Essays in Industrial Economics

by

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Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2008

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Abstract

This dissertation consists of three chapters in Industrial Economics. Chapter 1 is the product of joint work with Filippo Balestrieri. In chapter 1 we examine the use of lotteries among horizontal differentiated goods as a mechanism to price discriminate consumers. We use the linear city model to represent a market with two differentiated goods. We show that the optimal selling strategy for a multiproduct monopolist implies offering at least one lottery with probability $\frac{1}{2}$. This result is in stark contrast with the the optimal selling strategy of a single product monopolist that attempts to price discriminate consumers based on the probability of delivering the good. Riley and Zeckhauser (1983) show that the single good monopolist does not offer lotteries. We then examine the use of lotteries among the differentiated goods in the case of a market with two competing firms, each one producing one good. We define two different cases depending on whether the market is fully covered. We call fully covered market the case in which the equilibrium prices of the two firms are such that all consumers buy one good. We show that if the market is fully covered, no lotteries are offered in equilibrium. However, when the market is not fully covered the optimal selling strategy may include offering lotteries. With more than two firms selling differentiated goods, even in a fully covered market, lotteries can be used in equilibrium. In this case, firms might be worse off than in the case where no lotteries are provided.

Chapter 2 examines firms optimal pricing policy when they sell a storable good of repeated consumption to time-inconsistent consumers. Consumers with time-inconsistent preferences might struggle to make optimal consumption decisions over time. Sophisticate consumers, aware of their time-inconsistent preferences, often try to limit their consumption of certain goods by strategically rationing the quantities they purchase. On the other hand, naive consumers, unaware of their time-inconsistent preferences, may stockpile tempting goods at "home" not realizing that the higher availability of the good might lead them to overconsume the good.

It is shown that if consumers are time-consistent, quantity discounts don't increase the firms' profit. In contrast, if firms face naive time-inconsistent consumers, the optimal pricing policy is to use small quantity discounts as a device to increase sales. These consumers take advantage of quantity discount with the intention of saving on future purchases. However, after buying the good they can not resist and overconsume it. We also show that even if consumers are sophisticated, firms still use quantity discounts. Sophisticated time-inconsistent consumers realize that increasing the quantity purchased often leads them to overconsume the good. Hence, they require a significant quantity discount to increase the quantity purchased.

Offering a quantity discount leads them to stockpile the good "at home" and hence promotes overconsumption. Chapter 3 analyzes the use of exclusive dealing agreements to prevent the entry of rival firms. An exclusive dealing agreement is a contract between a buyer and a seller where the buyer commits to buy a good exclusively from the seller. Exclusive dealing agreements are one of the most common vertical restraints used by firms. Aghion and Bolton (1987) were the first to show that an incumbent seller may want to use exclusive dealing agreement that prevents the entry of a rival seller. They argue that an incumbent seller and a buyer sign an exclusive dealing contract in order to extract surplus from a more efficient entrant seller. We propose an alternative explanation for the use of exclusive dealing agreements to prevent entry when the buyer is a downstream distributor that also faces the threat of entry. The idea is that the entry of more efficient upstream seller, by decreasing the market power of the upstream firms, makes entry in the downstream market more attractive. This can lead to further entry in the downstream market and to an increase in the competition faced by the downstream firms. Since part of the bigger surplus created by the entry of a more efficient seller is captured by the downstream entrant firms, entry in the upstream market does not necessarily benefits the incumbent downstream firms.

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Acknowledgments

I am deeply indebted to my advisors, Glenn Ellison and Abhijit Banerjee, for they constant guidance, support and advice. Their passion for economic research, enthusiasm for understanding the world and intellectual openness were a source of inspiration throughout graduate school. There are no words to describe my personal and intellectual debt to them. I am also very grateful to Jean Tirole, Sergei Izmalkov, Muhamet Yildiz and the other faculty members of the MIT Economics Department for their continued commitment to excellence in teaching and mentoring.

I am thankful to Vivaldo Mendes and Vasco Santos and who encourage me to do a PhD in economics. I also want thank Filippo Balestrieri, my friend and coauthor, for his constant support and creativity. Many other friends contributed to this work and to make my life in Boston such a pleasant experience. In particular I want to thank my roommates Emre Kocatulum and Frantisek Ricka and my classmates Mauro Alessandro, Suman Basu and Ufuk Akcigit.

I am also thankful for the financial support of the Fundacao para a Ciencia e Tecnologia, the Portuguese National Science Foundation, ISCTE and the MIT Department of Economics .

To my Parents and Siblings

Chapter 1

Price discrimination through lotteries

1.1 Introduction

The online travel market have been recently affected by the appearance of innovative websites like hotwire.com or priceline.com. The impact of these new players on the market is interesting because of the peculiar products they offer. Consider, for example, how Hotwire operates in the market for hotel rooms. Apart from the standard menu of hotel rooms, hotwire provides the option to the consumers of paying a lower price for a room in a hotel whose identity is not revealed until after the payment is made. The only information the consumer may use to decide whether to accept this "blind offer" is the category of the hotel and the area where it is located. This mechanism can be described as lottery, whose prizes are the different hotel rooms that match the specified hotel category and location[1]. This innovative selling strategy has been applied successfully in the markets for hotel rooms, airplane tickets[2] and rental cars[3], but can

[1] The mechanism used by priceline.com is more complex. The consumer selects the hotel category and the area he is interested in. Then the consumer makes a bid that may be accepted or refused by a hotel that fits the charachteristics specified. Still the mechanism can be interpreted as requiring the consumer to select a lottery and bid for it.

[2] In the case of airplane tickets, the flight informations are revealed only after the payment. The offer specifies the number of stops and a frame of 6 or 12 hours for the departure time.

[3] Priceline offers the possibility for the consumer to choose the category of the car, an area where to pick up and leave the car. The consumer is uncertain about the rental company and the car assigned to him.

potentially be exported to many other markets with substitute goods.

Riley and Zeckhauser (1983) were the first to examine the use of lotteries as an attempt to screen consumers. They look at a single good monopolist that attempts to price discriminate consumers based on the probability of delivering the good. Consumers with a low valuation of the good might accept to pay less for a lottery with a lower probability of delivering the good, while consumers with a high valuation of the good will be more reluctant to accept such a risk. Interestingly, they show that the optimal selling strategy is to not offer lotteries. The monopolist should simply make a take-it-or-leave-it offer. The benefit obtained through better price discrimination is more than offset by the shift in the demand from the high price good to the low price lottery.

There are two important characteristics of the lotteries used by firms like Hotwire. First, the uncertainty is about which particular good is delivered and not about whether the good is delivered. This type of lottery can be seen as a mechanism to screen consumers with strong preferences for a particular good from consumers that are relatively indifferent among the different goods. Second, the lotteries are sold in a market of oligopolistic competition. A third party buys the goods from the firms and resells them through lotteries to the final consumers.

In this paper we start by examining the use of lotteries by a multiproduct monopolist. Then, to understand the emergence of firms like hotwire, we examine the introduction of lotteries in a market of oligopolistic competition.

We use a Hotelling model to represent a market with two differentiated goods. We show that if the transportation costs function is concave, the optimal selling strategy for a multiproduct monopolist includes just one lottery, where the consumer gets each good with probability 0.5. However, if the transportation costs function is convex, the optimal selling strategy includes a region of consumers, relatively indifferent between the goods, in which the monopolist offers a continuum of type contingent lotteries.

Hence, the optimal selling strategy for a multiproduct monopolist implies offering at least one lottery with probability $\frac{1}{2}$. This result is in stark contrast with the no lottery result for a single product monopolist in Riley and Zeckhauser [6]. The intuition is as follows. The single product monopolist by offering a lottery with probability q of delivering the good (and $1-q$ of not delivering the good) is damaging the good in the same proportion to all consumers. In contrast, the multiproduct monopolist by offering a lottery with probability $\frac{1}{2}$ of getting each differentiated good is damaging the good mostly for consumers with strong preferences for a particular good. If these consumers have also higher willingness to pay, the monopolist can increase its profits by offering a lottery with probability $\frac{1}{2}$.

Interestingly, the single lottery result with concave costs can be seen as extension of the no lottery result of Riley and Zeckhauser (1983) to the multiproduct case. When the transportation costs are concave, all consumers derive a higher utility of a lottery with probability $\frac{1}{2}$ than the consumer located in the middle of the segment, $x = \frac{1}{2}$. Intuitively, we can think as if the monopolist sells a lottery with probability $\frac{1}{2}$ to all consumers. Then, on top of this, he sells an additional option of trading the lottery for their favorite good. In each half of the segment line, the demand for this option is like the demand of a single good monopolist. Consumers located at the extreme of the segment have the highest willingness to pay for this extra option, while the consumer located at $x = \frac{1}{2}$ is willing to pay zero. Consider now the possibility of offering a lottery with probability q of delivering this option. Here, we can apply the Riley and Zeckhauser (1983) result to show that the monopolist does not offer lotteries of this extra option. Hence, we can conclude that $q = 0$. Offering to deliver this extra option with probability q, on top of a lottery with probability $\frac{1}{2}$ of getting each good, is formally equivalent to offering a lottery with probability $q + (1-q)\frac{1}{2}$ of getting each of the two goods. Hence, the optimal selling strategy is to offer only one lottery with probability $\frac{1}{2}$.

After showing that a multiproduct monopolist uses lotteries as a price discrimination device, we may think that in a market in which firms imperfectly compete against each other, there is room for the use of lotteries to screen consumers. In this market an external agent, like Hotwire, would buy the goods from the firms at a lower price and would resell them through a lottery.

We first examine the case of a market with two firms, each one producing one good, and we define two different cases depending on whether the market is fully covered when lotteries are not offered. We call fully covered market the case in which the equilibrium prices of the two firms are such that all consumers buy one good. On the contrary, if the market is not fully covered some consumers do not buy any good in equilibrium.

We show that if the market is fully covered, no lotteries are offered in equilibrium. Firms do not take advantage of the lottery to better price discriminate consumers. Intuitively, when there are only two firms, each one can block the creation of the lottery because an external agent is only able to sell lotteries if firms provide the good at a lower price through the lottery. Firms opt for not selling the good through the lottery because the benefit of better price-discrimination of consumers is more than offset by the increased competitive pressure resulting form the appearance of a new competing good, like the lottery, in the market.

However, when the market is not fully covered, using lotteries has the added benefit of allowing firms to sell to additional consumers. We show that in this case the optimal selling strategy may include offering lotteries.

With more than two firms, the results also change substantially. In this case each firm no longer has a power to veto the creation of lottery, since a lottery can always be created with all other firms' products. We show that, even in a fully covered market, an external agent might be able to sell lotteries. Interestingly, firms can be worse off than in the case where no lotteries are provided.

McAfee and Mcmillan (1998) extend Riley and Zeckhauser (1983) one lottery result to the multiproduct monopolist case using a multi-characteristics consumer model. Since the integration by parts technique developed by Mirrlees (1971), commonly used in screening problems, does not extend to the multiple dimensions, they restrict the consumers valuations' to satisfy a very specific hazard rate condition and the number of goods to two. Under these restricted conditions they show that lotteries don't increase the monopolist's profit. On the contrary, Thanassoulis (2004) shows that, in a setting with two goods and two dimensional types, lotteries can increase the monopolist's profits. However, he does not solve for the optimal monopolist selling strategy. In this paper we add to this literature by determining the optimal selling strategy with lotteries in an Hotelling model of horizontal differentiation and by examining the introduction of lotteries under oligopolistic competition.

The paper is organized in the following way. In Section 2 we set up the basic model. The multiproduct monopolist's optimal selling strategy is discussed in section 3. In section 4 we examine the possibility of lotteries being offered in markets of oligopolistic competition. Finally, section 5 concludes the paper.

1.2 Model

We consider a Hotelling model of horizontal differentiation. There are two goods, indexed by $i = \{A, B\}$, located at the two endpoints of a segment $[0,1]$. The marginal cost of producing each good is identical and - without loss of generality - is assumed to be equal to zero. First, we assume that both goods are sold by a multiproduct monopolist. Then, we consider the duopoly case in which each good is sold by a different firm. Firm A located in point 0 sells good A and firm B, located in point 1, sells good B.

There is a continuum of consumers with unit demand. We assume that the consumers are distributed uniformly along the segment. Each consumer's preference over the goods A and B is represented as a function of his location on the segment, which is private information.

A consumer located in x, with $x \in [0,1]$, has the following utility from buying good A

$$U(x) = V - c(x) - p_A$$

where p_A is the price of good A, V is a positive constant value, and $c(x)$ is a generic transportation cost function. We assume that $c(\cdot)$ and $c'(\cdot)$ are continuous functions, $c(0) = 0$, and $c'(x) > 0$[4]. Similarly, if he buys good B, his utility is

$$U(x) = V - c(1-x) - p_B$$

If a consumer does not buy any good, he gets utility 0. We assume first that V is high enough so that firms want to sell to all consumers.

1.3 Monopoly

In this section, we consider a monopolist selling two horizontal differentiated goods, A and B. We divide the segment $[0,1]$ in two sub-segments, $\left[0, \frac{1}{2}\right]$and $\left[\frac{1}{2}, 1\right]$. We consider the profit

[4] Different assumptions about the transportation cost function and the types distribution along the line can be made. To preserve our results, we need that the total consumer surplus is decreasing moving away from the endpoints of the segment.

maximization over the second sub-segment. The profit maximizing solution for the first sub-segment is symmetric.

Applying the Direct Revelation Principle, without loss of generality we can restrict to consider direct mechanisms, where each consumer reveals his location x to a fictitious mediator who then selects an outcome for him. The set of feasible outcomes we consider includes stochastic outcomes. We define an outcome as a combination of a probability q of getting good A, probability $1-q$ of getting good B and a corresponding price, $p(q)$. We refer to these probabilistic outcomes as lotteries[5].

The profit maximization problem of firm B can be formalized in the following way

$$\begin{array}{lll} & & \max_{p,q} \int_{\frac{1}{2}}^{1} p(x)\,dx \\ \text{s.t.} & \text{IR} & V - q(x)\,c(x) - (1-q(x))\,c(1-x) - p(x) \geq 0 \;\forall x \\ & \text{IC} & V - q(x)\,c(x) - (1-q(x))\,c(1-x) - p(x) \geq \\ & & \geq V - q(y)\,c(x) - (1-q(y))\,c(1-x) - p(y) \;\forall y, x \\ & & q(x) \geq 0;\; \forall x \in [0,1] \end{array}$$

The program is subject to the individual rationality constraint and incentive compatibility constraint of each consumer. The solution to the problem turns out to depend on the shape of the transportation cost function.

1.3.1 Concave Transportation Costs

In this section we solve the optimal selling strategy when the transportation costs are concave. In each half of the segment, the monopolist sells to consumers either their most favorite good or lotteries where they get their most favorite good with probability weakly higher than $\frac{1}{2}$. When the transportation costs are concave this model becomes like a standard model of price

[5]To simplify the problem we don't allow the monopolist to offer lotteries where consumers have a positive probability of getting no good. We can use the Riley and Zeckhauser's result to show that such a strategy is not optimal. If such a lottery were to be used, it would be because the benefit of decreasing the informational rents more than compensated the cost of selling this lottery at a lower price (due to the positive probability of getting no good). Notice that both the benefit and the cost are proportional to the probability. Hence, the probability of getting no good is either zero or one. We assume that V is high enough so that the firm wants to sell to all consumers.

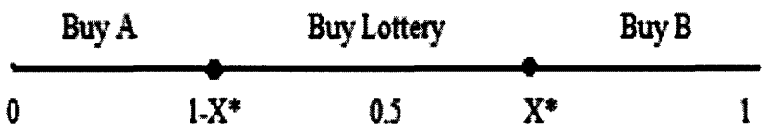


Figure 1-1:

discrimination where consumers can be ranked from the lowest type, in the middle of the segment, until the highest type, at the extreme of the segment. Consumers located closer to the extreme of each subsegment value their favorite good and all the lotteries target to their subsegment more than consumers located closer to the middle of the segment. The following proposition states that the monopolist offers only one lottery when the transportation costs are concave.

Proposition 1 *If the transportation cost function $c(x)$ is concave, then the firm sells the two basic goods and only one lottery. This lottery has probability $q = \frac{1}{2}$. There is a $x^* \in (\frac{1}{2}, 1)$ such that consumers of type $x \in [0, 1 - x^*]$ buy good A; consumer of type $x \in (1 - x^*, x^*)$ buy the lottery and consumers of type $x \in [x^*, 1]$ buy good B. The price of the lottery is $p_L = V - c\left(\frac{1}{2}\right)$ and the prices of goods are $p_A = p_B = V - \frac{1}{2}\left(c(x^*) - c(1 - x^*)\right) - c\left(\frac{1}{2}\right)$. Where x^* is given by the following condition*

$$x^* = 1 - \frac{c(x^*) - c(1 - x^*)}{c'(x^*) + c'(1 - x^*)} \tag{1.1}$$

Proof. Consider consumers in the sub-segment $\left[\frac{1}{2},1\right]$ of the segment of length one. Let $W(x)$ be the utility of consumer x

$$\begin{aligned} W(x) &\equiv V - q(x)\,c(x) - (1-q(x))\,c(1-x) - p(x) = && (1.2) \\ &= \max_y U(x,q(y)) \\ &= \max_y V - q(y)c(x) - (1-q(y))c(1-x) - p(y) && (1.3) \end{aligned}$$

Where $U(x,q(y))$ is the utility that consumer of type x derives from lottery $q(y)$. By the envelope theorem, the derivative of $W(x)$ with respect to x takes into account only the direct effect of x on W

$$\frac{\partial W(x)}{\partial x} = W'(x) = -qc'(x) + (1-q)c'(1-x) \tag{1.4}$$

To apply the Mirrlees' technique [5], we need the utility $W(x)$ to be an increasing function of x. This requires that

$$q < \frac{c'(1-x)}{c'(x)+c'(1-x)} \tag{1.5}$$

Given that $x \geq \frac{1}{2}$ and $c(\cdot)$ is concave, we necessarily have that $c'(1-x) \geq c'(x)$. This implies

$$\frac{c'(1-x)}{c'(x)+c'(1-x)} \geq \frac{1}{2} \;\; \forall x \geq \frac{1}{2}$$

where $\frac{c'(1-x)}{c'(x)+c'(1-x)} = \frac{1}{2}$ only at $x = \frac{1}{2}$. We show in the appendix that in the optimal mechanism we must have $q \leq \frac{1}{2}$. Therefore, we necessarily have that

$$q < \frac{c'(1-x)}{c'(x)+c'(1-x)} \;\; \forall x \in \left(\frac{1}{2},1\right]$$

Hence $W(x)$ is increasing in x. Integrating equation (1.4), we get

$$W(x) = \int_{\frac{1}{2}}^{x} \left(-qc'(z) - (1-q)c'(1-z)\right) dz + W\left(\frac{1}{2}\right) \tag{1.6}$$

By the individual rationality constraint of type $x = \frac{1}{2}$, $W\left(\frac{1}{2}\right) = 0$. Substituting $W(x)$ in (1.2) and rearranging the terms, we can express each price as

$$p(x) = V - qc(x) - (1-q)\,c\,(1-x) - \int_{\frac{1}{2}}^{x} \left(-qc'(z) - (1-q)c'(1-z)\right) dz$$

We can reformulate the monopolist's problem as

$$\max_{p,q} \pi = \int_{\frac{1}{2}}^{1} p\,(x)\,dx$$

$$\max_{q} \int_{\frac{1}{2}}^{1} \left\{ V - qc(x) - (1-q)\,c\,(1-x) - \int_{\frac{1}{2}}^{x} \left(-qc'(z) - (1-q)c'(1-z)\right) dz \right\} dx \qquad (1.7)$$

If we integrate by parts the last term we obtain that the last term

$$\begin{aligned} & \int_{\frac{1}{2}}^{1} \int_{\frac{1}{2}}^{x} \left(-qc'(z) - (1-q)c'(1-z))\right) dzdx \\ = & \int_{\frac{1}{2}}^{1} \left(-qf'(x) - (1-q)f'(1-x))\right)(1-x)\,dx \end{aligned}$$

Hence, expression (1.7) becomes

$$\max_{q} \int_{\frac{1}{2}}^{1} \left\{ V - qc(x) - (1-q)\,c\,(1-x) - \left(-qc'(x) - (1-q)c'(1-x)\right)(1-x) \right\} dx$$

The maximization of π with respect to the schedule $q\,(.)$ requires the term under the integral to be maximized with respect to each $q\,(x)$ for any x. Taking the first order conditions we get

$$-c(x) + c\,(1-x) + \left[c'(x) + c'(1-x)\right](1-x)\,dx\ \forall x \qquad (1.8)$$

Expression (1.8) does not depend on q. Hence, for each type x it is either positive or negative. If it is positive, the optimal solution is to assign to type x a lottery with the highest possible probability q, $q = \frac{1}{2}$. If it is negative, the solution is $q = 0$. Setting equation (1.8) equal to zero, we can determine the threshold value x^*.

$$x^* = 1 - \frac{c(x^*) - c(1 - x^*)}{c'(x^*) + c'(1 - x^*)} \tag{1.9}$$

In the optimal mechanism, types with $x \geq x^*$ get their most preferred good (good B) for sure (lottery with $q = 0$); while types with $x < x^*$ get a lottery with $q = \frac{1}{2}$. Notice that at $x^* = \frac{1}{2}$, the left side of the equation (1.9) is smaller than the right side of the equation, $(\frac{1}{2} < 1)$. For $x^* = 1$ the right side of the equation (1.9) is smaller than the left side (since $c(x^*) > c(1 - x^*)$). Therefore, given that c and c' are continuous functions, by the Bolzano's theorem, there is a solution x^* in the segment $\left(\frac{1}{2}, 1\right)$.

