total Profit | $\$ 69,465$ | $\$ 85,438$ | $\$ 57,915$ | $\$ 74,225$ | $\$ 98,036$ | $\$ 88,071$ | $\$ 31,652$ | $\mathbf{\$ 5 0 4 , 8 0 2}$ |

Figure 16 Actual Order Quantity, Fill Rate and Profit, Baseline, National Chain Level

The actual SupplierCo profit for week 31 's promotion was $\$ 278,008$. For illustration purposes, it was assumed that SupplierCo makes a profit margin of $20 \%$ on the sales price to RetailerCo. The profit was calculated using the following formula.

Actual Profit $=$ RetailerCo order quantity * wholesale price * Margin - rebate $*$ actual unit sales The actual SupplierCo profit for week 31 's promotion was $\$ 278,008$, and total RetailerCo and SupplierCo Profit is $\$ 504,802$.

## RetailerCo Profit Maximization ${ }^{1}$

Compared with actual promotion sales data in week 31, 2006, the RetailerCo profit maximization model can increase RetailerCo profit by $5.9 \%$. The model also increases SupplierCo's profit by $0.34 \%$. Table 7 shows a summary of the comparison between actual profit and optimal profit if RetailerCo orders each SKU according to the output of RetailerCo profit maximization model.

[^1]|  | Supplier <br> Profit | Retailer <br> Profit | Whole Supply <br> Chain Profit |  |
| :--- | ---: | :---: | :---: | ---: |
| Result of Actual Promotion | $\$ 278,008$ | $\$ 226,793$ | $\$$ | 504,802 |
| Result of Retailer Profit Max. Model | $\$ 278,947$ | $\$ 240,108$ | $\$$ | 519,055 |
| Improvement by model in \% | $0.34 \%$ | $5.9 \%$ | $2.8 \%$ |  |

Table 7 Comparison, Base Line and RetailerCo Profit Optimization Model
Table 8 shows the logics and detail behind the profit comparison in table 7.

|  | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-11 | SKU-12 | SKU-13 |  | Total Budget |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SupplierCo wholesale price Real demand in week 31 | $\begin{array}{\|cr} \hline \$ & 7 \\ & 42,416 \end{array}$ | $\begin{array}{lr} \hline \$ & 6 \\ & 33,484 \end{array}$ | $\begin{array}{lr} \hline \$ & 6 \\ & 25,832 \end{array}$ | $\begin{array}{lr} \hline \$ & 6 \\ & 32,860 \end{array}$ | $\begin{array}{lr} \hline \$ & 7 \\ & 58,080 \end{array}$ | $\begin{array}{lr} \hline \$ & 7 \\ 52,132 \end{array}$ | $\$$ 7 <br>  19,552 |  |  |
| Actual Order Q in week 31 | 40,300 | 33,384 | 24,576 | 31,200 | 55,820 | 50,120 | 18,496 | \$ | 1,707,411 |
| Optimal Order $\mathbf{Q}^{*}$ per Model | 39,527 | 37,941 | 25,290 | 31,538 | 51,771 | 49,887 | 18,640 | \$ | 1,707,408 |
|  |  |  |  |  |  |  |  | Total Profit |  |
| Actual RetailerCo Profit | \$22,879 | \$52,722 | \$33,634 | \$43,649 | \$33,508 | \$30,133 | \$10,270 | \$ | 226,793 |
| RetailerCo Profit if order $Q^{*}$ per model | \$20,234 | \$77,441 | \$37,276 | \$45,397 | \$19,659 | \$29,337 | \$10,763 | \$ | 240,108 |
| Actual SupplierCo Profit | \$46,587 | \$32,716 | \$24,281 | \$30,576 | \$64,528 | \$57,939 | \$21,381 | \$ | 278,008 |
| SupplierCo profit if order $\mathrm{Q}^{*}$ per model | \$45,693 | \$38,296 | \$24,987 | \$30,907 | \$59,847 | \$57,670 | \$21,548 | \$ | 278,947 |
| Actual whole supply chain profit | \$69,465 | \$85,438 | \$57,915 | \$74,225 | \$98,036 | \$88,071 | \$31,652 | \$ | 504,802 |
| Whole supply chain profit if order $\mathrm{Q}^{*}$ | \$65,927 | \$115,737 | \$62,263 | \$76,304 | \$79,506 | \$87,006 | \$32,311 | \$ | 519,055 |

Table 8 Profit Comparison between Baseline and RetailerCo Profit Optimization

A sensitivity analysis of the budget versus service level (Figure 17) and budget versus RetailerCo's expected optimal profit (Figure 18) shows that service level increases from $5.3 \%$ to $7.1 \%$, and RetailerCo profit increases from $\$ 148,000$ to $\$ 167,000$ when the budget increases from $\$ 1,500,000$ to $\$ 1,550,000$. This is because at this point the risk of lost sales is higher than the risk of excess inventory, hence, ordering more leads to a higher profit. Moreover, as the budget is further increased beyond $\$ 1,800,000$ the profitability starts decreasing. This is because an increase in the budget beyond a certain point results in a higher risk of excess inventory versus lost sales and thus a lower profit.


Figure 17 Budget vs. Service Level, RetailerCo Profit Optimization


Figure 18 RetailerCo Optimal Expected Profit

## Whole Supply Chain, RetailerCo and SupplierCo Total Profit Maximization

In the whole supply chain's profit maximization model, the entire channel's profit is maximized with the budget constraint. For the whole supply chain, the cost of the product is SupplierCo's cost and the selling price is RetailerCo's promotions sales price. With the budget $\$ 1,550,000$, the profit maximization model resulted in profit improvements by $37 \%$ compared with the baseline actual promotion data, increasing the whole supply chain profit to $\$ 690,000$.

|  | SupplierCo <br> Profit | RetailerCo <br> Profit | Supply Chain <br> Profit | Improvement <br> compared with <br> Baseline in $\%$ |
| :--- | :---: | :---: | :---: | :---: |
| Actual Profit in Week 31 | $\$ 278,008$ | $\$ 226,793$ | $\$ 504,802$ |  |
| Expected Profit if Order per <br> Retailer Profit Optimization Model | $\$ 278,947$ | $\$ 240,108$ | $\$ 519,055$ | $2.8 \%$ |
| Expected Profit if Order per Whole <br> Supply Chain Optimization Model |  |  | $\$ 690,224$ | $36.7 \%$ |

Table 9 Whole Supply Chain Profit Optimization, National Chain Level

The chart below shows that under the same budget constraint, the total supply chain profit based on whole channel optimization is higher than the total supply chain profit based only on RetailerCo's optimization.


