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AN ANALYSIS OF REVOLVING CREDIT AGREEMENTS

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ABSTRACT

This paper examines the pricing of intermediate term line commitments, often called revolving credit agreements. Their characteristics, covenants, and compensating balance features are discussed. The fixed portion of the line is described as a dual phased option; it behaves as a put or a call depending on whether bank borrowing is undertaken. Two valuation models, based on the use of the borrowing, are derived for infinitely-lived line commitments. The pattern of borrowing by the firm is shown to principally depend on the relative size of the fixed and variable costs of the line.

I. Introduction

Capital markets provide the bulk of long and intermediate-term funds for corporate investment. Often, however, firms also use commercial banks as a source of funds. The banking system provides economies of scale in terms of transactions costs to both large and small corporations. Small companies cannot always make efficient use of capital markets due to their relatively small demands or the cost of informing the market of these demands. Larger companies whose demands are for short periods of time, or which have a possibility of an impending change in financing, often can't justify the cost of a securities issue in the securities market.

Line commitments are one of the most widely used of the various arrangements through which banks supply funds to corporations. The demand for line commitments can be broadly broken into two separate, but sometimes overlapping, categories. The first is the short-term line commitment which is used to meet the need for funds due to flucuations in cash balances or working capital. The amount of line borrowing in this case should vary substantially during the existence of the line. The second category, the demand for intermediate-term line commitments is viewed as a way of changing the capital structure of the firm. Changes in borrowing should be rare over the life of the line agreement. Intermediate-term line commitments, often called revolving credit agreements, have experienced a dramatic increase in popularity over the last decade.

The purpose of this paper is to study revolving credit agreements. Section II examines the general terms of line commitments in detail. Section III characterizes line commitments as an unusual form of option. This section employs two simplifying assumptions, a flat term structure and an infinitely-lived line commitment, in order to solve two models of line commitment value. These models make it possible to examine two important questions: 1) what are the trade-offs between the fixed fee and the markup over prime in pricing the line, and 2) how do changes in the relevant variables affect the pattern of borrowing by the firm? Section IV concludes the study by exploring reasons for the existence of line commitments.

II. The Revolving Credit Agreement

A. Description of Line Commitments

The terms of line commitments vary from bank to bank and customer to customer. In this section a general model which incorporates the major elements of the terms of a line commitment is presented. Most terms which might be negotiated can be put into this model with minor re-definition.

A line commitment is a promise from a bank to loan up to \$L until the expiration date T. Between now and date T the firm, at its descretion, can borrow all or part of the line at $p(t)+\delta$ percent per period (prime plus a given risk premium), where the risk premium may be a function of the prime rate.[1] The gross dollar amount of the line being used at any time will be denoted by B and represents the face value of current borrowing. At date T any outstanding borrowing must be repaid.

In addition to paying interest on the amount of line borrowing, the firm often agrees to keep a certain level of deposits with the bank, called compensating balances. The required compensating balance is a proportion of both the size of the line commitment, L, and the amount of the line, B, currently being used or "drawn down." To simplify the analysis, only one proportion, α, is used so that required balances are $\alpha(L+B)$. It is assumed that a penalty is assessed whenever the actual compensating balances are below those stipulated by the terms of the contract. This penalty is of the form of ρ percent for every dollar the actual compensating balances, CB, are below the required balances [i.e., $\rho(\alpha(L+B)-CB)$ dollars]. Finally, a line fee of μ percent is paid on the amount of unused line borrowing.

The nature of compensating balances can differ greatly among loans. Often banks require that an average balance be kept over the course of a month. Consequently, the account can be used as transaction balances by the firm. In addition, these same compensating balances are often used to support other bank services such as lockboxes. On the other hand, some banks demand that net balances, net of reserve requirements, must always be kept above a certain level. It is assumed here that the firm is required to always keep at least $\alpha(L+B)$ dollars in compensating balances which do not also support any other bank services and earn no interest. Obviously when borrowing a rationally managed firm will not keep more balances than required since it could repay part of the outstanding loan with the excess.[2]

When the firm borrows \$B of the line, after adjusting for the compensating balances, net borrowing is only $B-CB \equiv N$ dollars. The firm may actually have negative net borrowing if no line borrowing is undertaken but required compensating balances are kept.