We can solve in a similar way the problem for the sub-segment $\left[0, \frac{1}{2}\right]$. Once we have determined that the optimal mechanism is a set of three lotteries, $q = \frac{1}{2}$, $q^B = 0$, and $q^A = 1$, their prices are determined to assure that types $x \in [1 - x^*, x^*]$ choose lottery $q = \frac{1}{2}$, all types $x \in [0, x^*]$ choose lottery $q^A = 1$, all types $x \in [x^*, 1]$ choose lottery $q^B = 0$. Notice that lotteries q^A and q^B give good A and good B with certainty, so their prices correspond to the price of good A (p_A) and good B (p_B). The price p of lottery $q = \frac{1}{2}$ is pinned down by the individual rationality constraint of the lowest type, $\left(x = \frac{1}{2}\right)$

$$p = V - c\left(\frac{1}{2}\right)$$

The price p_B (or p_A) is determined by making the incentive compatibility constraint of type $x = x^*$ (or $x = 1 - x^*$) binding

$$\begin{aligned} p_B &= V - c(x^*) - \left[V - \frac{1}{2}c(x^*) - \frac{1}{2}c(1 - x^*) - \left(V - c\left(\frac{1}{2}\right)\right)\right] \\ &= V - \frac{1}{2}\left[c(x^*) - c(1 - x^*)\right] - c\left(\frac{1}{2}\right) \end{aligned}$$

■

When the transportation costs are concave, consumers closer to the extremes of the segment attribute a higher value to a lottery with probability $\frac{1}{2}$ than consumers located closer to the middle of the segment, $x = \frac{1}{2}$. Those consumers also have a higher valuation of their favorite good than the more indifferent consumers. Hence, we can think of this case as a standard case of price discrimination where consumers closer to the extreme of the segment are the high types and consumers closer to middle of the segment are the low types. The one single lottery result can be seen as extension of the no lottery result of Riley and Zeckhauser [6] to the multiproduct case.

1.3.2 Convex Transportation Costs

When the transportation costs are convex it is no longer true that consumers at the extreme of the segments attribute a higher value to the lottery $q = \frac{1}{2}$ than consumers closer to the middle of the segment. Hence, we can no longer classify consumers as high value or low value consumers. Furthermore, we can no longer apply the Mirrlees' [5] integration by parts technique because when $c(x)$ is convex condition (1.5) does not necessarily hold.

The following proposition describes the monopolist's optimal mechanism when the transportation costs are convex.

Proposition 2 *If the transportation cost function $c(x)$ is convex and $V \geq \frac{1}{2}$, then there is an $x^* \in \left[\frac{1}{2}, 1\right]$ such that consumers of type $x \in [0, 1 - x^*]$ buy good A, consumers of type $x \in (1 - x^*, x^*)$ buy a continuum of type contingent lotteries and consumers of type $x \in [x^*, 1]$ buy good B. Goods A and B are sold at prices $p_A = p_B = V - c(x^*)$; a continuum of lotteries with probabilities $q = \frac{c'(1-x)}{c'(1-x)+c'(x)}$ for any $x \in \left[\frac{1}{2}, x^*\right]$ are sold at prices $p(q) = V - q + q^2$, and a continuum of lotteries with probabilities $q = \frac{c'x}{c'(1-x)+c'(x)}$ for any $x \in \left[1 - x^*, \frac{1}{2}\right]$ are sold at prices $p(q) = V + q + q^2$. x^* is given by the following condition*

$$1 - x^{**} = \frac{f(x^{**}) - f(1 - x^{**})}{f'(x^{**}) + f'(1 - x^{**})}$$

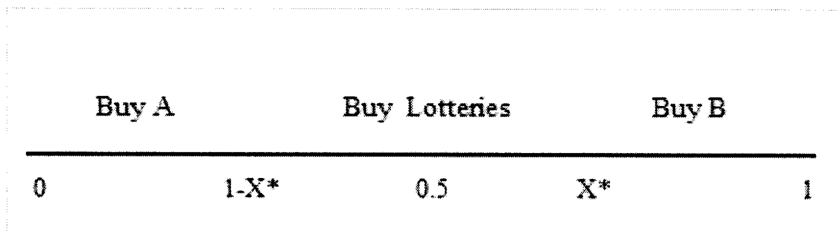


Figure 1-2:

Proof. We show in appendix that there is a intermediate region where both IR and IC constraints bind. From the local IC constraints we obtain

$$p'(x) = -q'(x)(f(x) - f(1-x))$$

The IR constraints are given by

$$p(x) = V - q(x) f(x) - (1 - q(x)) f(1-x)$$

Both conditions determine a sequence of lotteries and corresponding prices

$$\begin{aligned} q(x) &= \frac{f'(1-x)}{f'(1-x) + f'(x)} \\ p(x) &= V - \frac{f'(1-x)}{f'(1-x) + f'(x)} f(x) - \frac{f'(x)}{f'(1-x) + f'(x)} f(1-x) \end{aligned} \tag{1.10}$$

To determine the optimal mechanism, we restrict our analysis to the half-segment $\left[\frac{1}{2}, 1\right]$ [6] and we proceed through three stages: first, we show that the local IC constraint has to bind for all types and the IR constraint has to bind for at least one type; second, we prove that, given that f is convex, if the IR constraint binds for a type x^{**}, in the optimal mechanism it has to bind for all types $x \leq x^{**}$; third, we determine x^{**}, the threshold type such that, in the optimal

[6] Same analysis has to be repeated for the complementary sub-segment $\left[0, \frac{1}{2}\right]$.

mechanism, all $x < x^{**}$ buy the type contingent lottery $q(x)$ at price $p(x)$, and all type $x > x^{**}$ buy good B at price $p_B = V - f(x^{**})$. See appendix B. ■

1.4 Competition

We now consider the case of two competing firms, A and B, each selling only one good. We suppose that a third party, like Hotwire, buys the goods from the firms and resells them through a lottery[7]. We consider lotteries in which consumers get each good with probability $\frac{1}{2}$. Our main goal is to examine whether this lottery is provided in equilibrium. Notice that consumers only buy a lottery if its price is lower than the price of the goods, otherwise they prefer to buy their favorite good. Hence, a third party needs the agreement of the firms to be able to sell the goods through the lottery at a lower price. We assume that there is perfect competition among lottery providers and that selling the good through the lottery implies no additional costs to the firms. Then, we argue that our results also be extended to the case where the lottery providers have market power and charge firms a commission on the goods sold through the lottery.

The timing of the game is as follows. At time $t = 1$, lottery providers offer each firm the possibility of selling their goods through the lottery at the price p_L. The lottery is offered in the market if both firms accept the lottery . At time $t = 2$, after learning if the lottery is offered in the market, firms simultaneously choose their prices. Finally, at time $t = 3$ consumers make the purchasing decisions.

We consider two distinct cases. First, we examine a market where all consumers buy the good in the equilibrium with no lotteries. We say that this market is fully covered. Then, we consider a market where some consumers don't buy the good in equilibrium. We show that lotteries are only provided when the market is not fully covered. In this case, the introduction of lotteries leads to an increase in the quantity sold in equilibrium.

1.4.1 Fully Covered Market

First, we consider a market where no lotteries are provided. We obtain the following proposition.

[7] The lotteries could also be organized by the two firms.

Proposition 3 *If the market is fully covered, in the symmetric equilibrium the firms' optimal prices are given by*

$$p_A = p_B = \frac{g(0)}{g'(0)}$$

where $g^{-1}(x) = c(x) - c(1-x)$

Proof. See Appendix ■

We now introduce the possibility of a third agent to resell the goods of the firms through lottery at a price p_L. We may expect that the duopolist firms, as the monopolist firm, would want to use the lottery as a tool to price discriminate consumers. Interestingly, we show that, under general conditions, this is not the case.

Proposition 4 *If in a market with two firms and no lotteries the prices of the two goods are strategic complements, then in a market with lotteries the price of the lottery and the price of each good are also strategic complements.*

Proof. See appendix ■

Since prices are in general assumed to be strategic complemets, due to propostion 4 it is also natural to assume that the price of the lottery and price of each good are strategic suplements.

Proposition 5 *If the price of the lottery and the price of each good are strategic complements, a lottery where consumers get each of the two goods with equal probability is not offered in equilibrium.*

Proof. Let p_L denote the price of the lottery. A consumer is indifferent between buying good A directly from the seller and buying the lottery if

$$V - c(x) - p_A = V - \frac{1}{2}(c(x) + c(1-x)) - p_L$$

Which simplifies to

$$g^{-1}(x) = 2(p_L - p_A)$$

Demand for good A is given by

$$x = g(2(p_L - p_A)) \tag{1.11}$$

To show that no lottery is offered in equilibrium, we first assume the most favorable lottery to the firms. The lottery is either offered by a competitive intermediary with no commission charged, $C = 0$, or, alternatively, firms are able to organize the lottery between themselves at no cost. In both cases, firms split in two the profits from the lottery. Firm A maximizes

$$\underset{p_A}{Max}\; p_A g(2(p_L - p_A)) + \frac{p_L}{2}\left[1 - g(2(p_L - p_A)) - g(2(p_L - p_B))\right]$$

The optimal price of firm A, given the lottery price p_L is

$$p_A = \frac{g(2(p_L - p_A))}{2g'(2(p_L - p_A))} + \frac{p_L}{2} \tag{1.12}$$

Suppose that an intermediary offers to sell a lottery with probability $\frac{1}{2}$ of getting each good $i = \{A, B\}$ at a price equal to the equilibrium price in the market with no lottery, $p_L = \frac{g(0)}{g'(0)}$[8]. Substituting p_L in equation (3.11) we obtain that firms charge the same price as the lottery price, $p_A = p_L = \frac{g(0)}{g'(0)}$. Hence, if the intermediary sells a lottery at $p_L = \frac{g(0)}{g'(0)}$, the lottery would have no demand. Consider now the case in which the intermediary sells a lottery at a lower price $p_L < \frac{g(0)}{g'(0)}$. If the price of each good and the price of the lottery are strategic complements, then if the intermediary sets a lottery price lower than $\frac{g(0)}{g'(0)}$, each firm would decrease its price as well. This results in a drop in the industry profits. Therefore, at time t=1, firms would refuse to sell their goods sell through the lottery. If the agent instead chooses to set a price above $\frac{g(0)}{g'(0)}$, the firms' optimal reaction, in case they both accept the lottery at time t=1, is to choose a price equal to $\frac{g(0)}{g'(0)}$[9]. Again, in equilibrium, the lottery has no demand.

Consider now the case in which the lottery is provided by a intermediary with market power. Suppose this intermediary charges a commission C for each unit of the good sold through the lottery. In this case, the firms' optimal price, for a given lottery price, is weakly lower than in

[8] See section 4.1.

[9] If the lottery after the increase in price still had positve demand, each firm optimal reaction would be to increase its price as well, but by a lower amount. However, if both firms react in this way, the lottery would have no demand and the optimal price is the same as when no lottery is provided.

the case where no commission is charged. Hence, the lottery would have no demand. ■

In conclusion, firms don't take advantage of the lottery to better price discriminate among consumers, charging high value consumers a high price and letting low value consumers to buy the lottery at a low price. The proof relies on the fact that the lottery has zero demand when a lottery is introduced with the same price as the one firms charge without lottery. Intuitively, the lottery has two effects on firms' prices that offset each other. On the one hand, half of the profits of the lottery belong to each firm. This decreases by half the cost associated with losing each consumer when firms increase their price. On the other hand, a lottery is a closer substitute to good A than is good B. This makes firms' demand twice as elastic as without lottery. Hence, when the firm increases its price it loses twice as many consumers as without lottery. Since this second effect perfectly offsets the first effect, the firms' prices don't change when the lottery is introduced. Therefore, the lottery has zero demand in equilibrium.

1.4.2 Market Not Fully Covered

In this section, we consider a market in which some consumers do not purchase the good in equilibrium without lotteries. We show, as opposed to the case of the fully covered market, that a third agent can successfully sell lotteries of the two goods. In this case, the introduction of lotteries, by improving consumer screening, leads to an increase in the number of consumers that purchase the good. We focus on the case of the concave transportation costs function and the possibility of introducing a lottery with probability $\frac{1}{2}$ of getting each good.

In the Hotelling model, a market is not fully covered if the value V, that enters in the consumers' utility function is low when compared with the transportation costs. In this case consumers located close to the middle of the segment do not buy any good in equilibrium. The equilibrium with no lotteries is the following

Proposition 6 *If the market is not fully covered, in the symmetric equilibrium each firms sets its price equal to*

$$p_A = p_B = \widehat{p}$$

where $\widehat{p}$ solves the following condition

$$\widehat{p} = \frac{c^{-1}(V - \widehat{p})}{c^{-1\prime}((V - \widehat{p}))}$$

Proof. See Appendix ∎

The market is not fully covered *if* $V < V^*$, where V^* is defined as $V^* - c\left(\frac{1}{2}\right) - \widehat{p}(V^*) = 0$.

Consider now that a third agent sells a lottery with probability $\frac{1}{2}$ at a price p_L. We consider the case in which the lottery is offered at no cost by a competitive intermediary and is set at a price that maximizes the firms' profits. To simplify the problem we assume that V is high enough, $V > V^{**}$, so that it is optimal to sell to all consumers when the lottery is introduced.

Proposition 7 *If $V^{**} < V < V^*$, the lottery price is*

$$p_L^* = V - c(\frac{1}{2}) \tag{1.13}$$

Firms' prices are $p_A = p_B = \widehat{p}$. Where $\widehat{p}$ is determined by the following condition

$$\widehat{p} = \frac{g^{-1}(2(V - c(\frac{1}{2}) - \widehat{p}))}{2g^{-1\prime\prime}(2(V - c(\frac{1}{2}) - \widehat{p}))} + \frac{V - c(\frac{1}{2})}{2} \tag{1.14}$$

Proof. The optimal lottery price is set to make the individual rationality constraint of the consumer located at $x = \frac{1}{2}$ to bind. The firm A's optimal response to a given lottery price is given by equation (3.11). If we replace p_L using equation (1.13) we obtain condition (1.14) ∎

1.4.3 Model with Three Firms

When there are more than two firms in the market, the conditions under which an external agent can offer lotteries change significantly. From one side, lotteries still represent an opportunity for the firms to price discriminate the consumers on the base of their preferences. However, in this case no firm holds veto power on the lottery. If there are three firms in the market and one refuses to sell through the lottery, the third party could still organize a lottery among the goods of the other two firms. If the third firm does not accept to join the lottery, it will still face a "new market" with a lottery in place. In other words, the outside option for each firm is not the status quo without any lottery, but a "new market" with the other firms able to price discriminate.

In this section, we give an example where the firms join the lottery, even though their profit after joining the lottery is lower than the profit when no lottery is provided. We consider a simple model where the external agent has market power by assuming that he is the only provider of lotteries[10]. The external agent sets a final price p_L for the lottery with equal probability of getting each good, and asks the firms to join it. For each good sold through the lottery the external agent gets a commission fee C. The circular city model, Salop (1979), is the most commonly used extension of the Hotelling model to an arbitrary number of differentiated oligopoly firms. However, a distinguishing feature of the circular city model is that competition is localized. Each firm only competes with one other firm for the same customers. Since all other firms are not attractive for these consumers, they would only be interested in buying a lottery between those two firms. Therefore, in practice firms still have veto power on the creation of a lottery that is attractive to their customers. For this reason, we use instead an adapted version of the Spokes model, developed by Chen and Riordan (2007), with three firms [11]. The model has the following structure. Starting at the midpoint (center) add lines of length $\frac{1}{3}$ to form a network of 3 lines (spokes). Each spoke terminates at the center and originates at the other end. In order to have an equilibrium without lottery in this adapted Spokes model, we need to modify the linear transportation cost function $c(x) = tx$ in the following way

$$c(x) = \begin{cases} x \quad if \quad x < 1/3 \\ 2(2/3 - x) - 1/3 \quad if \quad x > 1/3 \end{cases}$$

Proposition 8 *In a market with no lotteries the firms' price are* $p_A = p_B = p_C = 2/3$, *and their profit is* 0.22

Consider now an external agent that allows the firms to sell the goods through a lottery,

[10] An alternative assumption would be that there is perfect competition in the market of lottery providers. The results are similar, but in this case lotteries are less likely to decrease firms' profits.

[11] We introduce one modification model Spokes model, by assuming that consumers derive positive utility from all goods.

with equal probability of getting each good, at a price p_L.

We consider that the lottery is offered by a monopolist third party, which charges a commission fee C.

Proposition 9 *i) There is an equilibrium in which the external agent offers a lottery at price* $p_L = 0.59$ *and a commission fee* $C = \frac{1}{3}$. *Firms choose to join the lottery and charge prices* $p_A = p_B = p_C = 0.76$. *The firms' profit is* 0.21.

ii) There is an equilibrium where no firms join the lottery

1.5 Conclusion

In this paper we show that a multiproduct monopolist uses lotteries over its goods to maximize its profits. The optimal selling strategy depends on the shape of transportation costs. If the transportation costs function is concave, the multiproduct monopolist offers only one lottery with probability $\frac{1}{2}$. However, if the transportation costs function is convex, the optimal selling strategy includes a region of consumers, relatively indifferent between the goods, in which the monopolist offers a continuum of type contingent lotteries.

