

Essays in Empirical Finance

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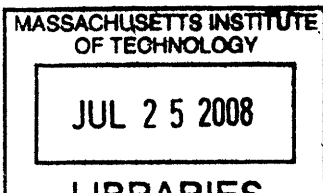
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Submitted to the Alfred P. Sloan School of Management on
May 23, 2008 in Partial Fulfillment of the Requirements for the Degree of Doctor of
Philosophy in Management

Abstract

The first chapter in my thesis investigates the association between selected hedge fund characteristics and persistence in performance over time. Analyzing TASS data from 1996-2006, I observe a positive correlation between persistence in good performance and fund size, as well as age. Furthermore, I find that more illiquid investment strategies exhibit significantly stronger persistence in good performance, both in the short and long run, even after controlling for illiquidity risk. These results indicate that higher fund size, age, and exposure to illiquidity are reflective of superior managerial skill. Finally, I note that funds with higher incentive fees display greater persistence in both good and bad (post-fee) performance in the long run. These findings are consistent with a scenario in which incentive fees are raised by both skilled and unskilled, (but lucky), fund managers in response to good past performance. Therefore, my analysis suggests that incentive fees for hedge funds may be endogenously determined.

The second chapter tests a simple explanation for momentum profits: systematic outperformance arises because certain stocks have persistently strong fundamentals which are not fully valued by the market. We find that “winner” portfolios have higher book-to-market ratios than “loser” portfolios, and the economic and statistical significance of momentum profits is markedly reduced when calculated above value benchmarks. A large component of the returns to relative strength portfolios may thus stem from such portfolios overweighting high value stocks, suggesting a close relation between the value and momentum anomalies.

The final chapter develops a measure of international financial contagion using a semi-structural approach. In particular, we work with a multi-country dynamic equilibrium setting, placing a constraint on portfolio volatility. The tightening of this constraint is a channel through which shocks are propagated globally in our model. We then derive a measure of the tightness of the constraint, or ‘contagion’, using cross-equation restrictions. We finally evaluate our measure of international contagion with regards to its predictability on global asset price co-movement, as well as on news about the recent sub-prime crisis. We find evidence that our contagion estimator is a strong measure of the sub-prime crisis in this regard.

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Hedge Fund Characteristics and Performance Persistence

Pavitra Kumar¹

May 23rd 2008

Abstract

This paper investigates the association between selected hedge fund characteristics and persistence in performance over time, both in the short and long run. Analyzing data from the TASS database over the period 1996-2006, I obtain evidence of a positive correlation between persistence in good performance and fund size, as well as age. Furthermore, I find that relatively more illiquid investment strategies exhibit significantly stronger persistence in good performance, both in the short and long run, even after controlling for illiquidity risk and short-term positive serial correlation. These results provide support for the argument that higher fund size, age, and exposure to illiquidity are reflective of superior managerial skill. Finally, I observe that funds with higher incentive fees display greater persistence in both good and bad (post-fee) performance in the long run. These findings are revealed to be consistent with a scenario in which incentive fees are raised by both skilled and unskilled, (but lucky), fund managers in response to good past performance. Therefore, my analysis sheds light on a potential mechanism through which incentive fees for hedge funds are endogenously determined. My investigation also highlights the differences between trends in performance persistence displayed by the mutual fund and hedge fund industries.

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1. Introduction

Over the past decade, there have been numerous studies of hedge fund performance that attempt to investigate persistence in excess returns. The issue of performance persistence is particularly significant in the hedge fund industry, given that investors are increasingly willing to pay extremely high fees to bet on persistent ‘alpha’, or outperformance of the market benchmark. Investors in hedge funds also often face substantial lockup periods, and as a result, it is crucial for them to be able to assess the future stability of funds’ performance in advance. This task, however, is very difficult, given the lack of transparency in the hedge fund industry. Indeed, investors can usually only observe past fund performance, and past good performance by itself is rarely a reliable indicator of future persistence in excess returns. Therefore, the main contribution of this paper is to analyze which hedge fund characteristics, as well as investment strategies, contribute to the strongest persistence in performance over time, both in a short run (two-period) and long run (multi-period) setting. As well as looking at the effect of different investment styles, the hedge fund features I choose to examine are size, (measured by a fund’s initial net asset value at the start of the sample period), age, (measured by the number of monthly performance observations reported by a hedge fund since the earliest performance observation within its fund family²), management fee and incentive fee structure.

My analysis has several useful implications for developing lucrative investment strategies in the hedge fund industry. In particular, a knowledge of which fund-specific factors are associated with long run persistence in good performance would be especially beneficial for investors in open-ended hedge funds, (or funds that remain open to new investors for a significant period of time after being set up). Another key motivation behind my study is to highlight the differences in performance persistence trends displayed in the mutual fund and hedge fund industries. For example, there is a far greater degree of persistence documented amongst hedge funds relative to mutual funds, (see Section 2 for a literature review). My major contribution is then to explain this

² As mentioned in Aragon (2006), many funds in the TASS database belong to the same management company, so it is important that a spinoff fund of a well-established management company is not classified as young.

finding by highlighting differences in characteristics between the mutual fund and hedge fund industries, (such as hedge funds employing more illiquid investment strategies and charging much higher incentive fees³), and showing how these particular characteristics affect persistence in both good and bad performance amongst hedge funds.

The main line of reasoning behind all my hypotheses is that superior (lower) hedge fund manager skill should be associated with better (worse) performance in any given period. Thus, since skill is itself an intrinsic quality that is likely to be persistent over time, I propose that superior (lower) managerial ability should be associated with a higher degree of persistence in good (bad) performance as well. I then hypothesize that my selected hedge fund characteristics reflect skill in various ways. Firstly, taking the characteristics size and age, it seems reasonable to expect that, on the one hand, managers of larger and older funds are more highly skilled since they are clearly capable of attracting higher capital inflows and keeping their funds alive for a longer period of time. This argument would generate the prediction of a positive monotonic relationship between size, as well as age, and persistence in good performance. On the other hand, decreasing returns to scale is a widely reported trend in the mutual fund and, to a lesser extent, the hedge fund industry as well. In the hedge fund industry in particular, the threat of scale diseconomies often forces funds to cap their size at a certain optimal level and set up smaller spinoff funds instead. This would at least partially eliminate the effects of decreasing returns to scale on their performance. Given the significantly higher skill required to ensure survival in the hedge fund industry, however, and also given far weaker evidence of scale diseconomies compared to the mutual fund industry, I propose that higher hedge fund size is primarily reflective of superior skill, rather than decreasing returns to scale. This would imply that even after controlling for size-capping, or allowing for scale diseconomies, one should still observe a positive (negative) relationship between size, (as well as age), and persistence in good (bad) performance. These trends are indeed confirmed by my results in both the two-period and multi-period

³ Hedge funds usually charge much higher performance-based fees, (generally a combination of ‘management fees’ on the assets, on average 1-2%, and ‘incentive fees’ charged as a percentage of net profits, on average around 20%), than mutual funds, which mostly do not charge performance-based incentive fees at all. Hedge funds also use investment techniques that are forbidden for mutual funds, including ‘short selling’ stock, using high levels of leverage, and investing in highly illiquid or esoteric securities which do not have to be marked-to-market daily, (a strict requirement for mutual fund portfolios).

settings. This provides a contrast to strong findings of decreasing returns to scale in the mutual fund industry, (see Section 3 for further details).

Regarding the effect of performance fees on persistence, I develop a scenario, realistic to the hedge fund industry, in which investors cannot distinguish between funds which have performed well in the past, and are either genuinely skilled in generating persistently high alpha, or have unskilled managers and have simply produced good performance due to luck. If one then assumes that investors are rational, and thus both classes of funds attract an inflow of new funds in response to similarly good past performance, they should also both have an incentive to raise performance fees to extract the highest possible investor surplus. Therefore, on a post-fee basis, one would expect to see funds with higher incentive fees generating stronger persistence in both good and bad performance over several periods, reflecting the skilled and unskilled hedge fund managers respectively. Strikingly, I do observe this relationship when analyzing multi-period persistence. This suggests that the level of incentive fees in the hedge fund industry could be endogenously determined in response to past performance. This prediction would not hold in the mutual fund industry, however, given that high incentive fees are exclusive to hedge funds.

Finally, I examine the effect of different investment strategies on performance persistence. Here, I refer to the evidence in Getmansky, Lo and Makarov (2004) that greater exposure to illiquidity produces more significant short-term positive serial correlation in monthly hedge fund returns. One would expect this trend to translate into greater overall persistence for more illiquid strategies in the short run setting. However, one might also expect that higher fundamental skill is required when implementing trading strategies in more illiquid and complex securities. As a result, I propose that, even after netting out the effect of higher illiquidity risk premia⁴ for some investment strategies, more illiquid funds should still generate significantly higher persistence in good performance, and this relationship should be discernable in the long run or multi-period setting as well. This is in fact what I find in the data. This prediction would again

⁴ See Pastor and Stambaugh (2003) and Aragon (2006) for evidence of pricing of illiquidity risk in the hedge fund industry.

not hold in the mutual fund industry, given that high illiquidity is far more prevalent in the hedge fund industry.

In summary then, my findings in this study are consistent with the concepts I develop to explain certain features of the hedge fund industry. Moreover, I observe that several significant trends in performance persistence hold in the multi-period setting as well as in the two-period framework. This is a very powerful result, given that persistence found in a multi-period setting is far less likely to be generated by chance.

I perform my analysis in this paper using monthly (post-fee) returns of hedge funds in the TASS Database from January 1996 to March 2006. I start by replicating the standard methodology of Agarwal and Naik (2000b). Based on the Agarwal-Naik model where the fundamental risk faced by hedge funds is determined by their investment style⁵, I construct two monthly measures of abnormal performance. I then test for overall persistence of risk-adjusted returns in a two-period framework using both non-parametric (contingency table based) and parametric (regression based) methods. In the multi-period setting, I use a ‘two-sample Kolmogorov-Smirnov’ (K-S) test to detect significant differences between the observed distribution of returns for individual hedge funds and the theoretical distribution under the assumption of no persistence. My specific contribution to the existing literature follows from sorting hedge funds into quintiles based on size and age. I also rank funds into three groups based on the level of their incentive and management fees⁶. I then examine two-period and multi-period persistence in abnormal returns of funds across the quintiles/groups for each characteristic, and across the eleven investment strategies classified by the TASS database. In addition, I develop a measure to analyze separately the impact of these characteristics and strategies on persistence in good performance and persistence in bad performance.

The rest of the paper is organized as follows. Section 2 contains a literature review on performance persistence in the hedge fund industry. In Section 3, I provide a more detailed description of my main hypotheses. Section 4 provides a description of my

⁵ The TASS database classifies funds into 11 different investment styles or ‘primary categories’. These are reported in Appendix A as: ‘Convertible Arbitrage’, ‘Long/Short Equity Hedge’, ‘Event Driven’, ‘Fund of Funds’, ‘Multi-Strategy’, ‘Global Macro’, ‘Emerging Markets’, ‘Managed Futures’, ‘Fixed Income Arbitrage’, ‘Equity Market Neutral’, and ‘Dedicated Short Bias’.

⁶ I classify funds into three categories instead of quintiles here in order to separate them into ‘low’, ‘moderate’ and ‘high’ fee categories.

sample. In Section 5, I examine the effect of the various characteristics of hedge funds on persistence in performance in the two-period setting. Section 6 extends the analysis to the multi-period framework. Section 7 concludes.

2. Literature Review

I firstly examine the literature on performance persistence in the mutual fund industry. In particular, Hendricks, Patel and Zeckhauser (1993), Elton, Gruber, Das and Hlava (1993) and Goetzmann and Ibbotson (1994) all find evidence of persistence in mutual fund excess returns. Brown and Goetzmann (1995) also obtain evidence of persistence that is mainly driven by repeat losers rather than repeat winners. Carhart (1997), however, uses a ‘four factor’ model, (the standard three factor Fama-French model with the addition of a performance attribution factor for momentum strategies), in order to examine mutual fund performance persistence between 1962 and 1993. He reports that mutual funds do not earn significantly positive risk-adjusted returns relative to his augmented model, thus providing evidence that common factors in stock returns almost completely explain away persistence. Nevertheless, as discussed by Berk and Xu (2004), even the Carhart model cannot justify the persistent underperformance of the worst performing mutual funds over the last ten years or so. Therefore, there does appear to be consistent evidence from the literature that suggests that any persistence in the mutual fund industry is primarily driven by consecutive losers. Apart from this finding though, there is no evidence of any other clear trends in persistence over the last few decades.

By contrast, findings of performance persistence in the hedge fund industry are far more widespread, although some of the standard methods used to test for persistence have produced conflicting results in the literature. The most common approach to examining persistence is to use non-parametric and parametric (or regression based) tests in a two-period setting, and a Kolmogorov-Smirnov (K-S) test in a multi-period framework. Brown, Goetzmann and Ibbotson (1999), for example, use an offshore hedge fund database and find that year-by-year cross-sectional regressions of hedge fund ‘alphas’, or excess returns, on past excess returns do not indicate performance persistence for the sample period 1989 to 1995. On the other hand, Agarwal and Naik (2000a) select

their sample from the broader HFR database. Using a multi-factor 8-index model to generate estimates of excess hedge fund returns between 1994 and 1998, they do obtain significant evidence of persistence in hedge fund performance at quarterly and half-yearly horizons.

Agarwal and Naik (2000b) extend their performance persistence tests from a two-period framework to a multi-period setting, now using a one-factor benchmark model and a sample period of 1982 to 1998. They observe that the degree of persistence decreases as the return horizon they use increases, and that persistence is mainly driven by consecutive losers rather than by winners. Furthermore, they note that the extent of persistence over several periods is not related to the investment strategy of a particular fund. Finally, the level of persistence found in the multi-period setting under a Kolmogorov-Smirnov test is significantly lower relative to the two-period framework, and there is a complete absence of persistence using a one-year return horizon, (at both the 5% and 10% significance levels).

Koh, Koh and Teo (2003) lend further support to these findings with their investigation of persistence in excess returns of Asian hedge funds, (both pre- and post-fee⁷), which they also find to diminish as the return horizon exceeds six months. Similar results are obtained by Capocci, Corgay and Hubner (2003), who find no persistence in annual risk-adjusted mean returns from the MAR database for the best and worst performing funds. In particular, in the hedge fund literature, only Caglayan and Edwards (2001), Baquero, ter Horst and Verbeek (2002) and Kouwenberg (2003) actually find any evidence of significant persistence at a yearly horizon.

Caglayan and Edwards (2001) conduct quite a different study of persistence in hedge fund performance over the period 1990 to 1998. They use a six-factor model to obtain estimates of Jensen alphas for individual hedge funds in the MAR database. They apply their analysis to hedge funds following eight different investment styles, and in contrast to Agarwal and Naik (2000b), report that the magnitude and persistence of

⁷ This follows the suggestion by Brown, Goetzmann and Ibbotson (1999) that persistence is displayed in pre-fee returns, and managers can be fully compensated for this through performance fees. However, given that fees are not paid during the year and are instead imputed, this adjustment may generate a spurious persistence in returns measured at horizons less than a year. Therefore, in order to test this, Koh, Koh and Teo (2003) and Agarwal and Naik (2000b) perform their persistence tests on both pre-fee and post-fee returns.

excess returns differ significantly across these strategies. They also find evidence of persistence amongst both repeat winners and losers over one-year and two-year horizons; this contradicts several of the findings in the mutual fund and hedge fund literature summarized so far.

Overall, one notes from a study of the literature that the most widely used non-parametric test for persistence over the last eight years is the cross-product ratio (CPR) test. (The test is based on a two-way winner and loser contingency table analysis). This approach is also used by Agarwal and Naik (2000a and 2000b), Caglayan and Edwards (2001), Kat and Menexe (2002), Koh, Koh and Teo (2003) and DeSouza and Gokcan (2004). Furthermore, looking at the various performance measures used in the hedge fund literature, one observes that the most common method of measuring excess returns is to use a one-factor benchmark model, (where fundamental risk is determined by the investment strategy of a hedge fund⁸), or to estimate relative risk-adjusted returns using a multi-factor model⁹. In my paper, I adopt the former approach of measuring abnormal returns for all hedge funds.

3. Hypotheses

As described in Section 1, my hypotheses are mainly derived from the reasoning that my selected hedge fund characteristics reflect managerial skill in various ways. One would expect superior (lower) hedge fund manager skill to be associated with better (worse) performance in any given period. Thus, since skill is itself intrinsic and likely to be persistent over time, I propose that superior (lower) ability should be associated with a higher degree of persistence in good (bad) performance as well.

There have been several past studies of the relationship between these hedge fund characteristics and performance. For example, De Souza and Gokcan (2003) perform

⁸ This method is used by Agarwal and Naik (2000b), Brown, Goetzmann and Ibbotson (1999), and Park and Staum (1998).

⁹ Baquero, ter Horst and Verbeek (2002) and Kat and Menexe (2002) estimate relative risk-adjusted returns for the hedge funds in their samples, (both of which are obtained from the TASS database). Caglayan and Edwards (2001), Capocci, Corhay and Hubner (2003), Kouwenberg (2003) and Boyson (2003b) follow the same procedure, using a multi-factor model.

regressions on the TASS database in order to examine which characteristics of hedge funds affect performance, (but not persistence in performance over time). They find that assets under management, a proxy for the size of a hedge fund, is positively related with performance. They also find that older funds outperform younger funds on average, suggesting a positive relationship between age of a fund and ability of managers to ensure survival. One can infer from these results that managers of larger and older funds are more likely to be highly skilled and capable of keeping their funds alive for a longer period of time. Thus, the natural implication is that one should observe a positive (negative) relationship between age, as well as size, and persistence in good (bad) performance.

On the other hand, decreasing returns to scale is also a trend that exists in the mutual fund and, to a lesser extent, the hedge fund industry as well. Chen et al (2002), for example, find that size significantly erodes performance in the mutual fund industry, and Agarwal, Daniel and Naik (2006) find that larger hedge funds with greater inflows are associated with poorer future performance, as well as a lower persistence in good performance. This latter result in particular is consistent with the argument that larger hedge funds with large money flows may find it difficult to generate high returns, since they may be unable to deploy their entire capital into certain trading strategies that are restricted by the size of the market. Therefore, these factors suggest that, at least beyond a certain threshold, one should observe a negative relationship between hedge fund size and persistence in good performance.

Thus, I have outlined two factors, the 'skill' effect and decreasing returns to scale, which generate opposite predictions about the expected relationship between size and performance persistence. In my investigation, I use two measures of fund size to address this issue. The first measure, for a given fund, is its initial net asset value at the start of the sample period. The second measure is the maximum initial net asset value across all funds in the same management company or fund family. I use this latter measure to adjust for the practice of 'size-capping', which would at least partially erase the effects of decreasing returns to scale on hedge fund performance. (In particular, in order to control for size-capping it is important not to classify spinoff funds in any given fund family as being smaller than the parent fund. My alternative measure of size specifically adjusts for

this). Given the significantly higher skill required to ensure survival in the hedge fund industry, however, and also given far weaker evidence of scale diseconomies compared to the mutual fund industry¹⁰, I propose that higher hedge fund size primarily reflects superior managerial skill, rather than decreasing returns to scale. One would therefore expect to see a positive association between size, (as well as age), and persistence in good performance, even after allowing for diseconomies to scale, or controlling for size-capping. In other words, this relationship between size and persistence should hold using *both* measures of size outlined above. This would provide a contrast to strong findings of decreasing returns to scale in the mutual fund industry.

I base my hypothesis about the relationship between incentive fees and performance persistence on a scenario in which managers raise fees in response to good past performance. Firstly, I note that De Souza and Gokcan (2003) find a positive correlation between incentive fees and performance in their hedge fund study. The standard justification for this result is that funds generally attract a higher inflow of funds after good performance¹¹, (as a reward for greater skill), and more skilled managers may consequently charge higher incentive fees to capture a greater surplus from investors. It is worth recognizing, however, that luck, as well as managerial skill, also potentially plays a significant role in determining performance of hedge funds over time. Therefore, it is natural to expect that investors, who are only able to observe past performance of hedge funds, cannot actually distinguish between funds which have performed well in the past and have genuinely skilled managers, and funds which have unskilled managers and have produced similarly good performance due to luck. Furthermore, if one assumes that investors supply capital competitively to funds, (and are fully rational, so that high fund performance is interpreted as a signal of high managerial ability), then this creates a

¹⁰ There is mixed evidence of decreasing returns to scale in the hedge fund industry. While Agarwal, Daniel and Naik (2006) find evidence of decreasing returns to scale in their study of flows and performance, several other studies, including De Souza and Gokcan (2003), Amenc, Curtis and Martellini (2003) and Getmansky (2004), observe that larger hedge funds significantly outperform smaller funds in terms of excess returns. Findings of decreasing returns to scale in the mutual fund industry, on the other hand, are far less ambiguous. See for example Chen et al (2002) who find that size significantly erodes performance in the mutual fund industry, and Berk and Green (2004) who use decreasing returns to scale as a key assumption in their model of rational mutual fund flows.

¹¹ See Agarwal, Daniel and Naik (2006) for an investigation of the relationship between fund flows and past performance. In particular, they find that funds with good recent performance do indeed experience higher money-inflows.

scenario in which capital flows are responsive to past performance as well. This is because, in equilibrium, capital will simply flow to investments in which it is expected to be most productive; thus, the better the performance of a fund in a given period, the higher the expected inflow of funds for the next period, and vice versa¹². Coupled with the assumptions described above, one would then expect both skilled, and unskilled but lucky funds to receive the same high inflow of funds from investors.

Following on from this discussion, it is intuitive to argue that the greater the inflow of new capital to a fund, the greater should be the incentives of managers to raise performance fees in order to extract the maximum surplus, (or alpha), from investors. Thus, the implication in this scenario is that both skilled, and unskilled but lucky funds should raise their incentive fees to the same high level, after realizing similarly good past performance. While the funds with skilled managers would be significantly more likely to continue to perform well over several periods, however, even on a post-fee basis, one would eventually expect the funds with lucky but fundamentally unskilled managers to report persistently below-average performance in the long run, after deducting their high fees. Therefore, the major prediction generated by this model is that higher incentive fees should be associated with a higher degree of persistence in both good and bad (post-fee) performance, and this relationship should be most discernable in a multi-period framework¹³.

Turning finally to the relationship between investment strategies and performance persistence, there appear to be mixed results in the hedge fund literature. For example, De Souza and Gokcan (2004) use both non-parametric and regression based approaches

¹² See Berk and Green (2004) for a rational framework in which this relationship holds.

¹³ There is further evidence in the data to support this hypothesis. In particular, I obtain all hedge fund data from the TASS database, which classifies funds into 'Live' and 'Dead' databases. The most common reason for death of a fund is prolonged poor performance, which is in turn most likely associated with poor managerial skill. Therefore, if my hypothesis is true, higher incentive fees can signal both higher skill, and poor skill in conjunction with luck. Consequently, one would expect higher incentive fees for any given fund to increase the probability of dying. I test this proposition by running a logistic regression, where the dependent variable takes the value 1 if a fund is in the Dead database, and 0 if it is still living. The regressor is the level of incentive fees across all funds in the sample. When performing this regression, I obtain a significantly positive coefficient on incentive fees at the 5% level. In particular, a one standard deviation increase in incentive fees, (calculated across all funds in the sample), generates a 0.2% increase in the probability of dying on average. The coefficient on incentive fees remains positive, although not statistically significant, when controlling for size, age, management fees and volatility of returns over the sample period, and also when including investment strategy dummies. Therefore, these findings do provide additional support for my hypothesis that higher incentive fees generate higher persistence in both good and bad performance over time.

to test for persistence in the HFR database. Using non-parametric methods, they obtain no evidence of persistence in Sharpe ratios and raw average returns at the three-year horizon across all hedge fund investment strategies. Regression based tests, however, display significant evidence of persistence in Sharpe ratios for ‘Convertible Arbitrage’ and ‘Equity Market Neutral’ strategies. Similarly, Kouwenberg (2003) finds evidence of persistence in Sharpe ratios and alphas amongst ‘Event Driven’, ‘Market Neutral’ and ‘Global’ funds in the late 1990s.

One line of reasoning which could explain why certain hedge fund investment strategies display significantly more persistence than others is related to the relative liquidity of the securities that they trade. In particular, Getmansky, Lo and Makarov (2004) investigate a similar topic in their paper, developing an econometric model of return smoothing in order to suggest that illiquidity exposure generates significant (positive) serial correlation in monthly hedge fund returns, at least in the short run, (i.e. for a lag of up to approximately six months). They then examine the distribution of estimated smoothing coefficients, (which act as a proxy for quantifying illiquidity exposure), across 17 different investment strategies. The authors observe that a certain group of investment categories, (comprising ‘Fixed-Income Directional’, ‘Convertible Arbitrage’, ‘Event Driven’, ‘Nondirectional/Relative Value’ and ‘Pure Emerging Markets’), display significantly more smoothed returns, and are thus much more likely to trade illiquid securities. On the other hand, another group of strategies comprising ‘US Equity Hedge’, ‘Global Equity Hedge’ and ‘Pure Managed Futures’, is shown to display less return-smoothing. These results are consistent with common intuition that these last three strategies, especially ‘Managed Futures’, involve relatively more liquid securities with well-established, not easily manipulated, markets.

Given that Getmansky, Lo and Makarov (2004) find that greater exposure to illiquidity produces more significant short-term positive serial correlation in hedge fund returns, one would also expect funds in more illiquid investment strategies to display higher overall persistence in a short run framework. Following this particular reasoning, the strategies in my sample that are likely to be most persistent are ‘Convertible Arbitrage’, ‘Event Driven’, ‘Fixed Income Arbitrage’ and ‘Emerging Markets’. The opposite result would be expected to hold true for the ‘Managed Futures’ category. One

would not, however, expect higher positive serial correlation to be a driving factor in the relationship between illiquidity exposure and long run performance persistence. This is because it is a short-term effect that disappears for horizons greater than six months. A more plausible version of this hypothesis in the multi-period setting, which accounts for underlying endogeneity in the selection of certain investment strategies by hedge funds, is that more highly skilled managers may signal their ability by choosing more complex and illiquid strategies. This would generate a positive relationship between illiquidity exposure and persistence in good performance in the long run, even after controlling for illiquidity risk. This is the final hypothesis that I test in my paper.

4. Sample Description

This paper uses the TASS database, which as of March 2006 lists 6542 funds, (including 2487 ‘Graveyard’ or ‘Dead’ funds, and 4055 ‘Live’ funds), with at least one monthly net return observation¹⁴. In my analysis, I consider monthly returns from January 1996 to March 2006.

As described in Fung and Hsieh (2000) and Liang (2000), which also use TASS for their analysis, there are numerous data biases that can arise when analyzing reported hedge fund returns from this database. These include survivorship bias¹⁵ and instant history bias¹⁶. Therefore, in replicating the analysis of Agarwal and Naik (2000b) on the TASS database, I follow their strategy of modifying the sample in order to minimize these potential biases, applying it to my analysis of monthly returns. Specifically, I include both live and dead firms in my sample, (from 1996-2006), in order to adjust for the survivorship bias problem. In order to mitigate the instant history bias, Fung and

¹⁴ In TASS, most returns are reported net of fees on a monthly basis. Returns are defined as the change in net asset value during the month, divided by net asset value at the beginning of the month.