The above description implies that the per-period payment of the firm due to the line, C, is:

$$C=\mu(L-B)+\rho[\alpha(L+B)-CB]+(p+\delta)B$$
$$=(\mu+\rho\alpha)L+[(p+\delta)-\mu+\rho\alpha]B-\rho CB. \qquad (1)$$

B. <u>A Discussion of Compensating Balances</u>

When compensating balances are actually being kept, the effective amount borrowed from the firm's viewpoint is gross borrowings minus compensating balances, \$N. However, this may not be the case from the bank's viewpoint because compensating balances do not necessarily go to the bank in case of default. Often they are considered as another asset of the firm and in bankruptcy are treated like all other assets. If there are other claimants of greater or equal seniority, then the bank could lose the gross borrowings, \$B, instead of just \$N. The bank should set the risk premium to reflect this possibility.

Compensating balances also affect the value of the bank's claim by changing the risk of the firm. The portion of the loan kept as compensating balances is essentially riskless. riskless. If instead this amount is invested in riskly assets, such as increasing the scale of the firm, then the risk of the firm is greater which decreases the value of the bank's claim. Therefore, even when compensating balances belong solely to the bank in case of default, the bank should consider the maximum possible borrowing, \$L, to be at risk since the firm can always elect to hold no compensating balances and pay the penalty.

What is a rational penalty from the bank's viewpoint? Suppose the firm decides to keep no compensating balances when not borrowing and pay the penalty. The bank will be indifferent between this and the firm borrowing $\$B=B'$ (where $B'=\alpha B'+\alpha L$, the amount at which the firm borrows just enough to cover required compensating balances) when $\rho=\frac{p+\delta-\mu}{(1-\alpha)}$. Setting the penalty at this level, coupled with the line being priced as having the maximum possible borrowing at risk, allows the payments of the firm to be written as:

$$C=\left[\mu+\frac{\alpha}{(1-\alpha)}(p+\delta-\mu)\right] L + \left[1+\frac{\alpha}{(1-\alpha)}\right][p+\delta-\mu]B$$

$$=(a_0+a_1 p)L+(b_0+b_1 p)B \qquad (1')$$

where

$$a_0 = \left[\frac{(\mu - 2\alpha\mu + \alpha\delta)}{(1-\alpha)}\right]$$

$$a_1 = \left[\frac{\alpha}{(1-\alpha)}\right]$$

$$b_0 = \left[\frac{1}{(1-\alpha)}\right](\delta - \mu)$$

$$b_1 = \frac{1}{(1-\alpha)}$$

At the derived level of the penalty, the firm is not indifferent to whether it holds compensating balances or not when borrowing. For the fixed risk premium its shareholders will never be worse off by keeping no compensating balances and paying the penalty as is shown in the next section. Therefore, it is assumed that net and gross borrowing are the same ($\$N=B$) without any loss. If compensating balances were allowed to support other bank services this would not be true.

C. Fixed Verses Variable Costs

The firm's payment of Equation (1') demonstrates that the bank has two degrees of freedom in setting the terms of the line: a fixed and a variable element.[3] The firm should approach the use of the line as it would any factor of production. It ignores the sunk cost [i.e., the fixed cost which are $(a_0+a_1p)L$] and makes decisions based on the marginal (i.e., variable) costs of borrowing given the line.

The firm borrows either all of the line, $\$B=L$, or none of the line, $\$B=0$, at any point in time; there are no so-called "partial markdowns."[4] Whenever the variable cost is less than a corresponding market rate on an "equivalent loan" the firm is getting an underpriced loan. Obviously the shareholders favor taking as much of this loan as possible. Conversely, when the market rate is less than the variable cost the firm is paying too much for the bank loan and, hence, won't use the line. This explains why, when borrowing, the firm does not keep compensating balances at the penalty discussed in the last section. When net is less than gross borrowing, $N<B, for a fixed risk premium, the firm is not taking the maximum amount of the underpriced loan.[5]

D. Line Commitment Covenants

Bank borrowing is similar to privately placed debt; special terms can be added to reflect the demands of the borrower.[6] Despite the individual nature of each line commitment, most employ standard positive and negative covenants. The positive covenants mostly relate to the reporting requirements of the firm. Negative covenants set limits on dividend payments, issue of other debt, disposition of assets, mergers, ect.[7]

Revolving credit agreements also contain a "materially adverse change" clause which states that if there is a significant change in the firm's fortunes, then the bank is relieved of the obligation of lending any additional money to the firm. Unfortunately, the effect of this clause is difficult to determine. To the author's knowledge, it has never been used by a bank to cancel a revolving credit agreement. Nevertheless, it does exist and can be used as leverage by the bank to force renegotiation if necessary. A rationale for this covenant is provided in the next section.