We then examine the use of lotteries in oligopolistic markets. We consider first a market with only two firms. Surprisingly, we show that if this market is fully covered, no lotteries are offered in equilibrium. However, when the market is not fully covered, lotteries can be sold in equilibrium.

With more than two firms, each firm no longer has a power to veto the creation of lottery, since a lottery can always be created with all other firms' products. We show that in this case even in a fully covered market, an external agent might be able to sell lotteries. Interestingly, firms can be worse off than in the case where no lotteries are provided.

1.6 Appendix

Proof of proposition 1 We show in this appendix that if $x \in \left[\frac{1}{2}, 1\right]$, then the optimal solution implies $q(x) \leq \frac{1}{2}$. Suppose an hypothetical optimal mechanism where $q(x') > \frac{1}{2}$ for some x', $x' > \frac{1}{2}$. We can have two different situations. First, $q(x)$ is weakly decreasing everywhere. In this case we must have that $q(x) > \frac{1}{2}$ for all $x < x'$. Hence, we can still apply the Mirrlees technique in the segment $[0, x']$. Applying this technique, we could show as in section (3) that $q(x = \frac{1}{2}) = \frac{1}{2}$, which is a contradiction.

The second situation is when $q(x)$ is strictly increasing somewhere. However, this cannot happen because it violates the incentive compatibility constraint of at least one type. If $q(x)$ is strictly increasing somewhere, we must have some y and z, in the same neighborhood, such that $y > z$ and $q(y) > q(z)$. Suppose without loss of generality that $y > z > \frac{1}{2}$. This implies that

$$u(y, q(z)) > u(y, q(y))$$

Hence, to satisfy type y's I.C. constraint we must have

$$p(z) - p(y) > u(y, q(x)) - u(y, q(y)) \tag{1.15}$$

Notice that $y > z$ and $q(y) > q(z)$ imply

$$u(y, q(z)) - u(y, q(y)) > u(z, q(z)) - u(z, q(y)) \tag{1.16}$$

Conditions (3.3) and (3.4) imply that we must have

$$p(z) - p(y) > u(z, q(z)) - u(z, q(y)) \tag{1.17}$$

However, condition (1.17) violates the I.C. constraint of type z. Hence, if $y > z$ and $q(y) > q(z)$ we cannot satisfy both type y and type z I.C. constraint.

Proof of proposition 2 The Spence-Mirlees condition is satisfied if$\frac{\partial}{\partial x}\left(\frac{\frac{\partial}{\partial q}U}{\frac{\partial}{\partial p}U}\right) > 0$. This implies we can consider only the local incentive compatibility constraints for all types. For a fixed price, any lottery provides each type a different level of utility. Given that f is assumed to be convex, the function that relates the utility from a lottery and the types x is not monotonic in x. Still, for a given lottery q, the types can be ranked in terms of the utility they can get from that lottery: We call the *lowest type for a lottery* q the type that gets the lowest utility from that lottery. In order to maximize the profit, at least one lottery used in the optimal mechanism is going to be priced in such a way that the IR constraint of its lowest type binds[12]. Define x^{**} this type. What we show now is that, in the optimal mechanism, the IR constraints of all types $x \leq x^{**}$ have to bind. We prove that by contradiction. If both the IC and IR constraints bind for all types $x \leq x^{**}$, the optimal mechanism would be a continuum of lotteries $q(x)$ and corresponding prices $p(x)$. Let's assume that there is a type $\widetilde{x} < x^{**}$ for which the IR constraint does not bind. Consider the case[13] in which $\widetilde{x}$ buys a lottery q such that $q < q(x)$. Given that we are assuming that $\widetilde{x}$ IR constraint does not bind, the price of that lottery has to be $p(q) < V - qf(x) - (1-q)f(1-x)$. Given that x^{**}'s IR binds, $q < q(x^{**})$, and there is a type x'', $\widetilde{x} < x'' \leq x^{**}$, such that $p(q) = V - qf(x'') - (1-q)f(1-x'')$. Define x''' as $p(x''') = p(q)$. Notice that x''' has to be lower than the value of x corresponding to the type that gets the highest utility from lottery q. Compare to the mechanism with $q(x)$ and $p(x)$, the new mechanism brings a gain equal to

$$\begin{aligned}&\left(x''' - x'\right)\left(V - qf\left(x''\right) - (1-q)f\left(1-x''\right)\right) + \\ &-\int_{x'}^{x'''}\left\{V - \frac{f'(1-x)}{f'(1-x)+f'(x)}f(x) - \frac{f'(x)}{f'(1-x)+f'(x)}f(1-x)\right\}dx\end{aligned}$$

and a loss

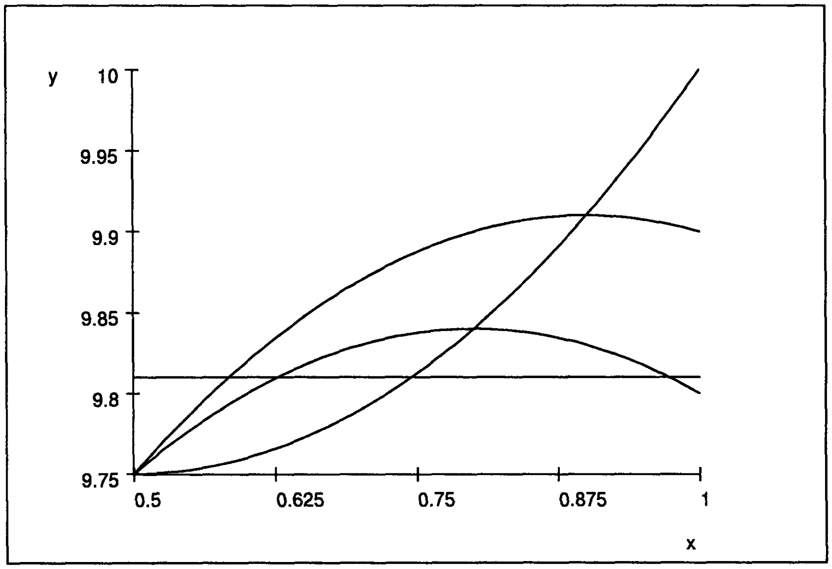
$$\begin{aligned}&\int_{x'''}^{x''}\left\{V - \frac{f'(1-x)}{f'(1-x)+f'(x)}f(x) - \frac{f'(x)}{f'(1-x)+f'(x)}f(1-x)\right\}dx + \\ &-\left(x'' - x'''\right)\left(V - qf\left(x''\right) - (1-q)f\left(1-x''\right)\right)\end{aligned}$$

where $x' = \max\left\{x : p(q) = V - qf(x) - (1-q)f(1-x) \text{ and } x < \widetilde{x}, \frac{1}{2}\right\}$. The gain comes from the fact that there is a subset of types that pays a higher price than $p(x)$ in order to buy a lottery with a probability q higher than $q(x)$. The loss comes from the fact there is also a subset

[12] Notice that, if f is concave, the lowest type fr any lottery q is $x = \frac{1}{2}$. If f is convex, the lowest type is a function of q: it varies with the lotteries we consider.

[13] This is the only relevant case.

of types that given the price $p(q)$ gets a level of utility such that it's not incentive compatible to charge them a price $p(x)$ for any lottery q' without violating the IR constraint of type x^{**}. If the function $\frac{\partial^2}{\partial x^2}\left(V - \frac{f'(1-x)}{f'(1-x)+f'(x)} f(x) - \frac{f'(x)}{f'(1-x)+f'(x)} f(1-x)\right) \geq 0$ in a neighborhood of $x = \frac{1}{2}$[14], then it's immediate to verify that the loss is bigger than the gain for any q if x^{**} is close enough to the neighborhood of $x = \frac{1}{2}$ where the function $V - \frac{f'(1-x)}{f'(1-x)+f'(x)} f(x) - \frac{f'(x)}{f'(1-x)+f'(x)} f(1-x)$ is convex.



For consumers located in the segment $[x^{**}, 1]$ only their IC constraint bind. Hence, in this region the price of a lottery with probability $q(x)$ of getting good A is

$$p(x) = V - [qf(x) + (1-q) f(1-x)] - \int_{x^{**}}^{x} \{-qf'(x) - (1-q)f'(1-x)\} dz$$

The monopolist maximizes

$$\pi = \int_{\frac{1}{2}}^{1} p(x) dx$$

[14]For example, if $f(x) = x^2$, then 1.10 is convex over all support $\left[\frac{1}{2}, 1\right]$.

Which is given by

$$\max_{x^{**},q}\int_{\frac{1}{2}}^{x^{**}}\left\{V-\frac{f'(1-x)\,f(x)}{f'(x)+f'(1-x)}-(1-\frac{f'(1-x)}{f'(x)+f'(1-x)})f(1-x)\right\}dx+$$
$$\int_{x^{**}}^{1}\{V-[qf(x)+(1-q)\,f(1-x)]\}\,dx-$$
$$\int_{x^{**}}^{1}\int_{x^{**}}^{x}\left\{f'(1-z)-q(f'(z)+f'(1-z))\right\}dz$$

After simplifying and integrating by parts, we obtain

$$\max_{x^{**},q}\int_{\frac{1}{2}}^{x^{**}}\left(V-\frac{f'(1-x)\,f(x)+f'(x)f(1-x)}{f'(x)+f'(1-x)}\right)dx+$$
$$\int_{x^{**}}^{1}\left\{V-[qf(x)+(1-q)\,f(1-x)]-\left(f'(1-x)-q(f'(x)+f'(1-x))\right)(1-x)\right\}dx$$

Notice that now only the second integral depends on $q(x)$. Maximizing the term under the second integral with respect to $q(x)$ for all $x>x^{**}$, we obtain

$$-f(x)+f(1-x)+\left[f'(x)+f'(1-x)\right](1-x) \tag{1.18}$$

Expression (1.18) does not depend on q. Hence, for each $x\in[x^{**},1]$ the expression is either positive or negative. If, for a given x, it is negative, then $q(x)=0$. If it is positive, then $q(x)$ is equal to the highest possible q, $q=q(x^{**})$. The first-order condition with respect to x^{**} is

$$V-\frac{f'(1-x^{**})\,f(x^{**})+f'(x^{**})f(1-x^{**})}{f'x^{**})+f'(1-x^{**})}-V+q(x^{**})f(x^{**})+ \tag{1.19}$$
$$(1-q(x^{**}))\,f(1-x^{**})+\left(f'(1-x^{**})-q(x^{**})(f'(x^{**})+f'(1-x^{**}))\right)(1-x^{**})=0$$

We guess and verify that expression (1.18) is negative for all $x \in (x^{**}, 1]$, and is equal to zero for $x = x^{**}$. In that case, $q(x) = 0$ for all $x \in (x^{**}, 1]$, and equation (1.19) simplifies to

$$\begin{aligned} 1 - x^{**} &= \frac{1}{f'(1-x^{**})} \frac{f'(1-x^{**}) f(x^{**}) + f'(x^{**}) f(1-x^{**})}{f'(x^{**}) + f'(1-x^{**})} - \frac{f(1-x^{**})}{f'(1-x^{**})} \\ \Leftrightarrow 1 - x^{**} &= \frac{f(x^{**}) - f(1-x^{**})}{f'(x^{**}) + f'(1-x^{**})} \end{aligned} \tag{1.20}$$

If we evaluate expression (1.18) at $x = x^{**}$ and substitute $1 - x^{**}$ by expression (1.20) we verify that expression (1.18) is indeed equal to zero.

Proof of Proposition 3 The consumer type x who is indifferent between buying good A or good B is determined by the equation

$$V - c(x) - p_A = V - c(1-x) - p_B \tag{1.21}$$

Equation (1.21) can be expressed as

$$g(x) = p_B - p_A$$

Hence, the indifferent consumer is given by

$$x = g^{-1}(p_B - p_A)$$

The profit maximization problem of firm A is

$$\max \pi = p_A g^{-1}(p_B - p_A)$$

After taking the first order conditions we get

$$p_A = \frac{g^{-1}(p_B - p_A)}{g^{-1\prime}(p_B - p_A)}$$

In the symmetric equilibrium, where $p_A = p_B$, we have

$$p_A = p_B = \frac{g^{-1}(0)}{g^{-1\prime}(0)}$$

Proof of proposition 4 We first derive the condition that guarantees that prices of the goods are strategic complements in a market without lotteries. The indifferent consumer is given by

$$V - c(x) - p_A = V - c(1-x) - p_B$$

which simplifies to

$$g^{-1}(x) = (p_L - p_A)$$

where $g^{-1}(x) = c(x) - c(1-x)$. Demand for good A is given by

$$x = g(p_B - p_A)$$

Firm A maximizes

$$\underset{p_A}{Max}\ p_A g(p_B - p_A)$$

The optimal price of firm A given the price of the other firm, p_B, is given by

$$p_A = \frac{g(p_B - p_A)}{g'(p_B - p_A)}$$

In order to determine the relation between p_A and p_B, we define the function

$$d(p_A, p_B) = p_A - \frac{g(p_B - p_A)}{g'(p_B - p_A)} = 0$$

and we apply the Implicit Function Theorem. We look for a function $p_B = h(p_A)$ such that $d(p_A, h(p_A)) = 0$. From the Implicit Function Theorem we get

$$h'(p_A) = -\frac{\frac{\partial d}{\partial p_B}}{\frac{\partial d}{\partial p_A}} = -\frac{-\frac{[g'(\cdot)]^2 - g''(\cdot)g(\cdot)}{g'(\cdot)^2}}{1 - \left(\frac{-[g'(\cdot)]^2 + g''(\cdot)g(\cdot)}{[g'(\cdot)]^2}\right)} = \frac{\frac{[g'(\cdot)]^2 - g''(\cdot)g(\cdot)}{g'(\cdot)^2}}{1 - \left(\frac{-[g'(\cdot)]^2 + g''(\cdot)g(\cdot)}{[g'(\cdot)]^2}\right)} = \frac{1 - \frac{g''(\cdot)g(\cdot)}{g'(\cdot)^2}}{2 - \frac{g''(\cdot)g(\cdot)}{[g'(\cdot)]^2}}$$

Hence, the prices of the goods are strategic complements if and only if

$$\frac{g''(\cdot)g(\cdot)}{g'(\cdot)^2} \leq 1 \quad or \quad \frac{g''(\cdot)g(\cdot)}{g'(\cdot)^2} \geq 2 \tag{1.22}$$

Next we derive the condition that guarantees that price of the lottery and the price of each of two goods are strategic complements. The indifferent consumer is given by

$$V - c(x) - p_A = V - \frac{1}{2}(c(x) + c(1-x)) - p_L$$

which simplifies to

$$g^{-1}(x) = 2(p_L - p_A)$$

where $g^{-1}(x) = c(x) - c(1-x)$. Demand for good A is given by

$$x = g(2(p_L - p_A)) \tag{1.23}$$

Firm A maximizes

$$\underset{p_A}{Max}\ p_A g(2(p_L - p_A)) + \frac{p_L}{2}\left[1 - g(2(p_L - p_A)) - g(2(p_L - p_B))\right]$$

The optimal price of firm A given the price of the lottery, p_L, is given by

$$p_A = \frac{g(2(p_L - p_A))}{2g'(2(p_L - p_A))} + \frac{p_L}{2}$$

In order to determine the relation between p_A and p_L, we define the function

$$d\left(p_L, p_A\right) = p_A - \frac{1}{2}\left(\frac{g(2(p_L - p_A))}{g'(2(p_L - p_A))} + p_L\right) = 0$$

and we apply the Implicit Function Theorem[15]. We look for a function $p_A = h\left(p_L\right)$ such that $d\left(p_L, h\left(p_L\right)\right) = 0$.

From the Implicit Function Theorem we get

$$\begin{aligned} h'\left(p_L\right) &= -\frac{\frac{\partial d}{\partial p_L}}{\frac{\partial d}{\partial p_A}} = -\frac{-\frac{1}{2}\left(\frac{2[g'(\cdot)]^2 - 2g''(\cdot)g(\cdot)}{g'(\cdot)^2} + 1\right)}{1 - \frac{1}{2}\left(\frac{-2[g'(\cdot)]^2 + 2[g^{-1\prime\prime}(\cdot)][g^{-1}(\cdot)]}{[g'(\cdot)]^2}\right)} \\ &= \frac{1 + \left(2 - \frac{2g''(\cdot)g(\cdot)}{[g'(\cdot)]^2}\right)}{2 + \left(2 - \frac{2g''(\cdot)g(\cdot)}{g'(\cdot)^2}\right)} = \frac{3 - \frac{2g''(\cdot)g(\cdot)}{[g'(\cdot)]^2}}{4 - \frac{2g''(\cdot)g(\cdot)}{g'(\cdot)^2}} = \frac{3/2 - \frac{g''(\cdot)g(\cdot)}{[g'(\cdot)]^2}}{2 - \frac{g''(\cdot)g(\cdot)}{g'(\cdot)^2}} \end{aligned}$$

Hence, the prices of the lottery and the price of each good are strategic complements if and only if

[15] In order to apply the Implicit function theorem, we need $\frac{\partial d}{\partial p_A} = 1 + \left(\frac{[g^{-1\prime}(2(p_L - p_A))][g^{-1\prime}(2(p_L - p_A))] - [g^{-1\prime\prime}(2(p_L - p_A))][[g^{-1}(2(p_L - p_A))]]}{[g^{-1\prime}(2(p_L - p_A))]^2}\right) \neq 0$, that is equivalent to $\frac{[g^{-1\prime\prime}(2(p_L - p_A))][[g^{-1}(2(p_L - p_A))]]}{[g^{-1\prime}(2(p_L - p_A))]^2} \neq -2$

$$\frac{g''(\cdot)g(\cdot)}{g'(\cdot)^2} \leq \frac{3}{2} \text{ or } \frac{g''(\cdot)g(\cdot)}{g'(\cdot)^2} \geq 2 \tag{1.24}$$

Condition (1.24) is verified by most well behaved transportation cost functions. For example, linear, quadratic and square root functions satisfy this condition[16]. Notice that condition (1.24) is satisfied whenever condition (1.22) is satisfied. This means that if the prices of the two goods are strategic complements, then the price of the lottery and the price of each good are also strategic complements.

Proof of proposition 5 Let us consider the profit maximization problem of firm A, located at 0. The consumer x who is indifferent between buying good A and not buying any good is

$$V - c(x) - p_A = 0$$
$$x = c^{-1}(V - p_A)$$

The value x defines the demand for good A, all types below x prefer good A than no good. Firm A's maximization problem is given by

$$\max \pi = p_A \left(c^{-1}(V - p_A)\right)$$

The first order condition gives

$$c^{-1}(V - p_A) - p_A c^{-1\prime}(V - p_A) = 0$$

[16]Intuitively, this condition imposes that the relative value of $g^{-1\prime\prime}(\cdot)$ cannot be too high. This avoids that the demand of the firm becomes very inelastic as the demand decreases. This guarantees that the firm does not want to increase its price when its demand falls in response to a decrease in the lottery price.