¹⁵ This is reported to be an annual value of 2.24 % by Liang (2000), and 3 % by Fung and Hsieh (2000). The bias arises from the fact that the TASS ‘Graveyard’ database only became active in 1994, so that funds dropped from the ‘Live’ database before 1994 are not listed in the ‘Graveyard’ database. Therefore, prior to 1994, failed funds are excluded because they no longer exist. This would generate an upwards ‘survivorship’ bias in mean returns, (and most likely a downward bias in return volatility as well). I control for this problem by including both live and dead funds in my analysis, for the post-1994 period 1996-2006.

¹⁶ This (upward) bias in returns is generated by the back filing of earlier returns for a fund that has been newly introduced into the database. It is estimated by Fung and Hsieh (2000) to be as high as 1.4 % for average annual hedge fund returns in TASS.

Hsieh (2000) and Agarwal and Naik (2000b) follow a standard procedure of excluding the first period's observation of returns for all hedge funds in the sample. In my analysis, however, I use a very short time-period of one month¹⁷. Therefore, merely excluding the first month of returns for all hedge funds would have a minimal impact on reducing the back filing bias in my sample. Consequently, in order to implement a more substantial adjustment for the instant history bias, I exclude the first year of returns for each hedge fund, (which is a more standard practice). I also exclude all funds with a return history of under two years, and all funds that report returns on a quarterly basis, do not report returns net-of-fees, or do not quote US dollars as their base currency.

After restricting my sample period to January 1996-March 2006, and completing the modifications outlined above, my final sample is composed of 4287 funds; 2611 live funds and 1676 dead funds. Table 1 contains a descriptive analysis of my sample, providing the numbers of live and dead funds in each of the 11 investment categories classified by the TASS database, as well as in each of the quintiles based on size and age. I also separate funds into 'Low', 'Moderate' and 'High'¹⁸ management and incentive fee groups; the number of live and dead funds in each of these groups is reported in Table 1 as well. Table 2 displays the total number of funds within each of the 11 investment strategies for each of the five size and age quintiles, (reported in Panel A), and for each of the three incentive and management fee groups, (shown in Panel B). Finally, Panel A of Table 3 reports the cross-sectional mean, median and standard deviation of size and age within each of the investment strategies.

5. Two-period Tests of Performance Persistence

In this section, I analyze performance persistence in a two-period or short run setting. As in Agarwal and Naik (2000b), I make the assumptions that excess returns are measured relative to a benchmark model where systematic risk is determined by a hedge fund's

¹⁷ My motivation for using such a short return period is that given the evidence in Agarwal and Naik (2000b) that the extent of performance persistence declines significantly with the return horizon, it appears that the effects of different hedge fund characteristics on persistence would be highlighted most clearly by using as short a return interval as possible.

¹⁸ These classifications are based on standard categorizations of high, moderate and low performance fees in the hedge fund literature. See Table 1 for further details.

investment strategy or ‘primary category’. In particular, as previously mentioned, TASS separates hedge funds into 11 different primary categories. One can classify these strategies further into ‘directional’ investment strategies, which exploit broad market movements, and ‘non-directional’ strategies, which exploit specific short-term market inefficiencies while hedging out as much market exposure as possible. (See the Appendix for classification and description of these investment styles). As can be inferred from the descriptions above, directional strategies are typically highly correlated with the market, whereas non-directional strategies are market-neutral and display low correlation with the market.

The motivation for using this particular benchmark model comes from the fact that different hedge fund strategies have been shown in the literature to imply significantly varying risk-return tradeoffs. As suggested by Brown, Goetzmann and Ibbotson (1999) and Agarwal and Naik (2000a), for example, hedge funds can be exposed to extremely different levels of fundamental risk depending on whether they follow a directional or non-directional strategy, and even depending on which specific investment category they fall into. Furthermore, the widely documented non-normality¹⁹ of hedge fund returns implies that any other standard linear multi-factor model would most likely be an inadequate measure of risk. Therefore, as in Agarwal and Naik, (2000a and 2000b), I benchmark funds’ performance based on the investment strategy they follow, (generating eleven benchmark measures). The major benefit of benchmarking funds’ performance based on investment strategy is that this allows one to control for potential differences in illiquidity risk premia across the different primary categories. Aragon (2006) provides evidence that illiquidity risk is priced in the hedge fund industry, so it is important to adjust for this when analyzing the relationship between illiquidity exposure and persistence in performance.

As a first measure of performance, I define the ‘alpha’ or excess return of a given hedge fund as its monthly return minus the equal-weighted average monthly return for all hedge funds in the sample following the same investment strategy. Using an equal-weighted average here allows me to control for the fact that some investment strategies, (in particular, the ‘Fixed Income Arbitrage’ category, as can be seen in Panel A of Table

¹⁹ See Fung and Hsieh (1997).

3), may contain outlier funds with substantial initial size. Therefore, a value-weighted average would place too high a weight on these outlier funds when calculating the benchmark. As a second measure, which I also adopt in my analysis, the monthly ‘appraisal ratio’²⁰ for a hedge fund is defined by Agarwal and Naik (2000b) as the alpha divided by the residual standard deviation resulting from a regression of the hedge fund’s returns (throughout the sample period) on the average monthly returns of all the hedge funds following that strategy. Panel B of Table 3 reports the cross-sectional mean, median and standard deviation of the time-series averages of alphas and appraisal ratios across all funds in each of the investment strategies classified by TASS. One notes from this table that although the cross-sectional means of the performance measures are generally negative for all the investment categories, (as well as overall), standard deviations are extremely high in comparison. In fact, the overall standard deviation of appraisal ratios per month is more than ten times higher than the absolute mean appraisal ratio across all hedge funds in the sample. Therefore, it is difficult to draw any significant inferences here about funds’ relative average performance across the investment strategies, as well as about their overall performance.

My analysis of persistence in a two-period setting involves two standard methods, parametric and non-parametric. In order to conduct the parametric test of persistence, I run a month-by-month cross-sectional regression of hedge fund alphas (appraisal ratios) on alphas (appraisal ratios) during the previous month. For each month, a significantly positive slope coefficient on past alpha (appraisal ratio) indicates persistence in performance. I then take a time-series average of the estimated slope coefficients for the month-by-month cross-section regressions, and perform t-tests on these averages²¹. I also conduct the same tests using standard errors corrected for serial correlation, or Newey-West standard errors; this is an important adjustment given findings of monthly serial return correlation in Getmansky, Lo and Makarov (2004).

On the other hand, the non-parametric method is based on constructing a two-way contingency table of winners and losers, where a particular hedge fund is defined as

²⁰ The appraisal ratio measure is thus adjusted for differences in volatility and leverage across hedge funds in different investment categories. It is effectively a measure of hedge fund alpha per unit of idiosyncratic risk. The use of the appraisal ratio as a performance measure can be dated back to Park and Staum (1998), who examine persistence in performance of hedge funds in the TASS database from 1986-1997.

²¹ See Fama and Macbeth (1973).

being a winner in a given month if its alpha (appraisal ratio) is greater than the median alpha (appraisal ratio) of all funds following the same strategy in that month. Individual hedge funds are then described as displaying ‘persistence’ if they are either winners or losers in two consecutive months, denoted respectively as WW and LL; (winners in the first period and losers in the second are denoted as WL and vice versa). One can detect persistence in alphas and appraisal ratios using a CPR (cross-product ratio) test, which compares the observed frequency distribution of consecutive wins and losses for each hedge fund with the expected frequency distribution under the null hypothesis of no persistence. In particular, the CPR is defined as $(WW*LL)/(WL*LW)$, where WW, LL, WL and LW now denote the total number of funds which are consecutive winners, consecutive losers, winners in the first period and losers in the second, and vice versa, summing across all the months in the sample period. Thus, the CPR is the ‘ratio of funds which show persistence in performance to the ones which do not’²², and under the null hypothesis of no persistence this ratio should be equal to one. One then constructs the Z statistic, which tests whether the logarithm of the CPR is significantly greater than zero, by taking the ratio of the logarithm of the estimated CPR to its standard error. This statistic has a standard normal distribution under the null²³.

In order to break this analysis down further and examine separately whether there is significant persistence in performance driven by repeat winners, (i.e. persistence in good performance), or persistence driven by repeat losers, I construct my own analogous ‘winner’ (‘loser’) CPR test. The purpose of these tests is to detect significant differences between the observed frequency distribution of consecutive wins (losses) for each hedge fund and the expected frequency distribution under the null of no persistence. The cross-product ratios used for these tests are, respectively, defined as (WW/WL) and (LL/LW) . (Corresponding standard errors and Z statistics are constructed analogously to those described above).

In Tables 4 and 9, I report the results of non-parametric and parametric analyses respectively, testing for persistence in both alphas and appraisal ratios across the whole sample of funds, (and also across the various primary categories, size and age quintiles

²² See Agarwal and Naik (2000b)

²³ See Agarwal and Naik (2000b) for further details on the construction of this test statistic.

and fee groups defined previously). I start by examining the results of the non-parametric persistence tests on my sample. In Table 4, one can see that for all hedge funds, we can reject the null hypothesis of no persistence in alphas and appraisal ratios at the 1% level of significance. Furthermore, a few cross-sectional trends emerge from the reported log cross-product ratios. Firstly, there is strong overall evidence of persistence, (in both alphas and appraisal ratios), across all primary categories at the 1% level of significance, except for in the ‘Managed Futures’ category. Moreover, testing for the significance of the differences between log cross-product ratios of funds in different strategies, (reported in Tables 5-7), one observes from Table 5 that the ‘Convertible Arbitrage’ and ‘Fixed Income Arbitrage’ strategies display the strongest evidence of overall persistence at the (one-sided) 1% level. In other words, both these strategies exhibit a significantly higher degree of overall persistence than all other primary categories at the 1% level, (excluding each other). I obtain similar results when I examine persistence in good performance, reported in Table 6, and persistence driven by losers, reported in Table 7. The finding of these trends for this recent sample period contrasts with the results of Agarwal and Naik (2000b), who claim that performance persistence is not related to investment strategy when analyzing hedge fund returns from 1982-1998.

These results provide evidence in support of my hypothesis that illiquidity exposure is positively associated with overall persistence in the two-period setting, even after controlling for differences in illiquidity premia across the various investment strategies. In particular, the ‘Convertible Arbitrage’ and ‘Fixed Income Arbitrage’ categories fall within the class of most illiquid hedge fund strategies identified by Getmansky, Lo and Makarov (2004). In addition, I observe that the primary categories which display the weakest evidence of overall persistence are those which trade in relatively more liquid securities, namely the ‘Managed Futures’ strategy. In this two-period test, however, when analyzing the relationship between illiquidity and persistence, it is difficult to distinguish between the effects of managerial skill and the fact that higher illiquidity is merely inducing higher short-term positive serial correlation in returns.

Turning to the analysis of hedge fund characteristics, and examining the significance of log cross-product ratios once again in Table 4, one can see that funds in all age (and size) quintiles exhibit significant evidence of overall persistence at the 1%

level, as well as persistence in wins and losses. Testing for the significance of the differences between log cross-product ratios across the age quintiles, one notes from Table 6 that the funds in the lowest age quintile exhibit the least significant evidence of persistence in wins compared to all other funds at the 1% level, followed by funds in the second lowest age quintile. Furthermore, there is a negative monotonic relationship between age and significance of persistence in losses displayed in Table 7, using appraisal ratios as the performance measure. In particular, funds in the lowest age quintile display the most significant persistence in losses relative to all older funds, (at the 1% level), followed by funds in the fourth and middle age quintiles, and then funds in the top two age quintiles. The generally positive (negative) relationship here between age and persistence driven by repeat winners (losers) supports my hypothesis that age is positively related with skill.

One observes similar trends across the size quintiles in my analysis. In particular, using appraisal ratios as the performance measure, one observes from Table 6 that funds in the lowest size quintile generate the weakest evidence of persistence in wins at the 1% level. The smallest (largest) funds also exhibit the most (least) significant evidence of persistence driven by repeat losers in Table 7. Once again, these findings provide evidence for my hypothesis that larger funds are associated with relatively stronger persistence in good performance. Furthermore, as described in Section 3, I repeat all these tests using an adjusted measure of size. In particular, for any given fund, I alternatively measure size as the maximum initial net asset value across all funds in the same management company or fund family. This measure specifically controls for the fact that hedge funds may try to eliminate the effects of decreasing returns to scale by ‘capping’ their size at a certain optimal level, and starting up smaller spinoff funds instead. The results using this new size measure, however, are very similar to those generated by the original measure of size, and are thus not reported in the paper. Therefore, my findings suggest that any effects of decreasing returns to scale are dominated by the fact that higher hedge fund size is mainly reflective of superior managerial skill²⁴.

²⁴ It is interesting to note here the trends in overall performance persistence displayed across the size and age quintiles as well. Examining Table 5, one observes that funds in the bottom four size (age) quintiles

Finally, reverse trends in performance persistence are displayed across the management fee and incentive fee groups in the two-period setting. One would expect this, given that hedge funds that charge high (low) incentive fees typically compensate by charging low (high) management fees at the same time. Specifically, one can see that funds in the low management fee group exhibit the strongest persistence in overall, poor and good performance, at the 5% level, relative to funds with high and moderate management fees. On the other hand, one notes from Table 6 that, using appraisal ratios, funds in the high incentive fee category exhibit significantly more persistence in good (overall) performance at the 5 (10) % level compared to both low and moderate incentive fee funds. Here, the generally positive²⁵ relationship between incentive fees and persistence in good performance is economically plausible. Indeed, this trend provides evidence that more skilled managers generate persistently better performance, and these managers may then set higher incentive fees in order to extract a higher surplus from investors. An alternative interpretation is that higher performance fees are actually required to attract more skilled managers. Either interpretation, however, supports the argument that the level of observed incentive fees in hedge funds is likely to reflect managerial skill and/or past performance.

In order to investigate the relationship between incentive fees and performance persistence further, as well as provide additional evidence to support my initial hypothesis that incentive fees are determined based on past performance, I devise a test to examine the channel of managerial fee setting in greater depth. In particular, changes in incentive fees for particular hedge funds over time are not directly observable, and it is commonly assumed that fees are set exogenously by funds at the time of their inception²⁶. However, I find significant evidence to suggest that, on average, managers of fund companies increase incentive fees charged by new spinoff funds when at least one fund in the family has performed persistently well so far²⁷. This is consistent with anecdotal

display significantly greater persistence in overall performance relative to funds in the top size (age) quintile at the 10 (5) % level. This indicates that the negative relationship between size (age) and persistence in poor performance is stronger overall than the positive relationship between size (age) and persistence in good performance.

²⁵ The relationship is not strictly monotonic.

²⁶ See Agarwal, Daniel and Naik (2007).

²⁷ In order to examine this, I take all the fund families listed in TASS with at least two or more funds that were started after January 1996; there are 456 families in total. I then take each company in turn and rank

evidence from the hedge fund industry that managers of a particular fund may find it contractually difficult to raise fees after realizing good performance, and therefore may have to increase fees on a spinoff instead. Following on from this, I then investigate the effects of incentive fees on persistence after controlling for this mechanism. My results from this new procedure in the two-period framework, using non-parametric analysis, are reported in Table 8. Here, I perform exactly the same tests as those described in Tables 4-7, using returns net of incentive fees, but now set the incentive fee level of any particular fund to be the lowest reported incentive fee across all funds within the same fund family or management company. This specifically adjusts for the fact that any spinoff funds may have had their fees increased in response to good past performance.

Examining the results in Table 8, one can see that the pattern of log cross-product ratios across the incentive fee groups reported in Panel A is similar to that observed in Table 4, for both performance measures. If, however, one examines the new results on the significance of the differences between log cross-product ratios, reported in Panels B and C of Table 8, one notes some changes. In particular, in Table 7, (using unadjusted incentive fees), we observe that the high incentive fee group displays more, although not significantly more, persistence in losers relative to the moderate and low fee groups. On the other hand, in Panel C of Table 8, we now see that funds in the high incentive fee group exhibit significantly less persistence in poor performance at the 1% (5%) level, relative to funds in the low (moderate) incentive fee group. The results on persistence in winners in Panel B of Table 8 show, however, that exactly as before, funds in the high

the funds within them by performance start date, (or the date on which they started reporting returns). For each performance start date in ascending order, I examine, firstly, whether there is at least one existing fund which has displayed significant persistence in good performance, (at the 5% level), up till that date. Here, I measure persistence in good performance using the two-period CPR test of persistence in winners. This is because I am not analyzing persistence of individual fund performance over the entire sample period. Secondly, I look at whether any of the new funds started on that date have an incentive fee that is strictly greater than the maximum incentive fee charged by all existing funds. For each fund family in the sample, I then calculate the proportion of performance start dates, (excluding the minimum performance date), for which we observe that both these criteria are met. This provides an approximate measure for each company of the likelihood that, each time they set up one or more spinoff funds, they will increase incentive fees charged by at least one of these new funds after there has been persistently good performance of at least one existing fund. Finally, I compute the cross-sectional average of this likelihood measure and test whether it is significantly greater than zero. Using appraisal ratios as the performance measure, I obtain a mean of 0.0834 and a standard deviation of 0.2357. Given the sample size of 456, this produces a t-statistic for the mean of 7.58, which is significantly greater than zero at the 5% level. Similarly, using alphas, I obtain a mean of 0.0939, a standard deviation of 0.2524, and a statistically significant t statistic of 7.96.

incentive fee category display significantly more persistence in good performance relative to funds in the low and moderate incentive fee groups.

Therefore, these findings indicate that there are actually two discernable effects coming into play in the two-period framework, which influence the relationship between incentive fees and persistence. Firstly, higher incentive fees are most likely associated with greater persistence in good performance because they are reflective of superior managerial skill, and we observe this relationship in the data. On the other hand, the positive (but insignificant) relationship which is observed between incentive fees and persistence in bad performance is most likely generated by the fact that hedge fund managers increase fees in response to past good performance, (via spinoff funds), and these managers include those that are unskilled but lucky. Thus, when I partially control for this effect across all hedge funds, using data on fees for spinoffs, one observes a reversal of the relationship between fees and persistence in poor (post-fee) performance, but no impact on the relation between fees and persistence in good performance. Overall, however, in the two-period setting, the positive relationship between incentive fees and persistence in good performance is most significant, whereas the positive relationship between incentive fees and persistence in poor performance is not. This indicates that the effects of managerial fee setting through spinoff funds are limited in the short run framework.

Having fully analyzed the trends revealed by non-parametric analysis of persistence in my sample, I now turn to the results of the parametric tests, (displayed in Tables 9 and 10). Here, I observe similarly strong evidence of persistence in alphas and appraisal ratios. For example, there is evidence of persistence in performance which is significant at the 1% level for the sample overall, using both performance measures. Furthermore, examining the significance of the differences between the coefficients across investment strategies in Table 10, one notes once again that the strongest evidence of persistence in alphas and appraisal ratios is displayed by funds in the 'Convertible Arbitrage' and 'Fixed Income' primary categories, as well as the similarly illiquid 'Event Driven' category. In particular, all three of these strategies generate significantly higher coefficients than six other primary categories at the 1% level, excluding each other. Examining appraisal ratios, I also calculate that, on average, a one standard deviation

increase in appraisal ratio in a given month leads to a 21%, 14.5% and 30.8% increase in appraisal ratio for the next month, for these three strategies respectively. Therefore, the statistical and economic significance of these findings closely supports my results in the non-parametric framework on the positive relationship between illiquidity exposure and overall persistence. Moreover, we find once more that all strategies, with the exception of the ‘Managed Futures’ category, display significant evidence of persistence overall at the 1% level using appraisal ratios, (5% level using alphas as the performance measure).

However, there are no significant trends displayed across the size, age, or performance fee groups when conducting the parametric tests. Results are very similar when testing positive persistence in above-average alphas, (persistence in good performance), and below-average alphas, (persistence in bad performance) separately²⁸. One reason why the results obtained from parametric analysis are marginally weaker overall than those generated by the non-parametric tests, could be that the latter approach of dividing funds into just two groups, (winners and losers), each period is a slightly coarser method of examining persistence in a two-period framework. Therefore, there is probably a higher degree of spurious persistence generated by the contingency table tests.

As an alternative to the parametric tests described above, which employ the Fama-Macbeth regression approach, I also conduct pooled time-series cross-sectional OLS regressions of alphas (appraisal ratios) each month on alphas (appraisal ratios) during the previous month, across funds with different strategies²⁹. The results are reported in Tables 11 and 12, with Newey-West standard errors, i.e. standard errors corrected for serial correlation. Generally, the inferences drawn from these new tests are similar to those drawn previously, (although results on persistence using appraisal ratios are less significant than before). For example, using appraisal ratios and examining Table

²⁸ These results are not reported in the paper. For the test of persistence in good performance, I run a month-by-month cross-sectional regression of hedge fund alphas (appraisal ratios) on alphas (appraisal ratios) during the previous month, conditional on alphas (appraisal ratios) in the previous month for any given fund being positive and above the median alpha (appraisal ratio) for all funds in the same investment strategy. The test of persistence in poor performance is defined analogously. I obtain similar results for both these tests compared to the test of overall persistence; i.e. ‘Convertible Arbitrage’, ‘Event Driven’ and ‘Fixed Income Arbitrage’ display the most significant persistence in both good and bad performance, and vice versa for the ‘Managed Futures’ strategy. No significant trends are observed across any of the quintiles/groups for the characteristics, however.

²⁹ Results are not reported across the quintiles/groups for the different characteristics, given the lack of significant trends observed across the coefficients.

12, which reports t-statistics from tests of the significance of the differences between coefficients across different investment strategies, one observes that the ‘Convertible Arbitrage’ category exhibits the strongest relative persistence at the 1% level, followed by the ‘Event Driven’ and ‘Emerging Markets’ strategies respectively. These results do not exactly match those from the parametric tests reported in Tables 9 and 10. However, given that the ‘Convertible Arbitrage’, ‘Event Driven’, and ‘Emerging Markets’ strategies all fall within the group of most illiquid strategies as classified by Getmansky, Lo and Makarov (2004), these findings do still provide significant evidence of a positive association between illiquidity exposure and overall persistence in the two-period setting. Importantly, these results hold even after controlling for illiquidity risk and serial correlation in monthly returns, providing stronger evidence for a positive link between illiquidity exposure and managerial skill.

In the next section, I extend my analysis to a multi-period framework and examine whether these relationships continue to hold. In particular, it will be especially powerful if I can show that several of the two-period results are also observed in the multi-period setting. This is because the increased power of a multi-period test is likely to diminish the significance of any kind of noise or spurious persistence observed in the two-period framework. Furthermore, if there are certain fund characteristics which are positively correlated with noise in the short run, (for example, one might expect smaller or younger funds to generate more noise or volatility in their monthly returns), then the test in the multi-period setting is a useful robustness check to confirm the validity of the trends displayed in the two-period framework.

6. Multi-period Tests of Performance Persistence

Following the procedure of Agarwal and Naik (2000b), I now analyze persistence of hedge fund performance in a multi-period framework. In order to do this, I use a ‘two-sample Kolmogorov-Smirnov’ (K-S) test to detect significant differences between the observed distribution of two and more consecutive wins and losses for individual hedge funds and the theoretical distribution of two and more consecutive wins and losses under

the assumption of no persistence³⁰. The results of the two-sample (two-sided) K-S test based on alphas and appraisal ratios are reported in Table 13. In particular, I calculate K-S test statistics and corresponding p-values from tests of persistence in wins and losses across the various investment strategies and characteristic groups. Here, the K-S test statistics measure the degree of persistence, or the maximum absolute difference between the observed and theoretical cumulative distributions under the null, while the p-values measure statistical significance. Therefore, I infer that persistence in, say, wins, is significantly higher in one category than another, if the K-S test statistic for the former category is higher, (indicating a higher degree of persistence), and also if its distribution of wins is significantly different from the latter category's distribution of wins at the 5% level, (indicating a significantly higher test statistic and degree of persistence). I do not report here the results of my tests of the significance of the difference between distributions of wins and losses across categories, but summarize the key findings in the following paragraphs.

Examining Table 13, I find that there is significant evidence of overall multi-period performance persistence at the 5% level, (examining the distribution of both wins and losses, and using both alphas and appraisal ratios as my performance measure). This result contrasts with that obtained by Agarwal and Naik (2000b), who find that the degree of persistence in a multi-period framework is significantly lower than that displayed in a two-period setting, for the earlier sample period of 1982-1998. Moreover, while Agarwal and Naik (2000a and 2000b) find that persistence over several periods is driven much more by repeat losers rather than by repeat winners, I obtain evidence of multi-period persistence driven almost equally by consecutive winners and losers. However, I do also observe that when I extend my analysis to the multi-period framework, there appears to be slightly weaker evidence of clear-cut trends in performance persistence across hedge fund characteristics and investment strategies. Nevertheless, I am still able to present several results that are consistent with those generated by tests in the two-period framework.

³⁰ Under the null hypothesis of no persistence, the distribution of wins and losses for any given hedge fund will be equivalent to a binomial distribution, with wins and losses being equally likely each period.

Firstly, looking across primary categories and using both performance measures, I find that the ‘Convertible Arbitrage’ and ‘Fixed Income Arbitrage’ strategies display the two highest K-S test statistics across all strategies, for tests of persistence in wins. These strategies also display significant overall persistence at the 5% level. Furthermore, I find that the distributions of consecutive wins for funds in ‘Convertible Arbitrage’ and ‘Fixed Income Arbitrage’ are significantly different from the distributions of wins for all other primary categories, (except for the ‘Event Driven’ strategy, and except for each other). Therefore, these results suggest that, as in the two-period framework, ‘Convertible Arbitrage’ and ‘Fixed Income Arbitrage’, which are two of the most illiquid strategies, display the most significant degree of persistence in good performance in the long run. The opposite result is true for ‘Managed Futures’, which is one of the very few strategies to display consistently insignificant evidence of persistence, and falls into the group of most liquid strategies classified by Getmansky, Lo and Makarov (2004).

In the multi-period framework, we can also now investigate further whether the higher observed persistence in good performance of more illiquid strategies is generated by superior managerial skill, or is merely a result of higher positive monthly serial correlation. In particular, Getmansky, Lo and Makarov (2004) find that this illiquidity-induced positive serial correlation is a short-term effect that disappears for horizons greater than six months. Consequently, in order to control for this effect, I examine multi-period persistence in wins over more than six consecutive periods, and compare the results across the different investment strategies. Results are reported in Panel A of Table 14. Here I observe that the ‘Convertible Arbitrage’ and ‘Fixed Income Arbitrage’ categories are two of only three strategies to display significant persistence in wins at the 5% level, and among those strategies, (which include the ‘Event Driven’ category), they display the highest K-S test statistics. In addition, I find that the distribution of consecutive wins for funds in the ‘Convertible Arbitrage’ strategy, followed by ‘Fixed Income Arbitrage’, differs most significantly from all other primary categories at the 5% level³¹. Therefore, these results indicate that even after controlling for illiquidity risk and the potential effect of higher short-term positive serial correlation in monthly returns, the

³¹ In other words, when comparing the significance of the difference in the distribution of wins across all investment strategies, the highest number of primary categories differ significantly from the ‘Convertible Arbitrage’ category, (followed by ‘Fixed Income Arbitrage’).