Figure 19 Chain Profit, Supply Chain Optimization vs. RetailerCo Optimization


Figure 20 Budget vs. Service Level, Whole Supply Chain Optimization

A sensitivity analysis of the budget versus service level shows that an increase in the budget from $\$ 1,250,000$ to $\$ 1,300,000$ results in a service level increase around $10 \%$, and a chain profitability increase over $\$ 37,000$. This is because at this point the risk of lost sales is higher than the risk of excess inventory, hence, ordering more quantity leads to a higher profit. However, as the budget increases from $\$ 1,500,000$ to $\$ 1,550,000$, the profitability becomes less sensitive and is increased by only $\$ 7,000$ and the service level increases in the range of $5 \%$. This is because an increase in the budget beyond a certain point results in a higher risk of excess inventory versus lost sales and thus a lower profit.

As shown in Table 9, the highest profitability is achieved when the entire chain i.e. both SupplierCo and RetailerCo profit is maximized. However, in order to achieve this optimized collaboration would need to take place between SupplierCo and RetailerCo.

### 5.3.1.3 Revenue Maximization Model

This subsection covers data analysis for the actual promotions data as well as the RetailerCo Revenue Maximization Model. It covers the scenario where the Revenue Maximization model is run to maximize RetailerCo's revenue from the sale of Brand A or Brand B products. The analysis shows that the RetailerCo revenue with the model is slightly higher than the baseline. However, sensitivity analysis of budget versus revenue and budget versus service level shows that having a budget beyond a certain point results in a constant service level and revenue.

## Baseline: Actual Promotion

For the promotion which was held in week 31, 2006, the budget constraint was $\$ 1,707,000$. The actual RetailerCo revenue was $\$ 1,394,000$. This was calculated by using the following formula:

Actual Revenue $($ Retailer $)=$ actual unit sales * retailer promotion price
The retailerCo revenue and service level for the SKU's under consideration are shown in the table below.

| SKU | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-11 | SKU-12 | SKU-13 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RetailerCo actual revenue in $\$ \mathrm{KK}$ | $\$ 221$ | $\$ 183$ | $\$ 135$ | $\$ 171$ | $\$ 306$ | $\$ 275$ | $\$ 102$ | $\$ 1,394$ |
| RetailerCo fill rate | $95 \%$ | $100 \%$ | $95 \%$ | $95 \%$ | $96 \%$ | $96 \%$ | $95 \%$ | $96 \%$ |

## Table 10 RetailerCo Actual Revenue, National Chain Level

The actual SupplierCo revenue for the promotion in week 31 year 2006 was $\$ 1,707,000$. This revenue was calculated using the following formula.

Actual Revenue $=$ Actual Unit Sales * Supplier wholesale price

## Revenue Maximization Model

In the retailerCo revenue maximization model, RetailerCo's revenue is maximized with the budget constraint. A sensitivity analysis of the RetailerCo expected optimal revenue versus budget constraint shows that an increase in the budget from $\$ 1,500,000$ to $\$ 1,550,000$ results in a service level increase in the range of $6 \%$ to $8 \%$ and RetailerCo revenue increase of over $\$ 32,000$.


Figure 21 Budget vs. Service Level, Revenue Maximization Model, National Chain Level

However, as the budget increases beyond the maximum specified by the model, which is $\$ 2,500,000$, the service level remains constant at almost $100 \%$ while the RetailerCo revenue continues to remain constant. Hence, increasing the budget beyond this point would not lead to any additional benefits and would be a wasteful use of resources.


Figure 22 Budget vs. Expected Optimal Revenue, National Chain Level

### 5.3.2 Distribution Center Level Optimization

This section discusses the data pre-processing activities as well as an analysis of the results after running the data through the Profit Maximization and Revenue Maximization models at the distribution center Level. The insights are similar in both the distribution center level and Nation wide levels.

### 5.3.2.1 Data Pre-Processing

One Brand B SKU and eleven Brand A SKUs promoted in week 41 year 2006 were considered for the purpose of analyzing the model. Due to unavailability of lost sales and demand forecast data, estimates were made based on historical rain checks and unit sales data, respectively.

Each SKU's estimated demand during promotion weeks is equal to actual promotion sales plus the estimated loss of sales. The lost sale for each SKU was estimated to be a percentage of rain checks redeemed by the customer after the promotion. A naïve demand forecast was generated by taking the mean and standard deviation of the estimated demand for four previous promotions at the same promotion price. This can be seen in the Table 10 below.

| SKU | SKU-1 | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-7 | SKU-8 | SKU-9 | SKU-10 | SKU-11 | SKU-12 | SKU-13 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Actual prom. Sale in unit |  |  |  |  |  |  |  |  |  |  |  |  |
| 2006-25 | 153,064 | 32,660 | 29,276 | 29,448 | 32,036 | 31,272 | 19,656 | 29,996 | 0 | 57,100 | 66,268 | 10,280 |
| 2006-29 | 117,640 | 39,244 | 36,004 | 36,144 | 37,236 | 34,968 | 23,228 | 36,632 | 6,212 | 66,616 | 75,276 | 12,112 |
| 2006-33 | 159,220 | 28,876 | 29,844 | 28,184 | 30,728 | 26,384 | 18,664 | 27,948 | 11,588 | 50,768 | 53,336 | 11,040 |
| 2006-37 | 126,760 | 27,100 | 28,136 | 26,568 | 27,656 | 25,308 | 14,748 | 26,184 | 11,640 | 46,132 | 51,384 | 9,296 |
| Estimated Loss of sales | 8\% | 5\% | 5\% | 5\% | 5\% | 5\% | 5\% | 5\% | 5\% | 5\% | 5\% | 5\% |
| Estimated real demand |  |  |  |  |  |  |  |  |  |  |  |  |
| 2006-25 | 165,309 | 34,293 | 30,740 | 30,920 | 33,638 | 32,836 | 20,639 | 31,496 | 0 | 59,955 | 69,581 | 10,794 |
| 2006-29 | 127,051 | 41,206 | 37,804 | 37,951 | 39,098 | 36,716 | 24,389 | 38,464 | 6,523 | 69,947 | 79,040 | 12,718 |
| 2006-33 | 171,958 | 30,320 | 31,336 | 29,593 | 32,264 | 27,703 | 19,597 | 29,345 | 12,167 | 53,306 | 56,003 | 11,592 |
| 2006-37 | 136,901 | 28,455 | 29,543 | 27,896 | 29,039 | 26,573 | 15,485 | 27,493 | 12,222 | 48,439 | 53,953 | 9,761 |
| Forecasted Demand |  |  |  |  |  |  |  |  |  |  |  |  |
| Mean | 150,305 | 33,569 | 32,356 | 31,590 | 33,510 | 30,957 | 20,028 | 31,700 | 10,304 | 57,912 | 64,644 | 11,216 |
| Std Dev. | 21,713 | 5,644 | 3,708 | 4,418 | 4,195 | 4,708 | 3,661 | 4,797 | 3,275 | 9,309 | 11,840 | 1,251 |

Table 11 Distribution Center Forecasted Demand, RetailerCo
The actual budget for the promotion was estimated to be the actual promotion sales in units multiplied by SupplierCo wholesale price to RetailerCo.