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Chapter 2

Quantity Discounts for Time-inconsistent Consumers

2.1 Introduction

Consumers with time-inconsistent preferences struggle to make optimal consumption decisions over time. Sophisticate consumers, aware of their time-inconsistent preferences, often try to limit their consumption of certain goods by strategically rationing the quantities they purchase. Many frequent smokers deliberately forgo significant per unit discounts by purchasing their cigarettes by the pack, instead of buying 10-pack cartons. They take advantage of the time delay between purchase and consumption, to avoid overconsuming the more tempting goods. Rationing quantities purchased helps them solve their self-control problems by limiting the stock they have at "home" and hence their consumption opportunities. Using laboratory experiments and a field survey at the retail level, Wertembroch (1998) provides evidence that many consumers forgo savings through quantity discounts for the opportunity to engage in self-control. On the other hand, naive consumers, unaware of their time-inconsistent preferences, may stockpile tempting goods at "home" not realizing that the higher availability of the good might lead them to overconsume the good.

In this paper we examine firms optimal pricing policy when they sell a storable good of repeated consumption to time-inconsistent consumers. We show that if consumers are time-consistent, quantity discounts don't increase the firms' profit. In contrast, if firms face naive time-inconsistent consumers, the optimal pricing policy is to use small quantity discounts as a device to increase sales. These consumers take advantage of quantity discount with the intention of saving on future purchases. However, after buying the good they can not resist and overconsume it. We also show that even if consumers are sophisticated, firms still use quantity discounts. Sophisticated time-inconsistent consumers realize that increasing the quantity purchased often leads them to overconsume the good. Hence, they require a significant quantity discount to increase the quantity purchased. Offering a quantity discount leads them to stockpile the good "at home" and hence promotes overconsumption.

Time-inconsistent preferences lead to too much current consumption relative to future consumption[1]. However, consumers are more likely to overconsume some goods than others. One example of goods that tend to be overconsumed are leisure goods. Leisure (investment) goods are goods that offer current benefits (costs) and future costs (benefits), like cigarettes. Consistent with our results, Wertembroch (1998) shows that quantity discounts are more prevalent and deeper for leisure goods than for investment goods.

In this paper, we assume that consumers have a quasi-hyperbolic discount function [Strotz 1956; Phelps and Pollak 1968; Laibson 1997], with a higher discount rate between the present and the next period than between any of the subsequent periods. Time-inconsistent agents are said to be sophisticated if they are aware of their preferences, while they are called naive if they believe they have time-consistent preferences even though they exhibit time inconsistent preferences [O'Donoghue and Rabin, 2001].

The interaction between consumers with biases and rational, profit maximizing firms is the central theme of the growing literature in behavioral industrial organization, surveyed in Ellison (2006). DellaVigna and Malmendier (2004) examine firms' optimal response when consumers have quasi-hyperbolic preferences. They consider a monopolistic firm selling a leisure or an investment good using a two-part tariff. The fixed fee is charged in the first period while in the second period consumers decide whether to consume based on the marginal price of the good. They show that when a firm faces consumers with self-control problems it prices leisure goods above the marginal cost. If these consumers are sophisticated, this pricing policy provides a commitment device by helping consumers to reduce the consumption of leisure goods. In case of

[1] For this reason, these type time-inconsistent preferences are often used to explain low savings

fully naive agents with self-control problems, this pricing strategy takes advantage of consumer's underestimation of future purchasing and consumption of leisure goods. Notice this result is in sharp contrast with the main result in our paper. While in DellaVigna and Malmendier (2004), the presence of time-inconsistent consumers leads the firms to distort upwards the marginal price of a leisure good, in our paper the presence of hyperbolic consumers leads to quantity discounts.

The key difference is that in our paper, as opposed DellaVigna and Malmendier (2004), consumers do not purchase the good for current consumption. Consider first the case of *sophisticated* consumers. The current self wants to avoid that his future self overconsumes the good. Since in DellaVigna and Malmendier (2004) consumers can purchase for immediate consumption, they know that next period they can purchase and overconsume the good. If the price was equal to the marginal cost of the good, they would consume even when the marginal cost is higher than the marginal benefit. Hence, they gain from paying a lower fixed fee today and a higher marginal price tomorrow that guarantees that they only consume the good when the benefit is above the marginal cost. In contrast, in our model consumers buy for future consumption. Hence, they can easily avoid overconsuming the good tomorrow by restricting the quantity they buy today. Increasing today the price above the marginal cost today, leads instead these consumers to buy too little of the good. In this paper we consider the case of storable goods of repeated consumption. In this case, if consumers are time-consistent, consumption depends only on the average price of the good. Hence, the firm can maximize its profit through simple linear pricing. Consider now sophisticated consumers with time-inconsistent preferences. These consumers realize that if they increase the amount purchased today they will consume more tomorrow, even when the benefit is lower than the average price. Hence, firms can persuade these consumers to buy and consume more by offering a quantity discount, even if the average price of the good is the same.

Let's now examine the case of *naive* time-inconsistent consumers. Consider first that consumers buy for current consumption as in DellaVigna and Malmendier (2004). Since they under-estimate their future consumption, it is not optimal for the firm to set the price equal to the marginal cost and extract the consumer surplus through the fixed fee. Consider now the case where consumers buy for future consumption and a storable good of repeated consumption. Naive consumers underestimate their willingness to consume the good in the next period. Hence, unless there is a quantity discount they buy and consume a relatively small amount of the good. However, if the firm offers a small quantity discount it can increase significantly sales because these consumers overestimate the benefit of taking advantage of a quantity discount. They think erroneously that they will save on future purchases and do not realize that they will end up overconsuming the good.

The paper is organizes as follows. In section 2 we set up the basic model. Consumer behavior is described in section 3. Section 4 examines the firm optimal pricing strategy. Finally, section 5 concludes the paper.

2.2 Model

A monopolistic firm sells a storable good of repeated consumption to consumers with quasi-hyperbolic preferences. The present value of the consumer utility, at time t, is

$$U_t = u_t + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u_\tau$$

with $\beta \leq 1$. Where the parameter β can be interpreted preference for immediate gratification. For $\beta = 1$, the consumer has standard exponential time-consistent preferences. If instead $\beta < 1$, the consumer has time-inconsistent preferences with an extra bias for now over the future. The discount factor between the current period and the next period is $\beta\delta$, while the discount factor between any two adjacent periods in the future is δ. We allow for consumers to underestimate the magnitude of their self-control problems. A partially naive hyperbolic consumer believes that his taste for immediate gratification is given by parameter $\hat{\beta}$, where $\beta \leq \hat{\beta} \leq 1$. There are two extreme cases of time-inconsistent consumers. A sophisticated consumer not only has time-inconsistent preferences but is aware of it ($\hat{\beta} = \beta$). A fully naive consumer has time-inconsistent preferences but believes he will behave like a time-consistent consumer in the future ($\beta < \hat{\beta} = 1$). The discount factor δ is considered to be strictly smaller than 1. However, to simplify we assume that δ arbitrarily close to 1.

We consider a good of repeated consumption that can be consumed every period by a infinitely lived agent. To examine quantity discounts it is sufficient to assume that in each period individuals can consume either one or two units of the good. The marginal utility of the first unit of the good v_1 has cumulative distribution function $F()$ with support $[0, 1]$ and F has a strictly positive density f over its support. The marginal utility of consuming a second unit of the good in the same period is a fraction k of the marginal utility of the first unit, $v_2 = kv_1$, $0 < k < 1$. To simplify we assume that the consumers' marginal utility is constant across periods. That is, in each period the marginal utility of consumption depends only on the number of units consumed in that period and is not affected by the consumption in the previous periods. We also assume for simplicity that the consumer can buy at most two units per period.

We consider goods for which there is a time delay between purchase and consumption. We model this delay by assuming that units of good purchased in one period can only be consumed in the following periods. In particular, we assume that a purchasing period is followed by a consumption period which in turn is followed by another purchasing period and so on[2]. There is a "storage cost" $s > 0$ of keeping the good until after next period, where s is an arbitrarily small number. Hence, unless there is a quantity discount consumers only buy and stockpile goods at "home" for next period consumption.

To simplify, we also assume a quasi-linear utility function. Let x_t, $x_t \in \{1, 2\}$, denote the consumption of the monopolistic firm's good in period t and m_t the consumption of all other goods. We normalize the price of m to be one. In each purchasing period the consumer has an exogenous income y that can be used to buy goods m and x to be consumed in the following consumption periods. We assume that $y > 2$[3].

In a purchasing period, when the consumer is "shopping", there is no consumption and the current utility is zero. Hence, the present value of the consumer utility is given by[4]

[2] Another way of modelling it would be to assume that in each period consumers can purchase and consume goods, but goods bought in the current period can only be consumed next period.

[3] This condition guarantees that for all prices of good x below 1, then even if consumer consumes two units of x he still has income left to consume a positive amount of good m next period.

[4] We assume that good x has a small weight on the budget constraint and that the consumer has enough resources to afford the consumption of the maximal amount of good x (two units) in all periods and to consume, in the current period, the maximal amount of m, $m = \theta$.

$$U_t = \beta \sum_{\tau=t+1}^{\infty} \left[\delta^{\tau-t}(v(x_\tau) + m_\tau)\right] \tag{2.1}$$

In a consumption period, the present value of the consumer utility is given by

$$U = v(x_1) + m_1 + \beta \sum_{\tau=t+1}^{\infty} \left[\delta^{\tau-t}(v(x_\tau) + m_\tau)\right] \tag{2.2}$$

We examine if the firm's optimal selling strategy involves offering a quantity discount when consumers buy two units in a given period. Let a denote the marginal price of the first unit and b, $b \leq a$, denote the marginal price of the second unit, when consumers buy two units in the same period. Finally, let d denote the average price implicit in the quantity discount, $d = (a + b)/2$. We assume that the firm can commit to future prices[5].

2.3 Consumer Behavior

In a consumption period, the agent consumes all the quantity he purchased of m. Consider now good x. The quantity consumed of good x, does not affect his future consumption of this good. Hence, if the consumer increases his current consumption of good x he would have to purchase more good x in the future than otherwise. This implies that there would be less available for consumption of other goods in the future. Hence, he decides to consume one extra unit of good x if its marginal utility, v_i, is higher than his *perceived* additional cost associated with consuming one additional unit today, βw. Where w is the additional cost of consuming one additional unit today. Additionally, the consumer can only consume good x today if he bought it in the previous periods.

Consider now the consumer in a purchasing period. This consumer can influence next period consumption by strategically rationing the quantities he purchases. He would like his future self to consume an extra unit of the good if its the marginal utility, v_i, is higher than the additional

[5]This is simplifying assumption. Given that this is a good of repeated consumption, this assumption is natural.

cost associated with consuming one additional unit today, w.

Assuming that the firm offer quantity discounts, the additional cost of consuming one additional unit today depends on the consumer purchasing behavior. Consider a consumer that purchases at most one good period. In this case, the cost of consuming one additional unit is the price of buying just one unit given by a. Consider now a consumer that is planning to take advantage of the quantity discounts and to buy two units every two purchasing periods and consuming just one unit per consumption period. Consider his decision whether or not to consume the second unit in a given period. Notice that the consumer can only consume two units if in the previous period he bought two units. If he does not consume the second unit today, he does not have to purchase the good in the next purchasing period. Hence the present value of his costs is

$$2d * (\delta^3 + \delta^7 ...) = \frac{2d\delta^3}{1 - \delta^4}$$

If he consumes the second unit today, but sticks to the original plan from next period on, he will have to purchase the good next period. Hence, the present value of his cost is

$$2d * (\delta + \delta^5 ...) = \frac{2d\delta}{1 - \delta^4}$$

Hence, the additional cost of purchasing the good today is given by

$$\underset{\delta \to 1}{Lim} \frac{2d\delta}{1 - \delta^4} - \frac{2d\delta^3}{1 - \delta^4} = \underset{\delta \to 1}{Lim} \frac{2d\delta}{1 + \delta^2} = d$$

Therefore, when the consumer buys two units every period and the discount factor is arbitrarily close to one, the additional cost of consuming one extra unit is average price of the good implicit in the quantity discount, d.

2.3.1 Time-consistent Consumers

Consider the purchasing behavior of a time-consistent consumer faced with quantity discounts. Notice that the time consistent consumer is never tempted to consume the good when that is not ex-ante optimal. Hence, he does not benefit by strategically rationing the quantities purchased. For this reason, if he decides to buy the good, it's always optimal to take advantage of the quantity discount and to buy two units of the good. If he consumes only one unit per period, he only needs to purchase the good every two periods. If instead he decides to consume two units per period he need to purchase two units of the good every consumption period.

A time-consistent consumer only consumes an extra unit of the good if the marginal benefit of that unit is higher than the average price of the good implicit in the quantity discount. Therefore, quantity discounts do not affect his consuming decisions beyond the effect they have on the average price.

Proposition 10 *If the average price of good is d and consumers are time consistent, the per period demand of the firm is given by*

$$2\left[1 - F(d/k)\right] + \left[F(d/k) - F(d)\right] \tag{2.3}$$

Proof. *If $v_1 < d$, consumers don't purchase the good. If $d < v_1 < d/k$. The first inequality means that they consume one unit per period while the second unit means that consuming the second unit is not optimal. Hence, consumers buy and consume one unit per period. If $v_1 > d/k$ consuming the second unit is also optimal and hence consumers buy and consume two units per period.* ■

2.3.2 Time-inconsistent Consumers

In this section, we consider consumers with quasi-hyperbolic preferences who are potentially naive. While they have future short-run discount factor $\beta \leq 1$, they believe that their future short-run discount factor is $\hat{\beta}$, $\beta \leq \hat{\beta} \leq 1$. In this case, being in possession of more than one unit of the good might lead to overconsumption. Hence, to prevent the temptation to overconsume consumers may want to forgo quantity discounts. We consider first the demand of time-inconsistent consumers when the firm does not offer quantity discounts.

Proposition 11 *If the firm charges a constant per unit price a and faces quasi-hyperbolic consumers, its per period demand is the same as if consumers are time-consistent*

Proof. *Since there are no quantity discounts, consumers only buy for next period consumption. Given that their preferences one period before consumption are the same as the time-consistent consumers, they want to consume exactly the same as time consistent consumers next period. Hence, their purchasing and consumption behavior is the same.* ■

Let's examine now the case in which the firm offers quantity discounts. Consider a consumer that takes advantage of quantity discount and buys two units every time he purchases the good. Suppose this consumer bought two units in the previous period and is deciding whether to consume or not a second unit in the current period. Assume that be believes that he will consume just one unit per period from next period on. The additional cost of consuming the second unit is d. A quasi-hyperbolic consumer is tempted to consume the second unit if the perceived cost of consuming that unit, βd, is lower than the marginal benefit of the second unit v_2. This is equivalent to $v_1 > \beta d/k$. However, if the consume is not fully sophisticated $\hat{\beta} > \beta$, he thinks that he is tempted to consumed the second unit if $v_1 > \hat{\beta} d/k$.

We will consider three different cases depending on the relative values of the parameter k, β and $\hat{\beta}$. First, we examine consumers with a relatively high hyperbolic discount factor, $\beta > k$. This implies that $\beta d/k > d$ and hence there are consumers that buy the good, $v_1 > d$, that are not tempted to consume two units of the good per period, $v_1 < \beta d/k$. These consumers manage to take advantage of the quantity discount and to consume only one unit per period. Next, we consider consumers with a low degree of time consistency but that think they have a relatively high degree of time consistency, $\beta \leq k < \hat{\beta}$. In this case, all consumers that buy two units end up also consuming two units. Some of these consumers, whose value v_2 lies in the interval $[d, \hat{\beta} d/k)$, buy two units thinking erroneously that they will consume just one unit per period. Finally, we look at the case of consumers with a relatively low degree of time consistency but that are aware of it, $\beta \leq \hat{\beta} \leq k$. In this case, since $\hat{\beta} d/k \leq d$, all consumers that buy the good realize that if they buy two units in one period they will consume two units in the following period.

It is also important to evaluate whether $\hat{\beta} d$ is smaller or higher than b. Consider first the case where $\hat{\beta} d < b$. Consumers whose marginal valuation of the second unit, v_2, lies in the interval $[\hat{\beta} d, b)$ do not take advantage of a quantity discount because they realize they would consume two units. Since $v_2 < b$, consuming the second unit is not ex-ante optimal. Given that

$v_2 \geq \hat{\beta}d$, these consumers are aware that they would consume two units in the next period if they were to buy two units today.

Consider now the alternative situation in which $\hat{\beta}d \geq b$. In this case, there are no consumers that do not take advantage of the quantity discount because in order to avoid overconsuming the good. Condition $\hat{\beta}d \geq b$ is equivalent to the following condition

$$a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b \tag{2.4}$$

Condition (2.4) implies a strictly positive quantity discount if the consumer is aware that he is not fully time-consistent, $\hat{\beta} < 1$. The next proposition describes the consumer's purchasing and consumption behavior when $\beta > k$ and the firm offers a quantity discount, $b < a$.

Proposition 12 *Assume $\beta > k$ and $b < a$*

1) If $v_1 \in [0, d)$ the agent does not purchase the good

2) If $v_1 \in [d, \beta d/k)$, the agent consumes one unit per period and takes advantage of the quantity discount by purchasing two units every two periods

3) If $v_1 \in [\beta d/k, \hat{\beta}d/k)$ the agent purchases and consumes two units per period

4) If $v_1 \in [\hat{\beta}d/k, 1]$

a) Consider $\hat{\beta} \geq 2\frac{k}{k+1}$ [6]

i) If $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ then for $v_1 \in [\hat{\beta}d/k, 1]$ the agent purchases and consumes two units every period

ii) If $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$ then for $v_1 \in [\hat{\beta}d/k, b/k)$ the agent purchases and consumes one unit every period, if $v_1 \in [b/k, 1]$ the agent purchases and consumes two units every period

b) Consider $\hat{\beta} < 2\frac{k}{k+1}$

[6] $\hat{\beta} \geq 2\frac{k}{k+1} \Leftrightarrow \frac{\hat{\beta}d}{k} \geq \frac{2d}{k+1}$, this condition guarantees that if $v_1 \geq \hat{\beta}d/k$ consuming two units per period is better than not consuming the good.

i) If $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ *or* $\frac{b}{k} < a$, *then for* $v_1 \in [\frac{\hat{\beta}d}{k}, \frac{2d}{1+k})$ *the agent does not purchase the good; if* $v_1 \in [\frac{2d}{1+k}, 1]$ *the agent purchases and consumes two units every period*

ii) If $\frac{\hat{\beta}/k}{2-\hat{\beta}/k}b \leq a < \frac{2-\hat{\beta}}{\hat{\beta}}b$ *and* $b/k \geq a$ *then for* $v_1 \in [\frac{\hat{\beta}d}{k}, a)$ *the agent does not purchase the good; if* $v_1 \in [a, b/k)$ *the agent purchases and consumes one unit every period; if* $v_1 \in [b/k, 1]$, *the agent purchases and consumes two units every period*

iii) If $a < \frac{\hat{\beta}/k}{2-\hat{\beta}/k}b$ *and* $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$ *then for* $v_1 \in [\frac{\hat{\beta}d}{k}, \frac{b}{k})$ *the agent purchases and consumes one unit every period; if* $v_1 \in [b/k, 1]$, *the agent purchases and consumes two units every period*

Proof. See appendix 1 ■

Notice that part 3) of proposition 11 refers to consumer that buy two units per period thinking that they will consume just one unit per period, while part 4) refers to consumers that realize that if they buy two units they will consume two units next period.