‘Convertible Arbitrage’ and ‘Fixed Income Arbitrage’ strategies still display the highest degree of persistence in good performance in the long run. This suggests that the positive relationship we observe in the multi-period framework between illiquidity and persistence in wins is most likely driven by an underlying fundamental factor such as skill.

Turning to the analysis of characteristics, one notes from Table 13 that funds across all size and age quintiles display significant evidence of multi-period persistence at the 5% level. One also observes that funds in the top size quintile display the highest p-value and lowest test statistic for the test of persistence in losses, using both performance measures. Furthermore, the distribution of consecutive losses for funds in the top size quintile is found to be significantly different from the distribution of losses for funds in each of the lower size quintiles. This indicates that the largest funds in the sample display significantly less persistence in bad performance relative to smaller funds. Similarly, I find that hedge funds in the bottom size (age) quintiles consistently generate significantly lower (higher) K-S statistics for tests of persistence in wins (losses), relative to funds in the top two size (age) quintiles. This suggests a significantly lower (higher) degree of persistence in wins (losses) for funds in the bottom size (age) quintile, relative to the largest (oldest) funds. Results are very similar using the adjusted measure of size described in Sections 3 and 5. These observations are thus consistent with previous results from contingency table analysis in the two-period framework.

A final notable trend that emerges from the multi-period analysis is associated, once again, with the relationship between incentive fees and persistence. Examining Table 13 and using appraisal ratios as the performance measure, one observes a positive monotonic relationship between the level of incentive fees and the K-S statistics from tests of persistence in both wins and losses. While the differences between these test statistics are not significant at the 5% level, these results still suggest that the strongest multi-period persistence, both in wins and losses, is exhibited by funds with the highest incentive fees. This trend supports my initial hypothesis that fees are raised in response to good past performance by both skilled, and unskilled but lucky funds.

I now analyze the relationship between incentive fees and performance persistence more rigorously by performing the same multi-period tests as described

above, but using the adjusted measure of incentive fees described in Section 5. The results from this analysis are reported in Panel B of Table 14. Here, I observe unchanged trends across the fee groups for the test of persistence in wins. However, examining persistence in losses across the incentive fee groups, I now find that the positive monotonic relationship between K-S statistics and fees is reversed, using both performance measures. In other words, the low fee category displays the largest test statistic relative to the high and moderate fee groups, and thus exhibits the highest persistence in losses. This constitutes a reversal of the positive relationship between fees and persistence in poor performance that we observed earlier. As in the two-period framework, these results thus reveal, once again, two channels that could affect the relationship between incentive fees and persistence; skill and luck. Therefore, one implication of my analysis is that my simple model of skill, luck, and endogenous determination of fees sheds light on an important determinant of long run performance persistence in the hedge fund industry.

7. Conclusion

The major result of this paper is that there exist hedge fund characteristics that are associated with persistence in performance over time. This is most likely because these characteristics are reflective, in various ways, of managerial skill and/or past performance. The findings in this paper contrast sharply with ambiguous evidence of performance persistence in the mutual fund industry.

Firstly, my analysis reveals that investment strategies with greater illiquidity exposure display significantly more persistence in good, as well as overall performance, even after controlling for short-term serial correlation and illiquidity risk. In particular, this result supports my hypothesis that more skilled hedge fund managers choose trading strategies in more complex and illiquid securities. The investment strategies that exhibit the most significant persistence both in the short and long run frameworks, and thus appear to send the strongest signal of skill, are the relatively illiquid ‘Convertible Arbitrage’ category, followed by ‘Fixed Income Arbitrage’. The opposite result holds for the relatively more liquid ‘Managed Futures’ strategy.

Secondly, there is evidence from the non-parametric tests, in both the short and long run settings, which indicates that the smallest (largest) hedge funds display the strongest signs of persistence in bad (good) performance, even after controlling for the effects of size-capping in response to decreasing returns to scale. The youngest (oldest) funds also exhibit the most significant persistence in bad (good) performance. Therefore, these results provide support for my initial hypotheses that higher age and size of hedge funds are primarily reflective of superior managerial skill.

Finally, my analysis reveals that there is a positive relationship between incentive fees and persistence in wins, generated by non-parametric tests in the two-period setting. This finding suggests that higher incentive fees reflect higher managerial skill. Furthermore, there is also evidence that high incentive fees generate the strongest persistence in good as well as bad performance in the multi-period framework. This result fits closely with the general predictions of the scenario that I develop in Section 3, where incentive fees are raised in response to good past performance by both skilled and unskilled, but lucky, fund managers. In particular, I find that, on average, management companies have a significantly positive probability of increasing incentive fees for spinoff funds after existing funds have realized persistently good performance. After I partially control for this effect, using data on fees for spinoffs, the positive relationship between incentive fees and persistence in poor performance reverses in the multi-period setting. Thus, one can conclude that this mechanism of endogenously determining incentive fees in the hedge fund industry is a potentially significant driver of long run performance persistence.

One very useful extension to this paper would be investigate whether one could develop a lucrative trading strategy by investing in (open-end) hedge funds which have performed well in the past, and also possess characteristics, or follow strategies, which I have shown to be associated with persistence in good performance. Consistent with the results from all my tests, this would imply investing in the most illiquid funds, or allocating capital to the funds that are the largest and oldest in addition to demonstrating prior good performance. This is an avenue of research that has not been explored so far in the literature, and thus could make an important contribution.

Other interesting extensions to the paper include examining more rigorously the determination of incentive fees in the hedge fund industry, and also studying the movement of individuals and managers across hedge funds, in order to provide further evidence for the endogenous selection of certain investment strategies. For example, given the findings in this paper, one would expect higher-ability workers and fund managers to migrate towards funds that follow more illiquid strategies, since they could better deploy their skills and might be able to demand higher compensation. Analyzing the movement of individuals across hedge fund strategies would therefore provide a very useful additional insight into the determinants of performance persistence in the hedge fund industry.

Appendix: Definition of Primary Categories in TASS.³²:

Primary Category	Definition	Type of Strategy
Convertible Arbitrage	A strategy of arbitraging the relative mispricing of related convertible securities (usually from the same issuer) in order to obtain low volatility returns.	Non-directional
Long/Short Equity Hedge	A strategy of investing in equity or equity-like instruments where net exposure (long minus short) is low.	Non-directional
Event Driven	A strategy that exploits mispricings arising in special situations or events such as mergers, restructurings, takeovers, and so on.	Non-directional
Fund of Funds	Capital is allocated to a variety of hedge funds and pooled investment vehicles which investors might not have access to otherwise.	Cannot be classified as a directional or non-directional strategy.
Multi-Strategy	Allocation of capital to a mixture of strategies simultaneously to realize short and long-term gains, and to capitalize on current investment opportunities	Cannot be classified as a directional or non-directional strategy.
Global Macro	A strategy which uses leverage and derivatives to exploit macroeconomic changes in global economies which affect securities, commodities, interest rates, exchange rates, and so on.	Directional

³² Descriptions are obtained from Agarwal and Naik (2000b) and Caglayan and Edwards (2001).

Primary Category	Definition	Type of Strategy
Emerging Markets	A strategy that focuses on investing in volatile emerging markets, capitalizing on economic changes.	Directional
Managed Futures	An arbitrage strategy that exploits relative mispricings between futures contracts and replicating portfolios of underlying securities.	Non-directional
Fixed Income Arbitrage	A strategy of holding long and short bond positions in cash and derivatives markets in order to exploit pricing discrepancies between related securities.	Non-directional
Equity Market Neutral	A strategy that employs both long and short positions in equity portfolios in order to hedge out market risk.	Non-directional
Dedicated Short Bias	A strategy similar to long/short equity hedge, except with significant net short exposure.	Directional

TABLE 1
Sample Description

Number of funds in the TASS Hedge Fund Live and Graveyard databases. Funds have at least two years of return history between January 1996 and March 2006, and the first 12 months of each fund's returns over time are excluded from this analysis. Size and age quintiles are displayed in decreasing order from highest to lowest, where 1 refers to the highest quintile, and 5 is the lowest. Size of a fund is measured as the fund's initial Net Asset Value at the start of the selected period during which it reports monthly returns. Age is defined as the number of monthly return observations reported by a fund since the date of the earliest return observation across all funds within its fund family. Management fees are defined as an annual percentage of the net assets managed by a fund, and the incentive fee is a percentage of a fund's annual net profits paid to managers in reward for good performance. 'High' management fees are between 2% and 8%, 'Moderate' management fees are between 1% and 2%, and 'Low' management fees are less than 1%. 'High' incentive fees are between 20% and 50%, 'Moderate' incentive fees are between 2% and 20%, and 'Low' incentive fees are less than 2%.

	Number of Funds		
	Live Funds	Dead Funds	Combined
Primary Category			
Convertible Arbitrage	85	77	162
Long/Short Equity Hedge	794	531	1325
Event Driven	289	127	416
Fund of Funds	673	283	956
Multi-Strategy	99	44	143
Global Macro	108	99	207
Emerging Markets	145	122	267
Managed Futures	154	210	364
Fixed Income Arbitrage	111	68	179
Equity Market Neutral	135	101	236
Dedicated Short Bias	18	14	32
Size (quintiles)			
1	475	382	857
2	607	251	858
3	527	330	857
4	495	363	858
5	507	350	857
Age (quintiles)			
1	642	211	853
2	593	269	862
3	481	372	853
4	493	363	856
5	402	461	863
Management Fee			
Low	113	135	248
Moderate	1939	1089	3028
High	559	452	1011
Incentive Fee			
Low	237	187	424
Moderate	528	287	815
High	1846	1202	3048
All Hedge Funds	2611	1676	4287

TABLE 2

Descriptive Statistics: Investment Strategies

Panel A reports the number of funds within each of the 11 investment strategies classified by TASS, in each of the five size and age quintiles. Size and age quintiles are displayed in decreasing order from highest to lowest, where 1 refers to the highest quintile, and 5 is the lowest.

Panel B reports the number of funds within each of the 11 investment strategies, in each of the three management fee and incentive fee groups.

PANEL A

	Size					Age				
	1	2	3	4	5	1	2	3	4	5
Primary Category										
Convertible Arbitrage	32	59	23	35	13	39	35	29	21	38
Long/Short Equity Hedge	341	208	209	333	234	244	214	300	293	274
Event Driven	96	123	87	68	42	91	78	82	88	77
Fund of Funds	99	165	235	191	266	241	230	169	186	130
Multi-Strategy	36	37	20	30	20	34	23	21	26	39
Global Macro	37	44	55	34	37	24	52	36	31	64
Emerging Markets	49	25	41	56	96	42	62	71	46	46
Managed Futures	107	66	79	39	73	97	79	50	69	69
Fixed Income	32	62	37	28	20	19	38	45	32	45
Equity Market Neutral	24	63	60	40	49	17	41	47	62	69
Dedicated Short Bias	4	6	11	4	7	11	10	3	2	6

PANEL B

	Inc.Fee			M.Fee		
	High	Mod.	Low	High	Mod.	Low
Primary Category						
Convertible Arbitrage	141	13	8	26	128	8
Long/Short Equity Hedge	1181	104	40	122	1145	58
Event Driven	376	18	22	73	320	23
Fund of Funds	170	513	273	218	668	70
Multi-Strategy	133	6	4	52	86	5
Global Macro	166	24	17	90	102	15
Emerging Markets	198	48	21	79	174	14
Managed Futures	285	65	14	272	66	26
Fixed Income	162	11	6	31	131	17
Equity Market Neutral	218	9	9	44	183	9
Dedicated Short Bias	28	4	0	5	25	2

TABLE 3**Cross-sectional Summary Statistics: Investment Strategies**

Panel A reports the cross-sectional mean, median and standard deviation of size and age within each of the 11 investment strategies classified by the TASS database. Panel B reports corresponding cross-sectional mean, median and standard deviation of time-series averages of alphas and appraisal ratios across all funds in each of the investment strategies.

PANEL A

	Size (millions \$)			Age (months)		
	Mean	Median	Std.dev	Mean	Median	Std.dev
Primary Category						
Convertible Arbitrage	1017	1128	1288	95	90	269
Long/Short Equity Hedge	1310	991	6008	89	76	280
Event Driven	1778	1118	8539	97	86	303
Fund of Funds	914	803	2043	103	96	297
Multi-Strategy	1132	1105	1926	91	77	281
Global Macro	1591	1026	5777	85	76	286
Emerging Markets	2279	1340	14339	90	83	239
Managed Futures	1724	1077	4825	108	92	381
Fixed Income	6490	1107	42969	82	82	224
Equity Market Neutral	987	1044	1793	76	65	242
Dedicated Short Bias	855	878	864	113	112	322
All Funds	1554	1026	10664	94	82	294

PANEL B

	Alpha (per month)			Appraisal ratio (per month)		
	Mean	Median	Std.dev	Mean	Median	Std.dev
Primary Category						
Convertible Arbitrage	-0.1068	-0.0685	0.5660	-0.1051	-0.0614	0.5672
Long/Short Equity Hedge	-0.1505	-0.0932	1.1159	-0.0800	-0.0284	0.3592
Event Driven	-0.0307	-0.0451	0.8960	-0.1663	-0.0447	0.9858
Fund of Funds	-0.0929	-0.0412	0.7214	-0.0719	-0.0320	0.4624
Multi-Strategy	-0.0733	-0.2373	1.1989	-0.8419	-0.1474	5.9151
Global Macro	-0.2317	-0.0463	1.3269	-0.0805	-0.0096	0.4056
Emerging Markets	-0.3650	-0.2766	1.8331	-0.1922	-0.0895	0.6165
Managed Futures	-0.3532	-0.1033	1.7748	-0.2495	-0.0317	2.5318
Fixed Income	-0.0629	0.0196	0.5771	0.0163	0.0113	0.3930
Equity Market Neutral	-0.0940	-0.0664	0.7422	-0.2061	-0.0363	2.1515
Dedicated Short Bias	-0.0131	-0.1807	0.8255	0.0053	-0.0378	0.2278
All Funds	-0.1485	-0.0674	1.1210	-0.1365	-0.0350	1.4844

TABLE 4

Winner and Loser Two-Way Contingency Test: Log Cross-Product Ratios

This table reports the logarithm of cross-product ratios from winner and loser two-way contingency table analysis. The 'Overall' CPR (cross-product ratio) is defined as $(WW*LL)/(WL*LW)$. The cross-product ratios for 'Winners' and 'Losers' are analogously defined as $(WW)/(WL)$ and $(LL)/(LW)$ respectively. Z tests are then used to examine whether the logarithm of each CPR is significantly greater than zero. *** indicates significance of the given log cross-product ratio at the 10% level, ** indicates significance at the 5% level, and * indicates significance at the 1% level.

	Log CPR: Overall		Log CPR: Winners		Log CPR: Losers	
	Alphas	Appraisal ratios	Alphas	Appraisal ratios	Alphas	Appraisal ratios
Primary Category						
Conv. Arbitrage	1.0357*	1.0239*	0.4920*	0.4958*	0.5437*	0.5281*
L/S Equity Hedge	0.2992*	0.3162*	0.1435*	0.1541*	0.1557*	0.1622*
Event Driven	0.7484*	0.7626*	0.3601*	0.3726*	0.3884*	0.3901*
Fund of Funds	0.4673*	0.4608*	0.2275*	0.2270*	0.2398*	0.2337*
Multi-Strategy	0.5706*	0.5333*	0.2547*	0.2422*	0.3159*	0.2911*
Global Macro	0.3156*	0.3174*	0.1385*	0.1438*	0.1771*	0.1736*
Emerging Markets	0.4966*	0.5408*	0.2390*	0.2613*	0.2577*	0.2796*
Managed Futures	0.0201	-0.0053	-0.0017	-0.0125	0.0218	0.0072
Fixed Inc. Arbitrage	1.0811*	1.0546*	0.5239*	0.5135*	0.5572*	0.5411*
Eq.Market Neutral	0.3906*	0.3424*	0.1714*	0.1564*	0.2191*	0.1860*
Dedicated Short Bias	0.3166*	0.3138*	0.1099**	0.1228*	0.2067*	0.1910*
Size (quintile)						
1	0.3856*	0.3857*	0.2205*	0.2277*	0.1651*	0.1580*
2	0.5004*	0.5051*	0.2715*	0.2742*	0.2288*	0.2310*
3	0.4420*	0.4313*	0.1563*	0.1498*	0.2858*	0.2815*
4	0.4641*	0.4619*	0.2352*	0.2344*	0.2288*	0.2275*
5	0.4147*	0.4298*	0.1261*	0.1442*	0.2887*	0.2855*
Age (quintile)						
1	0.3900*	0.3881*	0.1959*	0.2044*	0.1941*	0.1837*
2	0.4189*	0.4398*	0.2275*	0.2450*	0.1913*	0.1948*
3	0.5097*	0.4944*	0.2401*	0.2303*	0.2696*	0.2641*
4	0.5034*	0.4998*	0.2015*	0.1950*	0.3020*	0.3048*
5	0.4591*	0.4520*	0.1416*	0.1329*	0.3176*	0.3192*
Management Fee						
Low	0.4558*	0.4554*	0.2027*	0.2121*	0.2530*	0.2432*
Moderate	0.4676*	0.4714*	0.2286*	0.2331*	0.2391*	0.2383*
High	0.3401*	0.3384*	0.1869*	0.1850*	0.1533*	0.1533*
Incentive Fee						
Low	0.3965*	0.4102*	0.1824*	0.1874*	0.2141*	0.2228*
Moderate	0.4502*	0.4546*	0.2165*	0.2215*	0.2338*	0.2331*
High	0.4526*	0.4532*	0.2216*	0.2250*	0.2310*	0.2282*
All Hedge Funds	0.4417*	0.4430*	0.2092*	0.2134*	0.2324*	0.2296*

TABLE 5

Winner and Loser Two Way Contingency Test: Comparison of Overall Log Cross-Product Ratios

This table reports results from the test of the significance of the difference between overall log cross-product ratios of funds in different primary categories/quintiles/groups, using appraisal ratios as the performance measure. (Results using alphas as the performance measure are not reported here, given that they are very similar). Investment categories, size and age quintiles, and management and incentive fee groups are displayed along the columns. The figure reported in row i and column j is then the Z-statistic from the test of the significance of the following difference: the overall log cross-product ratio of funds in the category/quintile/group corresponding to column i, minus the overall log cross-product ratio of funds in the category/quintile/group corresponding to column j. A (one-sided) Z-statistic of 1.282, 1.645 and 1.96 corresponds to significance at the 10%, 5% and 1 % levels respectively. Bold print and *** indicates significance at the 10% level, bold print and ** indicates significance at the 5% level, and bold print alone indicates significance at the 1% level.

Conv. Arb	L/S Equity Hedge	Event Driven	Fund of Funds	Multi-Strategy	Global Macro	Emerg. Mkts	Manag. Futures	Fixed Inc. Arb.	Equity Market Neutral	Ded. Short Bias
0	14.96	4.97	11.64	7.56	11.59	8.70	19.28	-0.48	11.34	7.20
-14.96	0	-14.13	-6.04	-4.40	-0.03	-6.20	9.78	-15.48	-0.61	0.03
-4.97	14.13	0	9.10	4.21	8.94	5.17	19.13	-5.51	8.63	4.88
-11.64	6.04	-9.10	0	-1.4***	3.17	-2.13	13.54	-12.18	2.68	1.64
-7.56	4.40	-4.21	1.44***	0	3.45	-0.13	9.75	-8.00	3.09	2.20
-11.59	0.03	-8.94	-3.17	-3.45	0	-4.23	6.38	-12.03	-0.43	0.04
-8.70	6.20	-5.17	2.13	0.13	4.23	0	12.44	-9.20	3.83	2.42
-19.28	-9.78	-19.14	-13.54	-9.75	-6.38	-12.44	0	-19.72	-7.01	-3.45
0.48	15.48	5.51	12.18	8.00	12.03	9.20	19.72	0	11.79	7.50
-11.34	0.61	-8.63	-2.68	-3.09	0.43	-3.83	7.01	-11.78	0	0.30
-7.20	-0.03	-4.87	-1.6***	-2.20	-0.04	-2.42	3.45	-7.50	-0.30	0

Size (quintile)				
1	2	3	4	5
0	-4.6440	-1.6414***	-3.0406	-1.6292***
4.6440	0	2.5287	1.6275***	2.6480
1.6414***	-2.5287	0	-1.0677	0.0518
3.0406	-1.6275***	1.0677	0	1.1524
1.6292***	-2.6480	-0.0518	-1.1524	0
Age (quintile)				
1	2	3	4	5
0	-2.3121	-4.2811	-4.0099	-1.8558**
2.3121	0	-2.1217	-2.0919	-0.3487
4.2811	2.2127	0	-0.1736	1.1537
4.0099	2.0919	0.1736	0	1.2284
1.8558**	0.3487	-1.1537	-1.2284	0
Management Fee				
Low	Moderate	High		
0	6.5505	5.3207		
-6.5505	0	-1.0358		
-5.3207	1.0358	0		
Incentive Fee				
Low	Moderate	High		
0	0.1044	-1.5446***		
-0.1044	0	-1.6153***		
1.5446***	1.6153***	0		

TABLE 6

Winner and Loser Two Way Contingency Test: Comparison of Winner Log Cross-Product Ratios

This table reports results from the test of the significance of the difference between winner log cross-product ratios of funds in different categories/quintiles/groups, using appraisal ratios as the performance measure. A (one-sided) Z-statistic of 1.282, 1.645 and 1.96 corresponds to significance at the 10%, 5% and 1 % levels respectively. Bold print and *** indicates significance at the 10% level, bold print and ** indicates significance at the 5% level, and bold print alone indicates significance at the 1% level.

Conv. Arb	L/S Equity Hedge	Event Driven	Fund of Funds	Multi-Strategy	Global Macro	Emerg. Mkts	Manag. Futures	Fixed Inc. Arb.	Equity Market Neutral	Ded. Short Bias
0	10.22	3.31	7.86	5.52	8.16	5.98	13.47	-0.39	7.99	5.32
-10.22	0	-9.78	-4.31	-2.52	0.33	-1.18	7.17	-10.65	-0.08	0.49
-3.31	9.78	0	6.21	3.38	6.50	3.67	13.58	-3.76	6.28	3.81
-7.86	4.31	-6.21	0	-0.43	2.60	-1.3***	9.85	-8.30	2.26	1.6** *
-5.52	2.52	-3.38	0.43	0	2.22	-0.47	6.51	-5.88	1.96*	1.7**
-8.16	-0.33	-6.50	-2.60	-2.22	0	-3.14	4.36	-8.52	-0.31	0.30
-5.98	4.18	-3.67	1.29***	0.47	3.14	0	8.82	-6.38	2.86	2.08
-13.47	-7.17	-13.58	-9.85	-6.51	-4.36	-8.82	0	-13.83	-4.82	-2.06
0.39	10.65	3.76	8.30	5.88	8.52	6.38	13.83	0	8.35	5.56
-7.99	0.08	-6.28	-2.26	-1.96*	0.31	-2.86	4.82	-8.35	0	0.49
-5.32	-0.49	-3.81	-1.6***	-1.69**	-0.30	-2.08	2.06	-5.56	-0.49	0

Size (quintile)				
1	2	3	4	5
0	-2.5796	3.9466	-0.3823	4.3436
2.5796	0	6.0018	2.1268	6.4242
-3.9466	-6.0018	0	-4.1572	0.2562
0.3823	-2.1268	4.1572	0	4.5437
-4.3436	-6.4242	-0.2562	-4.5437	0
Age (quintile)				
1	2	3	4	5
0	-2.5827	-1.4732***	0.4761	2.9034
2.5827	0	0.8108	2.4617	4.4721
1.4732***	0.8108	0	1.6207***	3.7082
-0.4761	-2.4617	-1.6207***	0	2.2346
-2.9034	-4.4721	-3.7082	-2.2346	0
Management Fee				
Low	Moderate	High		
0	3.3647	1.7513**		
-3.3647	0	-1.9099**		
-1.7513**	1.9099**	0		
Incentive Fee				
Low	Moderate	High		
0	-0.3567	-1.9103**		
0.3567	0	-1.7516**		
1.9103**	1.7516**	0		

TABLE 7

Winner and Loser Two Way Contingency Test: Comparison of Loser Log Cross-Product Ratios

This table reports results from the test of the significance of the difference between loser log cross-product ratios of funds in different categories/quintiles/groups, using appraisal ratios as the performance measure. A (one-sided) Z-statistic of 1.282, 1.645 and 1.96 corresponds to significance at the 10%, 5% and 1 % levels respectively. Bold print and *** indicates significance at the 10% level, bold print and ** indicates significance at the 5% level, and bold print alone indicates significance at the 1% level.

Conv. Arb	L/S Equity Hedge	Event Driven	Fund of Funds	Multi-Strategy	Global Macro	Emerg. Mkts	Manag. Futures	Fixed Inc. Arb.	Equity Market Neutral	Ded. Short Bias
0	10.94	3.71	8.60	5.17	8.23	6.33	13.79	-0.29	8.05	4.86
-10.94	0	-10.20	-4.23	-3.70	-0.37	-4.58	6.66	-11.24	-0.78	-0.46
-3.71	10.20	0	6.66	2.57	6.15	3.64	13.48	-4.04	5.93	3.08
-8.60	4.23	-6.66	0	-1.6***	1.88**	-1.72**	9.30	-8.92	1.5***	0.68
-5.17	3.7	-2.57	1.61***	0	2.66	0.29	7.28	-5.43	2.41	1.4** *
-8.23	0.37	-6.15	-1.88**	-2.66	0	-2.84	4.65	-8.49	-0.30	-0.25
-6.33	4.58	-3.64	1.72**	-0.29	2.84	0	8.77	-6.63	2.55	1.3** *
-13.79	-6.66	-13.49	-9.30	-7.28	-4.65	-8.77	0	-14.06	-5.10	-2.83
0.29	11.24	4.04	8.92	5.43	8.49	6.63	14.06	0	8.32	5.04
-8.05	0.78	-5.93	-1.5***	-2.41	0.31	-2.55	5.10	-8.32	0	-0.07
-4.86	0.46	-3.08	-0.68	-1.4***	0.25	-1.3***	2.83	-5.04	0.07	0

Size (quintile)				
1	2	3	4	5
0	-2.9760	-6.3108	-3.8952	-6.7045
3.9760	0	-2.4593	0.1857	-2.7235
6.3108	2.4593	0	2.6906	-0.1891
3.8952	-0.1857	-2.6906	0	-2.9692
6.7045	2.7235	0.1891	2.9692	0
Age (quintile)				
1	2	3	4	5
0	-0.6972	-4.5813	-6.1774	-5.6256
0.6972	0	-3.8020	-5.4410	-5.0596
4.5813	3.8020	0	-1.8844**	-2.1410
6.1774	5.4410	1.8844**	0	-0.5309
5.6256	5.0596	2.1410	0.5309	0
Management Fee				
Low	Moderate	High		
0	5.8865	5.7586		
-5.8865	0	0.4458		
-5.7586	-0.4458	0		
Incentive Fee				
Low	Moderate	High		
0	0.5031	-0.2711		
-0.5031	0	-0.5303		
0.2711	0.5303	0		

TABLE 8

Winner and Loser Two-Way Contingency Test: Adjusted Incentive Fees

Panel A reports the logarithm of cross-product ratios from winner and loser two-way contingency table analysis across incentive fee groups. Incentive fees for any particular fund are now defined as the minimum incentive fee level across all funds in the same fund family. *** indicates significance at the 10% level, ** indicates significance at the 5% level, and * indicates significance at the 1% level.