III. Valuing Revolving Credit Agreements Using the Black-Scholes Methodology

A. Option Characteristics of Revolving Credit Agreements

When the firm is borrowing on the line it has a form of callable debt. When the firm is not borrowing it is paying for an option to put a security of the firm, callable debt, on the bank. The point at which the firm switches from borrowing to not borrowing defines a boundary. Once exercise occurs at the boundary, it is not the end of the option phase of the line. Rather, one form of the option is traded for the other; one or the other is "alive" until the expiration of the line commitment. The call, just like a call option on any debt, provides the ability to deprive the bondholders of any large gains. The put gives the firm the ability to sell callable debt and pay only straight debt rates for it marginally. By putting debt on the bank and recieving \$L for it, the firm forces the bank to share in any declines in firm value. The fixed payment is buying this option, in either form, for the firm.

To explore the properties of the boundary, the following notation is defined:

$C^{I} = (a_0 + a_1 p + b_0 + b_1 p)L$, fixed plus variable coupon payment.

$C^{II} = (a_0 + a_1 p)L$, fixed coupon payment.

$D^{I}(V,\tau) \equiv$ the market value of a τ period bond with coupon C^{I} and face value \$L.

$D^{II}(V,\tau) \equiv$ the market value of a τ period bond with coupon C^{II} and zero face value.

$DL(V,\tau) \equiv$ the market value of a τ period bond with a coupon of $(b_0 + b_1 p)L$ and face value \$L, given that the firm is also short D^{II}, which is of equal seniority. $(D^{I}=DL+D^{II})$.

$F^{I} \equiv$ the market value of the line commitment when the firm is borrowing on the line.

F^{II} ≡ the market value of the line commitment when the firm is not borrowing.

O^{I} ≡ the derived value of the implied call option when the firm is utilizing the line (i.e., $O^{I} = D^{I} - F^{I}$).

O^{II} ≡ the derived value of the implied put option when the firm is not borrowing (i.e., $O^{II} = D^{II} - F^{II}$).

The line can be characterized as a portfolio of options and debt. When the firm is borrowing on the line, the bank has a portfolio, F^{I}, comprised of 1) a long position in straight debt, DL, 2) a long position in the fixed cost, D^{II}, and 3) a short position in an option, O^{I}, to call the straight debt. The firm has the opposite position in each security. When no borrowing is done by the firm, the bank has a portfolio, F^{II}, with 1) a long position in the fixed cost, and 2) a short position in an option, O^{II}, to put the straight debt. The long position in the fixed cost has the same seniority as the straight debt; its size affects the value of the straight debt in an adverse fashion, as shown by Black and Cox (1976).

There is no contractually set call price in the terms of the line commitment. However, the firm can "call" the debt at any time by repaying the amount of the loan, \$L. If the firm is acting optimally, it will use the line when the market value of DL, the bank borrowing at the variable rate, is less than \$L and not use the line when it is greater. The boundary is the position of the firm's indifference to borrowing on the line or not. The firm is only concerned with the marginal cost of borrowing so the loan is not called when the total value of the line is equal to \$L. The option portion of the line commitment is a fixed cost, so it is only the value of the straight bank debt which matters (i.e., is $b_0 + b_1 p$ greater than the rate on an "equivalent loan"?).

Stated another way, the boundary is defined to be that position where the market value of the line commitment without borrowing, F^{II}, is equal to the market value of the line commitment with borrowing, F^{I}, minus \$L, the amount borrowed.

The variables which determine the use of line borrowing are those which affect the market value of the straight debt. It is well known from standard options analysis, Black and Scholes (1973), that as firm value increases, everything else equal, so does the market value of risky debt. Therefore, for large enough firm values the firm should move from using the line to alternative forms of financing (i.e., as $V \to \overline{V}$, $DL \to \$L$; for $V > \overline{V}$, $DL > \$L$ so the firm will no longer use the line). In addition, Merton (1974) has shown for discount bonds that as the riskless rate of interest increases, the risk premium on debt should decrease. Therefore, for large enough levels of the prime rate, the firm should cease using the line borrowing (i.e., as $p \to p$, $DL \to \$L$; for $p > p$, $DL > \$L$). Figure 1 shows these relationships. Line borrowing is being used in Region I and not used in Region II.