### 5.3.2.2 Profit Maximization Model

This subsection covers data analysis for the actual promotion data as well as the profit maximization model for traffic builder SKUs. It shows that while maximizing RetailerCo profit results in an increase in profitability for both RetailerCo and SupplierCo, the most optimal
scenario is where the profit of the entire chain is maximized. In both cases, the profit generated from the 'basket' of SKUs is maximized. Finally, sensitivity analysis of budget versus expected profit and budget versus service level shows the tradeoffs between having a budget which is too low or too high.

## RetailerCo Profit Maximization

The RetailerCo Profit Maximization Model is used to determine the optimal order quantity and expected optimal profit for each SKU under different budget constraints. For example, with the budget $\$ 4,000,000$, the optimal order quantity of each SKU is as below:

| SKU Number | SKU-1 | SKU-3 | SKU-4 | SKU-5 | SKU-6 | SKU-7 | SKU-8 | SKU-9 | SKU-10 | SKU-11 | SKU-12 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | SKU-13 | SKU |
| :--- |
| Optimal order Q* |

Table 12 Optimal Order Quantity with \$4M Budget, DC A

The RetailerCo maximum profit and SupplierCo profit are as below:

| RetailerCo Expected Optimal Profit | $\$$ | 320,797 |
| :--- | :--- | ---: |
| SupplierCo Profit | $\$$ | 628,406 |
| Budget Constraint | $\$, 000,000$ |  |

## Table 13 RetailerCo Maximized Profit with \$4M Budget, DC A

One of the benefits of the profit maximization model is to determine the appropriate budget for the promotion by comparing the tradeoff between budget versus service level and expected optimal profit. This is illustrated in Figure 23, which shows budget versus expected profit and Figure 24, which shows budget versus service level.

A sensitivity analysis of the budget versus service level and expected profit shows that a budget increase from $\$ 3,600,000$ to $\$ 3,700,000$ results in a service level increase ranging from $4 \%$ to $6 \%$ and a RetailerCo profitability increase by $\$ 17,000$ for the SKUs under promotion. This is because at this point the risk of lost sales is higher than the risk of excess inventory, thus
ordering more leads to higher profits. However, as the budget is further increased beyond $\$ 4,100,000$ the profitability starts decreasing. This is because an increase in the budget beyond a certain point results in a higher risk of excess inventory versus lost sales, and thus a lower profit.


Figure 23 RetailerCo Profit Maximization: Budget vs. Optimal Profit


Figure 24 RetailerCo Profit Optimization, Budget vs. Service Level

## Whole Supply Chain, RetailerCo and SupplierCo Profit Maximization

In RetailerCo and SupplierCo Profit Maximization model, the expected profit of the entire channel is maximized with the budget constraint. It is assumed that the cost of the product is SupplierCo's cost and the selling price is RetailerCo's promotions sales price. The profitability achieved by maximizing the entire chain is significantly higher than maximizing only RetailerCo. The profit improvements are in the range of $19 \%$ to $52 \%$, depending on the size of the budget. This can be seen in Figure 25 below.


Figure 25 Whole Supply Chain Profit, Chain Optimization vs. Retailer Optimization

### 5.3.2.3 Revenue Maximization Model

The Revenue Maximization Model is used to determine the optimal order quantity and expected profit for each SKU under different budget constraints. For example, with the budget $\$ 4,000,000$, the optimal order quantity of each SKU is as below:

| SKU Number | SKU-1 | SKU-3 | SkU-4 | SKU-5 | SKU-6 | SKU-7 | SKU-8 | SKU-9 | SKU-10 | SKU-11 | SKU-12 | SKU-13 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Optimal order Q $^{*}$ | 154,860 | 35,521 | 34,100 | 33,642 | 35,458 | 32,586 | 21,728 | 33,928 | 11,825 | 61,132 | 68,740 | 11,649 |

Table 14 Optimal Order Quantity, DC A Revenue Maximization

RetailerCo's maximum expected revenue and SupplierCo revenue are below:

| RetailerCo Expected Revenue | $\$$ | $2,824,605$ |
| :--- | :--- | :--- |
| SupplierCo Revenue | $\$$ | $4,000,000$ |
| Budget Constraint | $\$$ | $\mathbf{4 , 0 0 0}, 000$ |

Table 15 RetailerCo Maximum Expected Revenue and SupplierCo Revenue

One of the benefits of the revenue maximization model is to determine the appropriate budget for the promotion by comparing the tradeoff between the budget versus service level and expected revenue.

A sensitivity analysis of the budget versus service level shows that an increase in the budget from $\$ 3,600,000$ to $3,680,000$ results in a service level increase in the range of $4.3 \%$ to $5.6 \%$ and a RetailerCo Revenue increase of over $\$ 30,000$. However, as the budget increases beyond the maximum specified by the model, which is $\$ 6,000,000$, the service level remains constant at almost $100 \%$ while the RetailerCo revenue continues to remain constant. Hence, increasing the budget beyond this point would not lead to any additional benefits and would be a wasteful use of resources. SupplierCo revenues are almost linear with an increase in budget.

## RetailerCo Optimal Revenue under Different Budget Constriant



Figure 26 RetailerCo Optimal Revenue vs. Budget, DC A


Figure 27 Budget vs. Service Level

## 6 Recommendations, and Future Research

The objective of this section is to discuss recommendations as well as research opportunities for future theses. The recommendations can be categorized in the area of process integration, revenue and profit promotion optimization and end-to-end accountability and metrics.

### 6.1 Promotions Forecasting Process Insights

This subsection discusses process related improvements for the promotions planning and execution process based on interviews with SupplierCo and RetailerCo planners and benchmarking against standard industry processes.

### 6.1.1 As-is and To-be Promotions Planning Process

The SupplierCo and RetailerCo promotions planning and execution as-is process is benchmarked against the CPFR Retailer Event Collaboration standards, which were discussed in the literature review and a To-Be promotions planning and execution process is defined for SupplierCo and RetailerCo. Figure 28 below describes the activities under Retail Event Collaboration, the As-Is Promotions Planning and Execution Process and the To-Be Promotions Planning and Execution process.