Panel B reports results from the test of the significance of the difference between winner log cross-product ratios of funds in different incentive fee groups, using appraisal ratios as the performance measure. As before, the figure reported in row i and column j is the Z-statistic from the test of the significance of the difference between the winner log cross-product ratios of funds in the fee groups corresponding to columns i and j. Panel C reports analogous results from the test of the significance of the difference between loser log cross-product ratios of funds. Bold print and *** indicates significance at the 10% level, bold print and ** indicates significance at the 5% level, and bold print alone indicates significance at the 1% level.

PANEL A

	Log CPR: Overall		Log CPR: Winners		Log CPR: Losers	
	Alphas	Appraisal ratios	Alphas	Appraisal ratios	Alphas	Appraisal ratios
Incentive Fee						
Low	0.4221*	0.4292*	0.1671*	0.1714*	0.2550*	0.2578*
Moderate	0.4500*	0.4526*	0.2192*	0.2239*	0.2308*	0.2287*
High	0.4521*	0.4551*	0.2995*	0.2342*	0.2226*	0.2208*

PANEL B

Low	Moderate	High
0	-0.9997	-3.8421
0.9997	0	-3.2867
3.8421	3.2867	0

PANEL C

Low	Moderate	High
0	0.7547	2.2848
-0.7547	0	1.8457**
-2.2848	-1.8457**	0

TABLE 9
Parametric Tests

This table reports averages of estimated slope coefficients for month-by-month cross-section regressions of alphas (appraisal ratios) in each month on alphas (appraisal ratios) in the previous month. (All cross-sectional regressions are run with an intercept). T-tests are performed on the time-series averages of these coefficients, and the t-statistics for regressions in each category/quintile/group below are in parentheses. *** indicates significance at the 10% level, ** indicates significance at the 5% level, and * indicates significance at the 1% level, for a one-sided test.

	One-month alphas on one-month alphas		One-month appraisal ratios on one-month appraisal ratios	
Primary Category				
Convertible Arbitrage	0.2958*	(6.7235)	0.3708*	(9.0064)
Long/Short Eq. Hedge	0.0903*	(3.3901)	0.1375*	(4.4550)
Event Driven	0.2093*	(4.7825)	0.3126*	(7.0974)
Fund of Funds	0.1531*	(5.0270)	0.1777*	(5.5258)
Multi-Strategy	0.1762*	(3.9102)	0.2668*	(3.1603)
Global Macro	0.1124*	(4.0325)	0.0807*	(2.8204)
Emerging Markets	0.1408*	(4.4172)	0.2760*	(5.2748)
Managed Futures	0.0256	(0.8147)	0.0651	(0.6870)
Fixed Income Arbitrage	0.2548*	(6.9356)	0.3677*	(12.516)
Equity Market Neutral	0.0650*	(2.0607)	0.1488*	(4.4143)
Dedicated Short Bias	0.1053**	(1.8344)	0.1425*	(2.4517)
Size (quintile)				
1	0.0946*	(3.6075)	0.1496*	(5.4324)
2	0.1142*	(4.3238)	0.1951*	(6.9240)
3	0.0917*	(3.4615)	0.2170*	(5.1372)
4	0.1338*	(5.5065)	0.1884*	(6.8169)
5	0.1151*	(4.7681)	0.2723*	(2.3209)
Age (quintile)				
1	0.0943*	(3.9866)	0.2066*	(3.0911)
2	0.0993*	(3.8126)	0.1975*	(5.9264)
3	0.1333*	(5.0941)	0.1944*	(5.7676)
4	0.1249*	(4.2519)	0.1782*	(6.2344)
5	0.0887*	(2.8752)	0.1927*	(6.3202)
Management Fee				
Low	0.0945*	(3.8518)	0.2073*	(6.5452)
Moderate	0.1191*	(5.5029)	0.2021*	(6.9208)
High	0.0727*	(2.8390)	0.1997*	(2.4872)
Incentive Fee				
Low	0.1283*	(4.6612)	0.1618*	(5.0083)
Moderate	0.1090*	(4.9507)	0.2444*	(4.4748)
High	0.1033*	(4.7320)	0.2505*	(4.3101)
All Hedge Funds	0.1072*	(4.9922)	0.2265*	(4.7583)

TABLE 10

Parametric Tests: Comparison of Coefficients

This table reports results from the test of the significance of the difference between coefficients reported by funds in different primary categories, from the parametric tests in Table 9, using appraisal ratios as the performance measure. (Results using alphas as the performance measure are not reported here, given that they are very similar). Investment strategies are displayed along the columns. (Results for age, size and performance fee groups are not reported here given these are generally not significant at the 10% level). The figure reported in row i and column j is then the t-statistic from the test of the significance of the following difference: the coefficient generated by funds in the primary category corresponding to column i, minus the coefficient for the primary category corresponding to column j. A (one-sided) t-statistic of 1.282, 1.645 and 1.96 corresponds to significance at the 10%, 5% and 1 % levels respectively. Bold print and *** indicates significance at the 10% level, bold print and ** indicates significance at the 5% level, and bold print alone indicates significance at the 1% level.

Conv. Arb	L/S Equity Hedge	Event Driven	Fund of Funds	Multi-Strategy	Global Macro	Emerg. Mkts	Manag. Futures	Fixed Inc. Arb.	Equity Market Neutral	Ded. Short Bias
0	4.53	0.97	3.70	1.11	5.79	1.4***	2.96	0.06	4.17	3.20
-4.53	0	-3.26	-0.90	-1.4***	1.3***	-2.28	0.73	-5.40	-0.25	-0.08
-0.97	3.26	0	2.47	0.48	4.42	0.54	2.37	-1.04	2.95	2.33
-3.70	0.90	-2.47	0	-0.99	2.25	-1.6***	1.13	-4.36	0.62	0.53
-1.11	1.4***	-0.48	0.99	0	2.09	-0.09	1.6***	-1.13	1.3***	1.21
-5.79	-1.3***	-4.42	-2.25	-2.09	0	-3.28	0.16	-7.00	-1.5***	-0.95
-1.4***	2.28	-0.54	1.6***	-0.09	3.28	0	1.95**	-1.5***	2.04	1.7**
-2.96	-0.73	-2.37	-1.13	-1.6***	-0.16	-1.95**	0	-3.05	-0.83	-0.70
-0.06	5.40	1.04	4.36	1.13	7.01	1.5***	3.05	0	4.89	3.46
-4.17	0.25	-2.95	-0.62	-1.3***	1.5***	-2.05	0.83	-4.89	0	-0.09
-3.20	0.08	-2.33	-0.53	-1.21	0.95	-1.7**	0.70	-3.46	-0.09	0

TABLE 11

Parametric Tests: Pooled OLS Regressions for Investment Strategies

This table reports estimated slope coefficients for pooled time-series cross-sectional OLS regressions of alphas (appraisal ratios) in each month on alphas (appraisal ratios) in the previous month. (All cross-sectional regressions are run with an intercept). T-tests are performed after correcting standard errors for autocorrelation and heteroskedasticity, and the t-statistics for regressions in each primary category below are in parentheses. *** indicates significance at the 10% level, ** indicates significance at the 5% level, and * indicates significance at the 1% level, for a one-sided test.

	One-month alphas on one-month alphas		One-month appraisal ratios on one-month appraisal ratios	
Primary Category				
Convertible Arbitrage	0.2506*	(5.8562)	0.3755*	(12.575)
Long/Short Eq. Hedge	0.0708*	(5.4064)	0.1079*	(14.216)
Event Driven	0.0734*	(5.0560)	0.2705*	(5.5732)
Fund of Funds	0.0883*	(5.3052)	0.1718*	(18.043)
Multi-Strategy	0.0910*	(5.6968)	0.2586*	(5.0135)
Global Macro	0.0888*	(5.8989)	0.0170	(0.3288)
Emerging Markets	0.1006*	(6.5834)	0.2471*	(8.7558)
Managed Futures	0.0636*	(3.7860)	-0.2630	(-0.9840)
Fixed Income Arbitrage	0.0667*	(3.5562)	-0.2290	(-0.8770)
Equity Market Neutral	0.0621*	(3.2732)	-0.2550	(-0.9360)
Dedicated Short Bias	0.0634 **	(1.8344)	-0.2500	(-0.9300)
All Hedge Funds	0.0893*	(9.8465)	0.1711*	(4.7096)

TABLE 12

Parametric Tests: Comparison of Coefficients for Pooled OLS Regressions

This table reports t-statistics from the test of the significance of the difference between coefficients reported by funds in different primary categories, from the pooled OLS regression tests in Table 11, using appraisal ratios as the performance measure. Bold print and *** indicates significance at the 10% level, bold print and ** indicates significance at the 5% level, and bold print alone indicates significance at the 1% level.

Conv. Arb	L/S Equity Hedge	Event Driven	Fund of Funds	Multi-Strategy	Global Macro	Emerg. Mkts	Manag. Futures	Fixed Inc. Arb.	Equity Market Neutral	Ded. Short Bias
0	8.69	1.84**	6.50	1.96	5.99	3.13	2.38	2.30	2.30	2.31
-8.69	0	-3.31	-5.25	-2.89	1.7**	-4.76	1.39***	1.3***	1.33***	1.3** *
-1.8**	3.31	0	2.00	0.17	3.57	0.42	1.96	1.88**	1.9**	1.9**
-6.50	5.25	-2.00	0	-1.7**	2.94	-2.53	1.63***	1.53**	1.57***	1.6** *
-1.96	2.89	-0.17	1.65**	0	3.30	0.20	1.92**	1.83**	1.86**	1.9**
-5.99	-1.7**	-3.57	-2.94	-3.30	0	-3.90	1.03	0.92	0.98	0.98
-3.13	4.76	-0.42	2.53	-0.20	3.90	0	1.9**	1.81**	1.84**	1.8**
-2.38	-1.4***	-1.96	-1.6***	-1.92**	-1.03	-1.9**	0	-0.09	-0.02	-0.03
-2.30	-1.3***	-1.88**	-1.5***	-1.83**	-0.92	-1.81**	0.09	0	0.07	0.06
-2.30	-1.3***	-1.90**	-1.6***	-1.85**	-0.98	-1.84**	0.02	-0.07	0	-0.01
-2.31	-1.3***	-1.91**	-1.6***	-1.86**	-0.98	-1.84**	0.03	-0.06	0.01	0

TABLE 13

Kolmogorov-Smirnov Test for Multi-period Performance Persistence

This table reports K-S test statistics and p-values from the two-sided Kolmogorov-Smirnov test based on alphas and appraisal ratios. * indicates significance at the 5% level; (i.e. the actual distribution of two and more consecutive wins/losses is significantly different from the theoretical distribution at the 5% level, indicating multi-period performance persistence).

	Alphas				Appraisal Ratios			
	Wins		Losses		Wins		Losses	
	K-S	P-value	K-S	P-value	K-S	P-value	K-S	P-value
Primary Category								
Conv. Arb	0.1620*	0	0.1649*	0	0.1971*	0	0.1963*	0
Long/Short	0.0710*	0	0.0453*	0	0.0564*	0	0.0541*	0
Equity Hedge								
Event Driven	0.1145*	0	0.1128*	0	0.0965*	0	0.1150*	0
Fund of Funds	0.0665*	0	0.0665*	0	0.0701*	0	0.0660*	0
Multi-Strategy	0.0751*	0.0125	0.1042*	0.0001	0.0599	0.0808	0.1297*	0
Global Macro	0.0284	0.7119	0.0381	0.3532	0.0508	0.0910	0.0240	0.8800
Emerg. Mkts	0.0636*	0.0009	0.0856*	0	0.0680*	0.0003	0.0829*	0
M.Futures	0.0214	0.6474	0.0068	1	0.0119	0.9958	0.0071	1
Fixed Inc. Arb	0.1697*	0	0.1274*	0	0.1658*	0	0.1323*	0
Equity Market	0.0834*	0.0004	0.0681*	0.0057	0.0610*	0.0196	0.0443	0.1646
Neutral								
Ded. Sh. Bias	0.0715	0.5060	0.0614	0.6868	0.0900	0.2385	0.0880	0.2201
Size(quintile)								
1	0.0680*	0	0.0306*	0.0033	0.0717*	0	0.0270*	0.0133
2	0.0745*	0	0.0782*	0	0.0942*	0	0.0830*	0
3	0.0534*	0.00001	0.0732*	0	0.0474*	0.000187	0.0746*	0
4	0.0835*	0	0.0675*	0	0.0850*	0	0.0666*	0
5	0.0546*	0	0.0805*	0	0.0546*	0.000003	0.0836*	0
Age (quintile)								
1	0.0618*	0	0.0518*	0	0.0661*	0	0.0571*	0
2	0.0695*	0	0.0619*	0	0.0703*	0	0.0673*	0
3	0.0744*	0	0.0710*	0	0.0667*	0	0.0726*	0
4	0.0838*	0	0.0862*	0	0.0748*	0	0.0812*	0
5	0.0600*	0.0011	0.0905*	0	0.0530*	0.0058	0.0752*	0
Man. Fee								
Low	0.0503*	0.0314	0.0501*	0	0.0460	0.0583	0.1012*	0
Moderate	0.0767*	0	0.0697*	0	0.0767*	0	0.0678*	0
High	0.0573*	0	0.0501*	0	0.0612*	0	0.0463*	0
Inc. Fee								
Low	0.0738*	0	0.0565*	0.00014	0.0572*	0.0001194	0.0596*	0
Moderate	0.0709*	0	0.0649*	0	0.0643*	0	0.0623*	0
High	0.0740*	0	0.0728*	0	0.0731*	0	0.0773*	0
All Funds	0.0699*	0	0.0690*	0	0.0680*	0	0.0630*	0

TABLE 14

Kolmogorov-Smirnov Test for Multi-period Performance Persistence: Controlling for Short-term Serial Correlation and Adjusting Incentive Fees

Panel A reports results across investment strategies for the test of the difference between the observed distribution of 6 and more consecutive wins or losses, and the theoretical distribution of 6 and more consecutive wins or losses under the null of no persistence. * indicates significance at the 5% level.

Panel B reports results of the original K-S test, where incentive fees for a particular fund are now defined as the minimum incentive fee level charged across all funds in the same fund family. * indicates significance at the 5% level.

PANEL A

Primary Category	Alphas				Appraisal Ratios			
	Wins		Losses		Wins		Losses	
	K-S	P-value	K-S	P-value	K-S	P-value	K-S	P-value
Conv. Arb	0.3451*	0.0057	0.2186	0.1379	0.3315*	0.0256	0.2186	0.1912
L/S Eq. Hedge	0.0964	0.0755	0.1482*	0.0011	0.0734	0.3224	0.1980*	0
Event Driven	0.2285*	0.0006	0.3405*	0	0.2392*	0.0006	0.3581*	0
Fund of Funds	0.1112	0.0992	0.2207*	0	0.1068	0.1310	0.1186*	0.0467
Multi-Strategy	0.1353	0.8841	0.1857	0.5443	0.2154	0.4962	0.2684	0.1282
Global Macro	0.0861	0.9944	0.1720	0.3828	0.1740	0.5493	0.1349	0.7171
Emerg. Mkts	0.1211	0.5702	0.0924	0.8978	0.1239	0.5506	0.1606	0.1660
Manag. Futures	0.0421	1	0.0767	0.9874	0.2210	0.0688	0.0667	0.9920
Fixed Inc. Arb	0.2779*	0.0234	0.3230*	0.0040	0.2759*	0.0473	0.3925*	0.0027
Equity Neutral	0.1425	0.5231	0.1756	0.2753	0.1305	0.7414	0.1734	0.3306
Ded. Short Bias	0.3000	0.6490	0.2000	0.9748	0.3016	0.7899	0.4167	0.4710

PANEL B

Incentive Fee	Alphas				Appraisal Ratios			
	Wins		Losses		Wins		Losses	
	K-S	P-value	K-S	P-value	K-S	P-value	K-S	P-value
Low	0.0616*	0	0.0838*	0	0.0604*	0	0.0746*	0
Moderate	0.0610*	0	0.0662*	0	0.0628*	0	0.0660*	0
High	0.0743*	0	0.0624*	0	0.0751*	0	0.0637*	0

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Value and Momentum

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Abstract

This paper proposes and tests a simple explanation for momentum profits: systematic outperformance arises because certain stocks have persistently strong fundamentals which are not fully valued by the market. We find that “winner” portfolios have higher book-to-market ratios than “loser” portfolios, and the economic and statistical significance of momentum profits is markedly reduced when calculated above value benchmarks. Profits to a momentum strategy also disappear when removing firms with high (low) value from the winner (loser) portfolios. A large component of the returns to relative strength portfolios may thus stem from such portfolios overweighting high value stocks. Our results suggest a close relation between the value and momentum anomalies.

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1. Introduction

The momentum anomaly, first documented by Jegadeesh (1990), Lehmann (1990) and Jegadeesh and Titman (1993), remains one of the most puzzling phenomena to financial economists. It appears to violate the weakest form of financial market efficiency, since it suggests that investors can obtain excess returns by trading on past price information. While many anomalies disappear soon after they are published, momentum has continued to be prevalent many years after its initial discovery (Jegadeesh and Titman (2001)). Moreover, whereas excess returns to small and value stocks may be at least partly attributable to their higher risk (see, e.g., Fama and French (1995), Vassalou and Xing (2004)), it is less obvious that past winners should be riskier than the average stock.

Researchers have proposed a number of existing explanations for momentum. Behavioral theories include conservatism in expectations updating, (Barberis, Shleifer and Vishny (1998)), biased self attribution (Daniel, Hirshleifer and Subrahmanyam (1998)) and selective information conditioning (Hong and Stein (1999)). Rational explanations argue that the high returns of winner portfolios are expected by investors as compensation for risk. Conrad and Kaul (1998) and Berk, Green and Naik (1999) argue that stocks with high realized returns, selected by a momentum strategy, are those with high unconditional expected returns.² Johnson (2002) shows that momentum can arise from episodic shocks to dividend growth rates, and Chordia and Shivakumar (2002) point to business cycle explanations.³

This paper investigates an alternative, quite simple explanation for momentum. Certain stocks persistently outperform (underperform) because they are intrinsically of higher (lower) quality, in a way that is not immediately valued by the market. Put differently, high returns in a particular period result from the firm possessing attractive characteristics, and systematic undervaluation of these persistent characteristics also causes the firm to outperform in future periods.

² However, Grundy and Martin (2001) and Jegadeesh and Titman (2001) provide evidence contradicting this explanation.

³ Lesmond, Schill and Zhuo (2004) argue that momentum profits may not exist in the first place, as the strategy requires frequent trading in securities with high transaction costs. Korajczyk and Sadka (2005) disagree.

We hypothesize that strong fundamentals may be one such characteristic. This is motivated by the findings of Fama and French (1992), Lakonishok, Shleifer and Vishny (1994) and others, that firms with high fundamentals-to-price ratios typically outperform the market. Lakonishok et al., La Porta, Lakonishok, Shleifer and Vishny (1997) and Daniel and Titman (1997, 2006) provide evidence that this outperformance is due to the market systematically undervaluing the benefits of strong fundamentals, rather than the higher returns reflecting rational compensation for risk. Strong fundamentals can cause firms to systematically outperform, and thus exhibit upward momentum.

We commence by showing that momentum profits are somewhat weaker over 1999-2006, a later period than that examined by earlier studies. However, momentum profits are still strongly significant over the overall 1962-2006 period. For example, the monthly return to a strategy that sorts stocks into deciles based on their prior six-month return and holds them for the following six months is 0.93%, significant at the 1% level. The key result of the paper is to demonstrate that momentum profits become insignificant when calculated over stocks with similar book-to-market ratios. Returns to the 6m/6m are now halved to 0.48%. While this number is borderline significant, the returns for momentum strategies formed on different sorting and holding periods become statistically insignificant, in addition to their economically important fall in magnitude. While prior researchers have used different stock filters when constructing their momentum portfolios, our use of value benchmarks attenuates momentum profits regardless of the filters used. Similar attenuation occurs when calculating raw returns and removing high (low) value firms from the winner (loser) portfolio.

In sum, a sizable component of momentum profits appears to stem from the book-to-market characteristics of the long, (or winner) and short, (or loser) portfolios. Hence the source of value profits may also be a significant source of momentum returns. The implications for whether the momentum anomaly is behavioral or rational depends on one's beliefs on the source of excess returns to value stocks. The hypothesis that motivated this paper was that high book-to-market stocks have inherently strong fundamentals which are persistently undervalued by the market; in short, certain stocks systematically perform better (exhibit positive momentum) simply because they *are* better. This view is consistent with the previously cited evidence that returns to a value strategy stem from mispricing. Under this interpretation, momentum does not represent fair compensation for risk (as proposed by rational explanations) but is a behavioral

phenomenon. However, the underlying behavioral bias is ignorance of a firm's fundamentals, which is subtly different from delayed reaction to one-off items of news, as theorized by Barberis et al. (1998), Daniel et al. (1998) and Hong and Stein (1999).

An alternative view is that superior returns to value stocks stem from their greater risk (e.g. Fama and French (1995)). Thus, subtracting returns to value benchmarks removes the risk of a momentum portfolio, in turn suggesting that the high unadjusted returns to a relative strength portfolio also stem from compensation for risk. Regardless of whether excess returns to value stocks are rational or behavioral, our results point to a close interaction between the value and momentum anomalies, thus implying that a convincing explanation for value profits will also account for a large portion of momentum returns. However, we find that value profits are unaffected by subtracting returns to a momentum benchmark, suggesting that the returns to a value strategy stem from a different source to momentum profits.

Closest to our paper is a study by Asness (1997). He also considers both value and momentum, but seeks to see whether the momentum strategy is most profitable among high or low value stocks (and vice-versa), rather than whether a component of momentum profits stems from their value characteristics. He therefore does not construct benchmark-adjusted returns. Our explanation of momentum is similar in spirit to Chordia and Subrahmanyam (2006), who find that a significant component of momentum can be explained by earnings momentum.

This paper proceeds as follows. Section 2 examines the time trend in the profitability of momentum strategies, including the recent period of 1999-2006 which is not covered by existing studies. Section 3 demonstrates that momentum profits across all time horizons are significantly reduced when calculated in excess of value benchmarks. Section 4 concludes.

2. Momentum Profits, 1962-2006

This section replicates and extends earlier findings on the profitability of momentum strategies for the sample period 1962-2006. Jegadeesh and Titman's (1993) original study found significant profits to a momentum strategy over 1962-1989. Their subsequent 2001 paper found that profits continued to be strong in 1990-1998. In addition to being an

“out-of-sample” verification of their original findings, this result was particularly striking as most financial market anomalies disappear (or are significantly attenuated) soon after initial publication.

We start by examining whether momentum has continued to be profitable in the more recent 1999-2006 period. We largely follow the methodology of Jegadeesh and Titman (1993, 2001) to construct the momentum portfolios. We obtain return data from CRSP for all stocks traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) or Nasdaq. If a stock delists at any point during the sample period, we use the delisting return as the final return of the stock.⁴ Since the analysis in Section 3 requires calculation of book-to-market ratios, we exclude all stocks for which this ratio cannot be calculated. (The details of this calculation are in Section 3).

We first construct 3, 6, 9 and 12-month lagged returns for all stocks in the sample. If, for any month t and for any particular stock, any one of the (monthly) returns during the past J months is missing, then the J -month lagged return is also reported as missing at time t for that stock. For month t , stocks are then classified into the winner portfolio if their J -month lagged return is within the top decile of J -month lagged returns for all stocks at time t , and also if their market price is at least \$1 for month $t-1$. The loser portfolio for month t is analogously defined. Both winner and loser portfolios are equal-weighted.

While Jegadeesh and Titman (1993) do not use any kind of filters in constructing winner and loser portfolios, Jegadeesh and Titman (2001) exclude stocks priced below \$5, and all stocks with a market capitalization in the smallest NYSE decile. This is to ensure that their results are not driven by small or illiquid stocks, or by bid-ask bounce, but they state that these filters have little overall effect. Our baseline analysis uses a \$1 price filter and no market capitalization filter, as this is a middle ground between Jegadeesh and Titman (1993) and Jegadeesh and Titman (2001). However, we also report our results under these two alternative methodologies.

To increase the power of our tests, we construct overlapping portfolios. For J in $\{3,6,9,12\}$, K in $\{3,6,9,12\}$, and for each month t , we construct a momentum portfolio by buying the winner portfolio and shorting the loser portfolio for month t , based on performance over the past J months. This portfolio is then held for K months. Therefore,

⁴ This latter procedure is not followed by Jegadeesh and Titman (1993, 2001), but is followed in most other asset pricing papers. Eliminating this step makes little difference to our results.

in each month there will be K sub-portfolios: the portfolio newly constructed in the current month, plus the $K-1$ portfolios formed in the previous $K-1$ months. All are equally weighted to give the return for the momentum portfolio in month t , for the (J,K) strategy. For each value of J and K , we then calculate average returns to the momentum portfolio over time.

Table 1 illustrates the results under our baseline strategy of a \$1 price filter. Panel A shows that momentum profits over all horizons for the period 1999-2006 are significantly less than in the earlier periods studied by Jegadeesh and Titman (1993, 2001). Profits are not statistically significant at any horizon; indeed, in more than half of the (J,K) combinations, momentum profits are negative. It is not surprising that momentum profits have substantially declined since Jegadeesh and Titman (2001), given the substantial attention that the strategy has received by both academics and practitioners.

Despite the poor recent performance of the momentum strategy, profits remain highly significant over the entire 1962-2006 period. For example, the 6m/6m strategy earned an average of 0.93% per month, significant at less than the 1% level, and commensurate with the 1.17% monthly return reported by Jegadeesh and Titman (2001) for 1962-98.

Table 2 illustrates the time-trend in momentum profits for the 6m/6m strategy under different methodologies. To achieve an approximately equal split of the sample period, we divide the sample into 1962-1989 (studied by Jegadeesh and Titman (1993)) and 1990-2006. The “JT 2001” strategy excludes stocks priced below \$5 and stocks in the bottom decile by market capitalization. “JT 1993” employs no filters, but excludes Nasdaq stocks. “Baseline” includes Nasdaq stocks and uses a \$1 price filter, and “HLS 2000” is the methodology of Hong, Lim and Stein (2000). They sort firms into terciles rather than deciles, to ensure that momentum profits are not purely driven by firms at the extremes. The top (bottom) tercile consists of firms in the top (bottom) three deciles according to past returns, and the middle tercile contains the middle four deciles. They employ no filters and include Nasdaq stocks.