Two additional factors which affect the position of the boundary are not shown explicitly in Figure 1: the time to maturity, τ, and the magnitude of other claims against the firm which are of equal or greater seniority. In this case, the other claim is the fixed cost D^{II}. Unfortunately, these two factors are interrelated and it is not possible to say how they affect the boundary. This issue will be discussed in more detail in the section on aging of the line commitment.

The line commitment when borrowing is similar to callable debt,

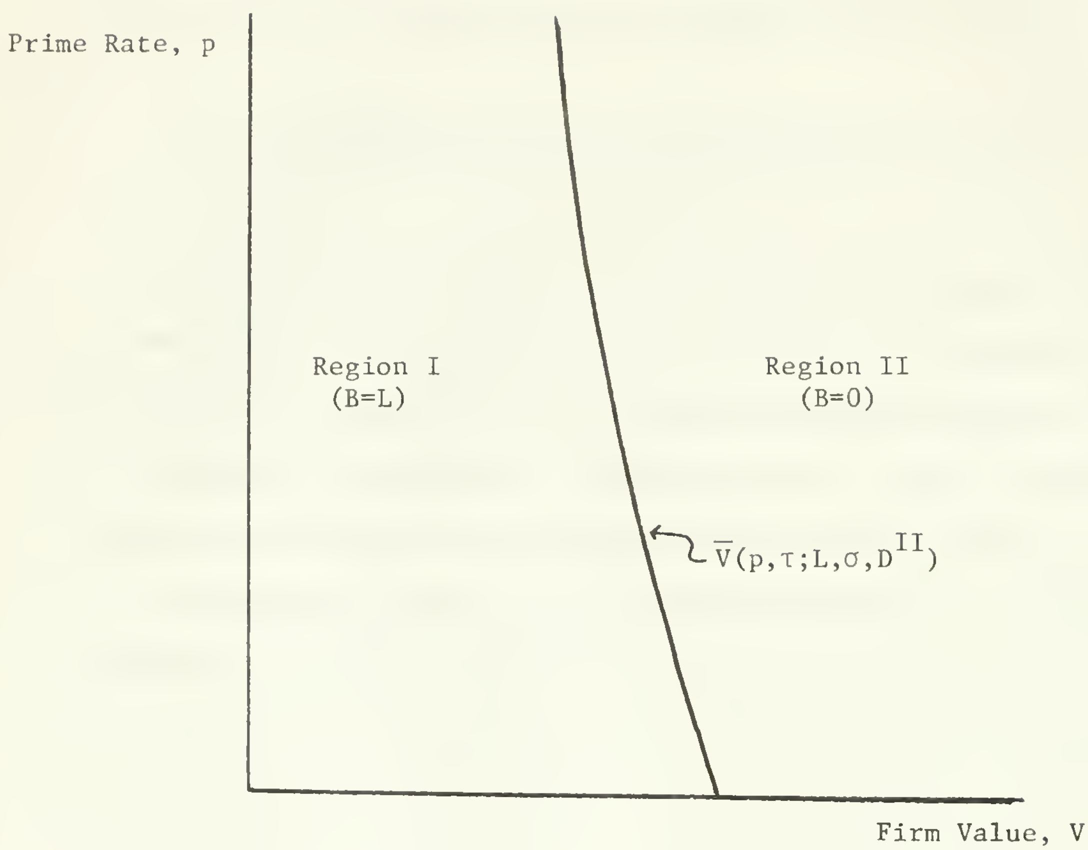

Figure 1- The line utilization boundary as a function of firm value and the prime rate. In Region I the firm is borrowing the maximum amount, \$L. In Region II the loan is "overpriced" so no line borrowing is undertaken.

although there are differences. First, the line commitment involves a variable interest rate loan instead of a fixed coupon rate. Second, unlike the call option implicit in callable bonds, the call associated with a fully utilized line changes into a put upon exercise. Third, in general no comparision can be made between the respective values of the line and callable bonds. The exercise price, $K(\tau)$, for callable bonds is given in the contract. The exercise price for line borrowing is a choice variable of the firm, determined by equity value maximization, and was shown to be $L. Even if callable bonds and the fully utilized line have the same exercise price, $K(\tau)=\$L$, the same coupon, $C^{CB}=C^{I}$, and the same maturity, no comparision can be made since exercise of the implied line call depends on the fixed cost, D^{II}. Given that the exercise points can't be related, in general, it is impossible to compare their values even over their common range of firm values. The value of each depends on the value of the implicit call option which in turn depends on the exercise point.