We examine next the consumer's behavior when $\beta < k < \hat{\beta}$

Proposition 13 *Consider* k *such that* $\beta < k < \hat{\beta}$ *and* $b < a$

1) If $v_1 \in [0, d)$ *the agent does not purchase the good.*

2) If $v_1 \in [d, \hat{\beta}d/k)$, *the agent purchases and consumes two units per period.*

3) If $v_1 \in [\hat{\beta}d/k, 1]$, *the same as proposition 11.4)*

Proof. See appendix 2 ■

Notice that in this case, as opposed to the case where $\beta > k$, there are no consumers that manage to take advantage of the quantity discount and consume just one unit per period as in part 2) of proposition 11.

We now consider consumers with a relatively high level of time inconsistency but degree of sophistication, $\beta \leq \hat{\beta} < k$.

Proposition 14 *Consider* $\hat{\beta} < k$ *and* $b < a$

1) If $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ or if $b/k < a$ then for $v_1 \in [0, \frac{2d}{(1+k)})$ the agent does not purchase the good; if $v_1 \in [2d/(1+k), 1]$, the agent purchases and consumes two units every period.

2) If $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$ and $b/k > a$ then for $v_1 \in [0, a)$ the agent does not purchase the good; if $v_1 \in [a, b/k)$ the agent purchases and consumes one unit every period; if $v_1 \in [b/k, 1]$, the agent purchases and consumes two units every period.

Proof. See appendix 3 ■

In this case, where consumers have a relatively high level of sophistication, there are no consumers that buy two units per period thinking that they will consume just one unit period as in part 3) of proposition 11.

Notice that proposition 13 part 2) refers to consumers who strategically rationing the quantities they purchase to engage in self control. This behavior is well documented on Wertembroch (1998).

2.4 Firm Behavior

In this section, we examine whether a monopolistic firm selling a good of repeated consumption wants to offer quantity discounts. Consumers can buy at most two units of the good per period. Some of the consumers might consume one unit per period while other might comsume two units per period. Hence, after two consumption periods, consumers have no more goods stockpiled at "home" and all of them need to purchased the good again. Therefore, we can think of the firm's problem as maximizing its profit over a two period horizon. Notice, however that this is not equivalent to a two period model since the demand is determined assuming infinitely lived consumers.

2.4.1 Time-consistent Consumers

Since the behavior of time-consistent consumers depends only on the average price, the firm can maximize its profit by choosing a simple per unit price. The firm's problem is given by

$$\underset{a}{Max}\ 2(a-c)\left[1-F(a/k)\right]+(a-c)\left[F(a/k)-F(a)\right]$$

Hence, the optimal price is given by the following condition

$$a = c + \frac{1 - F(\frac{a}{k}) + 1 - F(a)}{f(a) + \frac{1}{k} f(\frac{a}{k})} \tag{2.5}$$

2.4.2 Time-inconsistent Consumers

In this section we consider the firm's optimal selling strategy when it faces time-inconsistent consumers who are potentially naive. We consider three different cases depending on the relative values of parameters k, β and $\hat{\beta}$.

Optimal Pricing with Time-inconsistent but Sophisticated Consumers

We consider first the firm's profit maximization problem when consumers have a relatively high level of time inconsistency and are aware of that, $\beta \leq \hat{\beta} < k$.

If prices a and b, are such that $a \geq \frac{2-\hat{\beta}}{\hat{\beta}} b$ or $b/k < a$[7], then proposition 13 implies that the firm's demand is given by

$$2\left[1 - F(\frac{2d}{1+k})\right] \tag{2.6}$$

The demand function (2.6) is expressed in terms of the average price d because all consumers buy and consume two units per period.

If instead $a \leq \frac{2-\hat{\beta}}{\hat{\beta}} b$ and $b/k > a$, then from proposition 13 the firm's demand is given by

$$[1 - F(a)] + \left[1 - F(\frac{b}{k})\right] \tag{2.7}$$

Notice that the first term in expression (2.7) is the quantity demand for the first unit at marginal price a, while the last term is the demand for the second at marginal price b. If the firm chooses $a = b$, then its demand, given by condition (2.7), becomes equal to demand when the firm faces time-consistent consumers, given by expression (2.3).

The firm's problem is to choose the maxim of the following two problems

[7] Both these conditions imply that $a > b$.

$$
\begin{aligned}
&\underset{d}{Max}\ (d-c)2\left[1-F(2d/(1+k))\right] \qquad (2.8)\\
s.t.\ a \ &\geq\ \frac{2-\hat{\beta}}{\hat{\beta}}b \ \text{ or } \ b/k < a
\end{aligned}
$$

$$
\begin{aligned}
&\underset{a,b}{Max}\ (a-c)\left[1-F(a)\right]+(b-c)\left[1-F(\frac{b}{k})\right] \qquad (2.9)\\
s.t.\ a \ &<\ \frac{2-\hat{\beta}}{\hat{\beta}}b \ \text{ and } \ b/k > a
\end{aligned}
$$

Consider first problem (2.8). From the first order condition we obtain

$$
d = c + \frac{1-F(\frac{2d}{1+k})}{\frac{2}{1+k}f(\frac{2d}{1+k})} \qquad (2.10)
$$

The solution of problem (2.8) is a pair of prices a and b such that their average satisfies condition (2.10) and either $a > \frac{2-\hat{\beta}}{\hat{\beta}}b$ or $b/k < a$.

Consider now problem (2.9) without the restrictions. From the first order conditions we obtain that the optimal prices must verify

$$
\begin{aligned}
a^* &= c + \frac{1-F(a^*)}{f(a^*)} \qquad (2.11)\\
b^* &= c + \frac{1-F(\frac{b^*}{k})}{f(\frac{b^*}{k})\frac{1}{k}} \qquad (2.12)
\end{aligned}
$$

Assuming that the hazard rate of the distribution v_1 is increasing[8], conditions (2.11) and (2.12) uniquely determine the optimal prices of problem (2.9). We show in the appendix 4 that

[8]This is a assumption frequently made in the literature and is satisfied by many distributions such as the uniform, the normal, the exponential, the logistic and any distribution with non-decreasing density.

the maximum of this problem is always higher than the maximum of problem (2.8). We show also that conditions (2.11) and (2.12) imply that b^* is strictly smaller than a^*.

We saw that if consumers have a relatively high degree of time-inconsistency and are aware of that, $\beta \leq \hat{\beta} < k$, the optimal solution of the problem entails a quantity discount, $b < a$. In this case, since consumers are sufficiently aware that they are time-inconsistent, no consumers buy two units thinking they will consume just one unit. Hence, we have only two different kind of consumer behavior. Consumers whose valuation v_1 lies in the interval $[a, b/k)$ buy and consume one unit per period. These consumers recognize that if they take advantage of the quantity discount and buy two units, they will end up consuming two units in the next period. This is not optimal since $v_1 < b/k$. The second type are consumers whose valuation of the good v_1 is higher than b/k. These consumers buy and consume two units per period. While these consumers recognize that they will consume two units next period, they still find it better to buy and consume two units than buying just one unit at a higher price. Some of these last consumers, whose value v_1 lies in the interval $[b/k, d/k)$ would ideally like to buy two units and consume just one unit, but since they are time-inconsistent this is not an option available for them. Finally, for consumers whose valuation v_1 is higher than d/k, buying and consuming two units per period is the optimal time-consistent decision.

When $\beta \leq \hat{\beta} < k$, taking advantage of the consumer naivety is not the reason to offer quantity discounts. What instead motivates this selling strategy is the presence of consumers that are time-inconsistent and are aware of that. Since consumers are time inconsistent and unable to keep the good for future consumption, offering quantity discounts does not decrease the future sales of the firm. However, because consumers realize that increasing their purchases leads to non-optimal consumption behavior, they require a significant discount to purchase more than one unit.

Notice that the optimal prices a^* and b^* do not depend on β or $\hat{\beta}$ and thus do not converge to the optimal time-consistent price when $\beta \to 1$. The reason is that the optimal prices given in expression (2.11) and (2.12) are only valid for $\beta \leq \hat{\beta} < k < 1$.

In the next two sections we consider consumers who believe they have a relatively high degree of time consistency, $\hat{\beta} > k$. We will assume also that $\hat{\beta} > \frac{2k}{1+k}$. The case of $k < \hat{\beta} \leq \frac{2k}{1+k}$ leads to relatively similar results and is discussed in the appendixes 5 and 6.

Optimal Pricing with Time-inconsistent and Naive Consumers

We consider now consumers with a relatively high level of time-inconsistency and naivety, $\beta < k < \hat{\beta}$. If prices a and b, where $a > b$, do not satisfy condition (2.4), then proposition 12 implies that the demand of the firm is given by

$$[1 - F(d)] + \left[1 - F(\frac{b}{k})\right] + \left[F(\frac{\hat{\beta}d}{k}) - F(d)\right] \tag{2.13}$$

Notice that the first term in expression (2.13) is the demand for the first unit with a marginal price a, while the last two terms are the demand for the second unit which has a marginal price b. If instead prices a and b, satisfy condition (2.4), then by proposition 12 the demand of the firm is given by

$$2\,[1 - F(d)] \tag{2.14}$$

In the demand function (2.14) all consumers that purchase the good buy and consume two units per period at an average price d.

If instead the firm decides not to offer a quantity discount, $a = b$, his demand is given by expression (2.3). Comparing expressions (2.14) and (2.3) we can see that, for the same average price d, the quantity demanded when there is no discount is always smaller than the quantity demanded when there is a big enough discount. The intuition is as follows. When the firm does not offer quantity discounts, consumers have no incentives to buy more than they need for next period consumption. Hence, they buy the second unit only if the marginal benefit of the second unit is higher that the marginal price. If instead the firm offers a quantity discount, naive time-inconsistent consumers believe they can take advantage of the quantity discount and manage not to overconsume the good next period. Hence, they buy two units even if the marginal benefit of the second unit is smaller than the price. However, in the next period they cannot resist and consume two units. Thus, if consumers are significantly time-inconsistent and naive, the firm can expand the demand by offering quantity discounts. Therefore, the firm's optimal selling strategy is to offer a quantity discount.

The firm optimal solution are the prices that give the maximum of the following two problems

$$\max_{a,\, b} \; (a-c)\left[1-F(d)\right]+(b-c)\left[1-F(\frac{b}{k})\right]$$
$$+(b-c)\left[F(\frac{\hat{\beta}d}{k})-F(d)\right] \tag{2.15}$$
$$s.t. \quad a \;\leq\; \frac{2-\hat{\beta}}{\hat{\beta}}b \text{ and } a>b \tag{2.16}$$

and

$$\max_{d} \; (d-c)2\left[1-F(d)\right] \tag{2.17}$$
$$s.t. \; a \;>\; \frac{2-\hat{\beta}}{\hat{\beta}}b \text{ and } a>b \tag{2.18}$$

Notice that if the solution of the unrestricted problem (2.15) satisfies restriction (2.16), then this solution provides the profit maximization prices. The reason is that in problem (2.15) we can obtain any level of profit obtained in problem (2.17) by setting $b=\hat{\beta}d$ and choosing the same level of d. If the solution of the unrestricted problem (2.15) does not satisfy (2.16) then the optimal prices are the prices that maximize problem (2.17).

From the first order condition of problem (2.15) we obtain

$$b^* = c + \frac{k\left[F(\frac{a^*+b^*}{2}\frac{\hat{\beta}}{k})-F(\frac{b^*}{k})\right]}{f(\frac{b^*}{k})} \tag{2.19}$$

$$a^* = c + \frac{F(\frac{a^*+b^*}{2}\frac{\hat{\beta}}{k})-F(\frac{b^*}{k})}{f(\frac{b^*}{k})}\left[\frac{\beta f(\frac{a^*+b^*}{2}\frac{\hat{\beta}}{k})}{f(\frac{a+b^*}{2})}-k\right]+\frac{2\left[1-\;\; F(\frac{a^*+b^*}{2})\right]}{f(\frac{a^*+b^*}{2})} \tag{2.20}$$

Notice that conditions (2.19) and (3.11) only give the optimal prices if a^* and b^* verify restriction (2.16). Hence, under the conditions of problem (2.15) we have that $\frac{a^*+b^*}{2}\frac{\hat{\beta}}{k} < \frac{b^*}{k}$.

Therefore, condition (2.19) implies that $b < c$. This in turn implies that $b < a$. Hence, in this case, the firm optimal selling strategy also includes quantity discounts. Surprisingly, the price of the second unit is set below the marginal cost[9].

If prices a^* and b^* don't satisfy restriction (2.16), then the optimal prices are given by problem (2.17). From the first order condition of this problem we obtain

$$d^* = c + \frac{1 - F(d^*)}{f(d^*)} \tag{2.21}$$

Hence, the optimal prices are any prices, a and b, that satisfy conditions (2.21) and (2.18). In this case, there are more than one pair of prices, a and b, that satisfy these two conditions and maximize the firm profit. The reason is that if the price b is significantly lower than price a, as in condition (2.18), then all consumers that purchase buy two units per period. Thus, consumers decisions are determined only by average price and not by the individual value of the prices, provided that they satisfy condition (2.18).

We show in appendix 7 that when consumers have a high degree of naivety, with $\hat{\beta}$ close enough to one, the optimal solution is given by condition (2.21). Notice also that if $\hat{\beta} \to 1$, then condition (2.18) implies that b can become close enough to a. In the limit, when the consumer is fully naive and $\hat{\beta} = 1$, the firm can offer a very small discount, $a = b + \in$, where $\in$ is a small positive amount. When consumers are fully naive, they do not realize that taking advantage of quantity discounts might lead them to non-optimal consumption. Hence, they take advantage of any quantity discount no matter how small it is.

We conclude that when a firm is facing consumers with high level of time inconsistency and naivety, the optimal selling strategy is to offer quantity discounts. Notice that, as opposed to the previous section where the consumers were relatively sophisticated, we might have consumers

[9]The intuition is as follows. Consumers whose value v_1 lies in the interval $\left[d, \frac{\hat{\beta}d}{k}\right]$ or in the interval $\left[\frac{b}{k}, 1\right]$ always buy two units and thus their consumption decisions are only determined by the average price, d. Hence, for the monopolist any decrease of price b to these consumers is compensated by an increase in price of a of the same magnitude. Consumers whose value v_1 lies instead in the intermediate interval $\left[\frac{\hat{\beta}d}{k}, \frac{b}{k}\right]$ only buy one unit per period at the average price a. Notice that if b were higher than c, the monopolist could decrease b and increase a by the same magnitude, and there would be two positive effects in the profit. The monopolist would charge more to consumers in the intermediate interval. Additionally, it would sell the second unit to more consumers at a price that is still above the marginal cost. Hence, the impact on the profit would be unambiguously positive. For this reason, we can conclude that the optimal price b must be smaller than the marginal cost.

that buy two units of the good who would be better off if they had bought just one unit. When $d < v_1 < b/k$ consumers buy two units of the good to take advantage of the quantity discount but they plan to consume just one unit per period. However, after buying two units they cannot resist and consume two units per period.

Optimal Pricing with Moderately Time-inconsistent Consumers

Finally, we consider consumers with a relatively low level of time-inconsistency, $k < \beta \leq \hat{\beta} \leq 1$. In this case, we can have consumers that take advantage of quantity discount, by buying two units (every two periods) and are able to consume just one unit per period. The existence of these consumers can potentially limit the firm ability to offer quantity discounts.

If $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$, and $a > b$, proposition 11 implies that the firm demand is given by

$$\left[F(\frac{\beta d}{k}) - F(d)\right] + 2\left[F(\frac{\hat{\beta} d}{k}) - F(\frac{\beta d}{k})\right] + 2\left[1 - F(\frac{b}{k})\right] + \left[F(\frac{b}{k}) - F(\frac{\hat{\beta} d}{k})\right] \tag{2.22}$$

Notice that in expression (2.22) the first three terms are demand from consumers that buy two units of the good at an average price d, while the last term is the demand from consumers that buy just one unit at price a. This part of demand curve requires $\hat{\beta} < 1$, otherwise we could not have $\frac{b}{k} > \frac{\hat{\beta} d}{k}$.

If $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ proposition 11 implies that the firm's demand is given by

$$\left[F(\frac{\beta d}{k}) - F(d)\right] + 2\left[1 - F(\frac{\beta d}{k})\right] \tag{2.23}$$

The first term are consumers that buy two units of the good every two periods, while the second term are consumers that buy two units every period. Hence, they all buy the good at an average price d.

If the firm does not offer a quantity discount, $a = b$, his demand is given by expression (2.3). Comparing expressions (2.23) and (2.3) we can see that, for the same average price d, the quantity demand when there is no quantity discount is always smaller than the quantity demanded when there is a big enough quantity discount. If no quantity discount is offered, consumes whose value v_1 lies in the interval $\left[\frac{\beta d}{k}, \frac{b}{k}\right]$ buy and consume just one unit per period. However, if the firm offers a quantity discount, such that $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$, then those consumers will consume two units every period. Therefore, the firm can always increase its profits by offering a quantity discount.

The firm profit is the maximal of the two following problems

$$\begin{aligned}
& \max_{a,b} 2(d-c)\left[1 - F(\frac{b}{k}) + F(\frac{\hat{\beta}d}{k}) - \frac{1}{2}F(\frac{\beta d}{k}) - \frac{1}{2}F(d)\right] \\
& +(a-c)\left[F(\frac{b}{k}) - F(\frac{\hat{\beta}d}{k})\right] \qquad (2.24) \\
s.t. \quad a & < \frac{2-\hat{\beta}}{\hat{\beta}}b \text{ and } a > b \qquad (2.25)
\end{aligned}$$

and

$$\begin{aligned}
& \max_{d}(d-c)\left[F(\frac{\beta d}{k}) - F(d)\right] + 2(d-c)\left[1 - F(\frac{\beta d}{k})\right] \qquad (2.26) \\
s.t. \quad a & \geq \frac{2-\hat{\beta}}{\hat{\beta}}b \text{ and } a > b \qquad (2.27)
\end{aligned}$$

If we solve the unrestricted problem (2.24) and the optimal prices satisfy condition (2.25) then the optimal prices are given by problem (2.24), otherwise the optimal prices are given by problem (2.26). From the first order conditions of problem (2.24) we obtain

$$b = c + k\frac{F(\frac{\hat{\beta}}{k}\frac{a+b}{2}) - F(\frac{b}{k})}{f(\frac{b}{k})} \qquad (2.28)$$

$$a = c + 2\frac{2 - F(\frac{a+b}{2}) - F(\frac{\beta}{k}\frac{a+b}{2}) - 2\hat{\beta}\frac{f(\frac{\hat{\beta}}{k}\frac{a+b}{2})}{f(\frac{b}{k})}\left[F(\frac{b}{k}) - F(\frac{\hat{\beta}}{k}\frac{a+b}{2})\right]}{f(\frac{a+b}{2}) + \frac{\beta}{k}f(\frac{\beta}{k}\frac{a+b}{2})} \tag{2.29}$$

$$+\frac{k\left[F(\frac{b}{k}) - F(\frac{\hat{\beta}}{k}\frac{a+b}{2})\right]}{f(\frac{b}{k})} \tag{2.30}$$

Under the conditions of problem (2.24) we necessarily have that $\frac{\hat{\beta}}{k}\frac{a+b}{2} < \frac{b}{k}$. Hence, condition (2.28) implies that the price of the second unit, b, is smaller than the marginal cost, c. Therefore, we necessarily have that $a < b$ and hence the firm's optimal selling strategy in this case is to offer quantity discounts.

From the first order condition of problem (2.26) we obtain

$$d = c + \frac{2 - F(d) - F(\frac{\beta d}{k})}{f(d) + \frac{\beta}{k}f(\frac{\beta d}{k})} \tag{2.31}$$

In this case, the optimal prices would be any prices a and b whose average satisfies condition (2.31) and that satisfy (2.27) which imply that $a > b$.