| Phase | CPFR Retail Event Collaboration <br> Process | As Is Process |
| :---: | :--- | :--- |
| Strategy and <br> Planning Define scope and process of information <br> sharing Assign exception handling roles and <br> procedures <br> this phase <br> Communicate event details and updates <br> as they change |  |  |


| Phase | CPFR Retail Event Collaboration Process | As Is Process |
| :---: | :---: | :---: |
| Demand and Supply Management | - Develop and share event sales forecast estimates <br> - Develop and share event order plan estimates | - SupplierCo and RetailerCo determine aggregate dollar amount for the promotion at the chain level |
| Execution | - Place promotional order <br> - Deliver promotional quantities to third parties, retailer DCs or Stores <br> - Monitor store inventory and sales performance during the event | - SupplierCo DC planner reviews the DC level forecast and places Order <br> - SupplierCo delivers the Purchase Order to the RetailerCo DC |
| Analysis | - Trigger, communicate and resolve exceptions <br> - Communicate event performance results |  |



| Phase | To Be Process |
| :---: | :---: |
| Strategy and Planning | - Define detailed process for information sharing during promotions <br> - Define clear roles and responsibilities for promotions, which includes a key stakeholder from SupplierCo and RetailerCo accountable for promotional planning and execution <br> - Create product profiles across product categories <br> - Align promotion resources (e.g. budget) between SupplierCo and RetailerCo <br> - Collaboratively define promotions event calendar <br> - Define short term and long term strategic initiatives to improve promotional performance |
| Demand and Supply Management | - Share event sales forecast estimates <br> - Gather promotions data <br> - Collaborate on aggregate and SKU/Store level forecast <br> - Execute the optimization model to define budget and inventory allocations to products <br> - Collaboratively determine aggregate dollar amount for each promotion <br> - Communicate updates of promotions details |
| Execution | - Place promotions order <br> - Deliver promotions quantities to DC <br> - Monitor store inventory and sales performance during the event <br> - Maintain Scorecard for Out-of-Stock at store, DC level |
| Analysis | - Provide emergency shipments to stores which run out of inventory early in the promotion <br> - Collaboratively Perform post event analysis of promotions through weekly meetings <br> - Define recommendations to improve promotions process |

Figure 28 CPFR Retail Event Collaboration Process, Promotions Planning As-ls and To-Be processes

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Figure 29 To-Be Promotions Calendar

### 6.1.2 Promotions Process

Below are the recommendations based on interviews with SupplierCo and RetailerCo DC Planners:

In the current process, SupplierCo delivers the promotions purchase order to the RetailerCo Distribution Center, 4 weeks prior to the promotion. It is recommended that SupplierCo deliver the Purchase Order to RetailerCo closer to the promotion, allowing additional time for RetailerCo to make adjustments to the promotions order based on market changes and Promotions Ad Price or SKU changes. However, an analysis of the tradeoff between inventory savings and a potential increase in transportation costs would need to be conducted prior to implementing this recommendation.

Currently, the safety stock policy for all SKUs at the RetailerCo Distribution Center is based on the same target service level. There are opportunities for RetailerCo to improve their customer service and profitability by aligning the service level and safety stock policies for each SKU with its strategic importance. Another opportunity for RetailerCo is to improve its in-store service level for promotions by defining a safety stock policy for promotional SKUs.

The DC planner does not have access to inventory at the store level and does not take this into account while ordering promotional SKUs. Hence, 2 weeks prior to the promotion, when a store places an order to the DC , it makes a negative adjustment of the existing store inventory, which results in excess inventory at the DC. By providing the DC planner with access to store level inventory and defining a policy for reserving excess inventory at the store, there could be savings from holding less inventory at the DC.

### 6.2 Data Analysis Recommendations

The demand pattern analysis for Brand A and Brand B products led to the following recommendations, which could be applied to other categories:

- For forecasting purposes, aggregate demand for products which follow similar demand pattern
- Demand for new products stabilized within one year, this can be used as an input for forecasting new products
- Promotions demand is extremely sensitive to promotions price and they are negatively correlated. This should be taken into account while selecting SKUs for promotions.


### 6.3 Optimization Model Recommendations

This subsection covers key insights and results from the optimization model developed for budget allocation to SKUs in a given promotion.

One of the key insights from the profit maximization model is that optimizing the entire supply chain, which includes both SupplierCo and RetailerCo, yields the highest total profit as compared to maximizing only RetailerCo's profit. However, an incremental approach is recommended from an implementation standpoint. In the short term, it is recommended that the RetailerCo profit maximization model is implemented to achieve improvements for both SupplierCo and RetailerCo. In the long term, it is recommended to transition to implementing the optimization model which maximizes both SupplierCo and RetailerCo's profit with further research and analysis to define the collaboration relationship needed to foster this optimization. These might include some of the Supply Contracts described in the literature.

Currently, SupplierCo and RetailerCo tend to under budget promotions. Sensitivity analyses of budget versus service level, budget versus profit for the profit maximization, revenue maximization clearly shows that SupplierCo and RetailerCo could improve their profitability, as well as service level by increasing the budget for these promotions. A trade off between budget and service level should be evaluated between SupplierCo and RetailerCo prior to each promotion.

Based on analysis of previous promotions data, RetailerCo tends to under forecast for promotions which leads to Out-of-Stocks and lost sales. To determine the threshold of sales for Brand A and Brand B products, it is recommended to setup one test store with very high supplies of Brand A and Brand B products to determine what the threshold for maximum sales of these products would be. This will help SupplierCo and RetailerCo understand the true lost sales for Brand A and Brand B products.

### 6.4 Out-of-Stock Recommendations

According to the study of National Association of Convenience Stores (Gruen, Corsten and Bharadwaj, 2002), shoppers will be more likely to switch to another store when the product on the customer's planned shopping list is out-of-stock. The study reported that when a consumer faces an OOS in a planned purchase category, the shopper will permanently switch stores after an average of 2.4 such experiences.

According to the study by Gruen, Corsten and Bharadwaj (2002), about $70 \%$ of out-ofstocks are caused in the store, while $30 \%$ of out-of-stocks are due to upstream causes at the distribution center or headquarter level.


Figure 30 Root Causes of Out-of-Stocks

To track the root cause of Out-of-Stock, the retailer needs to use a scorecard both at the store and Distribution Center levels. At the store level, the store manager can monitor how often the Out-of-Stock took place and when the store runs Out-of-Stock. This will help the store manager understand the real demand and improve forecast accuracy. Below is a sample Store Level Out-of-Stock Scorecard.


Table 16 Store Level Out-of-Stocks Scorecard

At the Distribution Center level, the DC planner can estimate forecast accuracy by tracking how many stores are Out-of-Stock during the promotional week. For example, if there are 1,000 stores under one distribution center and only 10 stores are Out-of-Stock for one SKU, the root cause most likely rests at the store level. However, if there are more than 100 stores Out-of-Stock, the root cause is likely a process problem caused by a policy, forecast error or delivery schedule (Gruen, Corsten and Bharadwaj, 2002).

Below is a sample Out-of-Stock Scorecard which can be used at the Distribution Center.

| DC level Out of Stock Scorecard |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DC Name: | MA |  |  |  |  |  |  |
| \# of stores | 1000 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
| \# of store run out of inventory | 10 | 15 | 30 | 40 | 50 | 70 | 100 |
| $\%$, stores run out of inventory | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $5 \%$ | $7 \%$ | $10 \%$ |

Table 17 DC Level Out-of-Stock Scorecard

### 6.5 Future Research

The thesis provided insights into numerous opportunities for further research which could help improve promotional effectiveness through Supplier-Retailer Collaboration.

One area of opportunity is to determine the criteria for grouping SKUs together in to one basket and determine the true profitability of the basket per traffic builder SKU sold. This will allow SupplierCo and RetailerCo to evaluate which traffic builder SKUs to promote and how much to invest on promotions using the Profit Maximization Optimization Model.