As the table illustrates, momentum profits are highly significant for 1962-1989 across all strategies. As expected, the Hong et al. (2000) methodology produces the lowest profits, owing to the coarser sort. The profitability of all strategies declines for 1990-2006, with only JT 2001 producing statistically significant returns. For the entire

1962-2006 period, the profits from HLS 2000 are significant at the 10% level, and all strategies involving decile sorts produce returns significant at the 5% level or better.

3. Value and Momentum

Our hypothesis is that a significant part of momentum profits stems from the differential fundamentals of winners and losers. We therefore calculate the book-to-market (B/M) ratio of each stock, using the methodology of Fama and French (1992) and Daniel and Titman (2006).⁵ In each month, we calculate the average B/M ratio of all stocks in the winner portfolio. We then average this statistic across all months to obtain an average B/M ratio for a typical “winner” stock. We repeat the process for stocks in the loser portfolio, and conduct the analysis for all J in $\{3,6,9,12\}$.

Table 3 illustrates the results. For all values of J , there is a marked difference in the average B/M ratio of the winner and loser portfolios, in the region of 0.3-0.4⁶. Given that B/M is a significant determinant of cross-sectional returns (Fama and French (1992), Lakonishok et al. (1994)), it may indeed be the case that different B/M ratios at least partly explain the differential returns on winner and loser portfolios.

To test this, we employ the characteristics-matching method of Daniel, Grinblatt, Titman and Wermers (1997). In each month, we independently sort stocks into ten groups based on their B/M ratio. Each stock in the winner and loser portfolio is then matched with an equal-weighted portfolio of stocks in the same B/M decile, at the same point in time. We calculate abnormal returns for the winner and loser portfolios over and above their value benchmarks.

The results are shown in Table 4 for the baseline method, over the 1962-2006 period. For all values of (J,K) , momentum profits are markedly reduced. Out of the 16

⁵ Market value is at the end of the previous calendar year, calculated from CRSP. To calculate book value, we first calculate shareholders' equity. This is Stockholders' Equity (Compustat item 216) if it is not missing. If it is missing, we use Total Common Equity (item 60) plus Preferred Stock Par Value (item 130) if both of these are present. Otherwise, we use Total Assets (item 6) minus Total Liabilities (item 130) if both are present. Else, book-to-market is coded as missing and the observation removed from the sample. To obtain book value, we subtract from shareholders' equity the preferred stock par value, where we use redemption value (item 56), liquidating value (item 10), or carrying value (item 130), in that order, as available. If all are missing, we treat book-to-market as missing. Finally, if not missing, we add balance sheet deferred taxes (item 35) and subtract the FASB106 adjustment (item 330). All book values are recalculated each July and held constant through the following June.

⁶ The deciles of average B/M ratio across the whole sample of stocks are, in ascending order, 0.33, 0.49, 0.62, 0.73, 0.82, 0.93, 1.00, 1.05, 1.17 and 11.30.

(J,K) combinations, value-adjusted returns are negative for four combinations, and only significantly positive at the 5% level for three (6m/6m, 6m/9m and 9m/6m). These horizons featured highly significant raw momentum profits, and so it is not surprising that returns remain statistically significant after value adjustment. Even so, subtracting value benchmarks has an economically significant effect, reducing returns by approximately one half.

Table 5 conducts the same analysis for the 6m/6m strategy across the four different methodologies. (For HLS 2000, stocks are matched according to their value tercile). Over the entire 1962-2006 period, using value benchmarks renders momentum profits insignificant for both the JT 1993 and HLS 2000 methods. While profits remain significant for the other two methods, the monthly return falls by an economically significant 50 basis points. Value-adjusted returns for 1990-2006 are not significant under any methodology, and are insignificant for HLS 2000 even for the 1962-1989 period.

An alternative way to investigate whether momentum profits are driven by winner (loser) portfolios containing high (low) value stocks is to investigate the effect of removing such stocks from the portfolios. This analysis is conducted in Table 6. Panel A illustrates momentum returns under the baseline methodology, (results are similar for other methodologies), when removing stocks in the top (bottom) B/M decile from the winner (loser) portfolios. This has a similar effect to subtracting benchmark returns, with monthly returns dropping markedly, and becoming statistically insignificant in most cases. Panel B shows that removing stocks in the top (bottom) B/M tercile from the winner (loser) portfolios leads to returns being insignificant for all values of *(J,K)*.

Finally, we ask the reverse question: can higher returns to value stocks be explained by their momentum characteristics? In Table 7, Panel A, we calculate the raw returns to stocks in each B/M decile, and replicate previous findings on the significant premium earned by value stocks. Panel B calculates returns over and above momentum benchmarks, and finds that the returns to a value strategy are actually marginally enhanced by benchmark adjustment. Therefore, there is no evidence that any portion of the superior returns to a value portfolio stems from this portfolio overweighting past winners.

4. Conclusion

This paper investigates a simple explanation for momentum that does not require biased reactions to new information or time-varying risks. Instead, it proposes that certain firms persistently outperform the market because they have stronger intrinsic fundamentals. Simply put, certain firms consistently do well because they are better. We investigate the contribution to momentum profits of one characteristic previously linked to significant stock outperformance: value. We find that momentum profits are markedly reduced when calculated in excess of value benchmarks, regardless of the methodology used to formulate the momentum strategy. Indeed, in many cases, momentum profits lose their statistical significance. Similarly, a momentum strategy loses profitability when removing high (low) value firms from the winner (loser) portfolios. Both analyses suggest that the bulk of momentum profits stem from the strategy containing long (short) positions in firms with strong (weak) fundamentals. However, excess returns to value stocks cannot be explained by their momentum characteristics.

Although subtracting returns on value benchmarks has an economically meaningful effect on momentum profits, they remain positive in many cases and statistically significant in a few. While it is interesting that such a simple adjustment can explain a meaningful portion of momentum returns, it is not surprising that it cannot entirely explain away such a prominent and long-standing anomaly. The existing explanations proposed in previous research may explain why some momentum still prevails after adjusting for value. Moreover, the source of value returns, (in particular, whether they stem from mispricing or represent just compensation for risk), is itself under considerable debate. This paper's evidence suggests that the value and momentum anomalies may have a common source.

Table 1**Returns of Relative Strength (Momentum) Portfolios, Baseline Methodology**

The relative strength portfolios are formed based on J and K for the different strategies as indicated in the first column and row, respectively. All available stocks in the CRSP database, (for which yearly data on book-to-market ratio is available), are ranked in ascending order on the basis of J month lagged returns. An equally weighted portfolio of stocks in the highest (lowest) past return decile with prices exceeding \$1 is the winner (loser) portfolio. The relative strength or “momentum” portfolio takes a long (short) position in the winner (loser) portfolio for K months. The average returns to this strategy are presented in this table, with t -statistics in parentheses. Panel A presents results for the sample period of January 1999 to December 2006. Panel B presents results for the sample period of January 1962 to December 2006.

PANEL A: 1999-2006

J		K =	3	6	9	12
3	Winner – Loser		0.0002 (0.0162)	0.0056 (0.5920)	0.0021 (0.2389)	-0.0020 (-0.2960)
6	Winner – Loser		0.0063 (0.4886)	0.0052 (0.4365)	-0.0012 (-0.1188)	-0.0020 (-0.2389)
9	Winner – Loser		0.0001 (0.0056)	-0.0034 (-0.2962)	-0.0044 (-0.4496)	-0.0097 (-1.1454)
12	Winner – Loser		-0.0108 (-0.8809)	-0.0061 (-0.5698)	-0.0126 (-1.3223)	-0.0128 (-1.4487)

PANEL B: 1962-2006

J		K =	3	6	9	12
3	Winner – Loser		0.0022 (0.8414)	0.0059 (2.6751)	0.0061 (3.0299)	0.0059 (3.5308)
6	Winner – Loser		0.0071 (2.3212)	0.0093 (3.3906)	0.0089 (3.7519)	0.0064 (2.9764)
9	Winner – Loser		0.0082 (2.5813)	0.0098 (3.5447)	0.0074 (2.9057)	0.0041 (1.7557)
12	Winner – Loser		0.0078 (2.5533)	0.0072 (2.5508)	0.0043 (1.6250)	0.0013 (0.5320)

Table 2**Returns of Relative Strength Portfolios, Different Methodologies**

This table reports the average monthly returns of the relative strength portfolios, for the J=6, K=6 strategy, across various sample periods and methodologies. *t*-statistics are reported in parentheses. The “Baseline” strategy is the methodology described in Table 1. “JT 1993” employs no filters and excludes Nasdaq stocks. “JT 2001” includes Nasdaq stocks, but excludes stocks with a price below \$5 and a market capitalization in the lowest NYSE size decile. In “HLS 2000”, the winner (loser) portfolio is an equally weighted portfolio of stocks in the highest (lowest) past return tercile. This methodology employs no filters and includes Nasdaq stocks.

	1962-1989	1990-2006	1962-2006
Baseline	0.0105 (3.6622)	0.0063 (1.0973)	0.0093 (3.3906)
JT 1993	0.0083 (2.9665)	0.0017 (0.3682)	0.0062 (2.5520)
JT 2001	0.0137 (5.2830)	0.0153 (2.7301)	0.0147 (5.6518)
HLS 2000	0.0041 (2.1787)	0.0018 (0.4538)	0.0032 (1.7612)

Table 3**Average Book-to-Market Ratios of Relative Strength Portfolios, Baseline Methodology**

All available stocks in the CRSP database, (for which yearly data on book-to-market ratio is available), are ranked in ascending order on the basis of J month lagged returns and an equally weighted portfolio of stocks in the highest (lowest) past return decile is the winner (loser) portfolio. Stocks are only included in the winner and loser portfolios at each point in time if they have a price greater than or equal to \$1 at the beginning of the holding period. The column entitled "B/M" presents average values of the book-to-market ratio, over all months in the sample period, for the winner and loser portfolios, for all values of J . The sample period is January 1962 to December 2006.

	J=3	6	9	12
Winner	2.0778	1.9578	1.8465	1.7266
Loser	1.7916	1.6268	1.4934	1.3011

Table 4**Returns of Relative Strength Portfolios in excess of Book-to-Market Ratio Benchmarks,
Baseline Methodology**

All available stocks in the CRSP database are ranked in ascending order on the basis of J month lagged returns, and an equally weighted portfolio of stocks in the highest (lowest) past return decile with prices exceeding \$1 is the winner (loser) portfolio. Stocks are then independently sorted each month based on the value of their book-to-market ratio, and each stock within the winner and loser portfolio at every point in time is matched with an equal weighted portfolio of stocks in the same book-to-market ratio decile. This table reports the average monthly returns of the relative strength portfolios over and above the book-to-market ratio benchmarks, with t -statistics in parentheses. The sample period is January 1962 to December 2006.

J		K =	3	6	9	12
3	Winner – Loser		-0.0021 (-0.950)	0.0019 (0.9852)	0.0019 (1.0719)	0.0018 (1.2581)
6	Winner – Loser		0.0025 (0.9447)	0.0048 (2.0141)	0.0043 (2.1684)	0.0018 (0.9958)
9	Winner – Loser		0.0037 (1.3201)	0.0053 (2.2573)	0.0028 (1.3213)	-0.0005 (-0.2386)
12	Winner – Loser		0.0025 (0.9632)	0.0020 (0.8329)	-0.0009 (-0.4061)	-0.0038 (-1.8412)

Table 5

Returns of Relative Strength Portfolios, Different Methodologies, in excess of Book-to-Market Ratio Benchmarks

This table presents average monthly returns of the relative strength portfolios in excess of book-to-market ratio benchmarks, across various sample periods and methodologies, for the J=6, K=6 strategy. *t*-statistics are reported in parentheses.

	1962-1989	1990-2006	1962-2006
Baseline	0.0069 (2.9652)	0.0005 (0.1040)	0.0048 (2.2573)
JT 2001	0.0095 (4.0303)	0.0091 (1.8051)	0.0098 (4.1343)
JT 1993	0.0053 (2.4306)	-0.0005 (-0.1247)	0.0034 (1.6892)
HLS 2000	0.0019 (1.2288)	-0.0014 (-0.4023)	0.0006 (0.3930)

Table 6**Returns of Relative Strength Portfolios excluding High and Low Book-to-Market Ratio Stocks, Baseline Methodology**

All available stocks in the CRSP database are ranked in ascending order on the basis of J month lagged returns and an equally weighted portfolio of stocks in the highest (lowest) past return decile with prices exceeding \$1 is the winner (loser) portfolio. Stocks are then independently sorted each month based on the value of their book-to-market ratio. Panel A presents the average monthly returns of the relative strength portfolios when all stocks with book-to-market ratio in the top (bottom) decile are excluded from the winner (loser) portfolio. Panel B presents the analogous results when all stocks with book-to-market ratio in the top (bottom) tercile are excluded from the winner (loser) portfolio. t -statistics are reported in parentheses. The sample period is January 1962 to December 2006.

Panel A

J		K =	3	6	9	12
3	Winner – Loser		-0.0006 (-0.2129)	0.0036 (1.541)	0.0038 (1.8341)	0.0038 (2.199)
6	Winner – Loser		0.0046 (1.4389)	0.0068 (2.41)	0.0068 (2.7846)	0.0043 (1.9257)
9	Winner – Loser		0.0059 (1.7648)	0.0078 (2.7069)	0.0055 (2.115)	0.0023 (0.9496)
12	Winner – Loser		0.006 (1.8873)	0.0055 (1.9088)	0.0028 (1.036)	-0.0002 (-0.0842)

Panel B

J		K =	3	6	9	12
3	Winner – Loser		-0.0045 (-1.5826)	-0.0002 (-0.0691)	0.0005 (0.2228)	0.0007 (0.3884)
6	Winner – Loser		0.0003 (0.0815)	0.003 (1.0329)	0.0033 (1.2895)	0.0012 (0.501)
9	Winner – Loser		0.0017 (0.501)	0.0038 (1.2641)	0.0022 (0.8038)	-0.0005 (-0.2049)
12	Winner – Loser		0.0015 (0.4413)	0.0017 (0.557)	-0.0004 (-0.1507)	-0.003 (-1.1107)

Table 7**Returns of Book-to-Market Ratio-Sorted Portfolios.**

Panel A below presents average monthly returns of equal⁸-weighted portfolios of firms sorted into book-to-market ratio deciles. The ten portfolios of stocks are labeled below such that BM1 represents the portfolio with the lowest book-to-market ratio, and BM10 represents the portfolio with the highest book-to-market ratio. Average monthly returns are also reported for the strategy that buys portfolio BM10 and shorts portfolio BM1. This portfolio is labeled BM10 – BM1 below. *t*-statistics are reported in parentheses. The sample period is January 1962 to December 2006.

Panel B presents the average monthly returns of all portfolios over and above equal-weighted momentum benchmarks, (using 6-month lagged returns).

PANEL A		PANEL B	
BM1	0.0098	BM1	-0.0044
BM2	0.0093	BM2	-0.0037
BM3	0.0101	BM3	-0.0026
BM4	0.0118	BM4	-0.0009
BM5	0.0127	BM5	0.0002
BM6	0.0139	BM6	0.0014
BM7	0.0148	BM7	0.0021
BM8	0.0163	BM8	0.0034
BM9	0.0185	BM9	0.0051
BM10	0.0197	BM10	0.0063
BM10 – BM1	0.0099 (5.4506)	BM10 – BM1	0.0106 (6.9097)

⁸ Using value-weighted portfolios produces similar results.

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Measuring International Contagion during the Sub-prime Crisis: A Semi-Structural Approach

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Abstract

This paper develops a measure of international financial contagion using a semi-structural approach. In particular, we work with a multi-country dynamic equilibrium setting, placing a constraint on the portfolio volatility of one of the countries. The tightening of this constraint is a channel through which shocks are propagated internationally in our model. We then derive a measure of the tightness of the constraint using cross-equation restrictions from the model, rather than performing a full structural estimation. This measure is demonstrated to be a common factor that affects the covariance of all countries across the world at the same time, and thus constitutes our contagion estimator. We finally evaluate our measure of international contagion with regards to its predictability on asset price co-movement across the world, as well as on news about the recent sub-prime crisis. The major result of the paper is that we find evidence that our global contagion estimator is a strong measure of the sub-prime crisis in this regard.

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1. Introduction

Since the first quarter of 2007 the US equity markets have been hit with a sequence of shocks related to the sub-prime crisis. Despite the fact that prior to this crisis, developed countries seemed to have decoupled from one another, meaning that correlations of output, consumption, inflation, interest rates, and stock market returns had dropped significantly from the prior decade, the events after the summer of 2007 made it very clear that contagion was back, and with a vengeance. Measuring the degree of contagion has never been an easy task in applied work, however, and determining the relative importance of the different theories of contagion has been even more daunting. The main reason for this is that contagion is usually estimated using reduced form representations, which complicate the interpretation of coefficients as well as the separation between different theories of propagation. For example, from the computation of simple correlations to linear models, from copulas to GARCH models, from principal components to probit regressions, all are agnostic about the different theories of contagion¹. Furthermore, in most cases, different theories of why shocks are transmitted internationally are tested in a regression based framework, by introducing interaction terms as a regressor on the right hand side. Although this approach has taught us a great deal about the crises in the 1990s, it has had its limitations with regards to analyzing the strength of international linkages which generate contagion.

The purpose of this paper is twofold. Firstly, we develop an estimation method for international contagion that is semi-structural. As opposed to the standard methods in the literature, our estimation of the strength of global transmission mechanisms for shocks is based on cross-equation restrictions that arise from a formal model of financial constraints. We do not fit the model entirely, (which would constitute a fully fledged structural estimation), but rather use the restrictions derived from the model. We view this as the next (and necessary) step in the empirical contagion literature, disentangling the different channels of propagation before fitting a full structural model.

¹For principal component studies, see e.g., Calvo and Reinhart (1996) and Kaminsky and Reinhart (2007). For evidence of contagion based on reduced-form linear latent factor models, see e.g., Corsetti, Pericoli, and Sbracia (2005), Dungey, Fry, González-Hermosillo, and Martin (2005), and Rigobon (2003), and for a copula approach see e.g., Hartmann, Straetmans, and de Vries (2004) and Rodriguez (2007).

Secondly, we study the sub-prime collapse which originated in the US. Clearly there has been dramatic contagion across the world caused by the eruption of this crisis, and this has been exacerbated by the tightening of global financial constraints. In order to relate our measure of financial contagion to the actual contagion generated by the sub-prime meltdown, we evaluate our contagion measure with regards to its predictability on news and asset price co-movement in almost 50 stock markets in the world, around the time of the crisis.

For the first purpose of our study, the estimation of contagion, Sections 2 and 3 of the paper develop a dynamic equilibrium model of asset prices, in which contagion is transmitted via the (daily) tightening of portfolio volatility constraints². We assume that volatility constraints are placed on the portfolio of a representative financial institution in the base or 'Center' country, as in the model of Pavlova and Rigobon (2007). The reasoning behind this choice of constraint is that restrictions on volatility are very similar in spirit to limits on Value-at-Risk (VAR), which form an extremely important part of risk regulation for banks across the globe. In particular, the definition of VAR is the worst expected loss of a portfolio over a given horizon at a particular confidence level. Therefore, a constraint on VAR is equivalent to a limit on the downside risk of a portfolio, whereas a constraint on volatility simply restricts overall risk. The similarity between these two types of constraint is thus very clear; however, it is far more tractable to model restrictions on portfolio volatility rather than VAR.

The presence of these realistic financial constraints can then be directly tied to the contagious effects of crises such as the sub-prime collapse³. Indeed, the contagion measure which we derive from our model of volatility restrictions is demonstrated to be a common factor that affects the covariance of all countries across the world at the same time; this is explained

²Related literature on two-good two-country asset pricing models includes Helpman and Razin (1978), Cole and Obstfeld (1991) and Zapatero (1995), which do not feature portfolio constraints. Related work on portfolio constraints in the asset pricing literature includes, for example, Basak and Croitoru (2000), Basak and Cuoco (1998), Detemple and Murthy (1994), Detemple and Serrat (2003), Gallmeyer and Hollifield (2004) and Shapiro (2002). Also see Cass, Siconolfi, and Villanacci (2001) for analysis of a general equilibrium model with portfolio constraints. For more applied analyses, see Boyer, Kumagai, and Yuan (2006), Geanakoplos (2003), Gromb and Vayanos (2002), Mendoza and Smith (2002), and Yuan (2005).

³There are several papers in the empirical literature which argue that financial risk-regulation has indeed been largely responsible for contagion in developing and emerging markets over the last twenty years or so. For example, see Calvo (2002), Kaminsky and Reinhart (2000), and Van Rijckeghem and Weder (2003).

in greater detail in Section 4, which evaluates some of the implications of our model in the data. Therefore, our paper provides evidence which links actual international contagion to an increase in the tightening of financial constraints facing banks across the globe.

For the second purpose of our paper, namely our empirical analysis in Section 4, we collect information on the stock market indices of 49 countries, and their respective exchange rates relative to the US, (which we take as our 'Center' or constrained country; this makes sense given that the US was the country of origin of the sub-prime crisis). It is important to highlight that with the information we have, (at daily frequencies), we cannot fit the whole model. The reason is that we do not have enough moments in the data to pin down all the parameters. This is why we adopt a semi-structural approach.

Our model has two very important implications that distinguish our approach from the standard one used in the literature. Firstly, when there is a shock that tightens the constraint imposed on the Center country, the co-movement of all stock markets in the world is affected. In other words, a tightening of the constraint not only affects the co-movement between the Center country and any other country, but it also affects the co-movement between any two other countries in the world. The second implication from our model is that if all stock markets are measured in the same currency, then this change in the co-movement of any two stock markets has to be identical in magnitude across all possible pairs of countries in the world. This means that contagion cannot be measured in our model simply by looking at the increase in the correlation across countries after a shock to the constraint. Rather, it has to be the case that, as a result of a tightening of the constraint in the Center country, covariances across all countries increase by the exact same amount, and only this proportion of their co-movement can be associated with global financial contagion in our set-up.

Another main difference between our approach and the standard methodology in the literature is that we have to include as many countries as possible in our analysis. Contagion papers rarely do this; indeed, most of the time they analyze a subset of countries across the world. In our case, however, because of the implications of the model, contagion can only be measured from the co-movement implied by the joint shift in covariances of all countries.

Our approach is not exempt from criticism, given that our model has to make several

simplifying assumptions that are unlikely to be true in reality. Firstly, it assumes that consumers' utility has a log representation. Secondly, it assumes that markets are complete. Thirdly, we have an endowment economy without labor and capital, and our supply shocks are assumed to be independent across countries. Fourthly, our model is a real model where the demand shocks are represented by consumer expenditure shifts; therefore, it does not include nominal, monetary, fiscal, and other sources of shocks. As a result, it is clear that we are deriving our cross-equation restrictions from a very stylized setting.

Moreover, our empirical implementation requires us to extend the assumptions in our model even further. For example, we not only have to assume that supply shocks are independent across countries, but we also have to assume that their heteroscedasticity is independent across countries as well, (see Section 4). In other words, we are not only assuming independence of first moments, but for the estimation procedure we also need to assume independence of second moments. Furthermore, we end up implementing our contagion measure using a rolling window to estimate the variance-covariance matrix of all countries in the world. If daily options data on bundles of stock market indices and individual stock markets existed, we could at least estimate the implied variances of countries from financial markets at a daily frequency. Unfortunately, this information does not exist and we are left with having to use a rolling window to estimate variances and covariances, (and, in turn, our contagion measure), instead.

Despite the presence of several of these simplifying assumptions, the major result from our empirical analysis is that our measure of contagion has strong predictability effects on the daily number of news articles in world newspapers documenting the sub-prime crisis. There is also a high daily positive correlation between changes in our contagion estimator and changes in the co-movement of stock prices across the globe, following the sub-prime collapse. Therefore, our new contagion measure appears to be a strong measure of the crisis in this regard.

2. The Model

Our first objective in this paper is to model how a realistic example of a constraint on global financial institutions, (a restriction on volatility), affects the international co-movement of asset prices. Extending the model of Pavlova and Rigobon (2007), we initially work with a three country model, where all three countries are assumed to be identical in size. (The justification for this will be provided later on; in particular, we find that in both constrained and unconstrained environments, all three countries always hold an identical number of shares in the stocks of each country.) In this section and the next, we then specialize the model of Pavlova and Rigobon (2007) to consider a specific constraint on the volatility of the portfolio of one of the countries. We examine the implications of this constraint for the dynamics of stock prices and terms of trade at a partial and general equilibrium level.

2.1. The Economic Setting

The setting is almost exactly the same as in Pavlova and Rigobon (2007). We briefly recall their main assumptions and results here. We consider a continuous-time pure-exchange world economy with a finite horizon, $[0, T]$. We start with three countries in the world economy, indexed by $j \in \{0, 1, 2\}$. Countries 0, 1 and 2 are assumed to be identical in size. (For ease of reference, however, we refer to Country 0 as the Center country and Countries 1 and 2 as the Periphery Countries). Each Country j has a strictly positive output process modeled as a Lucas (1978) tree:

$$dY^j(t) = \mu_{Y^j}(t) Y^j(t) dt + \sigma_{Y^j}(t) Y^j(t) dw^j(t), \quad j \in \{0, 1, 2\}, \quad (1)$$

where μ_{Y^j} and $\sigma_{Y^j} > 0$ are the respective processes for the mean growth rate and volatility of output for Country j , and w^j is the independent Brownian motion representing an output or supply shock to Country j . The price of the good produced by Country j is denoted by p^j . We define a numeraire basket containing $\beta \in (0, 1)$ units of the good produced in Country 0 and $(1 - \beta)/2$ units of each of the remaining two goods and normalize the price of this basket to be equal to unity. We think of β as the size of the Center country relative to the world economy. Given our assumption that all countries are identical in size, we therefore set β

equal to $\frac{1}{3}$ in our model.

In order to complete markets, investment opportunities are represented by four securities. Each Country j issues a stock S^j , a claim to its output. All stocks are in unit supply. There is also the “world” bond B , which is riskless in units of the numeraire, and is in zero net supply. We define the terms of trade from the viewpoint of the Center country (Country 0): $q^1 \equiv p^1/p^0$ and $q^2 \equiv p^2/p^0$ are the terms of trade of the Periphery countries 1 and 2, respectively, with the Center country.