We show in appendix 7 that when β is close enough to one the firm optimal prices are given by conditions (2.27) and (2.31). Notice that if $\beta = 1$, condition (2.31) becomes equal to condition (2.5), which gives the optimal price when consumers are time-consistent.

We conclude that, even when the firm faces moderately time-consistent consumers and there are consumers that take advantage of quantity discount and consume just one unit per period, the firm optimal selling strategy is still to offer quantity discounts.

2.5 Conclusion

In this paper we have examined the problem of a profit-maximizing firm that sells a storable good of repeated consumption to time-inconsistent consumers with potentially naive expectations. In this case, if firms face time-consistent consumers, quantity discounts do not increase firms' profits. We show that if instead consumers are time-inconsistent, the optimal selling strategy is to offer quantity discounts. Naive time-inconsistent consumers take advantage of any small quantity discount in order to save on future purchases. However, the higher availability of the good after the purchase leads them to consume more than they had planned. Sophisticated time-inconsistent consumers realize that taking advantage of the quantity discount leads them to consume more that they would like. Hence, they will require a significant quantity discount to increase the quantity purchased. These results are also consistent with the empirical evidence[10] that quantity discounts are deeper and more prevalent for vice goods than for virtue goods.

2.6 Appendixes

Appendix 1 (Proof of proposition 11) 1) If the marginal valuation of the first unit is lower than the average price, $v_1 < d$, then the optimal decision is to not consume the good and hence the agent does not purchase the good.

2) Consider now v_1 such that $d \leq v_1 < \beta d/k$. The first inequality means that it is optimal to consume at least one unit per period. The second inequality means that the agent is not tempted to consume the second unit. If he were to consume the second unit, then he would have to buy another unit at average price d. Hence, these consumers take advantage of the quantity discount and purchase two units every two periods and consume just one unit per period.

3) When $\beta d/k \leq v_1 < \hat{\beta} d/k$, the second inequality means that the consumer does not realize that he is tempted to overconsumption which leads him to take advantage of the quantity discount. The first inequality means that he will consume two units next period if he were to buy two units in the current period. Consumers whose valuation $v_1 \geq \hat{\beta} d/k$, realize that if they buy two units this period they will consume two units next period.

a) Consider $\hat{\beta} \geq 2\frac{k}{k+1}$. This condition is equivalent to $\frac{\hat{\beta} d}{k} \geq \frac{2d}{k+1}$. This inequality implies that if $v_1 > \frac{\hat{\beta} d}{k}$ then $v_1 > \frac{2d}{k+1}$. This last condition means that the benefit of consuming two goods is higher than the cost.

i) We also have that $a \geq \frac{2-\hat{\beta}}{\hat{\beta}} b$ which is equivalent to $b/k \leq \hat{\beta} d/k$. This second inequality implies that if $v_1 \geq \hat{\beta} d/k$ then $v_1 \geq b/k$. This condition means that the benefit of consuming

[10] See Wertembroch (1998)

the second unit is higher than its cost. Hence, if $v_1 \geq \hat{\beta}d/k$ the consumer buys and consumes two units per period.

ii) Consider now that $b/k > \hat{\beta}d/k$. Since $\hat{\beta}d/k \geq 2d/(1+k)$, we have that $2d/(1+k) < b/k$. This last condition implies that $a < 2d/(1+k)$. Therefore, $a < \hat{\beta}d/k < b/k$. The first inequality implies that if $v_1 \geq \hat{\beta}d/k$ it is optimal to buy and consume at least one unit of the good. The second inequality implies that if $\hat{\beta}d/k \leq v_1 < b/k$ it is not optimal to buy the second unit, because the individual realizes that he will consume the second unit when that is not optimal. Hence, if $\hat{\beta}d/k \leq v_1 < b/kv_1$ the agent buys and consumes just one unit per period. Consider now v_1 such that $b/k \leq v_1 \leq 1$, the first inequality means that buying and consuming the second unit is now optimal.

b) Consider now $\hat{\beta} < \frac{2k}{k+1}$. This is equivalent to $\frac{\hat{\beta}d}{k} < \frac{2d}{k+1}$. Hence, even if $v_1 \geq \frac{\hat{\beta}d}{k}$ we cannot guarantee now that $v_1 \geq \frac{2d}{k+1}$.

i) The condition $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ is equivalent to $b/k \leq \hat{\beta}d/k$. This condition together with $\frac{\hat{\beta}d}{k} < \frac{2d}{k+1}$ implies that $b/k < \frac{2d}{k+1} < a$. Consider v_1 such that $\hat{\beta}d/k \leq v_1 < 2d/(1+k)$. The second inequality means that the consumer does not want to buy and consume two units. Since we also have that $v_1 < a$, the consumer does not want also to purchase one unit. Consider now that $2d/(1+k) \leq v_1 < 1$, the first inequality means that buying and consuming two units is better than not buying the good. Since this also implies that $v_1 > b/k$, buying the second unit is optimal. Hence, the consumer buys and consumes two units per period.

ii) Condition $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ is equivalent to $b/k \geq \hat{\beta}d/k$ and condition$\frac{\hat{\beta}/k}{2-\hat{\beta}/k}b < a$ is equivalent to $a > \hat{\beta}d/k$. This implies that $b/k > a > \hat{\beta}d/k$. If $\hat{\beta}d/k < v_1 < a$ the agent does not purchase the good. Consider now v_1 such that $a < v_1 < b/k$, the first inequality means that is optimal to consume at least one unit while the second implies that consuming two units is not optimal. Hence, the agent purchases and consumes one unit every period. if $b/k < v_1 \leq 1$, the first inequality means that is now also optimal to consume the second unit. Thus, the consumer buys and consumes two units per period.

iii) We have that $b/k > \hat{\beta}d/k$ and $a < \hat{\beta}d/k$. Hence, we have that $b/k > \hat{\beta}d/k > a$. Consider $\hat{\beta}d/k < v_1 < b/k$. The first inequality implies that $v_1 > a$, hence consuming at least one unit is optimal. The second inequality means that consuming the second unit is not optimal. Hence, the agent purchases and consumes one unit every period. Consider $v_1 > b/k$, now consuming the second unit is also optimal and thus the consumer buys and consumes two units every period.

Appendix 2 (Proof of proposition 12) 1) The same as proposition 11.1)

2) Consider $d < v_1 < \hat{\beta}d/k$. Since $d < v_1$, these consumers want to buy at least one unit. $v_1 < \hat{\beta}d/k$ means that these consumers do not realize that they are tempted to overconsumption and thus they take advantage of the quantity discount and buy two units each time they buy the good. However, since $\beta < k$, we have that $v_1 > \beta d/k$. This implies if these consumers buy two units they will consume two units in the following period.

3) The same as proposition 11.4)

Appendix 3 (Proof of proposition 13) Since $\hat{\beta} < k$, all consumers that may buy the good, $v_1 > d$, are aware that if they buy two units they will consume two units, $v_1 > \hat{\beta}d/k$.

1) The condition $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ is equivalent to $b/k \leq \hat{\beta}d/k$. This condition, given $\hat{\beta} < k$, implies that $b/k < d < \frac{2d}{k+1} < a$. Consider v_1 such that $0 < v_1 < 2d/(1+k)$. The second inequality means that is not optimal to buy and consume two units. Since that inequality also implies that $v_1 < a$, it is not optimal also to buy and consume one unit. Hence, the consumer does not purchase the good. If $v_1 \geq \frac{2d}{k+1}$, this means that the benefit of buying both units is higher that the cost. Since this is also implies that $v_1 > b/k$, it is better to buy and consume two units than to buy and consume just one unit. Hence, if $v_1 \geq \frac{2d}{k+1}$the consumer buys and consumes two units.

If $b/k < a$, then we also have that $b/k < \frac{2d}{k+1} < a$. Hence, the consumption decisions are the same as when $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$.

2) The condition $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$ is equivalent to $b/k > \hat{\beta}d/k$. Since $\hat{\beta}d/k < d$, we also have that $\hat{\beta}d/k < a$. Hence, In this case, we have that $\hat{\beta}d/k < d < a < b/k$. Consider consumers with valuation $v_1 < a$. For these consumers we also have that $v_1 < b/k$. These last two inequalities imply that consuming one or two units is not optimal. Consider now v_1 such that $a \leq v_1 < b/k$, the first inequality means that is optimal to consume at least one unit while the second inequality implies that consuming the second unit is not optimal. Hence, the agent purchases and consumes one unit every period. Consumers with value v_1 is such that $b/k < v_1$

≤ 1, buy and consume two units per period. The first inequality means that is now also optimal to consume the second unit.

Appendix 4 Assuming that the hazard rate of the distribution of v_1 is increasing, conditions (2.11) and (2.12) provide unique solutions to the optimal prices a^* and b^*. We show next that these conditions together with the increasing hazard rate condition imply that $b^* < a^* < b^*/k$. Multiplying both sides of equation (2.12) by $\frac{1}{k}$ we obtain that

$$\frac{b^*}{k} = \frac{c}{k} + \frac{1 - F(\frac{b^*}{k})}{f(\frac{b^*}{k})} \tag{2.32}$$

If $c > 0$ and given that $k < 1$, we necessarily have that.

$$\frac{b^*}{k} > a^* \tag{2.33}$$

Condition (2.33) together with condition that the hazard rate is increasing, implies that the right on side of equation (2.12) is smaller than the right on side of equation (2.11) and hence we must also have that $b^* < a^*$ .The optimal prices a^* and b^* also verify the condition that $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$

$$a^* < \frac{2-\hat{\beta}}{\hat{\beta}}b^* \Leftrightarrow a^* < \frac{(2-\hat{\beta})k}{\hat{\beta}}(b^*/k)$$

This last inequality is necessarily verified because $\hat{\beta} < 1$, $k > \hat{\beta}$[11] and $a^* < b^*/k$. Hence, the solution of problem (2.9) without the restrictions is the same as the solution of the problem (2.9) with the restrictions. Because of this we can also show that the maximum of problem (2.9) is always strictly higher that the maximum of problem (2.8). In problem (2.9) by choosing $a = b/k$ we could always obtain the same value of profit as in problem (2.8) for the same level of the average price, d. hence the fact that in problem (2.9) we choose $a^* < b^*/k$, implies that the profit with this solution is strictly higher than the profit that can be obtained under the conditions of problem (2.8).

[11]This is the initial condition of this problem.

Appendix 5 In this appendix we determine the optimal prices when $\beta < k < \hat{\beta}$ and $\hat{\beta} < 2\frac{k}{k+1}$. If $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$ and $a < b/k$, then by proposition 12 the firm's demand is given by

$$2\left[1 - F(\frac{b}{k})\right] + 2\left[F(\hat{\beta}d/k) - F(d)\right] + [F(b/k) - F(a)] \tag{2.34}$$

In the expression (2.34) the first two term are consumers that buy two units at an average price d. The last term are consumer that buy only one unit at a price a. The firm's maximization problem is given by

$$\max_{a,b} 2(\frac{a+b}{2} - c)\left[1 - F(\frac{b}{k})\right] + 2(\frac{a+b}{2} - c)\left[F(\hat{\beta}\frac{a+b}{2k}) - F(d)\right] + (a-c)\left[F(\frac{b}{k}) - F(a)\right] \tag{2.35}$$

The optimal prices satisfy the following conditions

$$a = c + \frac{2\left[1 - F(\frac{a+b}{2}) + F(\frac{a+b}{2}\frac{\hat{\beta}}{k}) - F(a)\right] + (k - \hat{\beta})\left[F(\frac{b}{k}) - F(a)\right]}{f(\frac{a+b}{2})f(\frac{b}{k}) - f(\frac{b}{k})\, f(\frac{a+b}{2}\frac{\hat{\beta}}{k})\beta/k + 2f(\frac{b}{k})f(a) - \hat{\beta}f(\frac{a+b}{2}\frac{\hat{\beta}}{k})f(a) + kf(\frac{a+b}{2})f(a)}$$

and

$$b = c + \frac{\left[\beta f(\frac{a+b}{2}\frac{\hat{\beta}}{k}) - kf(\frac{a+b}{2})\right](F(\frac{b}{k}) - F(a)) + 2kf(a)(1 - F(\frac{b}{k}) + F(\frac{a+b}{2}\frac{\hat{\beta}}{k}) - F(\frac{a+b}{2}))}{f(\frac{a+b}{2})f(\frac{b}{k}) - f(\frac{b}{k})\, f(\frac{a+b}{2}\frac{\beta}{k})\hat{\beta}/k + 2f(\frac{b}{k})f(a) - \hat{\beta}f(\frac{a+b}{2}\frac{\hat{\beta}}{k})f(a) + kf(\frac{a+b}{2})f(a)}$$

These two conditions imply that the optimal prices must satisfy $a \leq b/k$ and that $a > b$. Hence, in this case we also have a quantity discount. Notice that here we have four types of customers whose valuation v_1 is higher than d. Customers whose value v_1 lies in the interval $\left[d, \frac{\hat{\beta}d}{k}\right]$ buy and consume two units per period, because they don't realize they will overconsume the good. Customers whose value lie in the interval $[d, a]$ don't buy the good, because they realize they would overconsume the good. Customers whose value lie in the interval $[a, b/k]$

buy only one unit of the good, because they know that if they buy the second unit they will consume it and that is not optimal. Finally, consumers whose value lies in the interval $[b/k, 1]$ buy two units despite knowing they will consume two units next period.

If $a > \frac{2-\hat{\beta}}{\hat{\beta}}b$ proposition 12 implies that the firm's demand is given by

$$2\left[1 - F(\frac{2d}{1+k})\right] + 2\left[F(\hat{\beta}d/k) - F(d)\right] \tag{2.36}$$

In this case, the firm maximization problem is given by

$$\max_{d}(d-c)2\left[1 - F(\frac{2d}{1+k})\right] + (d-c)2\left[F(\hat{\beta}d/k) - F(d)\right]$$

The optimal average price is given by

$$d = c + \frac{1 - F(\frac{2d}{1+k}) + F(\hat{\beta}d/k) - F(d)}{\frac{2}{1+k}f(\frac{2d}{1+k}) + f(d) - \frac{\hat{\beta}}{k}f(\hat{\beta}d/k)} \tag{2.37}$$

The optimal prices a and b also must satisfy conditions

$$a > b/k \quad \text{or } a > \frac{2-\hat{\beta}}{\hat{\beta}}b$$

Either of these conditions imply a positive quantity discount.

Appendix 6 In this appendix we determine the optimal prices when $\beta > k$ and $\hat{\beta} < 2\frac{k}{k+1}$. Consider now that $\hat{\beta} < 2\frac{k}{k+1}$. If $a < \frac{2-\hat{\beta}}{\hat{\beta}}b$ and $b/k > a$ proposition 11 implies that the firm's demand is given by

$$\left[F(\frac{\beta d}{k}) - F(d)\right] + 2\left[F(\frac{\hat{\beta}d}{k}) - F(\frac{\beta d}{k})\right] + \left[F(\frac{b}{k}) - F(a)\right] + 2\left[1 - F(\frac{b}{k})\right] \tag{2.38}$$

Notice that in expression (2.38) the first three terms are consumers that buy two units each time they purchase the good at an average price of d, while the last term are consumers that buy just one unit per period at a price a.

If instead $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$ or $b/k < a$ proposition 11 implies that the firm's demand is given by

$$\left[F(\frac{\beta d}{k}) - F(d)\right] + 2\left[F(\frac{\hat{\beta} d}{k}) - F(\frac{\beta d}{k})\right] + 2\left[1 - F(\frac{2d}{1+k})\right] \tag{2.39}$$

The firm chooses a and b such that $a \geq \frac{2-\hat{\beta}}{\hat{\beta}}b$. Hence, the relevant part of the firm's demand is given by expression (2.39). The firm's maximization problem is given by

$$\begin{aligned} & \max_d (d-c)\left[F(\frac{\beta d}{k}) - F(d)\right] + 2(d-c)\left[F(\frac{\hat{\beta} d}{k}) - F(\frac{\beta d}{k})\right] \\ & +2(d-c)\left[1 - F(\frac{2d}{1+k})\right] \\ s.t.\ a \geq\ & \frac{2-\hat{\beta}}{\hat{\beta}}b \text{ or } b/k < a \end{aligned}$$

The optimal average price satisfies the following condition

$$d = c + \frac{2 - F(\frac{\beta d}{k}) - F(d) + 2F(\frac{\hat{\beta} d}{k}) - 2F(\frac{2d}{1+k})}{\frac{\beta}{k} f(\frac{\beta d}{k}) + f(d) - 2\frac{\hat{\beta}}{k} f(\frac{\hat{\beta} d}{k}) + \frac{4}{1+k} f(\frac{2d}{1+k})}$$

Appendix 7 As $\hat{\beta} \to 1$, problem (2.15) because of restriction (2.16) is only valid for a arbitrarily close to b. Notice that the solution of problem (2.15) given by conditions (2.19) and (3.11) requires that $b < c$. Therefore, the profits of the firm would become arbitrarily close to zero. Hence, the optimal solution is not given by problem (2.15) but instead by problem (2.17).

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Chapter 3

Exclusive Dealing and Entry

3.1 Introduction

Exclusive dealing agreements are one of the most common vertical restraints used by firms. An exclusive dealing agreement is a contract between a buyer and a seller where the buyer commits to buy a good exclusively from this seller. The idea that exclusive dealing agreements could prevent the entry of more efficient sellers was initially dismissed by economists. The incumbent seller would not find such an agreement profitable because his potential gain from exclusion, the monopoly profit, is lower than what he has to pay the buyer to induce him to sign an exclusive dealing agreement, the monopoly profit plus the deadweight loss. This argument assumes that buyers are final consumers. However, most exclusive agreements are made between firms, namely between producers and distributors.

In this paper, we propose an explanation for the incumbent firms to sign an exclusive contract that prevents the entry of a more efficient upstream seller. The idea is that the entry of more efficient upstream seller, by decreasing the market power of the upstream firms, makes entry in the downstream market more attractive. This can lead to further entry in the downstream market and to an increase in the competition faced by the downstream firms. Since part of the bigger surplus created by the entry of a more efficient seller is captured by the downstream entrant firm, entry in the upstream market no longer necessarily benefits the incumbent downstream firms.

Since Aghion and Bolton (1987), a number of papers have shown why an incumbent seller may want to use exclusive dealing agreements to deter the entry of a rival seller. Aghion and Bolton (1987) argue that an incumbent seller and a buyer might agree to sign an exclusive dealing contract in order to extract surplus from a more efficient entrant seller. If the contract can be renegotiated, the entrant firm would have to pay in order to be able to trade with the buyer. In their model entry is not intentionally blocked and doesn't always occur. Exclusion is a "side effect" of trying to extract the maximal surplus from the entrant when its production costs are unknown. However, once the entrant firm's costs are observable, exclusion doesn't survive three-way renegotiation. In Eric B. Rasmussen et al. (1991) and Ilya R. Segal and Michael D. Whinton (2000) exclusive dealing is used to deter entry when there are economies of scale and the entrant needs to supply enough consumers to cover its fixed costs. When buyers sign an exclusive dealing agreement, they impose a negative externality on other buyers by making entry less likely. The incumbent seller takes advantage of this externality to persuade each buyer to sign an exclusive dealing agreement.