Information systems and availability of accurate data enable effective decision making. SupplierCo and RetailerCo should define data requirements for successful promotions
collaboration, which include SKU level promotions forecast data as well as promotions forecast accuracy at the DC and Store level.

Another area of opportunity is to develop a model which maximizes SupplierCo's profit during promotions. This will help SupplierCo internally assess the right SKU mix to achieve maximum profitability.

One of the key insights from the Profit Maximization model was that maximizing the entire supply chain leads to the highest combined profit for both SupplierCo and RetailerCo Further research would need to be conducted on developing an optimized collaborative relationship between SupplierCo and RetailerCo to achieve maximum profits.

Finally, the scope of the thesis was to conduct promotions analysis and optimization at the DC level. To improve service level at the stores, further research should be conducted in store segmentation and improving promotional effectiveness at the store level.

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## Appendix A

## Derivations

## Introduction

To solve the multi-product newsboy problem with budget constraint, a Lagrange multiplier approach is applied. Under the budget constraint, retailer needs to optimally allocate the dollars to each SKU in order to maximize profit or revenue. The derivations is based on Book, Inventory Management and Production Planning and Scheduling (Silver, Pike and Peterson ,1998)

## 1 Maximize Retailer's Revenue (Per Silver et al, 1998)

For SKU i, the following notations are used
$Q_{i}$ : order quantity
$Q_{i}{ }^{*}$ : optimal order quantity under
$R_{i}$ : the retailer sale price
$C_{i}$ : the supplier's wholesale price
$S_{i}$ : Salvage value
$L i$ : the penalty of loss sales
$x_{i}$ : the demand of product
$f_{i}(x)$ : probability density function of demand, SKU i
$F i(x)$ : cumulative density function of demand, SKU i
$F_{i}(x)$ is differentiable, strictly increasing and $0 \leq \mathrm{F}(0) \leq 1$

## Objective function:

$\operatorname{Max} \sum_{i}^{N} \operatorname{profit}\left(Q_{i}\right)=$
$\sum_{i}^{N}\left[R_{i} \int_{0}^{Q_{i}} x_{i} f_{i}(x) d x_{i}+R_{i} Q_{i} \int_{Q_{i}}^{\infty} f_{i}(x) d x_{i}+S_{i} \int_{0}^{Q_{i}}\left(Q_{i}-x_{i}\right) f_{i}(x) d x_{i}-C_{i} Q_{i}-L_{i} \int_{Q i}^{\infty}\left(x_{i}-Q_{i}\right) f_{i}(x) d x_{i}\right]$
Subject to
$\sum_{i}^{N} C_{i} Q_{i} \leq$ Budget
Apply the Lagrange multiplier M

$$
\begin{aligned}
W & =\sum_{i}^{N} \operatorname{profit}\left(Q_{i}\right)-M\left(\sum_{i}^{N} C_{i} Q_{i}-\text { Budget }\right) \\
W & =\sum_{i}^{N}\left[R_{i} \int_{0}^{Q_{i}} x_{i} f_{i}(x) d x_{i}+R i Q_{i} \int_{Q_{i}}^{\infty} f_{i}(x) d x_{i}+S_{i} \int_{0}^{Q_{i}}\left(Q_{i}-x_{i}\right) f_{i}(x) d x_{i}-C_{i} Q_{i}-L_{i} \int_{Q_{i}}^{\infty}\left(x_{i}-Q_{i}\right) f_{i}(x) d x_{i}\right] \\
& -M\left(\sum_{i}^{N} C_{i} Q_{i}-B u d g e t\right)
\end{aligned}
$$

According to Leibniz's Rule,
$\frac{d}{d y} \int_{g(y)}^{h(y)} f(x, y) d x=\int_{g(y)}^{h(y)} \frac{\partial f(x, y)}{\partial y} d x+f[h(y), y] \frac{d h(y)}{d y}-f[g(y), y] \frac{d g(y)}{d y}$
So,
$\frac{d}{d Q} \int_{0}^{Q} x f(x) d x==Q f(Q)$
$\frac{d}{d Q} \int_{0}^{Q} Q f(x) d x=\int_{0}^{Q} f(x) d x+Q f(Q)=F(Q)+Q f(Q)$
To find the optimal $M$, apply the partial differentiation and find the solution of $\frac{\partial W}{\partial Q_{i}}=0$
$\frac{\partial W}{\partial Q_{i}}=\left(R_{i}-S_{i}+L_{i}\right) Q_{i} f\left(Q_{i}\right)+\left(S_{i}-R_{i}-L_{i}\right)\left[F_{i}\left(Q_{i}\right)+Q_{i} f_{i}\left(Q_{i}\right)\right]+\left(R_{i}-C_{i}+L_{i}\right)-M C_{i}$
$\frac{\partial W}{\partial Q_{i}}=0$
$\left(S_{i}-R_{i}-L_{i}\right) F_{i}\left(Q_{i}\right)+\left(R_{i}-C_{i}+L_{i}\right)-M C_{i}=0$
$F_{i}\left(\right.$ Demand $\left.\leq Q_{i}^{*}\right)=\frac{\left(R_{i}-C_{i}+L_{i}\right)-M C_{i}}{\left(R_{i}-S_{i}+L_{i}\right)}=\frac{R_{i}+L_{i}-(M+1) C_{i}}{R_{i}-S_{i}+L_{i}}$
$Q i^{*}=F_{i}^{-1}\left[\frac{R_{i}+L_{i}-(M+1) C_{i}}{R_{i}-S_{i}+L_{i}}\right]$
So, the optimal order quantity of $\mathrm{Q} i$ is the inverse function of cumulative probability function $F_{i}(x)$ with the parameter equal to $\frac{R_{i}+L_{i}-(M+1) C_{i}}{R_{i}-S_{i}+L_{i}}$.

## 2 Maximize Retailer's Revenue

The objective of the revenue maximization model is to maximize total revenue of SKUs under promotion with a budget constraint. The total revenue depends on retailer's promotion price, the supplier's wholesale price and the demand of each SKU. The budget constraint is a product of the quantity ordered and the retailer's cost for the product.