B. Development of Valuation Model

To derive an expression for the value of the line commitment the following general market and behavioral assumptions are needed:

A1 - Investor utility is an increasing function of wealth.

A2 - Firm management maximizes the value of equity.

A3 - There are no transactions costs on capital market transactions.

A4 - Capital structure irrelevance holds and will be achieved by the assumption of no taxes.

A5 - There are no restrictions on borrowing or short selling.

The assumptions necessary to obtain explicit valuation are:

A6 - The value of the firm, V, follows an Ito process with a constant variance rate (i.e., $dV=(\gamma V-C)dt+\sigma dz$, where C is the instantaneous net payments of the firm).

A7 - Trading in the capital markets takes place continuously.

A8 - The firm issues only two types of claims against its assets: 1) equity and 2) the bank line commitment. No continuous payments are made to equity.

A9 - The riskless rate is equal to the prime rate, p=r.

A10 - The instantaneous riskless rate, r, is a constant. This implies that the payments of the firm of Equation (1') can be written as $C=\Phi L+(r+\Delta)B$, where Φ is the fixed cost and Δ is the risk premium on borrowing.

A11 - At any firm value, the firm always invests in the same homogeneous set of assets.

The above assumptions imply that the value of the bank line commitment, $F(V,\tau)$, must satisfy the partial differential equations:

In Region I (where B=L),

$$\frac{1}{2}\sigma^2V^2F^I_{VV}+(rV-C^I)F^I_V-rF^I+F^I_\tau+C^I=0 \tag{2.1}$$

In Region II (where B=0),

$$\frac{1}{2}\sigma^2V^2F^{II}_{VV}+(rV-C^{II})F^{II}_V-rF^{II}+F^{II}_\tau+C^{II}=0 \tag{2.2}$$

where, from A10, the payments of the firm are

$$C^I=(\phi+r+\Delta)L$$

$$C^{II}=\phi L$$

The equality of the prime rate and the riskless rate is probably the most objectionable of the assumptions. Usually the prime rate is at least 100 basis points above the Treasury bill rate, the rate typically associated with the riskless rate. However, this spread seems to vary. The causes for this spread are not clear but probably reflect the existence of transactions costs, the absence of state taxes on Treasury bills, and the risk associated with lending to even the best of the bank's customers.

The prime rate is also as a rule less variable than the Treasury bill rate. Banks do not seem to be willing to change the prime rate too often for reasons which will not be pursued here. However, assumption A10 is not critical to the analysis. It is made because it is not clear that any replacement would be better or add any insights.

Assuming a constant riskless rate is also noticeably incorrect, but reasonably innocent. Fortunately, changes in firm value are probably the major factor in determining the pattern of use of revolving credit agreements. Since most line commitments have variable rate loans, changes in term structure won't have the effect they do on many securities, especially fixed coupon callable debt. The loss from using assumption A11 is in not capturing the effect of a changing riskless rate on the appropriate risk premium.[8]

In examining the properties of line commitments, two polar models, based on the use of the line borrowing, are developed. The gain from examining two cases is in understanding the rationale for the covenants and restrictions on revolving credit agreements. The definition of the models uses the following accounting identity which holds at every period in time:

New Investment + Payouts = Net New Financing + Earnings.

Earnings depend on past investment and effect the line only through their effect on firm value. Payouts always include payments to the bank, but can also include payments to shareholders. Since there are no taxes, dividends and share repurchase are equivalent and collectively represent the dividend policy of the firm. Line borrowing represents net new financing.

C. Model 1-Fixed Firm Payout Policy

The Fixed Firm Payout Policy (FFPP) Model assumes that the firm, in addition to the required payments to the line, is limited by covenant to a set level of dividends to shareholders. To simplify the analysis, the contracted level is taken to be zero dividends. Based on the firm's accounting identity, all net new financing is used for new investment. The value of equity as a function of firm value under the FFPP model is shown in Figure 2.[9]

As firm value increases, the firm moves from borrowing in Region I to not borrowing in Region II, at firm value $\overline{V}$. The firm contracts its assets -negative new investment- to repay the loan; the firm sells \$L, the loan amount, of its assets to repay the bank. Conversely in moving from Region II to Region I, at the firm value $V^{\dagger}$, the firm will expand its assets -new investment- by the amount of the line borrowing. Together $\overline{V}$ and $V^{\dagger}$ define the boundary. Since they move in lockstep (i.e., $\overline{V}=V^{\dagger}+L$), the discussion of Section IIIA is unchanged.