Two recent papers, Fumagalli and Motta (2007) and Simpson and Wickelgren (2007), examine the effect of downstream competition among the buyers on the ability of the incumbent seller to exclude the entry of a more efficient rival firm. Interestingly, these two papers reach almost opposite results. Fumagalli and Motta (2007) argue that intense downstream competition makes exclusion impossible, because one single free buyer that buys the good at a lower price is able to increase its demand sufficiently to make entry profitable. Simpson and Wickelgren (2007) claim that the result obtained in Fumagalli and Motta is based on the fact that the buyers who sign an exclusive contract exit the market if there is one free buyer who becomes a downstream monopolist. Hence, the gain from rejecting the exclusive contract exceeds whatever side payment the incumbent is willing to offer for him to sign an exclusive contract. Instead, Simpson and Wickelgren argue that, if downstream competition is intense, buyers are forced to pass to their customers most of the benefits of buying from a more efficient seller. Hence, the gain for the buyer from not signing an exclusive dealing contract is substantially lower than the sum of the monopoly profit plus the deadweight loss. Consequently, the incumbent seller might be able to induce buyers to sign an exclusive contract. Finally, Comanor and Rey (2000) consider the case where the more efficient entrant distributor can only trade with one of the upstream firms. It is assumed that he faces much higher costs when dealing with the other upstream firm. In this case, the incumbent distributor can monopolize the market by signing an exclusive dealing with the only upstream producer with whom the entrant distributor can potentially trade.

The structure of the paper is as follows. Section 2 presents the basic model and provides the general conditions for exclusive dealing. In section 3, we assume that firms compete in quantities in the downstream market. The case of price competition in the downstream market is analyzed in section 4. Finally, section 5 concludes the paper.

3.2 The Model

In the upstream market there is one incumbent producer, denoted IP, that produces an input at a constant marginal cost, c_I. A more efficient rival producer, EP, may enter the upstream market selling an identical good. The $EP's$ marginal cost of producing the input, c_E, is lower than the marginal cost of the incumbent, $c_E < c_I$. However, the EP faces a fixed sunk cost, E_P, to enter the market. In case of entry in the upstream market, firms are assumed to compete a la Bertrand. This is a common assumption in this literature which focuses on the case where an incumbent producer is trying to avoid being replaced by a more efficient entrant producer.

In the downstream market there is an incumbent distributor, denoted ID. There is also one entrant distributor, denoted ED, which has sunk a cost of entry of E_D. Without loss of generality, distributors are assumed to have zero marginal cost of reselling the good bought in the upstream market. For simplicity, we also assume that, in case of entry, downstream firms face symmetric demand functions. We derive general conditions under which the incumbent firms sign an exclusive dealing agreement, without assuming any particular form of competition in the downstream market. We then apply our results to both the case where downstream firms compete in quantities and to the case where they compete in prices with differentiated goods.

We assume that upstream firms charge two-part tariffs to the downstream firms

$$T_k^j = A_k^j + w_k^j q_k^j \qquad j = I, E;\ k = I, E$$

Where A is the fixed fee, w is the per unit price, I and E refer, respectively, to the incumbent and the entrant firm, while the producers and distributors are denoted by j and k. q_k^j denotes the quantity sold by producer j to the the distributor k. Allowing firms to charge two-part tariffs has the advantage of avoiding double marginalization. We can thus focus exclusively on the effect of exclusive dealing on the entry of new firms, without adding another distortion. We also assume that upstream firms make a take-it-or-leave-it offer to the downstream firms[1]. If there is an exclusive dealing agreement between the incumbent firms, then, in case of entry, the EP can only sell to the ED. When there is no exclusive dealing agreements, each distributor can buy from both producers. We assume the tie-breaking rule that if the input prices charged by the producers are equal, only the lower cost producer sells the input. Producers can price discriminate between the buyers. For example, if the incumbent firms have signed an exclusive dealing agreement, then the IP may want to charge a different two-part-tariff to the ID, from the two-part tariff it charges to the ED, who has also the option of buying from the EP.

The timing of the game is as follows. At time t_1, the IP offers a payment x to the ID if he agrees to sign an exclusive dealing contract. Then, the ID decides whether to accept the contract. If a contract is signed it cannot be breached afterwards[2]. At time t_2, the EP decides whether to enter the market after observing if the incumbent firms have signed an exclusive dealing contract. Then, at time t_3, the ED makes the entry decision, after observing if an exclusive dealing contract was signed by the incumbent firms and if there was entry in the upstream market[3]. At time t_4, active firms simultaneously choose their prices (quantities).

We will need the following notation:

- Let π_i denote the net profit of firm i, where $i = IP, EP, ID, ED$.

- Let $\pi_i(a, b)$ denote the profit (gross of fixed fee and entry cost) of the downstream firm i, $i = ID, ED$, when there is competition in the downstream market and the ID faces an input price a and the ED faces an input price b.

- Let $\pi_{ID}(a)$ denote the profit (gross of the fixed fee) of the ID when he is the monopolist in the downstream market and faces an input price a.

[1]Our main results would still hold if we had assumed linear pricing or downstream firms with bargaining power.

[2]Here, it is not relevant if the exclusive contract include or not any commitment to prices since we are assuming that two-part tariffs can be charged.

[3]We could have instead assumed that both entrant firms make simultaneously entry decisions. This would imply that we could obtain an extra equilibrium. In this equilibrium both entrant firms would not enter because each of them expects the other entrant firm to not enter the market.

We look for sub-game perfect equilibria and examine the impact of the threat of entry in the downstream market. We show that when there is no threat of entry in the downstream market, the incumbent firms don't sign an exclusive dealing agreement and the entrant producer always enters the market whenever that is efficient. By contrast, when there is also the possibility of entry in the downstream market, the incumbent firms may decide to sign an exclusive dealing agreement that prevents the entry of new firms.

This result would be still valid had we assumed that the incumbent producer sells the input to many local monopolists distributors, instead of selling to just a single distributor. However, if the downstream incumbent firms compete against each other, we can apply the Simpson and Wickelgren's result to show that exclusive dealing is possible, even if there is no threat of entry in the downstream market. This is because distributors are forced to pass to their customers part of the benefits of buying from a more efficient seller. However, if producers are allowed to implement resale-price maintenance policies, then Simpson and Wickelgren (2007) is no longer valid. In this case, the incumbent firms can use exclusive dealing agreements to prevent the entry of more efficient producers only when there is also a threat of entry in the downstream market.

To solve the model, we consider first the case where the incumbent firms do not sign an exclusive dealing at time t_1. Then, we consider the case where the incumbent firms sign an exclusive dealing agreement.

3.2.1 No Exclusive Dealing

In this section, we look at the case where the incumbent firms do not sign an exclusive dealing agreement at time t_1. The following proposition describes the optimal prices charged by producers.

Proposition 15 ***[Price decisions at time*** t_4***. No exclusive dealing]*** *The two-part tariffs chosen by the producers depend on the market structure of the upstream and downstream markets.*

A) *If at time* t_2 *the EP does not enter the market, then the IP sets a two part tariff which depends on the type of competition in the downstream market* *i) If there is a monopoly in the downstream market, the IP chooses the following two-part tariff*

$$w_I = c_I \text{ and } A_I = \pi(c_I)$$

ii) If there is competition in the downstream market, the IP chooses the following two-part tariff

$$w_I = w_I^* > c_I \text{ and } A_I = \pi(w_I^*)$$

B) If at time t_2 the EP enters the upstream market, then only the EP sells the input and its two-part tariff depends on whether the ED enters in the downstream market

i) If the ED does not enter the market, then the EP's two-part tariff is given by

$$w_E = c_E \text{ and } A_E = \pi(c_I) - \pi(c_E)$$

ii) If the ED enters the downstream market, then the EP's two-part tariff is given by either

$$w_E = c_I \text{ and } A_E = 0 \tag{3.1}$$

or

$$w_E = w_E^*, \quad c_E < w_E^* < c_I \quad \text{and } A_E = \pi_{ED}(w_E, w_E) - \pi_{ED}(w_E, c_I)^4 \tag{3.2}$$

where w_I^* is given by condition (3.18) in the appendix and w_E^* is determined in the same way.

Proof. See appendix ■

Consider first the case where EP does not enter the market. If there is also a monopoly in the downstream market, then the IP sets the per unit price equal to the marginal cost in order to maximize the industry profit. Then, he captures the industry profit by setting the fixed fee equal to the profit of the ID. If, instead, there is competition in the downstream market, each distributor when he chooses the price does not take into account the externality on the other

[4] Which is equal to $A_E = \pi_{ID}(w_E, w_E) - \pi_{ID}(c_I, w_E)$

distributor's profit. Therefore, in order for the distributors to choose prices that maximize the industry profit, the IP needs to set the marginal price of the input above the marginal cost.

Consider now the case where EP enters the market. When there is a monopoly in the downstream market, the EP sets the price per unit equal to the marginal cost and then chooses the fixed fee to leave the ID indifferent between buying from him or buying from the IP. If, instead, there is competition in the downstream market, the EP no longer set its price equal to the marginal cost because that leads to prices below the level that maximize the industry profit[5]. In the appendix, we show that if competition in the downstream market is strong enough and the difference in the marginal cost of the producers is small enough, then the optimal two-part tariff implies $w_E = c_I$. For simplicity, we focus on this case in the remaining part of the paper[6].

At time t_3, the ED decides if it enters the market, after observing whether the EP entered the market at time t_2. The next proposition addresses the ED's optimal entry decision.

Proposition 16 ***[Entry decision of the ED at time t_3. No exclusive dealing]*** *The ED's entry decision at time t_3 depends on the market structure in the upstream market*

A) *If at time t_2 the EP does not enter the upstream market, then at time t_3 the ED does not enter the downstream market.*

B) *If at time t_2 the EP enters the upstream market, then at time t_3 the ED enters the downstream market if*

$$\pi_{ED}(c_I, c_I) \geq E_D$$

Proof. See Appendix. ■

When the EP enters the market, the stronger competition in the upstream market decreases the market power of the producers and makes entry in the downstream market more likely.

Next, we examine the EP's decision of whether to enter the upstream market at time t_2. To make this decision the EP takes into account what he expects the ED will do in the following period.

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[5] Since upstream firms sell homogeous imputs, the lower cost entrant producer takes all the market in case of entry. If the competition in the downstream market is strong enough the entrant producer optimal unit price is $w_E = c_I$ and the fixed fee is $A_E^f = 0$.

[6] The main results of the paper are still valid when the two-part tarif is instead given by condition (3.2).

Proposition 17 *[Entry Decision of the EP at time t_2. No exclusive dealing]*

i) If $\pi_{ED}(c_I, c_I) < E_D$, then the EP enters the market if

$$\pi_{EP} = \pi(c_E) - \pi(c_I) \geq E_P \tag{3.3}$$

ii) If $\pi_{ED}(c_I, c_I) \geq E_D$, then the EP enters the market if

$$\pi_{EP} = 2(c_I - c_E)D_i(c_I, c_I) \geq E_P \tag{3.4}$$

In case i), if condition (3.3) holds, the profits of the producers are $\pi_{IP} = 0$ and $\pi_{EP} = \pi(c_E) - \pi(c_I)$. The profits of the distributors are $\pi_{ED} = 0$ and $\pi_{ID} = \pi(c_I)$. If condition (3.3) does not hold, the profits are $\pi_{IP} = \pi(c_I)$ and $\pi_{EP} = \pi_{ID} = \pi_{ED} = 0$.

In case ii), if condition (3.4) holds, the profits of the producers are $\pi_{IP} = 0$ and $\pi_{EP} = (c_I - c_E)2D(c_I, c_I)$. The profits of the distributors are $\pi_{ID} = \pi_{ID}(c_I, c_I)$ and $\pi_{ED} = \pi_{ED}(c_I, c_I)$. If condition (3.4) does not hold, the profits are $\pi_{IP} = \pi(c_I)$ and $\pi_{EP} = \pi_{ID} = \pi_{ED} = 0$.

where $D_i(a, b)$ denotes the equilibrium demand of distributor firm i, $i = ID, ED$, when the distributors, ID and ED, face, respectively, the input prices a and b and maximize their profits

Proof. This proposition follows directly from propositions 14 and 15. ■

If $\pi_{ED}(c_I, c_I) < E_D$, then the ED does not enter the market even if the EP enters the market. This implies that the EP only enters the market if its profit, when he faces a monopoly in the downstream market, is higher than its entry costs. If, instead, $\pi_{ED}(c_I, c_I) \geq E_D$, then the ED enters the market if the EP also enters the market. Hence, the EP enters the market if the profit, when he faces competition in the downstream market, is higher than the entry cost. The EP is more likely to enter when there is also entry in the downstream market. In this case, the EP's profits are higher because the higher competition between the distributors leads to a higher quantity being sold for the same input price.

3.2.2 Exclusive Dealing

In this section, we consider the case where the incumbent firms sign an exclusive dealing at time t_1. Exclusive dealing agreements change the results of the previous section only when there is entry in the upstream market. Hence, in this section we only examine this case.

Proposition 18 *[**Price decisions at time** t_4. **Exclusive dealing**] If the incumbent firms sign an exclusive dealing agreement and the EP enters the upstream market at time t_2, then the producers' optimal prices depend on whether the ED enters the market at time t_3*

A) *If the ED does not enter the downstream market, then the EP does not sell the input and the IP's two-part tariff is*

$$w_I = c_I \text{ and } A_I = \pi(c_I)$$

B) *If the ED enters the downstream market, in equilibrium the IP sells to the ID, while the EP sells to the EP. The IP's two-part tariff is*

$$w_I = c_I \text{ and } A_E = \pi_{ID}(c_I, c_E)$$

The EP's two-part tariff is

$$w_E = c_E \text{ and } A_E = \pi_{ED}(c_I, c_E) - \pi_{ED}(c_I, c_I)$$

Proof. See appendix. ■

Notice that, despite having signed an exclusive dealing agreement, the IP still finds it optimal to charge the ID a unit price equal to its marginal cost. This is because it can capture the profits of the ID through the fixed fee. The lower cost EP sells only to the ED. It charges the ED a price equal to the marginal cost, $w_E = c_E$. Since the ED can alternatively buy the input from the IP at $w_I = c_I$, the fixed fee charged by the EP leaves the ED indifferent between buying from the two upstream firms.

At time t_3, the ED decides whether to enter the market after observing if the EP has entered the market at t_2.

Proposition 19 *[Entry decision of the ED at time t_3. Exclusive dealing] If the incumbent firms sign an exclusive dealing agreement and the EP enters the upstream market, then, at time t_3, the ED enters the downstream market if and only if*

$$\pi_{ED}(c_I, c_I) \geq E_d$$

Proof. This follows directly from proposition 17 ■

Notice that, assuming the EP has entered the market, propositions 15 and 17 imply that the decision of the ED of whether to enter the market is not affected by the existence of an exclusive dealing agreement between the incumbent firms. Therefore, the exclusive dealing agreement only affects the ED's entry decision through the impact it might have on the EP's decision to enter the upstream market.

Finally, at time t_2, the entrant producer realizes that if it enters the market, then he can only sell the input to the ED.

Proposition 20 *[Entry decision of the EP at time t_2. Exclusive dealing] If an exclusive dealing agreement is signed by the incumbent firms, then the EP enters the market if and only if the following two conditions hold*

$$\pi_{ED}(c_I, c_I) \geq E_D \tag{3.5}$$

$$\pi_{ED}(c_I, c_E) - \pi_{ED}(c_I, c_I) \geq E_U \tag{3.6}$$

If conditions (3.5) and (3.6) hold, the profits of the producers are $\pi_{IP} = 0$ and $\pi_{EP} = \pi_{ED}(c_I, c_E) - \pi_{ED}(c_I, c_I)$ and the profits of the distributors are $\pi_{ED} = \pi_{ID} = \pi_{ED}(c_I, c_I)$. If one of the conditions (3.5) and (3.6) does not hold, the profits are $\pi_{IP} = \pi_{ID}(c_I) - \pi_{ID}(c_I, c_I)$, $\pi_{ID} = \pi_{ID}(c_I, c_I)$ and $\pi_{EP} = \pi_{ED} = 0$.

Proof. This follows directly from proposition 17 and 18. ■

Condition (3.5) guarantees that the ED enters the market at time t_3 if the EP also enters the market at time t_2. Condition (3.6) guarantees that the EP enters the market if he expects that the ED also enters. In this case, the EP enters the market if the joint profit of the entrant firms less the profit obtained by the ED is higher than the entry cost of the EP.

Notice that the EP and the ED benefit from each other entering the market. It can happen that the sum of the profits of the entrant firms is higher than the sum of their entry costs and none of the entrant firms enters the market because one of them has profit lower than the entry costs. We consider next the case where the entrant firms are allowed to subsidize each other's entry. We assume that at time t_2 before EP decided to enter the market, each entrant firm can offer the other a payment in case it enters the market.

Proposition 21 *If the incumbent firms sign an exclusive dealing agreement and the entrant firms can subsidize each other's entry, then the entrant firms enter the market if and only if*

$$\pi_{ED}(c_I, c_E) \geq E_U + E_D$$

Proof. This follows directly from perfect information about entry costs. ■

3.2.3 Incumbent Firms Decision to Sign an Exclusive Dealing Contract

We now consider the incumbent firms' decision of whether to sign an exclusive dealing agreement at time t_1. The IP can offer an amount x to the ID if he agrees to sign an exclusive dealing contract. There are two different cases to consider under free competition. First, the EP enters the market but the ED does not enter the market. Second, both firms enter the market. In this case, the entry in the upstream market leads to further entry in the downstream market[7]. We show that while in the first case incumbent firms don't sign an exclusive dealing agreement, in the second case they might sign an exclusive dealing agreement if it prevents the entry. Hence,

[7]Notice that ED never enters in the market at time t_3 when EP does not enter the market at time t_2.

exclusive dealing agreements are only used when there is a threat of entry both in the upstream and in the downstream market.

The next proposition considers the case where there is no threat of entry in the downstream market.

Proposition 22 *[**No exclusive dealing**] If the following two conditions hold*

$$\begin{aligned} i)\ \pi_{ED} &= \pi_{ED}(c_I, c_I) < E_D \\ ii)\ \pi_{EP} &= \pi(c_E) - \pi(c_I) \geq E_P \end{aligned} \tag{3.7}$$

then the incumbent firms don't sign an exclusive dealing agreement. The EP enters the market and the ED does not enter the market.

Proof. Under the conditions of this proposition, we can apply proposition 15 to show that the ED does not enter the downstream market, while proposition 16 implies that the EP enters in the upstream market if no exclusive dealing agreement is signed. In this case, the profits of the incumbent firms are

$$\pi_{IP} = 0 \text{ and } \pi_{ID} = \pi(c_I)$$

If the incumbent firms sign an exclusive dealing agreement, then, by proposition 19, the EP does not enter the market at time t_2. In this case, the profit of the incumbent firms are

$$\pi_{IP} = \pi_{ID}(c_I) \text{ and } \pi_{ID} = 0$$

Notice that the joint profit of the incumbent firms is the same in both cases. The maxim amount x the IP is willing to offer to the ID if he signs an exclusive dealing agreement, $\pi(c_I)$, is equal to the increase in the ID's profit when there is entry in the upstream market. Here, we assume the tie-breaking rule that if $x = \pi(c_I)$, the ID decides not to accept the exclusive dealing agreement. This tie-breaking rule is the reasonable assumption here because we have also assumed the tie-braking rule that if the producers two-part tariffs provide the same surplus to the distributor, he opts for the lower cost producer. Had we not assume this tie-breaking rule, the lowest cost producer would always be willing to make an offer that is slightly better than the other producer. ■

We conclude that if there is no threat of entry in the downstream market, then the incumbent firms do not sign an exclusive dealing agreement. Intuitively, in order to induce the ID to sign an exclusive dealing agreement, the IP has to compensate him for the loss he suffers from not buying from a more efficient producer. However, since the loss suffered by the IP in case of entry is smaller than the gain of the ID, the incumbent firms don't sign an exclusive dealing agreement.