A representative consumer-investor of each country is endowed at time 0 with a supply of the stock market of his country; the initial wealth of agent i is denoted by $W_i(0)$. Each consumer i chooses nonnegative consumption of each good $(C_i^0(t), C_i^1(t), C_i^2(t))$, $i \in \{0, 1, 2\}$, and a portfolio of the available risky securities $x_i(t) \equiv (x_i^{s^0}(t), x_i^{s^1}(t), x_i^{s^2}(t))^\top$, where x_i^j denotes a fraction of wealth W_i invested in security j . For the sake of tractability, preferences of consumer i are represented by a time-additive log utility function defined over consumption of all three goods:

$$E \left[\int_0^T u_i(C_i^0(t), C_i^1(t), C_i^2(t)) dt \right],$$

where

$$\begin{aligned} u_0(C_0^0, C_0^1, C_0^2) &= \alpha_0 \log C_0^0(t) + \frac{1-\alpha_0}{2} \log C_0^1(t) + \frac{1-\alpha_0}{2} \log C_0^2(t), \\ u_1(C_1^0, C_1^1, C_1^2) &= \frac{1-\alpha_1(t)}{2} \log C_1^0(t) + \alpha_1(t) \log C_1^1(t) + \frac{1-\alpha_1(t)}{2} \log C_1^2(t), \\ u_2(C_2^0, C_2^1, C_2^2) &= \frac{1-\alpha_2(t)}{2} \log C_2^0(t) + \frac{1-\alpha_2(t)}{2} \log C_2^1(t) + \alpha_2(t) \log C_2^2(t). \end{aligned}$$

We set the preference weight on the domestically-produced good, α_i , to be greater than $1/3$ (and less than 1), for each Country i . This is in order to generate a home bias in preferences. We also include demand shifts as a key source of uncertainty in our model⁴. In particular, an increase in α_i in our model represents a demand shift towards domestically produced goods in Country i . We take each α_i to be a martingale (i.e., $E[\alpha_i(s)|\mathcal{F}_t] =$

⁴Demand shifts are modeled along the lines of Dornbusch, Fischer, and Samuelson (1977). In particular, in the absence of demand shocks, free trade in goods would imply extremely high correlation of stock markets and therefore irrelevancy of financial structure, as established by Helpman and Razin (1978), Cole and Obstfeld (1991) and Zapatero (1995).

$\alpha_i(t)$, $s > t$), and hence can be represented as

$$d\alpha_1(t) = \sigma_{\alpha_1}(t)^\top dw(t), \quad d\alpha_2(t) = \sigma_{\alpha_2}(t)^\top dw(t).$$

We keep the preference parameter of the Center country, α_0 , fixed, in order to keep the focus on the Periphery Countries.

2.2. Countries' Optimization and Benchmark Unconstrained Equilibrium

In an environment with no portfolio constraints, we have the following results reported from Pavlova and Rigobon (2007).

Proposition 1. (i) *We can define a representative agent who is endowed with the aggregate supply of securities and consumes the aggregate output. His utility is given by*

$$U(C^0, C^1, C^2; \lambda_0, \lambda_1, \lambda_2) = E \left[\int_0^T u(C^0(t), C^1(t), C^2(t); \lambda_0, \lambda_1, \lambda_2) dt \right],$$

with

$$u(C^0, C^1, C^2; \lambda_0, \lambda_1, \lambda_2) = \max_{\sum_{i=0}^2 C_i^j = C^j, j \in \{0,1,2\}} \sum_{i=0}^2 \lambda_i u_i(C_i^0, C_i^1, C_i^2),$$

where $\lambda_i > 0$, $i = 0, 1, 2$, are the weights on consumers 0, 1, and 2, respectively. These weights are constant in the unconstrained economy, but will be stochastic in the economy with portfolio constraints. In the unconstrained case, these weights are simply the inverses of the Lagrange multipliers on the consumers' intertemporal budget constraints. Since in equilibrium these multipliers, and hence the weights, cannot be individually determined, we adopt a normalization $\lambda_0 = 1$.

(ii) *Terms of trade are given by the relevant marginal rates of substitution processes*

$$q^1(t) = \frac{u_{C^1}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \frac{Y^0(t)}{Y^1(t)}, \quad (2)$$

$$q^2(t) = \frac{u_{C^2}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)}{u_{C^0}(Y^0(t), Y^1(t), Y^2(t); \lambda_1, \lambda_2)} = \frac{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \frac{Y^0(t)}{Y^2(t)}. \quad (3)$$

(iii) *The prices of the stocks of the Center and the Periphery countries are given by*

$$S^0(t) = \frac{1}{\beta + \frac{1-\beta}{2} q^1(t) + \frac{1-\beta}{2} q^2(t)} Y^0(t)(T-t), \quad (4)$$

$$S^1(t) = \frac{q^1(t)}{\beta + \frac{1-\beta}{2} q^1(t) + \frac{1-\beta}{2} q^2(t)} Y^1(t)(T-t), \quad (5)$$

$$S^2(t) = \frac{q^2(t)}{\beta + \frac{1-\beta}{2} q^1(t) + \frac{1-\beta}{2} q^2(t)} Y^2(t)(T-t). \quad (6)$$

(iv) Wealth distribution is constant, determined only by the initial shareholdings:

$$\frac{W_1(t)}{W_0(t)} = \lambda_1 \quad \text{and} \quad \frac{W_2(t)}{W_0(t)} = \lambda_2. \quad (7)$$

(v) In the unconstrained equilibrium, all agents hold an equal number of shares of stocks S^0 , S^1 , and S^2 . This number is given by $\frac{\lambda_i}{\lambda_0 + \lambda_1 + \lambda_2}$, where $\lambda_0 = 1$. No shares of the bond are held in equilibrium.

(vi) The joint dynamics of the terms of trade and the three stock markets in the benchmark unconstrained economy, (suppressing the drift term), are given by

$$\begin{bmatrix} \frac{dq^1(t)}{q^1(t)} \\ \frac{dq^2(t)}{q^2(t)} \\ \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} \\ \frac{dS^2(t)}{S^2(t)} \end{bmatrix} = I(t)dt + \underbrace{\begin{bmatrix} a(t) & b(t) & 1 & -1 & 0 \\ \tilde{a}(t) & \tilde{b}(t) & 1 & 0 & -1 \\ -X_{\alpha_1}(t) & -X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2} \frac{M(t)}{q^1(t)} & \frac{1-\beta}{2} \frac{M(t)}{q^2(t)} \\ a(t) - X_{\alpha_1}(t) & b(t) - X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2} \frac{M(t)}{q^1(t)} & \frac{1-\beta}{2} \frac{M(t)}{q^2(t)} \\ \tilde{a}(t) - X_{\alpha_1}(t) & \tilde{b}(t) - X_{\alpha_2}(t) & \beta M(t) & \frac{1-\beta}{2} \frac{M(t)}{q^1(t)} & \frac{1-\beta}{2} \frac{M(t)}{q^2(t)} \end{bmatrix}}_{\Theta_u(t)} \begin{bmatrix} d\alpha_1(t) \\ d\alpha_2(t) \\ \sigma_{y^0}(t)dw^0(t) \\ \sigma_{y^1}(t)dw^1(t) \\ \sigma_{y^2}(t)dw^2(t) \end{bmatrix}$$

The drift term I and quantities X_{α_1} , X_{α_2} , M , a , \tilde{a} , b , and \tilde{b} are defined in Pavlova and Rigobon (2007).

2.3. Volatility Constraints

Here, we specify that the Center country now faces a constraint on the volatility of its stock portfolio. We make the plausible assumption that the constraint is placed on the portfolio of a representative financial institution, such as a commercial bank, in this country. As described in Section 1, the reasoning behind this choice of constraint is that a restriction on volatility is very similar to a limit on portfolio Value-at-Risk (VAR), but is far more tractable to model. In particular, VAR is a risk measure used by banks across the world, which calculates the worst expected loss of a portfolio over a given horizon at a particular confidence level. Therefore, a constraint on VAR is a limit on the downside risk of a portfolio, whereas a volatility constraint simply restricts overall risk⁵.

⁵We recognize here that we do not model separate sets of investors or economic agents in Country 0 who are constrained, and do not get into exact details as to why a VAR constraint, or its proxy, a volatility constraint, may be imposed. It is important to stress, however, that VAR limits are a very realistic example of financial restrictions facing institutions across the globe, given that they function as a tool for controlling risk. For example, according to the Basel Committee on Banking Supervision (2001) report, the Basel Committee stipulates that international commercial banks should use VAR as an internal risk measure,

Suppose we then have the following, exogenously set dynamic⁶ constraint on the volatility of Country 0's portfolio x_0 , (where σ represents the volatility matrix of all stock returns in the economy).

$$x_0(t)^\top \sigma(t) \sigma(t)^\top x_0(t) \leq \phi(t). \quad (8)$$

Equation (8) thus provides us with the exact form of the constraint we are going to use when analyzing the properties of general equilibrium in the economy with volatility limits. We assume a daily⁷ horizon here for changes in the degree of the volatility constraint, $\phi(t)$, over time.

3. Equilibrium in the Economy with Volatility Constraints

We now describe general equilibrium under the specific volatility constraint considered in Section 2.3. As described in Pavlova and Rigobon (2007), in a constrained economy, the only change to be made to Proposition 1 is that the representative agent must now be defined using stochastic weighting processes, (introduced by Cuoco and He (2001)), with these stochastic

subject to the fulfillment of certain sets of conditions and the approval of their supervisory committees. Under these circumstances, banks have no choice but to comply with this regulation, allowing us to justify the imposition of a VAR or volatility restriction on a representative financial institution, or commercial bank, in Country 0.

⁶We can rely on a wealth of evidence from current practices to support the assumption of a stochastic limit on portfolio VAR or volatility. For example, dynamic VAR constraints are often used by banks and firms to help set position limits for traders, as well as to trim the risk of different business units of a corporation. These VAR limits are stochastic because they are usually tied to movements of the market. For instance, if market volatility increases, more stringent controls on risk may have to be imposed via tighter VAR constraints, so that financial positions are scaled down appropriately. (On the other hand, if market volatility jumps up suddenly, a risk manager may alternatively want to relax his VAR limits to avoid a liquidation under difficult market conditions; thus, the implementation and flexibility of dynamic restrictions always depends on the judgement of the risk manager).

⁷Relating this to actual practice, we note that dynamic VAR limits, while used widely to control risk across all financial institutions, firms and banks, are best suited to fast-paced trading environments where turnover is rapid, leverage is high, and portfolio positions change several times a day, (see Jorion (2000)). These environments are most prevalent in commercial banks, which usually have high turnover and liquidity in their portfolios, and thus need to adjust risk exposure frequently. Furthermore, a daily horizon is most commonly used by commercial banks across the world in order to calculate trading VARs, (with the added advantage that a daily VAR is consistent and easily comparable with daily profit and loss measures). Therefore, we can justify our assumption here that a daily volatility constraint, (as a proxy for a daily VAR limit), is placed on the portfolio of a representative commercial bank in Country 0

weights representing the effects of the new constraint⁸. The following results are reported from Pavlova and Rigobon (2007).

Proposition 2. (i) *In an equilibrium with the portfolio constraint, the weighting processes λ_1 and λ_2 are the same up to a multiplicative constant, which we denote as λ . The dynamics of λ are given below:*

$$d\lambda(t) = \lambda(t)[r(t) - r_0(t) + m(t)^\top(m_0(t) - m(t))]dt - \lambda(t)(m_0(t) - m(t))^\top dw(t). \quad (9)$$

where m_0 and r_0 are, respectively, the effective market price of risk and interest rate faced by Country 0 due to the portfolio constraint⁹, and m and r are, respectively, the vector market price of risk and interest rate of the unconstrained Periphery Countries.

(ii) *When such an equilibrium exists, the joint dynamics of the terms of trade and three stock markets in the economy with the portfolio constraint are given by*

$$\begin{bmatrix} \frac{dq^1(t)}{q^1(t)} \\ \frac{dq^2(t)}{q^2(t)} \\ \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} \\ \frac{dS^2(t)}{S^2(t)} \end{bmatrix} = I_c(t)dt + \begin{bmatrix} A(t) \\ \tilde{A}(t) \\ -X_\lambda(t) \\ A(t) - X_\lambda(t) \\ \tilde{A}(t) - X_\lambda(t) \end{bmatrix} \Theta_u(t) \begin{bmatrix} \frac{d\lambda(t)}{\lambda(t)} \\ d\alpha_1(t) \\ d\alpha_2(t) \\ \sigma_{Y^0}(t)dw^0(t) \\ \sigma_{Y^1}(t)dw^1(t) \\ \sigma_{Y^2}(t)dw^2(t) \end{bmatrix}$$

where $\lambda(t) \equiv \lambda_1(t)$, I_c , A , \tilde{A} and X_λ are reported in Pavlova and Rigobon (2007), and the unconstrained dynamics matrix $\Theta_u(t)$ is as defined in Proposition 1.

The only departure from the corresponding expression in the benchmark economy is in the addition of the first, $d\lambda/\lambda$, term. Therefore, a movement in λ represents a tightening or a loosening of the portfolio constraint; this will be explored in greater detail later in this section. Furthermore, given the definition of λ in part (iv) of Proposition 1 in terms of the Periphery countries' relative wealth, an increase (decrease) in lambda also reflects a wealth

⁸See, for example, Basak and Croitoru (2000), Basak and Cuoco (1998), and Shapiro (2002). For the original solution method see Negishi (1960).

⁹The optimization problem of the Center under portfolio constraints is formally equivalent to a dual problem with no constraints but with the Center facing a fictitious 'tilted' investment opportunity set (Cvitanic and Karatzas (1992)). Cvitanic and Karatzas show that the tilt in the fictitious investment opportunity set is characterized by the multipliers on the portfolio constraints. Consequently, Country 0's market price of risk under the constraint, m_0 , and interest rate, r_0 (derived in the Appendix), differ from those faced by the unconstrained investors in the Periphery; these are denoted respectively as m and r . Given that the difference in investment opportunity sets faced by the Center and Periphery countries is due to the presence of the portfolio constraint on Country 0, it is clear that all investors will only hold the same portfolio in the unconstrained case, where the tilted market price of risk $m_0(t)$ coincides with $m(t)$, and $r_0(t)$ coincides with $r(t)$.

transfer in the world economy to (away from) the Periphery countries. This mechanism of endogenous wealth transfers is the channel through which changes in λ affect the tightness of the portfolio constraint in our model¹⁰.

We can now define all equilibrium quantities in our constrained setting. The set of equations required to do this is presented in the following proposition.

Proposition 3. *When equilibrium exists, the equilibrium market price of risk processes faced by the Center and the Periphery are related as follows:*

$$\begin{aligned} \text{When } \quad x_0(t)^\top \sigma(t) \sigma(t)^\top x_0(t) &= m(t)^\top m(t) \leq \phi(t), \\ m_0(t) &= m(t), \quad \psi(t) = 0, \quad (\text{Constraint not binding}), \end{aligned} \quad (10)$$

otherwise,

$$\begin{aligned} m_0(t) &= \frac{m(t)}{1 + 2\psi(t)}, \\ (1 + 2\psi(t))^2 &= \frac{m(t)^\top m(t)}{\phi(t)}, \quad (\text{Constraint binding}), \end{aligned} \quad (11)$$

where ψ is the multiplier on the volatility constraint, and σ is the volatility matrix of the three stock returns in the economy; see Pavlova and Rigobon (2007). Furthermore,

$$\begin{aligned} \sigma_{Y^0}(t) i_0 &- \frac{\left(\lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2} \right) (m(t) - m_0(t)) - \frac{\lambda_1(t)}{2} \sigma_{\alpha_1}(t) - \frac{\lambda_2(t)}{2} \sigma_{\alpha_2}(t)}{\alpha_0 + \lambda_1(t) \frac{1-\alpha_1(t)}{2} + \lambda_2(t) \frac{1-\alpha_2(t)}{2}} \\ &= X_\lambda(t) (m(t) - m_0(t)) + X_{\alpha_1}(t) \sigma_{\alpha_1}(t) + X_{\alpha_2}(t) \sigma_{\alpha_2}(t) \\ &\quad + \frac{1-\beta}{2} M(t) (q^1(t) + q^2(t)) \sigma_{Y^0}(t) i_0 - \frac{1-\beta}{2} M(t) q^1(t) \sigma_{Y^1}(t) i_1 \\ &\quad - \frac{1-\beta}{2} M(t) q^2(t) \sigma_{Y^2}(t) i_2 + m_0(t). \end{aligned} \quad (12)$$

where $i_0 \equiv (1, 0, 0)^\top$, $i_1 \equiv (0, 1, 0)^\top$, and $i_2 \equiv (0, 0, 1)^\top$.

Equations (10)–(11) are the complementary slackness conditions from the constrained portfolio optimization of the Center country. When the portfolio volatility constraint is not binding, the market price of risk faced by the Center coincides with that faced by the Periphery countries. Therefore, the portfolio of the Center is equivalent to the Periphery

¹⁰For other papers which explore contagion as a wealth effect, see Kyle and Xiong (2001) and Cochrane, Longstaff, and Santa-Clara (2008).

β	$\frac{1}{3}$	ϕ	0.003		
α_0	0.75	$Y^0(t)$	1.0	$\sigma_{y^0}(t)$	0.1
α_1	0.75	$Y^1(t)$	1.0	$\sigma_{y^1}(t)$	0.1
α_2	0.75	$Y^2(t)$	1.0	$\sigma_{y^2}(t)$	0.1
$\lambda_1(t) \in [0.75, 1.25]$					
$\lambda_2(t) \in [0.75, 1.25]$					
$\sigma_{\alpha_1}(t)$	(0, 0.2, 0)				
$\sigma_{\alpha_2}(t)$	(0, 0, 0.2)				

Table 1: Parameter choices

countries' unconstrained portfolios. When the constraint is binding, however, there is now a difference between the market prices of risk faced by the Center and the Periphery, expressed in (11). Equation (12) is derived from market clearing in the consumption goods.

Finally, given (10)–(11), we can derive an analogous result to (v) of Proposition 1 in the constrained environment. This provides us with the justification for our assumption that all three countries are identical in size in our model:

Corollary 4. *In the constrained equilibrium, all agents still hold an equal number of shares of stocks S^0 , S^1 , and S^2 . Their holdings of the bond, however, are different, although by assumption, net demand = net supply = 0 in equilibrium.*

The relationship between λ and the multiplier on the volatility constraint ψ , (or in other words, the link between λ and the tightness of the portfolio volatility constraint), can be expressed in a simple plot, Figure 1. Here, we fix all time-dependent/state variables in our model, except for the wealth shares of the Periphery countries, $\lambda_1(t)$ and $\lambda_2(t)$. The horizontal axes measure λ_1 and λ_2 and the vertical axis measures ψ . The parameters used in the analysis are summarized in Table 1.

From Pavlova and Rigobon (2007), the reasoning behind the choice of these parameters is described as follows. In our model, as previously mentioned, all three countries in the world are identical in size, so in the numeraire consumption basket they each represent $\frac{1}{3}$ of the world. We set 75 percent as the share of expenditures on the domestic good, which is a conservative estimate given the share of the service sector in GDP. It is also found that in the data, demand shocks are positively correlated with domestic supply innovations. Therefore, we assume that a demand shift in Country j has a positive loading on w^j and zero loadings

on the remaining Brownian motions.

We can observe from Figure 1 that roughly along the diagonal of the horizontal plane, where λ_1 is equal or close to λ_2 in magnitude, the volatility constraint does not bind, and the multiplier ψ is zero. This is our unconstrained region. As λ_1 and λ_2 diverge from each other, however, the constraint does become more binding and the multiplier increases. This means that the constraint on the volatility of the Center country's portfolio becomes tighter as λ_1 and λ_2 become increasingly different, or as the relative wealth shares of the two other countries in the world diverge. These results make sense; indeed, given that we are imposing a volatility constraint on the portfolio, or, equivalently, on the overall wealth of Country 0, it is natural to expect that this constraint would become more (less) binding as the wealth flows of Country 0 to and from Countries 1 and 2 diverge from (converge to) each other, or as Countries 1 and 2 diverge (converge) in relative wealth. Given that in equilibrium λ_1 and λ_2 are the same up to a multiplicative constant, which we denote as λ , Figure 1 thus displays how movements in λ_1 , λ_2 , and therefore λ , tighten or loosen our portfolio constraint. In particular, as λ_1 and λ_2 diverge (converge), the absolute magnitude of λ will diverge from (converge to) 1, and this will consequently make the volatility constraint tighter (looser) in our model.

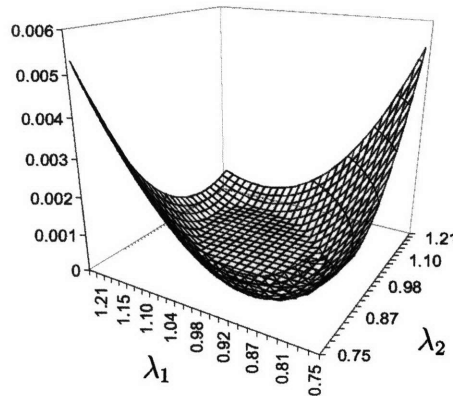


Figure 1: Value of the multiplier on the portfolio volatility constraint ψ

4. Empirical Estimation

In this section we estimate the contagious effect derived from the US sub-prime crisis, (now taking the US to be the Center or constrained country in our model). The sub-prime crisis has had small output effects from the beginning of 2007 until February 2008. However, it has had a large effect on financial markets. One of the implications of the sub-prime meltdown is the tightening of cash constraints in the US financial system, which can be interpreted as a tightening of the portfolio risk constraint in our model, or a change in λ . Of course, this is an oversimplification. Nevertheless, by taking our model literally and estimating it in the data during the sub-prime collapse, we are in effect measuring the contagion generated by such a crisis due to the tightening of portfolio constraints.

Our model is extremely difficult to estimate in the data given that we would need to observe both output and consumption, as well as asset prices, at high frequencies. In particular, as discussed in the introduction, the effects of the crisis on stock markets have been relatively short lived. Therefore, these effects would be undetectable at the lower frequencies at which output is typically observed. In this section, we thus make some simplifications to the model in order to take it to the data.

We extend our model to include $N+1$ countries. Manipulating part (ii) of Proposition 2, we can express the joint dynamics of all relative asset prices in the economy as follows:

$$\begin{bmatrix} \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} - \frac{dq^1(t)}{q^1(t)} \\ \frac{dS^2(t)}{S^2(t)} - \frac{dq^2(t)}{q^2(t)} \\ \vdots \\ \frac{dS^n(t)}{S^n(t)} - \frac{dq^n(t)}{q^n(t)} \end{bmatrix} = \tilde{I}(t)dt + \begin{bmatrix} -X_\lambda(t) \\ -X_\lambda(t) \\ -X_\lambda(t) \\ \vdots \\ -X_\lambda(t) \end{bmatrix} \left| \Theta_v(t) \right. \begin{bmatrix} \frac{d\lambda(t)}{\lambda(t)} \\ \{d\alpha_1(t), \dots, d\alpha_n(t)\}' \\ \{\sigma_{\gamma^0}(t)dw^0(t), \dots, \sigma_{\gamma^n}(t)dw^n(t)\}' \end{bmatrix}$$

where matrix $\Theta_v(t)$ is defined below as:

$$\begin{bmatrix} -X_{\alpha_1}(t) & \dots & -X_{\alpha_n}(t) & \beta M(t) & \frac{1-\beta}{2} M(t)q^1(t) & \frac{1-\beta}{2} M(t)q^2(t) & \dots & \frac{1-\beta}{2} M(t)q^n(t) \\ -X_{\alpha_1}(t) & \dots & -X_{\alpha_n}(t) & \beta M(t) - 1 & \frac{1-\beta}{2} M(t)q^1(t) + 1 & \frac{1-\beta}{2} M(t)q^2(t) & \dots & \frac{1-\beta}{2} M(t)q^n(t) \\ -X_{\alpha_1}(t) & \dots & -X_{\alpha_n}(t) & \beta M(t) - 1 & \frac{1-\beta}{2} M(t)q^1(t) & \frac{1-\beta}{2} M(t)q^2(t) + 1 & \dots & \frac{1-\beta}{2} M(t)q^n(t) \\ \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots \\ -X_{\alpha_1}(t) & \dots & -X_{\alpha_n}(t) & \beta M(t) - 1 & \frac{1-\beta}{2} M(t)q^1(t) & \frac{1-\beta}{2} M(t)q^2(t) & \dots & \frac{1-\beta}{2} M(t)q^n(t) + 1 \end{bmatrix}$$

$\underbrace{\hspace{10em}}_{\text{demand}}$
 $\underbrace{\hspace{10em}}_{\text{supply}}$

and where the vectors $\{d\alpha_1(t), \dots, d\alpha_n(t)\}'$ and $\{\sigma_{y^0}(t)dw^0(t), \dots, \sigma_{y^n}(t)dw^n(t)\}'$ represent all the demand and supply shocks in the economies.

We make several assumptions here regarding the demand and supply shocks in this economy. Firstly, we have already assumed that the supply shocks or Brownian motions are independent across countries. Secondly, we have assumed that the demand shocks are linear functions of the supply shocks that have different loadings on each shock. Furthermore, we note that $\Theta_v(t)$ has exactly the same loading for all countries on the demand shocks, while the loadings on the supply shocks are different across all countries. This means that the product of $\Theta_v(t)$ and the demand and supply shocks is a linear function that will have different loadings on every supply shock for each of the $N+1$ countries in the economy. Finally, any change in (λ) , (equivalent to a tightening or loosening of the portfolio volatility constraint), depends on both the supply shocks and also on changes in the degree of the constraint.

Tracking the impact of any particular supply shock becomes, therefore, a very difficult empirical exercise, given that the total effect is a convolution of supply, demand, and portfolio constraint dynamics. In order to tackle this problem, let us consider a change in the tightness of the portfolio constraint, or a change in (λ) , when all the country supply shocks are zero. In this case, an increase in the tightening of the constraint shifts λ , and this change is reflected equally in all relative asset prices, (given that the loading on changes in λ , $-X_\lambda(t)$, is the same for all countries in the model). Thus, an alternative to estimating the impact of a tightening of the constraint is to find periods in time in which just the constraint shifts. The impact of the constraint will then be equal to the average change in all relative asset prices at that time. This method involves making a very strong assumption, however, given that supply shocks are unlikely to be zero at any point in time.

Nevertheless, the same intuition can be used if we think about shifts in second moments instead. As can be inferred from part (i) of Proposition 2, shocks to the degree of tightening of the constraint change the conditional heteroscedasticity of the λ process. Let us now assume that shocks to the output supply for all countries are homoscedastic. If this is the case, then any heteroscedasticity observed across relative asset prices has to be explained by changes in the portfolio constraint factor. Therefore, just estimating this heteroscedasticity

would be a measure of the contagious effect of the portfolio constraint. Once again, the assumption that all supply shocks are homoscedastic is a strong one. However, given we have assumed that the supply shocks are independent across countries, if we are instead willing to make the assumption that these shocks are heteroscedastic, but their conditional heteroscedasticity is independent across countries as well, then we can estimate the impact of the portfolio constraint as the average change in the heteroscedasticity of all moments of relative stock prices.

More precisely, if a change in the tightening of the portfolio constraint shifts $\frac{d\lambda(t)}{\lambda(t)}$, then all variances and covariances across all countries will shift exactly by a factor of $X_{\lambda}^2(t) \cdot \text{var}\left(\frac{d\lambda(t)}{\lambda(t)}\right)$. If a supply shock is heteroscedastic, on the other hand, then some variances and covariances between relative asset prices will increase in response, while others will decrease. Furthermore, the magnitude of the increases and decreases will vary across countries as a result of any supply shock. However, if the heteroscedasticity of supply shocks is independent across countries, (meaning that the average change in the variance of supply shocks across all countries is equal to zero), then the average change in covariance of relative asset prices coming from supply shocks will be equal to zero as well.