For SKU i, the following notations are used
$Q_{i}$ : order quantity
$Q_{i}{ }^{*}$ : optimal order quantity under
$R_{i}$ : the retailer sale price
$C_{i}$ : the supplier's wholesale price
$x_{i}$ : the demand of product
$f_{i}(x)$ : probability density function of demand, SKU i
$F_{i}(x)$ : cumulative density function of demand, SKU i
$F_{i}(x)$ is differentiable, strictly increasing and $0 \leq \mathrm{F}(0) \leq 1$

Objective function
MAX $\sum_{i}^{N} \operatorname{Re}$ venue $\left(Q_{i}\right)=\sum_{i}^{N}\left[R_{i} \int_{0}^{Q_{i}} x_{i} f_{i}(x) d x_{i}+R_{i} Q_{i} \int_{Q_{i}}^{\infty} f_{i}(x) d x_{i}\right]$

Subject to
$\sum_{i}^{N} C_{i} Q_{i} \leq B u d g e t$

Apply the Lagrange multiplier $\mathbf{M}$
$W=\sum_{i}^{N} \operatorname{revenue}\left(Q_{i}\right)-M\left(\sum_{i}^{N} C_{i} Q_{i}-B u d g e t\right)$
$W=\sum_{i}^{N}\left[R_{i} \int_{0}^{Q_{i}} x_{i} f_{i}(x) d x_{i}+R_{i} Q_{i} \int_{Q_{i}}^{\infty} f_{i}(x) d x_{i}\right]-M\left(\sum_{i}^{N} C_{i} Q_{i}-\right.$ Budget $)$

According to Leibniz's Rule
$\frac{d}{d y} \int_{g(y)}^{h(y)} f(x, y) d x=\int_{g(y)}^{h(y)} \frac{\partial f(x, y)}{\partial y} d x+f[h(y), y] \frac{d h(y)}{d y}-f[g(y), y] \frac{d g(y)}{d y}$
so,
$\frac{d}{d Q} \int_{0}^{Q} x f(x) d x==Q f(Q)$
$\frac{d}{d Q} \int_{0}^{Q} Q f(x) d x=\int_{0}^{Q} f(x) d x+Q f(Q)=F(Q)+Q f(Q)$
To find the optimal $M$, apply the partial differentiation and find the solution of $\frac{\partial W}{\partial Q_{i}}=0$

$$
\begin{aligned}
& \begin{array}{l}
\frac{\partial W}{\partial Q_{i}}=R_{i} Q_{i} f_{i}\left(Q_{i}\right)+R_{i}-R_{i}\left[F_{i}\left(Q_{i}\right)+Q_{i f} f_{i}\left(Q_{i}\right)\right]-M C_{i} \\
\quad=R_{i}-R_{i} F_{i}\left(Q_{i}\right)-M C_{i}
\end{array} \\
& R_{i}-R_{i} F_{i}\left(Q_{i}\right)-M C_{i}=0 \\
& F_{i}\left(\text { Demand } \leq Q_{i}^{*}\right)=\frac{R_{i}-M C_{i}}{R_{i}} \\
& Q_{i}^{*}=F_{i}^{-1}\left(\frac{R_{i}-M C_{i}}{R_{i}}\right)
\end{aligned}
$$

So, the optimal order quantity of $\mathrm{Q} i$ is the inverse function of cumulative probability function $F_{i}(x)$ with the parameter equal to $\frac{R_{i}-M C_{i}}{R_{i}}$

The formula of retailer's expected revenue (Silver, Pyke and Peterson, Inventory Management and Production Planning and Scheduling, 1998, Chapter 10):
(Retailer's Promotion Price) * $\left\{\mathrm{Q}-\sigma *\left[\mathrm{z}^{*} \Phi(\mathrm{z})+\Phi(\mathrm{z})\right]\right\}$
Where
$\mathrm{z}=\frac{x-u}{\sigma}$, which is the z -value of the standard normal distribution.
$\Phi(\mathrm{z})$ is the PDF of the standard normal distribution.
$\Phi(\mathrm{z})$ is the CDF of the standard normal distribution.

## Appendix B

## Manual, How to Solve Multi-Product Newsboy Problem with Budget Constraint by Spreadsheet <br> Introduction

This manual will show the methodology and the steps of how to set up the Excel model and use Excel build-in functions to solve the Multi Product Newsboy Problem with a Budget Constraint (MPNBC). The methodology and solution of the basic single product newsboy inventory problem is first introduced. This is then developed to introduce the MPNPBC model.

## 1 Single Product Newsboy Inventory Problem ${ }^{1}$

### 1.1 The definition of single product newsboy inventory problem

Let us consider the situation faced by the owner of a newsstand on the corner of street. Each day, the newsboy must decide how many papers to be ordered at the wholesale price from the supplier. Then, during the day, the newsboy sells the papers at a retail price which is higher than the wholesale price.

At the end of each day, the newsboy either has unsold papers left at hand or can not meet the demand by running out of the papers earlier. How many papers should the newsboy order to balance the overage cost, associated with each copy that is not sold, and underage cost, associated with each demand that cannot be met?

The main features of this type of problem are:

- Single period problem - ordering decision made every period
- Demand is stochastic instead of deterministic
- Order is placed before demand materializes
- Only one order each period
- Overage cost for holding too many and underage cost for holding too few
- Examples

1. Christmas tree vendor
2. Fashion products
3. Perishable products

### 1.2 Single Product Newsboy Problem Model Basics

To solve the newsboy problem for a retailer and its supplier, the assumption is that the demand follows normal distribution. The following variables are used:

## Inputs

$Q$ : order quantity
Q*. optimal order quantity which can maximize newsboy profit
$R$ : the retailer sale price
$C$ : the supplier's wholesale price
$S$ : Salvage value
$L$ : the penalty of lost sales
$x$ : the demand of product
$f(x)$ : probability density function of demand
$F(x)$ : cumulative density function of demand
$F(x)$ is differentiable, strictly increasing and $0 \leq \mathrm{F}(0) \leq 1$

## Intermediate variables

Overage cost: cost per unit of excess inventory at the end of the period.
Overage $\cos t=$ wholesale price - salvage value

Underage cost: cost per unit of insufficient inventory at end of the period.
Underage $\cos t=$ retail price - wholesale price + penalty of lost sales

[^2]Underage $\cos t$ plus overage $\cos t$ $=$ retail price - salvage value + penalty of lost sales

## Decision variables

Q : order quantity by retailer

The formulation of optimal order quantity for the retailer ${ }^{1}$
Let $Q^{*}$ denote the optimal order quantity. The optimal solution to the newsboy problem has the solution:
$\mathrm{F}\left(\mathrm{Q}^{*}\right)=\mathrm{P}\left(\right.$ demand $\left.\leq Q^{*}\right)=\frac{\text { Underage Cost }}{\text { Underage Cost }+ \text { Overage Cost }}$
The ratio of $\frac{\text { Underage Cost }}{\text { Underage Cost + Overage Cost }}$ is called as the critical ratio.
The critical ratio should be between $(0,1)$ to find the optimal order quantity.