Capital structure irrelevance states that $f^i=V-F^i$, where f^i is the value of the firm's equity. Rewriting Equations (2) in terms of equity value gives:

$$\frac{1}{2}\sigma^2V^2f^i_{VV}+(rV-C^i)f^i_V-rf^i+f^i_\tau=0. \qquad (2')$$

The boundary conditions for equity in Region I are:

$$f^I(0,\tau)=0 \quad (3.a) \qquad\qquad f^I(V,0)=\mathrm{Max}[V-L,0] \quad (3.b)$$

The boundary conditions for equity in Region II are:

$$f^{II}(V,0)=V \quad (3.c) \qquad\qquad f^{II}/V\leq 1 \quad (3.d)$$

The shared boundary conditions are:

$$f^I(\overline{V},\tau)=f^{II}(V^{\dagger},\tau) \quad (3.e) \qquad\qquad f^I(\overline{V},\tau)=f^{II}(V^{\dagger},\tau) \quad (3.f)$$

where

$$V^{\dagger}\equiv\overline{V}-L$$

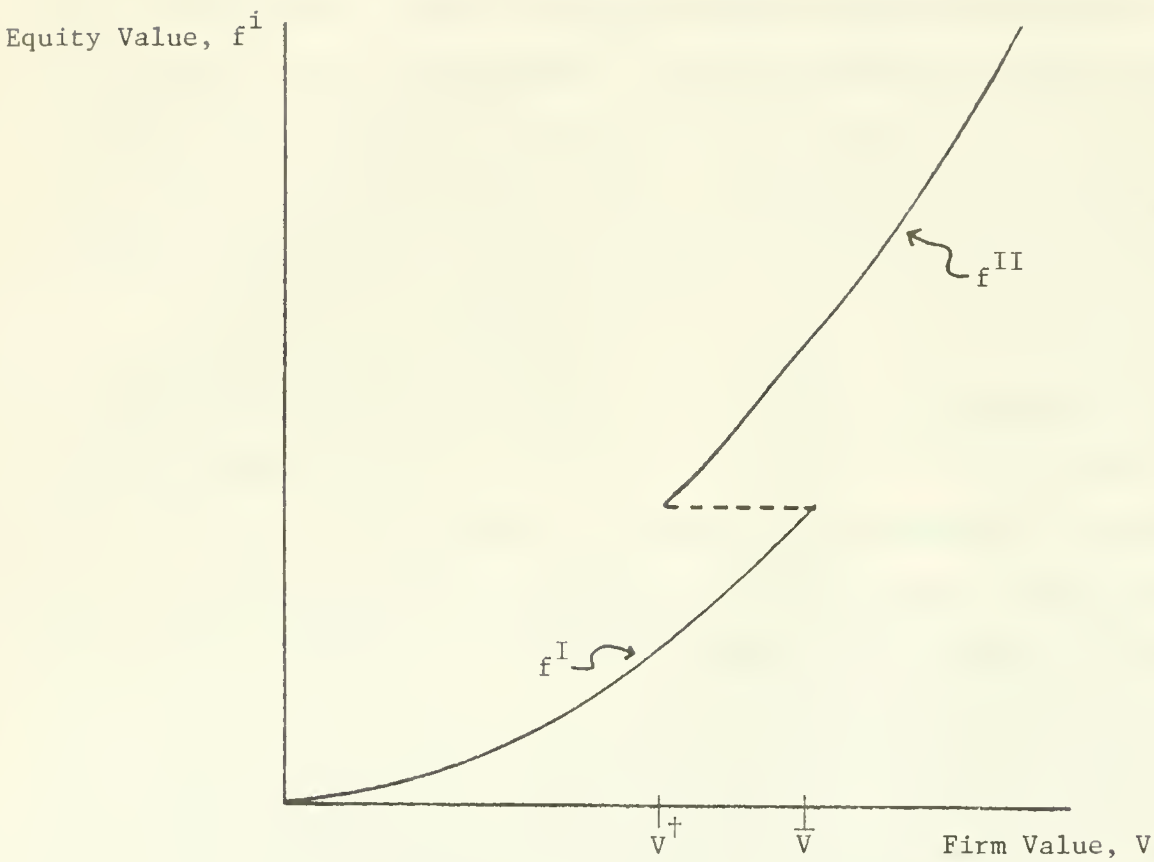

Figure 2 - Equity value as a function of firm value for a firm with a Fixed Firm Payout Policy (i.e., zero dividends in this case). Equity value when borrowing, f^I, is reduced by the loan amount, \$L, when line borrowing is repaid at $\bar{V}$. $\bar{V}=V^\dagger-L$.