In summary then, the change in the tightening of the portfolio constraint implies a shift in second moments that is common to all pairs in the data, assuming that all stock markets are measured in the same currency. Supply shocks do not share this property. We use this difference to implement our estimator of contagion. Denote the covariance matrix of all relative stock prices as

$$\Omega_t = \text{covar} \begin{bmatrix} \frac{dS^0(t)}{S^0(t)} \\ \frac{dS^1(t)}{S^1(t)} - \frac{dq^1(t)}{q^1(t)} \\ \frac{dS^2(t)}{S^2(t)} - \frac{dq^2(t)}{q^2(t)} \\ \vdots \\ \frac{dS^n(t)}{S^n(t)} - \frac{dq^n(t)}{q^n(t)} \end{bmatrix}$$

then, the average change in all the elements of Ω_t is proportional to the change in variance of $\frac{d\lambda(t)}{\lambda(t)}$, induced by a change in the tightening of the portfolio constraint. The average covariance change of relative asset prices is thus our new contagion measure at any point in

time.

Although the assumptions required to rationalize this measure of contagion are strong, (namely, output shocks are independent and their conditional volatility is also independent across countries), its implementation is extremely simple. The first step is to compute all stock markets in one common currency, (we use the US dollar as our base). Secondly, we estimate the covariance matrix displayed above at a daily frequency using a rolling window; the justification for this methodology is provided shortly. (Given that we have assumed a dynamic daily portfolio constraint, we need to use a time horizon of one day in our contagion estimation procedure). Thirdly, we estimate the daily change in the average of all the elements in this covariance matrix in order to identify changes in the constraint, and thus contagion, over time.

4.1. Description of the Data

The data we have collected on global stock markets and exchange rates¹¹ in order to estimate our covariance matrix originates from DataStream 4.0. Daily stock market data was obtained from the "DataStream Total Market" index for each country listed in the database, (there were 51 countries in total reporting this index), both in US dollars and local currency, from February 9th 2007 to February 11th 2008. The US dollar and local currency values of the stock market index for each country were then used to calculate individual exchange rates. For countries which did not report the "DataStream Total Market" index in both US dollars and their local currency, daily spot exchange rates were downloaded instead.

The full list of countries reporting daily stock market index data from DataStream comprises, (in alphabetic order): Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China, Columbia, Cyprus, Denmark, Finland, France, Germany, Greece, Hong Kong, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, South Korea, Luxembourg, Malaysia, Mexico, Netherland, New Zealand, Norway, Pakistan, Peru, Philippines,

¹¹Strictly speaking, given that we are taking our common currency to be the US dollar, in order to compute the global covariance of relative asset prices we need daily data on the terms of trade, or the relative price of a representative consumption basket, for all other countries in the world relative to the US. It is not possible, however, to obtain data on global terms of trade at a daily frequency, meaning that we have to use daily exchange rate data instead

Poland, Portugal, Romania, Russia, Singapore, Slovenia, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Taiwan, Thailand, Turkey, United Kingdom, United States of America and Venezuela.

Two of these countries, Columbia and Venezuela, were dropped from our sample due to inadequate exchange rate data, leaving us with 49 countries in total.

4.2. Contagion Measure

We implement our contagion estimator from February to October 2007. This is a period of time during which news about the sub-prime crisis dominated financial markets in the US. We estimate the daily covariance matrix for relative asset prices across all countries, (measuring stock prices in US dollars), using a rolling window of 5 days. (We also estimate the covariance matrix using 10 and 20 day rolling windows and the results are qualitatively the same. Our preferred specification uses the highest frequency possible.)¹² Using the rolling window we then compute the change in the average of all the elements of the covariance matrix from day to day. We depict the results in Figure 2. We also present the conditional daily standard deviation of the US stock market, computed using the same 5 day rolling window, for discussion purposes only. All daily stock market and exchange rate data is measured in logs.

It is interesting to compare our measure of contagion to the stock market standard deviation in the US. Clearly, the sub-prime crisis was a shock which originated in the US, and therefore it may initially make sense to use the US stock market volatility as a measure of the size of that shock. However, that would be incorrect, given that the US stock markets have been hit by a multitude of shocks which may not have been propagated with the same intensity to other countries. For instance, the implicit expected monetary policy response to the sub-prime crisis has been accounted for in the US markets, but might not have

¹²One important question is if we can compute the daily implied variance-covariance matrix for relative asset prices from the data. Unfortunately we cannot, and thus have to use a rolling window to estimate covariances instead. In particular, we may have daily information about implied volatility in some countries from options on the stock market indices, but we certainly do not have information that we could use to estimate daily covariances. It is also worth highlighting that we cannot estimate the conditional variance-covariance matrix in our case either. The reason is that we are allowing the covariance terms across countries to be as unrestricted as possible, and are just using their average shift to estimate contagion.

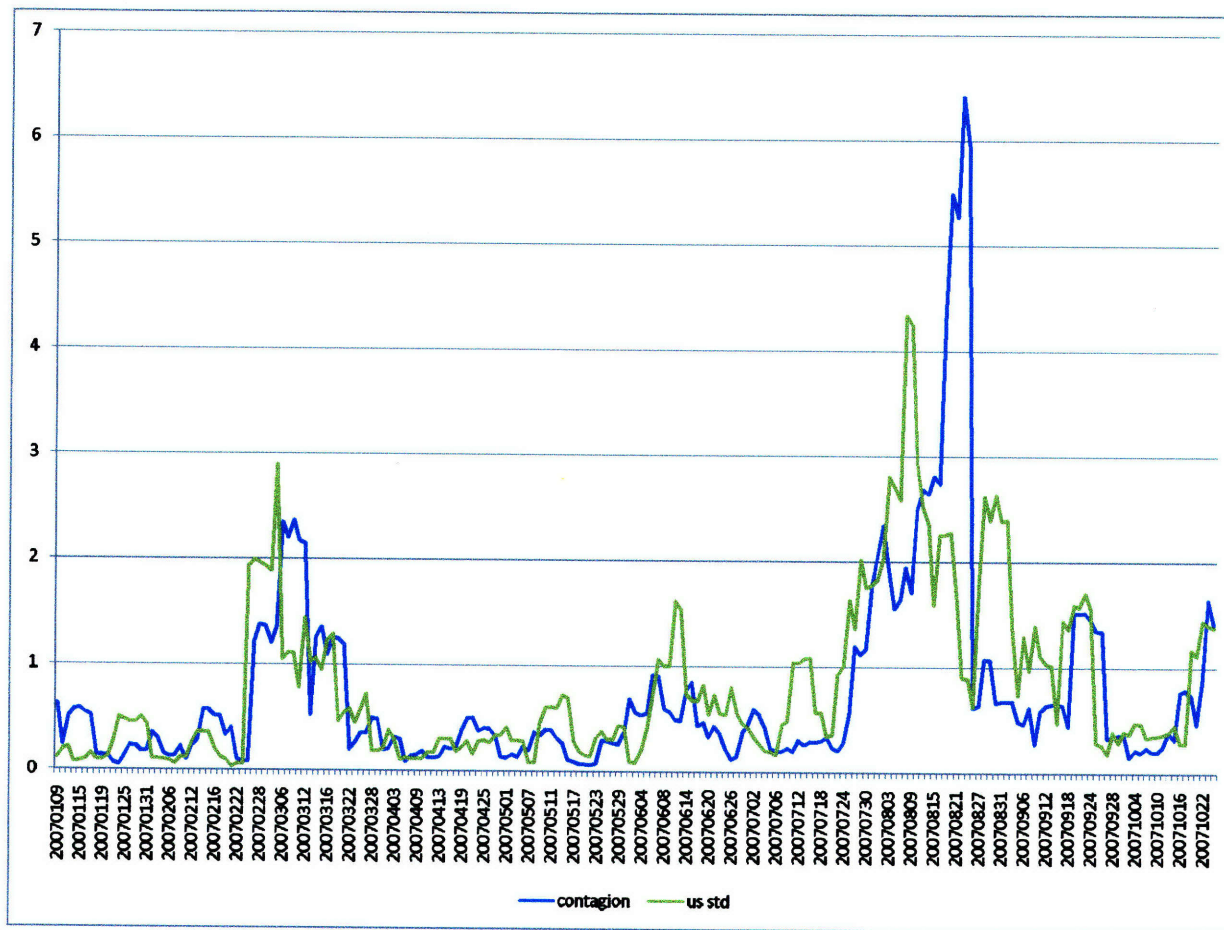


Figure 2: Contagion Measure and Rolling Standard Deviation of US Stock Market.

been reflected in other countries. Furthermore, fiscal packages in the US will have affected US asset prices, but might not have been accounted for to the same degree elsewhere in the world. Our estimator of contagion, however, captures exclusively the increase in co-movement across asset prices that is compounded from all sources of shocks in the US; these include the automatic responses, and the surprise responses as well, of fiscal and monetary policy.

Interestingly, even though in the figure our measure of contagion and the US stock market standard deviation appear to be correlated, their correlation in levels is only 52.9 percent, and -2.8 percent in changes. In other words, these variables do indeed appear to be measuring very different aspects of the US sub-prime crisis.

In Figure 2 we can clearly see two major instances in which the contagion measure increased significantly. One event starts in February 26th, 2007, and the other starts after July 19th, 2007. In Figure 3 we present all 49 stock market prices measured in dollars and normalized to have a price of one on February 26th of 2007; the scale for all stock market prices is displayed on the left vertical axis of the graph. The corresponding scale for the contagion measure, represented by the separate thick line in the chart, is displayed on the right vertical axis. The idea here is to observe the co-movement of stock prices globally, and how it increases, when our contagion estimator increases. We notice that except for a few countries, all stock markets in the world collapse together immediately following the February 26th event, and our contagion measure increases sharply at the same time. The opposite trend can be observed around the beginning of the second week of March, when stock markets start to pick up again and diverge from each other, and our contagion measure drops at the same time. Therefore, there appears to be a strong positive correlation here between changes in the contagion estimator and co-movement of stock prices across the globe, following the outbreak of the sub-prime crisis.

Figure 4 presents the same exercise for the July 19th event. In the first case in February, our measure of contagion increases from an average of 0.29 to a maximum of 2.37. During the July 19th event, the average is 0.39 and increases to 6.42. Because the same methodology is used throughout, we can therefore say that the second event is more contagious, in the

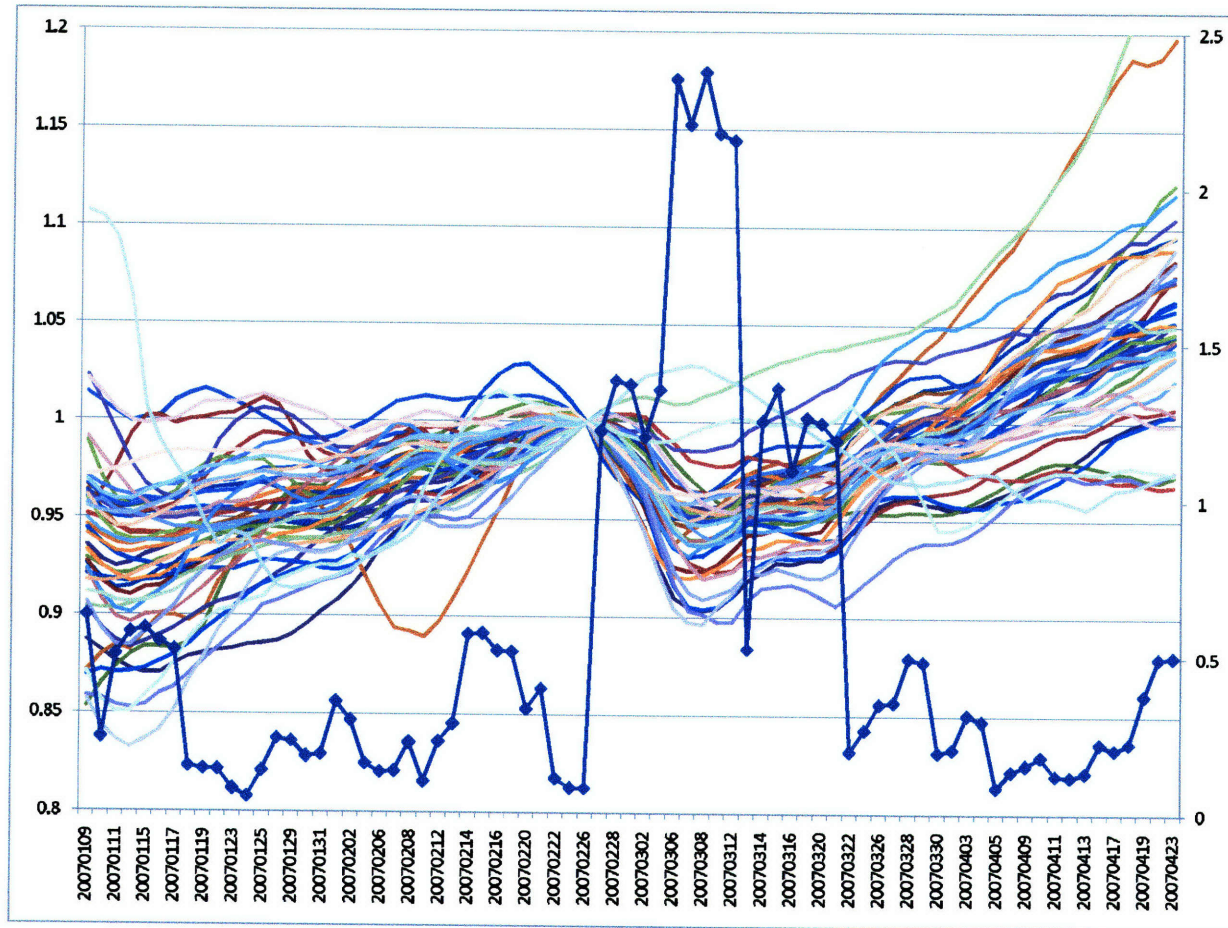


Figure 3: Sub-prime Crisis: February Contagious Event.

sense that the sub-prime shock should increase the degree of co-movement across countries much more. Indeed, not surprisingly, asset prices do appear to co-move with higher intensity after the shock in July 19th, than after the corresponding shock in February. Furthermore, we observe that the same positive relationship between changes in the contagion estimator and co-movement of stock prices holds for the July contagious event as well, now magnified to an even greater extent.

4.3. Sub-prime News

An alternative measure of the “size” of the sub-prime shock is to compute the number of news articles that have appeared in world newspapers about the sub-prime crisis. We obtained this data from the Lexis-Nexis Academic search engine, using the search prompt “subprime” OR “sub-prime”, for four distinct media groups: US Newspapers and Wires, Major World Newspapers, Wire Services Stories (All Wires), and News, All (English, Full Text). The total number of articles and/or wires found in accord with these prompts was recorded on a daily basis from January 1st 2007 to February 19th 2008. In Figure 5 we present the actual numbers of articles that mention the sub-prime crisis from these four different sources of news. Obviously the series are highly correlated and present a very clear pattern: when the sub-prime crisis becomes an important issue, then newspapers tend to report more articles about the crisis. ¹³

Figure 6 presents the number of news articles (in logs) together with our measure of contagion and the rolling standard deviation of the US stock market. (The scale for the contagion measure and the US stock market standard deviation is displayed on the left vertical axis of the graph, and the scale for the news is on the right vertical axis). Once again, we focus on the period February to October 2007. As can easily be seen, there is a significantly positive correlation between changes in our estimator of contagion and changes in the actual number of daily news articles reporting the sub-prime crisis. For instance, the simple daily correlation between our contagion measure and the number of news articles is

¹³We are presenting the data only for the weekdays. During the weekends there is a significant decline in the number of news articles reported; hence, there is weekly seasonality that does not appear in the figure. We decided to drop those observations as opposed to including them as news for the following Monday.

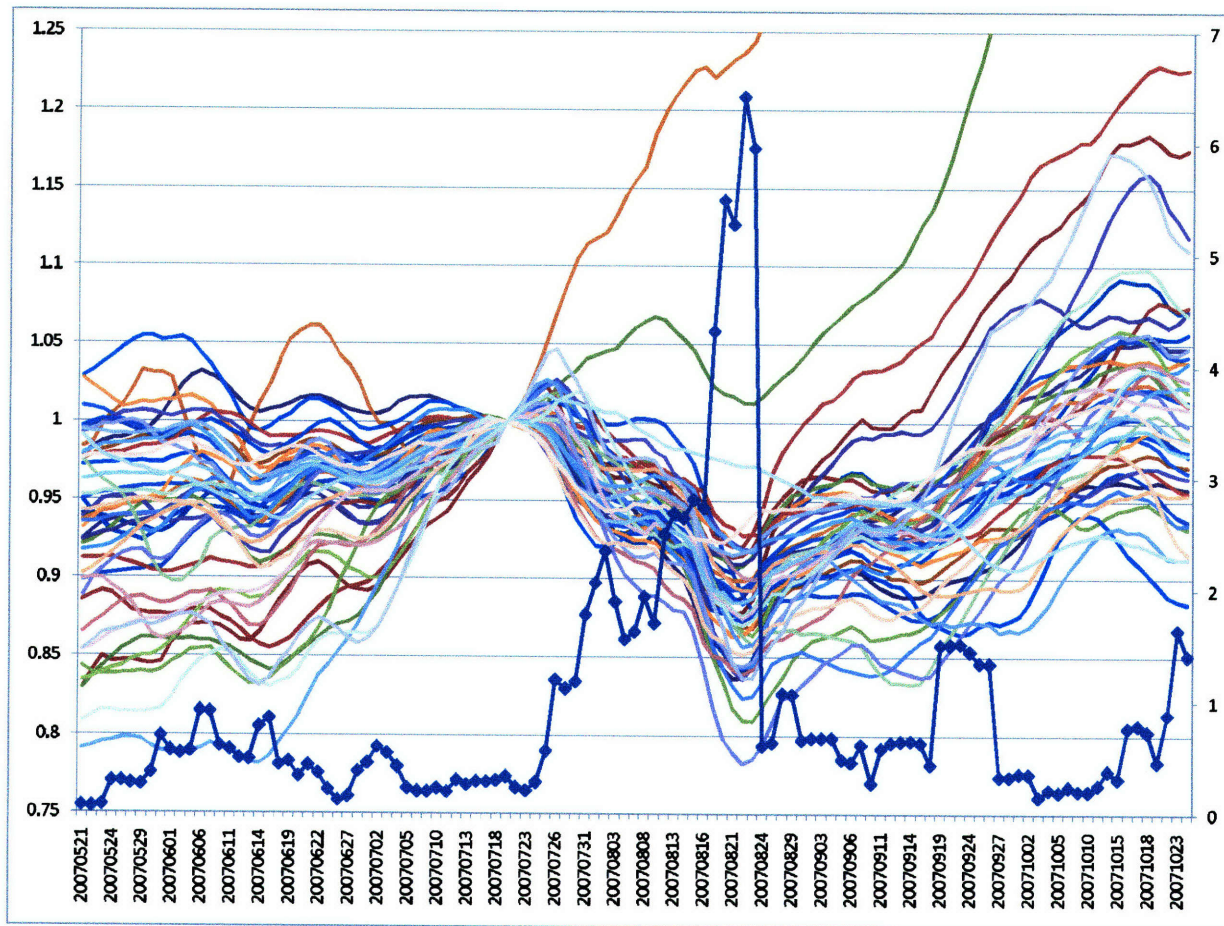


Figure 4: Sub-prime Crisis: July Contagious Event.

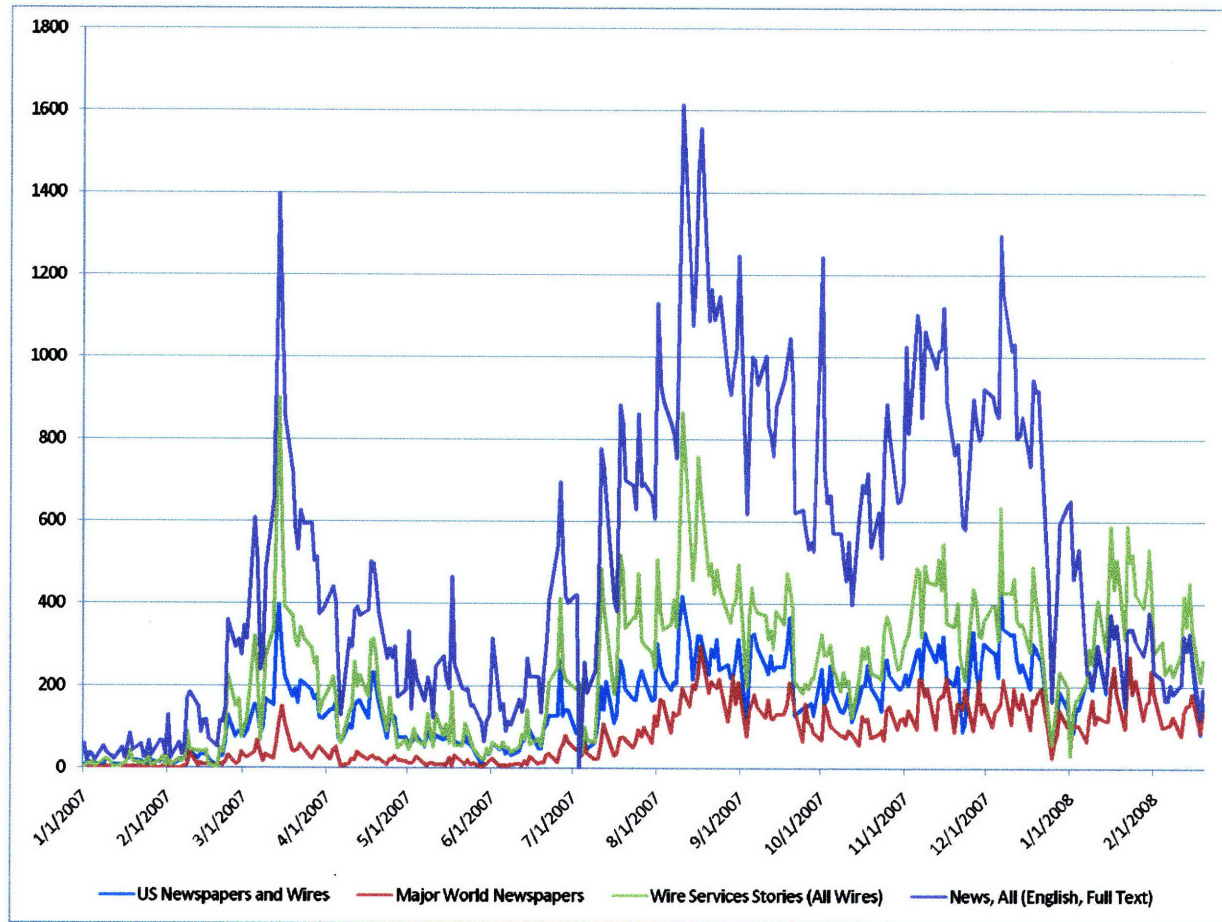


Figure 5: News from Four Different Sources. January 2007 - February 2008.

11.8 percent, (in first differences), but the corresponding correlation in changes between the US stock market standard deviation and the number of news articles (contagion measure) drops to only 2.6 (-2.8) percent.

Following on from this, we also specifically evaluate our contagion measure's predictive power on the news reported, and its changes, at different horizons in a regression based framework. In particular, one would expect that when our contagion estimator increases, there should be an increase in the number of news articles documenting the sub-prime crisis. We measure news in changes of logs and contagion in changes of our estimator. The results from the regression over the period February to October 2007, (with leads and lags of our contagion measure included for robustness), are presented in Table 2. Here, one observes that very few of the coefficients on changes in the contagion measure are statistically or economically different from zero. Indeed, just about less than 5 percent of the coefficients are statistically different from zero at the 95 percent confidence level. However, in Table 3 we present the p-values from the F-test that all the coefficients on contagion are zero. As can be seen, these hypotheses are strongly rejected after at least one lag of the contagion measure is included in the regression. Furthermore, the higher the number of leads and lags included in our specification, the stronger the total cumulative effect of contagion generally becomes. For example, one can see that the p-values from the F-test monotonically decrease as the number of lags increases, (keeping the number of leads constant at zero). Moreover, as the number of leads and lags increase together from 1 to 10, p-values again decrease, and the sum of coefficients on contagion monotonically increases from 0.056 to 0.292. This means that when we include 10 leads and lags of the contagion estimator in our regression specification, the total cumulative effect of a one percentage increase in contagion, on any day, is a 0.29 percentage increase in the (log) number of news articles from that day to the next. The corresponding percentage increase in (log) news with only 1 lead and lag of contagion is, however, much lower, at around 0.06 percent.

The inclusion of lags and leads of the contagion measure in our estimation is not entirely unreasonable, especially given that we estimate contagion using a rolling window. In particular, some of the stock markets across the world may register the effects of contagion a day

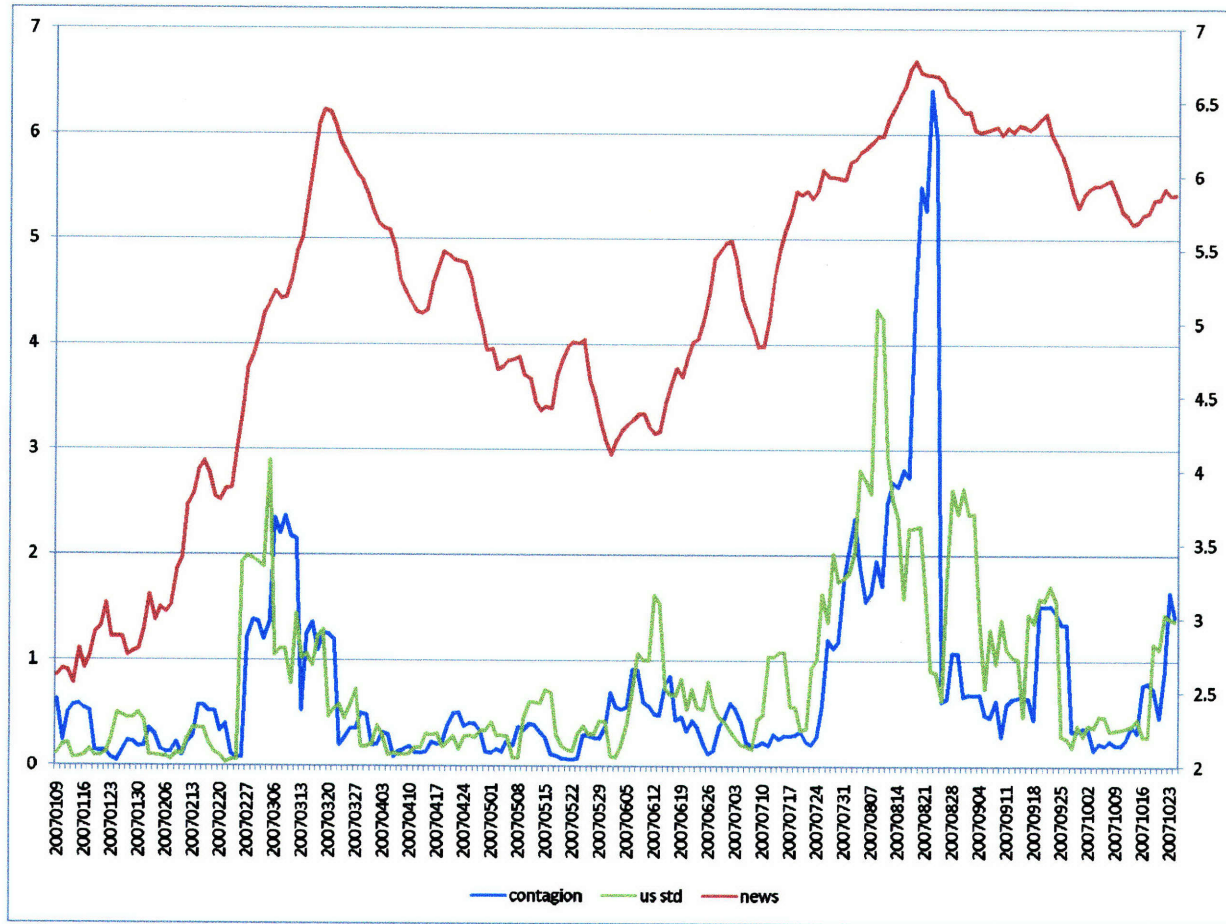


Figure 6: Contagion Measure, Rolling Standard Deviation of US Stock Market and Sub-prime Crisis Number of News.