If renegotiation was allowed, the incumbent firms would sign an exclusive dealing contract in order to extract surplus from the EP. This contract would have a penalty of breaching it equal to the EP net profits. Thus, the entrant producer would always enter the market and would have zero profits. This is the result obtained by Aghion and Bolton (87) for the case with no uncertainty about the entrant costs.

We now show that if there is a threat of entry in both markets, then the incumbent firms may want to sign an exclusive dealing agreement.

Proposition 23 *[No exclusive dealing] The incumbent firms sign an exclusive dealing agreement that prevents the entry of the new firms if and only if the following conditions hold*

$$\begin{aligned}
\pi_{ED} &= \pi_{ED}(c_I, c_I) \geq E_D \\
\pi_{EP} &= 2(c_I - c_E)D(c_I, c_I) \geq E_P \\
\pi_{EP} &= \pi_{ED}(c_I, c_E) - \pi_{ED}(c_I, c_I) < E_P
\end{aligned}$$

Under these conditions, the IP offers the ID an amount $x = \pi_{ID}(c_I, c_I)$ to sign an exclusive dealing agreement.

Proof. By proposition 15 and 16 part ii) both entrant firms enter the market if the incumbent firms don't sign an exclusive dealing agreement. In this case, the profit of incumbent firms are

$$\pi_{IP} = 0 \text{ and } \pi_{ID} = \pi_{ID}(c_I, c_I)$$

If the incumbent firms sign an exclusive dealing agreement, then by proposition 18 and 19 both entrant firms stay out of the market. The profits of the incumbent firms profits are

$$\pi_{IP} = \pi(c_I) \text{ and } \pi_{ID} = 0$$

Since we must have that[8]

$$\pi(c_I) > \pi_{ID}(c_I, c_I)$$

the IP is willing to offer an amount $x = \pi_{ID}(c_I, c_I)$ to the ID if it signs an exclusive dealing contract. ■

Entry of a more efficient producer and the increasing competition in the upstream market, implies that upstream firms are not able to extract as much surplus from downstream firms. The main difference between propositions 21 and 22, is that, in the second case, increasing competition in the upstream market also leads to entry in the downstream market. This implies that entry of a more efficient producer no longer increases the joint profits of the incumbent firms. Now part of the bigger surplus created by the entry of a more efficient entrant producer is captured by the entrant distributor and by the consumers through the increasing competition in the downstream market.

Next, we look at one condition that guarantees that incumbent firms sign an exclusive dealing agreement for some level of the fixed costs.

Proposition 24 *There is a non-empty set of entry costs, E_P and E_D, that leads the incumbent firms to sign an exclusive dealing agreement if and only if the following condition holds*

$$\pi_{ED}(c_I, c_E) < \pi_{ED}(c_I, c_I) + 2(c_I - c_E)D(c_I, c_I) \tag{3.8}$$

Proof. See appendix ■

Proposition 23 means that if the joint profits of the entrant firms under exclusive dealing are

[8] Firms are assume to compete a la Cournot or compete in prices with differentiated and substitute goods.

lower than the sum of their profits under free market, then the three conditions of proposition 22 can be satisfied for some level of the entry costs. Hence, this condition guarantees that for some level of the entry costs, the incumbent firms want to sign an exclusive dealing agreement.

If we assume that the entrant firms can subsidize each other entry, then we can be more specific about the conditions that lead to exclusive dealing.

Proposition 25 *If the entrant firms are allowed to subsidize each other entry, then the incumbent firms sign an exclusive dealing agreement if and only if the following two conditions hold*

$$\pi_{ED}(c_I, c_E) < E_U + E_D \leq \pi_{ED}(c_I, c_I) + 2(c_I - c_E)D(c_I, c_I) \tag{3.9}$$

$$E_D \leq \pi_{ED}(c_I, c_I) + 2(c_I - c_E)D(c_I, c_I) - \pi(c_E) + \pi(c_I) \tag{3.10}$$

Proof. See appendix. ■

Condition (3.9) means that the sum of the entry costs is higher than the sum of their profits under exclusive dealing but lower than the joint profits obtained under free market. Condition (3.10) guarantees that ED also enters the market under no exclusive dealing when firms can subsidize each other entry.

We have seen that the necessary condition for exclusive dealing is that the sum of the entrant firms' profits are higher under free market than under exclusive dealing. However, while this is in general the case, this is not always true and depends on the type of competition in the downstream market. There are two factors that have opposite effects on the joint profit of the entrant firms when we move from exclusive dealing to free market:

i) In a free market, the EP can also sell to the ID. This tends to increase the joint profit of the entrant firms under free market.

ii) Under exclusive dealing the EP charges the ED a unit price equal to its marginal cost. However, under free market the EP sells to both firms and charges them a price higher than its marginal cost, $w_E > c_E$. Therefore, the ED faces an input price higher than the marginal cost of the EP. This has a direct negative effect on the joint profit of the entrant firms. Furthermore, if the downstream firms compete in quantities this also has a negative strategic effect, since quantities are strategic substitutes. Hence, in this case, the overall impact of charging $w > c_E$ is negative. However, if downstream firms compete in prices, the strategic effect is positive. In this case, this overall effect can be either positive or negative.

Due to this trade-off, unless we specify the demand function and the type of competition in the downstream market, we cannot determine exactly when the conditions of propositions 23 and 24 hold. For this reason, in the next section we consider the case where firms compete in quantities in the downstream market. Then, in section 4, we look at a standard model of price competition.

3.3 Quantity Competition in the Downstream Market

In this section, we consider that firms compete in quantities in the downstream market. To simplify, we assume that in the downstream market the inverse demand function is linear and given by

$$\begin{aligned} p &= 1 - Q \\ Q &= q_I + q_E \end{aligned}$$

where q_I and q_E are, respectively, the quantities offered in the downstream market by the ID and the ED. We first consider the case where the incumbent firms do not sign an exclusive dealing agreement. Then, we examine the case of exclusive dealing.

No Exclusive Dealing Consider first that incumbent firms have not signed an exclusive dealing agreement and both entrant firms enter the market. Then, assuming the difference in the marginal costs of the upstream firms is small enough, proposition 16 implies that the EP charges an input price equal to the marginal cost of the IP, $w_E = c_I$[9].

The quantity sold by each downstream firm is

[9] We verify this condition in the end of this section.

$$q = \frac{1 - c_I}{3}$$

The final price of the good is

$$p = \frac{1 + 2c_I}{3}$$

The EP takes all the market and obtains a profit of

$$\pi_{EP} = \frac{2}{3}(1 - c_I)(c_I - c_E)$$

The profit of each downstream firms is

$$\pi_{ID} = \pi_{ED} = \frac{1}{9}(1 - c_I)^2$$

Exclusive Dealing Consider now that the incumbent firms sign an exclusive dealing agreement and both entrant firms enter the market. Proposition 17 part B) implies that IP sells to ID and EP sells to ED. It also implies that both producers charge an input price equal to their marginal cost, $w_I = c_I$ and $w_E = c_E$.

The quantity sold by the incumbent firms and the entrant firms is respectively

$$q_I = \frac{1 - 2c_I + c_E}{3}, \qquad q_E = \frac{1 - 2c_E + c_I}{3}$$

The joint profit of the incumbent firms and of the entrant firms are

$$\pi_I^{ex} = \frac{(1 - 2c_I + c_E)^2}{9}, \quad \pi_E^{ex} = \frac{(1 - 2c_E + c_I)^2}{9}$$

Proposition 23 says that the set of entry costs, E_U and E_D, that leads to exclusive dealing is not empty if

$$\pi_E^{ex} < \pi_{EP} + \pi_{ED}$$

Which is equivalent to

$$\frac{(1-2c_E+c_I)^2}{9} < \frac{2}{3}(1-c_I)(c_I-c_E) + \frac{1}{9}(1-c_I)^2$$

This simplifies to

$$c_E < c_I < \frac{2}{3}c_E + \frac{1}{3} \tag{3.11}$$

Condition (3.11) also guarantees that the IP's optimal input price is $w_E = c_I$, as we have assumed at the beginning of this section.

We conclude that if downstream firms compete in quantities and the demand function is linear, then exclusive dealing might be used to deter the entry of new firms. In order to have exclusive dealing the EP needs to be more efficient than the IP, but not much more efficient. Notice that the joint profit of the incumbent firms when they prevent the entry of the entrant firms is higher than when there is no exclusive dealing and the entrant firms enter the market

$$\pi_I^{ex} = \pi_{ID}(c_I) = \frac{1}{4}(1-c_I)^2 > \pi_{ID}(c_I, c_I) = \frac{1}{9}(1-c_I)^2$$

3.4 Price Competition in the Downstream Market

In this section, we consider that downstream firms compete in prices. We assume a linear city model where consumers have unit demands and are uniformly distributed along a segment of length one. Consumers are assumed to have linear transportation costs of t per unit of length. The ID is located at $x = 0$, while the ED, in case it enters the market, is located t $x = 1$. A consumer located at x, derives the following surplus of buying from the ID

$$V - p_I - tx$$

Where V is the consumer surplus gross of price and transportation costs.

We assume that there is actual competition in the downstream market in case of entry. For that we need to impose the following condition

$$t < v - c_I$$

No Exclusive Dealing To verify if condition (3.8) holds (the necessary condition for exclusive dealing), we start by computing the joint profits of the entrant firms in case of no exclusive dealing. Notice that, in this case, the optimal two-part tariff part charged by the EP is given by

$$w_E = c_I \text{ and } A_E = 0$$

The EP has demand equal to 1 and its profit is

$$\pi_{EP} = c_I - c_E \tag{3.12}$$

The profit of the entrant distributor is given by

$$\pi_{ED} = \frac{t}{2} \tag{3.13}$$

Hence, the joint profits of the entrant firms with no exclusive dealing is

$$c_I - c_E + \frac{t}{2} \tag{3.14}$$

Exclusive Dealing Consider now the case where incumbent firms sign an exclusive dealing agreement. Proposition 17 implies that in equilibrium the IP sells to the ID while the EP sells to the ED. Producers charge the following marginal prices

$$w_I = c_I \text{ and } w_E = c_E$$

The joint profit of the entrant firms with exclusive dealing is

$$\begin{aligned}
\pi_{ED}(c_I, c_E) &= \frac{t}{2} + \frac{c_I - c_E}{3} + \frac{(c_I - c_E)^2}{18t} \quad \text{if } t \geq \frac{c_I - c_E}{3} \\
\pi_{ED}(c_I, c_E) &= c_I - c_E - t \quad \text{if } t < \frac{c_I - c_E}{3}
\end{aligned} \tag{3.15}$$

Looking at expressions (3.14) and (3.15) we conclude that, unless t=0, the joint profit of the entrant firms under exclusive dealing is always lower than their joint profit under free market. Therefore, if there is product differentiation in the downstream market, then there is always a non-empty set of entry costs, E_U and E_D, such that the incumbent firms sign an exclusive dealing agreement to prevent the entry of the more efficient rival producer.

Notice that, as the degree of product differentiation increases, the difference between the joint profit of the entrant firms under free market and their joint profit under exclusive dealing also increases. Hence, assuming entrant firms are allowed to subsidize each other entry, proposition 24 implies that the interval of the sum of the entry costs that guarantees the use of exclusive dealing becomes bigger. We can also easily check that, as t increases, condition (3.10) also becomes more easily satisfied. Therefore, we can conclude that exclusive dealing becomes more likely as the degree of product differentiation increases.

However, if the degree of product differentiation is high enough so that in case of entry the downstream firms do not compete against each other and become local monopolists, then the incumbent firms do not use an exclusive dealing agreement. In this case, the reason they don't sign an exclusive dealing agreement is not because it is not effective, but instead because the entry of the new firms no longer decreases the joint profits of the incumbent firms. This is because the entrant distributor is no longer competing with the incumbent distributor.

3.5 Conclusion

In this paper, we argue that the entry in upstream market of a more efficient producer, by increasing the competition among upstream firms and reducing their market power, might lead to further entry in the downstream market. In this case, the entry of a more efficient producer no longer necessarily increases the joint profits of the incumbent firms. This is because part of the bigger surplus created by the more efficient upstream firm is now appropriated by the entrant distributor and by the consumers, due to the increase in competition in the downstream market. Therefore, the incumbent firms may sign an exclusive dealing agreement to prevent the entry of the new firms.

We provide general conditions for the incumbent firms to use exclusive dealing agreements as a mechanism to prevent the entry of rival firms. Then, we looked at two different types of competition in the downstream market. When downstream firms compete in quantities, incumbent firms may sign exclusive dealing agreements if the cost advantage of the entrant producer is neither very small nor very big. If, instead, firms compete in prices in a linear city model, incumbent firms may want to use exclusive dealing agreements whatever the degree of product differentiation. Furthermore, exclusion becomes more likely as the product differentiation increases.

3.6 Appendix

Proof of proposition 14 We prove proposition 14 for the case where downstream firms compete in prices. The proof for the case where firms compete in quantities is similar.

A i) See Tirole (1988) page 176.

ii) Consider first the non-integrated structure, with the incumbent producer and two independent downstream firms. Retailer i maximizes

$$\max_{p_i} \ (p_i - w_I) * q_i(p_i, p_j)$$

where p_i and p_j denote the prices of firm i and firm j and $q_i(p_i, p_j)$ denotes the demand of firm i given prices p_i and p_j, with $i = ID, ED$, $j = ID, ED$, and $i \neq j$.

The first order condition of retailer i, $i = ID, ED$, is

$$q_i(p_i, p_j) + \ (p_i - w_I) * \frac{\partial q_i(p_i, p_j)}{\partial p_i} = 0 \tag{3.16}$$

Consider now the profit maximization problem of the integrated structure

$$\max_{p_i,\, p_j}(p_i - c_I) * q_i(p_i, p_j) + (p_j - c_I) * q_j(p_i, p_j)$$

The first order condition in order to p_i, $i = ID, ED$, is

$$q_i(p_i, p_j) + (p_i - c_I) * \frac{\partial q_i(p_i, p_j)}{\partial p_i} + (p_j - c_I) * \frac{\partial q_j(p_i, p_j)}{\partial p_i} = 0 \tag{3.17}$$

We assume that the good are substitutes, $\frac{\partial q_j(p_i,p_j)}{\partial p_i} < 0$. If, in the case of the non-integrated structure, the producer chooses $w = c$. Then, the prices that satisfy the two conditions given by (3.16), one for each firm, are lower than the prices that satisfy the two equations given by condition (3.17), one for the price of each good. Notice that the prices given by condition (3.16) increase with w. Hence, to make the downstream firms choose the prices that maximize the industry profit, we need $w > c$. From conditions (3.16) and (3.17) we obtain that the optimal input price is

$$w_I^* = c - \frac{(p^* - c_I) * \frac{\partial q_j(p^*,p^*)}{\partial p_i}}{\frac{\partial q_i(p^*,p^*)}{\partial p_i}} \tag{3.18}$$

Where p^* is the solution of the system of two equation given by condition (3.17)

B) i) Given the two-part tariff charged by the entrant firm, $w_E = c_E$ and $A_E = \pi(c_I) - \pi(c_E)$, we have that $\pi_{ID} = \pi(c_I)$. The IP cannot offer the ID any profitable two-part tariff that increases the ID payoff. Assume that IP offers the following two-part tariff $w_I = c_I$ and $A_I = 0$. Given this, the EP also does not want to deviate. It chooses the fixed fee such that the ID's payoff is $\pi(c_I)$. Then, following Tirole (1998) he sets $w_E = c_E$ to maximize the industry profit.

ii) We can determine the two equations given by a condition equivalent to condition (3.17) for the case of the entry firm, where c_I is replaced by c_E. These equations determine the input price that leads the competing downstream firms to choose prices that maximize the industry profit. Consider first that the solution of these equations given by condition (3.17) is $w_E^* < c_I$.

Then, in equilibrium, the EP chooses the marginal price w_E^* and each downstream firm obtains a gross profit of $\pi_{ED}(w_E^*, w_E^*)$. The equilibrium fixed fee, given by condition (3.2), leaves each downstream firm indifferent between buying from the EP and the IP. The equilibrium profit of each downstream firm is $\pi_{ED}(w_E^*, c_I)$. Assume that the IP offers a two-part tariff with $w_I = c_I$ and $A_I = 0$. The downstream firms don't want to deviate and buy from the IP, because they obtain the same profit of $\pi_{ED}(w_E^*, c_I)$. IP does not want to deviate because the only two-part-tariffs that downstream firms would accept would leave him with negative profits. Consider now that $w_E^* > c_I$. This is not an equilibrium since the IP can offer $w_I = c_I$ and $A_I = 0$ and the downstream firms would prefer to buy from the IP. Hence, we have that $w_E^* = c_I$ and $A_E = 0$. Notice if we allow the EP to sign an exclusive dealing with the downstream firms, then we could sustain $w_E^* > c_I$ in equilibrium. However, this would not change the qualitative results obtained in this paper.

Proof of proposition 15 **A)** Part A of the proposition follows directly from proposition 14. If the IP is a monopolist in the upstream market, then it charges each distributor a fixed fee equal to the distributor's gross profit. Hence, distributors have zero net profits and thus the ED does not enter the market.

B) If the EP enters in the market, by proposition 14, $\pi_{ED} = \pi_{ED}(c_I, c_I)$. Hence ED enters the market if $\pi_{ED}(c_I, c_I) \geq E_D$.

Proof of proposition 17 **A)** Given the exclusive dealing agreement signed between the IP and the ID, the ID cannot buy from the EP. This implies that IP can charge a fixed fee that leaves the ID indifferent between buying and not buying. If the ED does not enter, the ID is a monopolist distributor. Following Tirole (1988) page 176, the two-part tariff that maximizes the IP's profit is $w_I = c_I$ and $A_I = \pi(c_I)$.

B) If the ED enters the market, then there is a duopoly in the downstream market. Given the exclusive dealing agreement, IP chooses a two-part tariff that leaves the ID indifferent between buying and not buying. Hence, in this equilibrium we must have that $A_I = \pi_{ID}(w_I, w_E)$. Once again, following Tirole (1988) page 176, the two part tariff that maximizes the IP profit is $w_I = c_I$ and $A_I = \pi_{ID}(c_I, w_E)$. The ED can buy from both firms. Following Tirole (1988) page 176, the EP chooses $w_E = c_E$. We can also see that if $A_E = \pi_{ED}(c_I, c_E) - \pi_{ED}(c_I, c_I)$, then the ED would be indifferent between accepting this offer and buying from the IP at $w_I = c_I$ and $A_I = 0$. Notice that each producer's two-part tariff is a best response to the other producer's two-part tariff.

Proof of proposition 24 By proposition 20, the second inequality in condition (3.9) implies that if there is an exclusive dealing agreement, the entrant firms do not enter the market. Condition (3.10) implies that under free market the ED enters the market if the EP also enters the market, since its profit plus what the EP is willing to subsidize ED for entering the market is higher than ED's entry costs. The first inequality in expression (3.9) together with condition (3.10) guarantees that they enter the market with no exclusive dealing.

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