Normally, this set of boundary conditions would be sufficient to solve for the equity value. In this case an additional condition is needed to solve for the boundary points. The boundary is at the discretion of the firm's management, and they will choose it so that equity value is maximized. This is the condition used to solve for $\overline{V}$, and thereby, $V^{\dagger}$.

In spite of the appearance of Figure 2, conditions (3.e) and (3.f) are met so the solution will have an equity value which is continuous and has a continuous first derivative everywhere. These conditions are from Merton (1977) and the discussion in Section IIIA. They insure that dominance does not occur. In Figure 2, it appears that there are two possible equity values for the range of firm values $V{:}V^{\dagger},\overline{V}$. In reality, there is only one possible equity value for a given firm value. If $V=\overline{V}-\varepsilon$ (on f^{I}, where ε is some arbitrarily small amount), then for $V=\overline{V}-L-\varepsilon$, which would be the firm value after the loan is repaid, f^{II} is not defined. Similarly, if at $V=V^{\dagger}+\varepsilon$ (on f^{II}), then f^{I} is not defined at $V=V^{\dagger}+L+\varepsilon$. It is possible to change "paths" only at the boundary.

The assumption on other types of claims against the firm, A8, can be relaxed with no substantive change in the analysis. The pricing of the line would change to reflect the added risk, but the pattern of line use would remain essentially the same. Likewise, contrary to the no dividend assumption, the firm could pay a small dividend without major change to the model. Of course the firm will always pay the largest allowed dividend in order to maximize equity value.[10] In reality, most revolving credit agreements contain provisions against the issuance of additional debt from sources other than the bank. One exception is line commitments used to back up commercial paper which are discussed in the next section. Most agreements also allow for the continued payment of dividends at a "historical" level.

A possible explanation for partial markdowns can be given for this model. The expansion of assets may not be realized immediately due to frictions in the real goods markets, such as construction costs, transportation, ect. If so, and if short term investments available to the firm yield less than the marginal cost of borrowing on the line, then the firm will gradually draw down the line. It is also probable that firms do not actually sell assets when repaying line borrowing. They simply do not reinvest as machinery depreciates and use these funds to repay the loan.

C.1 Solution of FFPP Model

To solve explicitly for the value of the line commitment, one additional assumption must be made:

A13 - The line commitment is infinitely-lived, $\tau=\infty$.

This reduces the equity value, Equations (2'), to ordinary differential equations in firm value. It also eliminates boundary conditions (3.b) and (3.c). The cost of A13 is in determining the effect of time on the value of the line commitment. This cost is offset by the ability to obtain a closed-form solution and thereby more easily determine the effect of the other variables.

As an aid in interpreting the solution of line value, it is useful to present a related security, the infinitely-lived risky coupon bond derived in Merton (1974) and Ingersoll (1976). The value of this security is given as:[11]

$$D(V,\infty)=\frac{C}{r}\,[1-P(a,ad)]+VP(a+1,ad) \qquad (4)$$

where C is the instantaneous coupon payment

$$a=\frac{2r}{\sigma}$$

$$d=\frac{C/r}{V}$$

P( ) is the incomplete gamma function.[12]

The solution to Equations (2') and (3) is

$$f^{I}=j[V-D^{I}(V,\infty)] \tag{5.1}$$

and

$$f^{II}=V-[1-J]\frac{C^{II}}{r}-JD^{II}(V,\infty) \tag{5.2}$$

This implies that the value of the line commitment is:

$$F^{I}=[1-j]V+jD^{I}(V,\infty)$$

and

$$F^{II}=[1-J]\frac{C^{II}}{r}+JD^{II}(V,\infty)$$

where D is risky perpetual debt with coupon C .