Leads	Lags	Sum of Coefficients	p-values
0	0	0.02655	9.2%
0	1	0.03295	4.9%
0	2	0.03813	2.6%
0	3	0.02892	1.0%
0	4	0.03795	0.5%
0	5	0.02946	0.3%
0	6	0.04685	0.0%
0	7	0.06538	0.0%
0	8	0.07547	0.0%
0	9	0.10635	0.0%
0	10	0.13024	0.0%
1	1	0.05558	0.4%
2	2	0.08389	0.0%
3	3	0.09956	0.0%
4	4	0.15045	0.0%
5	5	0.15696	0.0%
6	6	0.16997	0.0%
7	7	0.18833	0.0%
8	8	0.20214	0.0%
9	9	0.27612	0.0%
10	10	0.29216	0.0%

Table 3: Total cumulative effect of changes in our contagion measure on the change in (log) news, February - October 2007.

later than the shock originated in the US, and global newspapers also usually report their news a day later than the corresponding market suffered the shock. For all these reasons, it is not surprising that the significance and effect, (i.e. the size of the coefficient on the contagion measure), increase with the horizon used. Another interesting aspect of the estimation here is that the contagion coefficient which is usually significant is the contemporaneous one. For instance, in Figure 7 we present the estimated coefficients on the contagion measure with two 95 percent confidence bands. We show the case with 10 leads and 10 lags of the contagion estimator; the negative numbers to the left on the x-axis represent the leads, while the 10 coefficients to the right, (positive on the x-axis), are the lags of the contagion measure. We observe here that (only) the contemporaneous coefficient on contagion is statistically different from zero at the 5 percent level. This suggests that when we include both 10 leads as well as 10 lags of contagion, an increase in contagion at any point in time produces a statistically significant increase in the number of (log) news articles reported about the sub-prime crisis at the same point in time. Moreover, when we compute the F-test for this specification, the significance of the total cumulative effect of contagion is tremendously high, as we already saw in Table 3.

5. Conclusion

In conclusion, our paper develops a measure of international financial contagion which is derived from the cross-equation restrictions of a model with portfolio constraints. This semi-structural estimation procedure generates a contagion measure which, under certain assumptions about the first and second moments of output shocks, reflects the average change in the joint covariance of all countries in the world with each other. Therefore, our estimator of contagion here differs from standard contagion measures which just focus on calculating the covariance between a 'constrained' country, or the country which suffers an initial shock, (in the case of the sub-prime crisis, this would be the US), and other countries in the world.

In our empirical tests, we demonstrate that changes in our new global contagion estimator are positively correlated with changes in the number of daily news articles documenting the

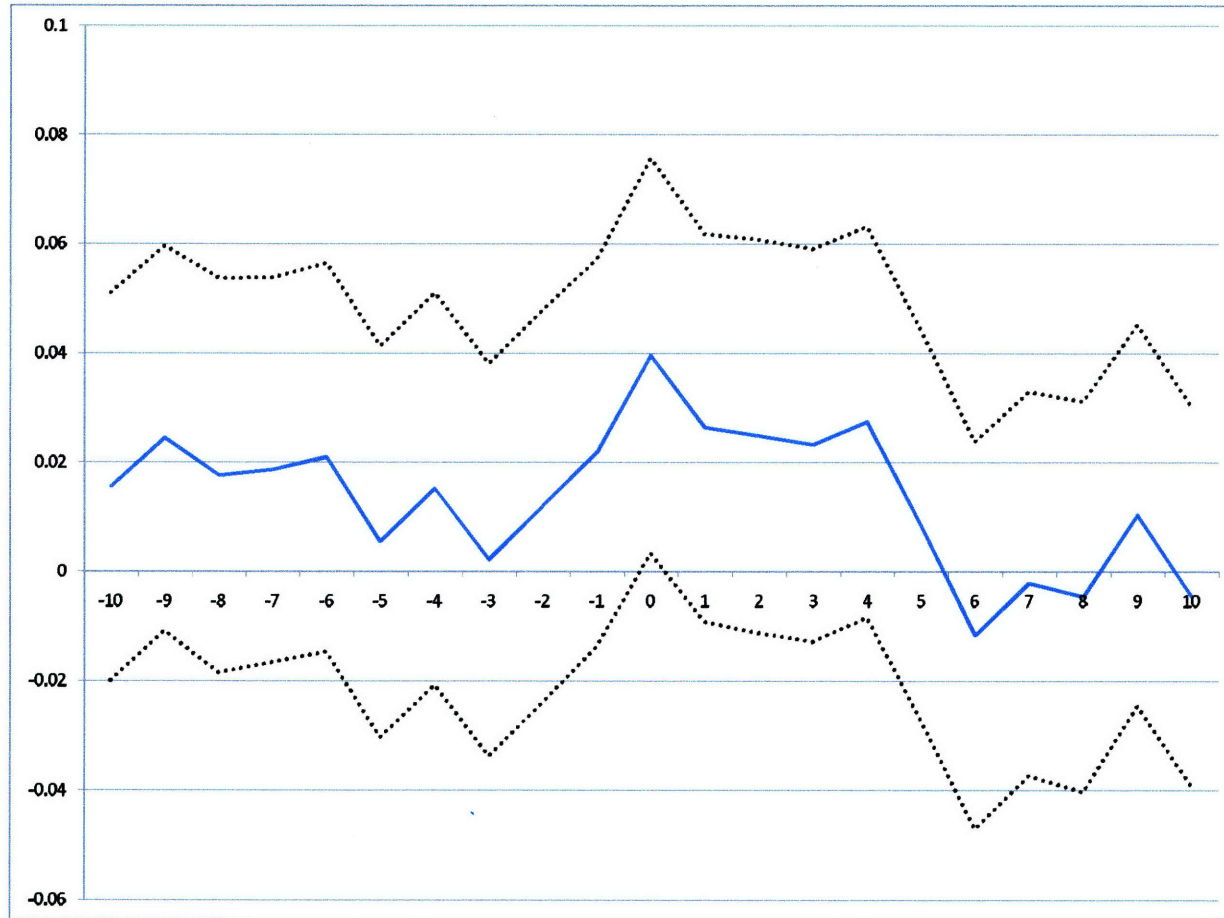


Figure 7: Estimated coefficients on contagion measure for the case with 10 leads and 10 lags, February - October 2007. Dotted lines represent the 95 percent confidence bands.

sub-prime crisis. This correlation is far higher than the corresponding correlation between changes in our contagion measure and changes in the standard deviation of the US stock market, and also between changes in the number of news articles and changes in the standard deviation of the US stock market. These results indicate that the US stock market variance does not measure the degree of the sub-prime crisis to the same extent as either our contagion measure or the volume of news. Furthermore, there is evidence here that our estimator of contagion is itself a strong measure of the sub-prime crisis. These findings are further reinforced by the fact that there appears to be a positive relationship between changes in our contagion measure and co-movement of stock prices across the globe following the sub-prime collapse. Finally, we obtain the result that our estimator of contagion has a highly significant total cumulative effect on changes in the number of news articles reporting the crisis. For example, when we include up to ten leads and lags of the contagion measure, in a regression of changes in news articles on changes in the contagion measure, the null hypothesis that all coefficients are equal to zero is rejected very strongly at the 5 percent level. Moreover, the contemporaneous coefficient on contagion is significantly positive at the 5 percent level.

In sum, these findings reveal that our estimator of contagion is indeed a robust measure, and even predictor, of the sub-prime crisis. The implication from our empirical analysis is then that we can explain the spread of international financial contagion due to the sub-prime collapse, at least to a certain extent, via the tightening of financial constraints facing banks or institutions in the US. This result, as well as the new estimation procedure for contagion introduced in the paper, represents our most important contribution to the literature.

Appendix: Proofs

Proof of Propositions 1 and 2 See Pavlova and Rigobon (2007)

Proof of Proposition 3 Proposition 3 can be proved as follows. Let us consider the partial optimization problem of Country 0, where the representative agent is maximizing his expected utility over trading strategies $x_0(t)$, subject to the constraint in (8). In particular, from Pavlova and Rigobon (2007), we can represent the objective function of country 0 in the form

$$E \int_0^T [\log W_0(t) - K(t)] dt.$$

where $K(t) = \alpha_0 \log(p^0(t)(T-t)) + \frac{1-\alpha_0}{2} \log(p^1(t)(T-t)) + \frac{1-\alpha_0}{2} \log(p^2(t)(T-t))$. The investor of country 0 takes prices in the good markets p^j , $j = 0, 1, 2$ as given, and hence from his viewpoint $K(t)$ is exogenous at any time t . Therefore, the constrained maximization problem for the Center Country reduces to the following:

$$\max_{W_0(t)} E \left[\int_0^T \log W_0(t) dt \right] \quad (\text{A.1})$$

$$\text{subject to } x_0(t)^\top \sigma(t) \sigma(t)^\top x_0(t) \leq \phi(t). \quad (\text{A.2})$$

Since we know that, for Country 0:

$$dW_0(t) = W_0(t)[(r(t)1 + x_0(t)^\top (\mu(t) - r(t)1)dt + (x_0(t)^\top \sigma(t))dw(t)], \quad (\text{A.3})$$

(where $\mu(t)$ and $\sigma(t)$ are, respectively, the vector of expected stock returns and the volatility matrix of the investment opportunity set), then it follows from (A.3) that:

$$d \log W_0(t) = [(r(t)1 + x_0(t)^\top (\mu(t) - r(t)1) - \frac{1}{2}|x_0(t)^\top \sigma(t)|^2)dt + x_0(t)^\top \sigma(t)dw(t)] \quad (\text{A.4})$$

$$W_0(t) = W_0(0)e^{\int_0^t (r(t)1 + x_0(t)^\top (\mu(t) - r(t)1) - \frac{1}{2}|x_0(t)^\top \sigma(t)|^2)dt + \int_0^t x_0(t)^\top \sigma(t)dw(t)} \quad (\text{A.5})$$

Therefore, if we assume in addition that $x_0(t)^\top \sigma(t) \in (H^2)^3$, (the set of square-integrable processes), so that $E \left[\int_0^T x_0(t)^\top \sigma(t)dw(t) \right] = 0$, then substituting (A.4) into (A.1) implies that (A.1) reduces to the following problem:

$$\max_{W_0(t)} \left[\int_0^T x_0(t)^\top (\mu(t) - r(t)1) - \frac{1}{2}|x_0(t)^\top \sigma(t)|^2 dt \right] \quad (\text{A.6})$$

$$\text{subject to } x_0(t)^\top \sigma(t) \sigma(t)^\top x_0(t) \leq \phi(t) \quad (\text{A.7})$$

where ψ is the multiplier on the volatility constraint above (so that $\psi = 0$ when the constraint is not binding).

The first order conditions from (A.6) at time t imply that:

$$x_0(t) = \frac{\sigma^{-1}(t)^\top m(t)}{1 + 2\psi} \quad (\text{A.8})$$

where $m(t) = \sigma(t)^{-1}(\mu(t) - r(t))$.

Now using the fact, from Pavlova and Rigobon (2007), that the countries' optimal portfolios of risky assets are given by:

$$x_0(t) = (\sigma(t)^\top)^{-1} m_0(t), \quad x_i(t) = (\sigma(t)^\top)^{-1} m(t), \quad i \in \{1, 2\}.$$

we can solve for $m_0(t)$ by reexpressing (A.8) as:

$$m_0(t) = \sigma(t)^\top x_0(t) = \frac{m(t)}{1 + 2\psi} \quad (\text{A.9})$$

Substituting (A.8) into (8), (with the inequality constraint binding now as an equality), we can solve for $\psi(t)$:

$$(1 + 2\psi(t))^2 = \frac{m(t)^\top m(t)}{\phi(t)} \quad (\text{Constraint binding}) \quad (\text{A.10})$$

$$\psi(t) = 0 \quad (\text{Constraint not binding}) \quad (\text{A.11})$$

Therefore, to summarize, in the case where the constraint is not binding, $\psi(t) = 0$, and therefore $m_0(t) = m(t)$. This is reported in (10). In the case where the constraint is binding, (A.9) and (A.10) give us the complementary slackness conditions presented in (11). Equation (12) follows from market clearing, coupled with the investors' first-order conditions.

Q.E.D.

Proof of Corollary 4. We first present the proof of Corollary 4, and show that the analogous result in the unconstrained environment, i.e. result (v) of Proposition 1, is just a special case of this. Consider Country 0's portfolio. From (iii) of Proposition 1, and expressions for optimal consumption allocations and sharing rules for aggregate endowment from Pavlova and Rigobon (2007), we have:

$$\begin{aligned} S^0(t) &= p^0(t)Y^0(t)(T - t) \\ S^1(t) &= p^1(t)Y^1(t)(T - t) \\ C_0^0(t) &= \frac{1}{p^0(t)(T - t)} \alpha_0 W_0(t) = \frac{Y^0(t)}{\alpha_0 + \lambda_1 \frac{1 - \alpha_1(t)}{2} + \lambda_2 \frac{1 - \alpha_2(t)}{2}} \alpha_0 \\ C_0^1(t) &= \frac{1}{p^1(t)(T - t)} \frac{1 - \alpha_0}{2} W_0(t) = \frac{Y^1(t)}{\frac{1 - \alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1 - \alpha_2(t)}{2}} \frac{1 - \alpha_0}{2} \end{aligned}$$

Therefore, after some manipulation:

$$\frac{W_0(t)}{S^0(t)} = \frac{1}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

$$\frac{W_0(t)}{S^1(t)} = \frac{1}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

Following the exact same procedure for the portfolios of Countries 1 and 2, we can report the following results for the Countries' wealth as a proportion of stock prices. (Here, we just present results for S^0 and S^1 for simplicity - the expressions for the Countries' wealth as a proportion of the stock price S^2 are derived analogously):

$$\frac{W_1(t)}{S^0(t)} = \frac{\lambda_1}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

$$\frac{W_1(t)}{S^1(t)} = \frac{\lambda_1}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}}$$

$$\frac{W_2(t)}{S^0(t)} = \frac{\lambda_2}{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)}$$

$$\frac{W_2(t)}{S^1(t)} = \frac{\lambda_2}{\frac{1-\alpha_0}{2} + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \alpha_2(t)}$$

Therefore, letting $N_j^i(t)$ be the number of shares Country i holds in stock j , then we have:

$$N_j^i(t) = \frac{x_i(t)^\top i_j W_i(t)}{S^j(t)} \quad i = 0, 1, 2, \quad j = 0, 1, 2, \quad (\text{A.12})$$

Using the securities market clearing equation for all stocks j , we also have:

$$N_j^0(t) + N_j^1(t) + N_j^2(t) = 1 \quad j = 0, 1, 2 \quad (\text{A.13})$$

Therefore, applying (A.12) and (A.13) for stocks 0 and 1, (i.e. for $j = 0, 1$), we have:

$$1 = \frac{x_0(t)^\top i_0 + x_1(t)^\top i_0 \lambda_1 + x_2(t)^\top i_0 \lambda_2}{\alpha_0 + \lambda_1 \frac{1-\alpha_1(t)}{2} + \lambda_2 \frac{1-\alpha_2(t)}{2}} \quad (\text{A.14})$$

$$1 = \frac{x_0(t)^\top i_1 + x_1(t)^\top i_1 \lambda_1 + x_2(t)^\top i_1 \lambda_2}{\frac{1-\alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1-\alpha_2(t)}{2}} \quad (\text{A.15})$$

Now, from the expression above, (in the proof of Proposition 3), for countries' optimal portfolios of risky assets, and equation (11), we can easily derive the following expression for Country 0's portfolio in terms of the portfolio of the periphery countries, (letting $x(t) = x_1(t) = x_2(t)$ denote the portfolio held by each of the Periphery countries, given that these will always be identical in both the constrained and unconstrained cases):

$$x_0(t) = \frac{x(t)}{1 + 2\psi} \quad (\text{A.16})$$

Therefore, using (A.16), (A.14) and (A.15) can be reexpressed as:

$$1 = \frac{x_0(t)^\top i_0 (1 + (\lambda_1 + \lambda_2)(1 + 2\psi))}{\alpha_0 + \lambda_1 \frac{1 - \alpha_1(t)}{2} + \lambda_2 \frac{1 - \alpha_2(t)}{2}} \quad (\text{A.17})$$

$$1 = \frac{x_0(t)^\top i_1 (1 + (\lambda_1 + \lambda_2)(1 + 2\psi))}{\frac{1 - \alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1 - \alpha_2(t)}{2}} \quad (\text{A.18})$$

Using the above two equations, it follows directly that:

$$\frac{x_0(t)^\top i_0}{\alpha_0 + \lambda_1 \frac{1 - \alpha_1(t)}{2} + \lambda_2 \frac{1 - \alpha_2(t)}{2}} = \frac{x_0(t)^\top i_1}{\frac{1 - \alpha_0}{2} + \lambda_1 \alpha_1(t) + \lambda_2 \frac{1 - \alpha_2(t)}{2}} \quad (\text{A.19})$$

The left hand side of (A.19), is equal to $N_0^0(t)$, and the right hand side is equal to $N_1^0(t)$, i.e. we have shown that Country 0 holds the same number of shares in stocks 0 and 1. This argument can be easily extended to show that Country 0 holds the same number of shares in all stocks, (using the securities market clearing equation for S^2 and manipulating in the same way). The same logic applies to Countries 1 and 2. The proof of the analogous result for the unconstrained setting is then just a special case of this argument with $\psi = 0$. Therefore, in both the unconstrained and constrained settings, all countries hold an equal number of shares in stocks 0, 1 and 2.

Furthermore, in the unconstrained case, since $\psi = 0$, it can also be shown very easily from manipulating equations (A.12) and (A.13), (for all i and j), that for all Countries i and Stocks j :

$$N_j^i(t) = \frac{\lambda_i}{\lambda_0 + \lambda_1 + \lambda_2}, \quad \lambda_0 = 1 \quad (\text{A.20})$$

In other words, when there is no constraint, the number of shares of each stock purchased by any given Country is equal to its relative wealth share in the world economy. It follows that in the benchmark equilibrium, agents in each Country invest all their wealth in their respective stock portfolios. Therefore, no shares of the bond are traded in equilibrium. We do not arrive at the same result in the constrained case, however, since we have the extra term ψ appearing in equation (A.13) for all i and j , (implying that equation (A.20) no longer holds). In this scenario, therefore, agents have different holdings of the bond in equilibrium.

Q.E.D.

Derivation of r_0 (for Country 0) Equations (10) and (11) are derived at a partial equilibrium level. The partial-equilibrium constrained optimization problem of country 0 is

an example of the class of problems considered by Cvitanić and Karatzas, (Cvitanić and Karatzas (1992)). In particular, as a result of the volatility constraint (8), portfolio values x_0 are restricted to lie in a closed, convex, non-empty subset $K \in R^3$. We can define the constraint set K as follows:

$$K = \{x_0(t); x_0(t)^\top \sigma(t) \sigma(t)^\top x_0(t) \leq \phi(t)\}.$$

As previously mentioned, the problem of Country 0 facing a portfolio constraint is equivalent to a fictitious problem, based on a different financial market with no constraints. In particular, this fictitious market is tilted, or rather, the (constrained) Center country's effective interest rate and the market price of risk, r_0 and m_0 , are tilted to reflect the extent to which the country's investments are constrained. Let this tilt be captured by a three-dimensional vector of multipliers, $\nu(t)$, so that for each process $\nu(t)$, (in the set of processes adapted to $\{\mathcal{F}_t; t \in [0, T]\}$), we define a fictitious market M^ν , in which the three stocks and the global bond are traded. Define the support function, $\delta(\nu(t))$ as follows:

$$\delta(\nu(t)) = \sup_{x_0(t) \in K} (-x_0(t)^\top \nu(t))$$

We can also define the effective domain of the support function, \tilde{K} , as :

$$\tilde{K} = \{\nu(t); \delta(\nu(t)) \leq \text{infinity}\}.$$

Now we can define the following tilted processes in the fictitious, unconstrained market :

$$r_0(t) = r(t) + \delta(\nu(t)) \quad (\text{A.21})$$

$$m_0(t) = m(t) + \sigma^{-1}(t)\nu(t) \quad (\text{A.22})$$

$$d\xi^\nu(t) = -\xi^\nu(t)[(r(t) + \delta(\nu(t)))dt + (\sigma^{-1}(t)\nu(t) + m(t)^\top)dw(t)] \quad (\text{A.23})$$

Given the quadratic form of our constraint, and the definition of $\delta(\nu(t))$ and K above, we can easily solve for $\delta(\nu(t))$ from a simple constrained minimization problem. We obtain:

$$\delta(\nu(t)) = (\phi(t)\nu(t)^\top (\sigma(t)\sigma(t)^\top)^{-1}\nu(t))^{1/2} \quad \nu(t) \in \tilde{K}. \quad (\text{A.24})$$

In the fictitious market characterized by $\nu(t)$, Country 0 will maximize his expected utility of consumption subject to his budget constraint, (with state price density process $\xi^\nu(t)$ replacing $\xi_0(t)$). The dynamic portfolio choice problem in the fictitious market without constraints, M^ν , can be equivalently expressed in a static form:

$$Q^\nu = \sup_{C_0^0, C_0^1, C_0^2} E \left[\int_0^T u_0(C_0^0(t), C_0^1(t), C_0^2(t)) dt \right] \quad (\text{A.25})$$

$$\text{subject to} \quad E[\xi^\nu(T)W_0(T)] \leq W_0(0). \quad (\text{A.26})$$

where Q^ν is the value function defined in market M^ν , (and we define $\tilde{\psi}$ to be the multiplier on the wealth constraint above). In addition, from Cvitanić and Karatzas, we also know that for any admissible choice of ν , the value function Q^ν gives an upper bound for the optimal value function of the original, constrained problem. Therefore, if we minimize the value function of the fictitious problem, Q^ν , with respect to ν , (and the minimum corresponds to a feasible trading policy), then we have found an optimal portfolio policy under the constraint on Country 0. (Cvitanić and Karatzas outline sufficient conditions under which the solution of the minimization problem exists and leads to the optimal portfolio policy, i.e. conditions under which there is no duality gap).

Referring to the proof of Proposition 1, we can substitute Country 0's log utility function into (A.25) to obtain the following equivalence relation between the minimization problems:

$$\min_{\nu \in \tilde{K}} Q^\nu = \min_{\nu \in \tilde{K}} E \left[\sup_{W_0(t)} \int_0^T \log W_0(t) dt - W_0(T) \tilde{\psi} \xi^\nu(T) \right] = \min_{\nu \in \tilde{K}} \tilde{U}(\tilde{\psi} \xi^\nu(T))$$

We can now form the Bellman equation for this problem, where the value function, $Q(t, y)$ is a function of t and state variable y , $Q(T, y) = \tilde{U}(y)$, and $y = \tilde{\psi} \xi^\nu(t)$. (Here, we have suppressed the dependence of y on t). Therefore, using (A.23) to obtain the drift of $\xi^\nu(t)$, (which in turn determines the drift of y over time), the standard Bellman equation for this problem is as follows:

$$\min_{\nu \in \tilde{K}} (Q_t - y Q_y (\delta(\nu(t)) + r(t)) + \frac{1}{2} Q_{yy} y^2 (m(t) + \sigma^{-1}(t) \nu(t))^2) = 0 \quad (\text{A.27})$$

Since we have a log utility function for Country 0, (which is a special case of a power function with curvature/elasticity of intertemporal substitution set equal to 1), the Value function $Q(t, y)$ solving (A.27) will take the following standard form:

$$Q(t, y) = \log y f(t)$$

for some function of time, $f(t)$.

In particular, for all power functions, $y Q_y$ and $y^2 Q_{yy}$ will be constant, so the optimal $\nu(t)$ from (A.27) will not depend on the state variable y . Therefore, equation (A.27) reduces to the minimization problem below:

$$\min_{\nu \in \tilde{K}} ((m(t) + \sigma^{-1}(t) \nu(t))^2 + \delta(\nu(t))) = 0 \quad (\text{A.28})$$

Therefore, let $\nu^*(t)$ be the solution to (A.28) so that:

$$\nu^*(t) = \operatorname{argmin}((m(t) + \sigma^{-1}(t) \nu(t))^2 + \delta(\nu(t)))$$

Substituting (A.24) into the above, we have, equivalently:

$$\nu^*(t) = \operatorname{argmin}((m(t) + \sigma^{-1}(t)\nu(t))^2 + (\phi(t)\nu(t)^\top(\sigma(t)\sigma(t)^\top)^{-1}\nu(t))^{1/2})$$

A nonzero solution to this minimization problem will exist if and only if the original volatility constraint,(8), is binding, (so that, obviously, $\nu^* = 0$ when Country 0 is unconstrained and does not face a fictitious set of investment opportunities).

Let us consider the case where (8) is binding. From (A.22), we have :

$$\nu^*(t) = \sigma(t)(m_0(t) - m(t)) \quad (\text{A.29})$$

Substituting (A.9) into (A.29), we obtain the following expression for ν^* in terms of the multiplier $\psi(t)$:

$$\frac{-2\psi(t)}{1 + 2\psi(t)} \sigma(t) m(t) = \nu^*(t) \quad (\text{A.30})$$

From (A.22) and (A.24), we then have:

$$r_0(t) = r(t) + (\phi(t)\nu^*(t)^\top(\sigma(t)\sigma(t)^\top)^{-1}\nu^*(t))^{1/2} \quad (\text{A.31})$$

When (8) is not binding:

$$\nu^*(t) = 0 \quad (\text{A.32})$$

$$m_0(t) = m(t) \quad (\text{A.33})$$

$$r_0(t) = r(t) \quad (\text{A.34})$$

Q.E.D.

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