The optimal $Q^{*}$ is the inverse function of $F(x)$
$\mathrm{Q}^{*}=\mathrm{F}^{-1}\left(\frac{\text { Underage Cost }}{\text { Underage Cost }+ \text { Overage Cost }}\right)$
If demand is normally distributed, the optimal order quantity can be found by Excel build-in function NORMINV:

$$
Q^{*}=N O R M I N V\left(\frac{\text { Underage Cost }}{\text { Underage Cost }+ \text { Overage Cost }}, \mu, \sigma\right)
$$

Or

$$
Q^{*}=N O R M I N V\left(\frac{\text { retail price }- \text { wholesale price }+ \text { penalty of lost sales }}{\text { retail price }- \text { salvage value }+ \text { penalty of lost sales }}, \mu, \sigma\right)
$$

[^3]
### 1.3 The example

Suppose the newsboy faces the daily demand and the price as below, what is the optimal order quantity for newsboy to maximize his expected profit?

| Demand, $\boldsymbol{\mu}$ |  |
| :--- | ---: |
| Demand, $\boldsymbol{\sigma}$ | 100 |
| Retail price | $\$$ |
| Wholesale price | 2.00 |
| Salvage value | $\$$ |
| Penalty of lost sales | $\$ .50$ |
|  | 0.50 |

According to the formulation mentioned above, the critical ratio is $70 \%$ and the newsboy optimal order quantity is 110 per day.

| Underage cost | $\$$ | 2.30 |
| :--- | :--- | :--- |
| Overage cost | $\$$ | 1.00 |
| underage + overage | $\$$ | 3.30 |
| Critical ratio |  | $70 \%$ |
| $Q^{*}$ | $\mathbf{1 1 0}$ |  |
| $\mathbf{Q}^{*}=\operatorname{NORMINV}(70 \%, 100,20)$ |  |  |

The chart below clearly shows the relationship between demand distribution function and the optimal order quantity.


Figure 31 Optimal Solution of Single Product Newsboy Problem ${ }^{1}$

[^4]
## 2 Multi-Product Newsboy Problem with Budget Constraint (MPNPBC)

### 2.1 The definition of MPNPBC

The difference between single product newsboy problem and MPNPBC is that the newsboy needs to order more than one type of newspaper and has a budget constraint. In reality, many businesses not only need to order multiple products in single period but also face several resource constraints. For example, the manager of a sea food restaurant needs to decide the order quantities of different sea food, such as lobster, shrimp and crab, with the constraint of budget and storage space. The garment manufacturer may need to decide what quantity of each style good should be produced prior to the short selling season under the constraint of product capacity. Prior to Christmas, the retailer needs to decide what quantity of each Christmas gift should be order under a budget constraint.

### 2.2 Single Product Newsboy Problem Model Basics

There are two assumptions to solve MPNPBC problem:

- Each product's demand follows a normal distribution
- Each product's demand is independent from any other product

The model of MPNPBC follows the same logic as the single product newsboy problem, but a Lagrangian multiplier M is added to optimally allocate the dollars among different SKUs to optimize the order quantity of each SKU.

### 2.3 Spreadsheet Model

### 2.3.1 The Layout of the Spreadsheet Model

The layout of spreadsheet model is shows in Figure 32, and the formulation of each cell in the column D and H is show in Figure 35. The formulation of each cell in column E \& F can be duplicated from column D .

All the cells highlighted in yellow are input data into the model. All the other cells are calculated by Excel. The model includes four parts:

1 The demand of each SKU, mean, and standard deviation
2 The price of each SKU and critical ratio of each SKU,
3 The budget range under which the model can work properly
4 The optimal solution, which includes optimal order quantity, service level and the retailer's optimal expected profit.

It is important for the users to know two conditions in order to run the model properly. Firstly, the critical ratio of each SKU must be between $(0,1)$. Since the solution of order quantity is the inverse cumulative probability function of the normal distribution with the parameter equal to critical ratio, INVNORM(critical ratio, $\mu, \sigma$ ), the critical ratio must be between 0 and 1 . The equation of critical ratio is shown below.
critical ratio $=\frac{\text { retail price }- \text { wholesale price }+ \text { penalty of lost sales }}{\text { retail price }- \text { salvage value }+ \text { penalty of lost sales }}$
The user should carefully check the input price to make sure the critical ratio is between $(0,1)$. Next, the budget should be between the lower boundary and higher boundary. When the budget is too low, the optimal order quantity for the SKU with long tail demand may be negative; when the budget is too high, each SKU can achieve $100 \%$ service level without consuming all the budget. The MAX budget and MIN budget have been calculated by the model.

Under the given budget, the model will try to find the optimal M , allocate the dollars to each SKU, and maximize the retailer profit. After the model finds each SKU's optimal order quantity, the retailer expected maximum profit ${ }^{1}$ is calculated at the bottom the model.

$$
\begin{aligned}
E[P(Q)]= & (\text { retailer price }- \text { salvage value }) \times \text { mean }-(\text { Wholesale }- \text { salvage }) \times Q^{*} \\
& -(\text { retailer price }- \text { salvage value }+ \text { penalty of lost sales }) \times \sigma \times G(Z)
\end{aligned}
$$

Where $G(z)$ is unit normal loss function, where

$$
\begin{aligned}
G(z) & =f(z)-z *[1-F(z)] \\
& =\operatorname{NORMDIST}(z, 0,1,0)-z^{*}[1-\operatorname{NORMDiST}(z, 0,1,1)]
\end{aligned}
$$

[^5]$\begin{array}{llllll}C & \text { D } & \text { E } & \text { F } & \text { G }\end{array}$

Multiple SKU Promotion Inventory Optimization Model - Maximizing Retailer Profit


Figure 32 Spreadsheet Model of Multi-Product Newsboy Problem with Budget Constraint

### 2.3.2 How to Run the Model

Through $\mathbf{M}$ in the model, the spreadsheet model builds the relationship between the budget and each SKU's optimal order quantity. Here, Excel's build-in function Goal Seek is used to find the $M$ which can optimally allocate the budget to each SKU.

Goal Seek can be used when you know the result of a formula, but need to find the value of one variable which would lead to that result. Goal Seek is found under the Tools menu, and is perfect for reverse calculations, such as mortgage or loan queries.

How the budget can be changed to $\$ 20,000$ by Goal Seek function is described below and shows in figure 33.

Choose Goal Seek function, Tools menu $\rightarrow$ Goal Seek
Set cell: the cell that contains the formula that you want to settle is called the Set cell; here, it is cell D10, (Cost of order).

To value: the value you want the formula to change to is called To value; here it is the budget $\$ 20,000$, and the value must be manually input.

By Changing Cell: the part of the formula that you wish to change is called By Changing Cell, here, it is cell F5, (M).

The Set cell MUST always contain a formula or a function, whereas the Changing Cell must contain a value only. Not a formula or function.

Goal Seek must always be activated when you click on the Set cell as the Set cell will always be the formula that you wish to settle.


Figure 33 How to Use Goal Seek Function to Solve the Problem

Once you set up the Goal Seek, you need to click OK. As soon as you select OK you will see that Goal Seek re-calculates your formula. Figure 34 shows the optimal solution under new budget $\$ 20,000$. Then, you then have two options, OK or Cancel. If we select OK the new term will be inserted into our Worksheet. If you select Cancel, the Goal Seek box will disappear, and your Worksheet will be in its original state.


Figure 34 Optimal Solution Under New Budget \$20,000

Figure 35 also shows the formula of each cell in the model. Due to the limit of space, only SKU1 and SKU-3 formulas are showed in figure 35. Since the formula of SKU-2 and SKU-4 have the same formula as SKU-1, reader can duplicate the SKU-1 formula and get SKU-2 and SKU-4 formula.


Figure 35 The Formulation of Model

Multiple SKU Promotion Inventory Optimization Model - Maximize RetailerCo Profit

|  |  |  | M |  | 0.12597 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| RetailerCo Holding Cost (\%) |  | 20\% |  |  |  |  |  |  |  |  |  |
| RetailerC0 Expected Profit | \$ | 205,522 |  |  |  |  |  |  |  |  |  |
| SupplierCo Expected Profit | \$ | 279,028 |  |  |  |  |  |  |  |  |  |
| Budget Constraint | \$ | 1,707,408 |  |  | Idijet | \$ | 1,940,405 |  |  |  |  |
| Cost of order | \$ | 1,707,408 |  |  | dijet | 5 | 1,220,070 |  |  |  |  |
| Min M |  | 0.4858 | 0.8413 |  | 0.8230 |  | 0.8413 | 0.4858 | 0.4858 | 0.4858 | 0.4858 |
| Min Budget Quantity | \$ | 166,030 | \$223,189 | \$ | 132,285 | \$ | 185,345 | \$216,076 | \$216,356 | \$ 80,789 |  |
| Max M |  | 0.22688 | 0.11611 |  | 0.12183 |  | 0.11611 | 0.22688 | 0.22688 | 0.22688 | 0.1161 |
| Max Budget Quantity | \$ | 313,862 | \$258,166 | \$ | 200,671 | \$ | 215,025 | \$411,532 | \$393,939 | \$147,210 |  |
| SKU Number |  | SKU. 3 | SKU4 |  | SKU.5 |  | SKU6 | SKU-11 | SKU-12 | SKU-13 | Total |
| Optimal order $0^{*}$ |  | 39,527 | 37,941 |  | 25,290 |  | 31,538 | 51,771 | 49,887 | 18,640 | 254,594 |
| Forecasted Demand, $\boldsymbol{\mu}$ |  | 39,464 | 36,836 |  | 22,804 |  | 30,600 | 51,688 | 49,812 | 18,612 |  |
| Demand, $\sigma$ |  | 5,128 | 1,664 |  | 3,916 |  | 1,412 | 6,780 | 6,160 | 2,304 |  |
| Underage Cost |  | 2.53 | 4.41 |  | 4.32 |  | 4.41 | 2.53 | 2.53 | 2.53 |  |
| Underage Cost + Overage Cost |  | 5.02 | 5.90 |  | 5.86 |  | 5.90 | 5.02 | 5.02 | 5.02 |  |
| Critical Ratio |  | 0.50 | 0.75 |  | 0.74 |  | 0.75 | 0.50 | 0.50 | 0.50 |  |
| RetailerCo Promotion Revenue |  | $\$ 5.49$ | $\$ 5.49$ |  | $\$ 5.49$ |  | $\$ 5.49$ | $\$ 5.49$ | $\$ 5.49$ | $\$ 5.49$ |  |
| RetailerC0 Traffic Builder Benefit |  | \$2.00 | \$2.00 |  | \$2.00 |  | \$2.00 | \$2.00 | \$2.00 | \$2.00 |  |
| SupplierCo Rebate |  | \$0.25 | \$0.25 |  | \$0.25 |  | \$0.25 | \$0.25 | \$0.25 | \$0.25 |  |
| Prom Price + Rebate + Benefit |  | \$7.74 | \$7.74 |  | \$7.74 |  | \$7.74 | 97.74 | $\$ 7.74$ | \$7.74 |  |
| SupplierCo Wholesale Price |  | $\$ 7.03$ | \$6.15 |  | \$6.19 |  | $\$ 6.15$ | \$7.03 | \$7.03 | \$7.03 |  |
| RetailerCo Salvage Value |  | $\$ 5.43$ | $\$ 5.43$ |  | $\$ 5.43$ |  | $\$ 5.43$ | $\$ 5.43$ | $\$ 5.43$ | $\$ 5.43$ |  |
| RetailerCo Customer Goodwill |  | \$2.00 | \$2.00 |  | \$2.00 |  | \$2.00 | \$2.00 | \$2.00 | \$2.00 |  |
| RetailerCo Lost Sales Penalty |  | \$2.71 | \$3.59 |  | \$3.55 |  | $\$ 3.59$ | $\$ 2.71$ | \$2.71 | \$2.71 |  |
| Retailerco Service Level |  | 50.5\% | 74.7\% |  | 73.7\% |  | 74.7\% | 50.5\% | 50.5\% | 50.5\% |  |
| Expected Optimal Profit Calculation |  |  |  |  |  |  |  |  |  |  |  |
| Z |  | 0.0122 | 0.6640 |  | 0.6349 |  | 0.6640 | 0.0122 | 0.0122 | 0.0122 |  |
| $G(Z)$, unit normal loss function |  | 0.3929 | 0.1518 |  | 0.1593 |  | 0.1518 | 0.3929 | 0.3929 | 0.3929 |  |

The formula of $G(Z): G(Z)=f(Z)-Z^{*}[1-F(Z)]=\operatorname{NORMDIST}(Z, 0,1,0)-Z^{*}[1-\operatorname{NORMDIST}(Z, 0,1,1)]$

| Retailerco expected profit | $\$ 17,806$ | $\$ 56,283$ | $\$ 29,801$ | $\$ 46,714$ | $\$ 23,195$ | $\$ 23,097$ | $\$ 8,626$ | $\$ 205,522$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

The frmula of $\mathrm{E}[\mathrm{P}(\mathrm{Q})]=(\text { price }- \text { salvage })^{*}$ mean $-\left(\right.$ cost - salvage) ${ }^{*} \mathrm{Q}^{*}-$ (price - salvage + lost sales) ${ }^{*} \sigma^{*} \mathrm{G}(Z)$

| SupplierCo Revenue | $\$$ | 277,872 | $\$ 233,337$ | $\$$ | 156,546 | $\$$ | 193,956 | $\$ 363,949$ | $\$ 350,707$ | $\$ 131,040$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |

Figure 36 Layout of RetailerCo Profit Optimization Spreadsheet Model


[^0]:    ${ }^{1}$ Rain-checks are the sales at promotional price in non-promotional week.

[^1]:    ${ }^{1}$ The layout of RetailerCo profit maximization spreadsheet model is in figure 36, appendix B

[^2]:    ${ }^{1}$ Edward A. Silver, David F. Pyke and Rein Peterson, Inventory Management and Production Planning and Scheduling, Chapter 10

[^3]:    ${ }^{1}$ Edward A. Silver, David F. Pyke and Rein Peterson, Inventory Management and Production Planning and Scheduling, Chapter 10

[^4]:    ${ }^{1}$ CDF: Cumulative distribution function

[^5]:    ${ }^{1}$ Edward A. Silver, David F. Pyke and Rein Peterson, Inventory Management and Production Planning and Scheduling, Chapter 10