$$j \equiv \frac{b+BD_{V}^{II}(V^{\dagger},\infty)}{D_{V}^{II}(V^{\dagger},\infty)+b-bD_{V}^{I}(\bar{V},\infty)}$$

$$J \equiv \frac{1-B+BD_{V}^{I}(\bar{V},\infty)}{D_{V}^{II}(V^{\dagger},\infty)+b-bD_{V}^{I}(\bar{V},\infty)}$$

$$b \equiv \frac{\frac{C^{I}}{r}-D^{II}(V^{\dagger},\infty)}{\bar{V}-D^{I}(\bar{V},\infty)}$$

$$B \equiv \frac{V^{\dagger}-\frac{C^{II}}{r}}{\bar{V}+D^{I}(\bar{V},\infty)} .$$

The variable j is positive since the value of the line when borrowing, F^{I}, must always be less than or equal to the firm's value. J is also positive since the value of the line when not borrowing, F^{II}, can never have a value greater than riskless debt with the same coupon.

The boundary points, $\bar{V}$ and $V^{\dagger}$, are found by minimizing either F^{I} or F^{II} with respect to $\bar{V}$. The equation which the boundary points must satisfy

is:
$$\frac{C^I}{r}\left[\frac{V^\dagger}{\bar{V}}\right]^2 [P(a,ad^I)-P(a+1,ad^I)] = \left[L + \frac{C^{II}}{r}\right] [1-P(a+1,ad^I)] - \frac{C^I}{r} [1-P(a,ad^I)] \qquad (6)$$

where

$$d^I = \frac{\frac{C^I}{r}}{\bar{V}} \quad \text{and} \quad d^{II} = \frac{\frac{C^{II}}{r}}{V^\dagger}$$

The partial derivitives of V are:

$\bar{V}_L > 0$ (7.a) $\qquad \bar{V}_\Delta < 0$ (7.b) $\qquad \bar{V}_\phi > 0$ (7.c)

From Equation (6) it is difficult to determine the impact of changing either the variance or the riskless rate. Computer simulations show:

$\bar{V}_\sigma \gtrless 0$ (7.d) $\qquad \bar{V}_r \gtrless 0$ (7.e)

The first three results are intuitively appealing. The firm value at which the firm exercises increases as the amount borrowed, \$L, increases; for a fixed risk premium, it is only at higher firm values that the value of the straight bank debt, DL, reaches \$L. An increase in the risk premium on the borrowing portion of the line commitment, Δ , causes exercise at lower firm values. An increase in the fixed cost, Φ, increases the "drag" on the firm. This lowers the value of the straight debt, DL, for every firm value in Region I and thereby increases $\bar{V}$.

The upper inequality of (7.d) holds for small values of variance and is akin to the argument for $\bar{V}_L$; as variance increases for a fixed risk premium it will only be at larger firm values that the value of the straight debt reaches \$L. The lower inequality represents an offsetting factor. As variance increases so does the value of the implied option, whether it is in its put or call phase, which is being "paid for" by the fixed payment, C^{II}, in both regions. The value of the fixed payments, D^{II}, is dropping in both regions as variance increases. The combination is dropping more quickly in Region II causing the boundary to move out.[13]

The explanation of (7.e) is similar. For an increasing riskless rate at low values, the present value of the risk premium on the line is decreasing causing the boundary to occur at higher firm values.[14] This is offset at a certain level of r by the relatively greater decrease in the value of the fixed commitment in Region I.

The comparitive statics of line commitment value are:

$$F^i_L > 0 \quad (8.a) \qquad F^i_\Delta > 0 \quad (8.b) \qquad F^i_\phi > 0 \quad (8.c)$$

$$F^i_r > 0 \quad (8.d) \qquad F^i_\sigma < 0 \quad (8.c) \qquad F^i_V > 0 \quad (8.f)$$

$$. \qquad F^i_{VV} < 0 \quad . \quad (8.g)$$

These derivatives are conditional on the boundary, $\overline{V}$, remaining constant. Several of the variables can move $\overline{V}$ opposite to the direction of change in F^i. For example, for a given firm value and a change in risk premium, Δ, the firm could move from Region I into Region II where the value of the line is definitely lower since the firm is no longer borrowing. Obviously both effects must be taken into account.

None of the results are suprising. All signs in Region I are the same, except for r, as they would be for D^I, straight risky debt with the same coupon. The reason the riskless rate has the unexpected sign reflects that line borrowing is a variable rate security. As the riskless rate increases so does the interest payment for borrowing.

Computer simulations show that the magnitude of all derivitives (8) is less than or equal to those of risky coupon debt, except for F^{II}_V. For the line in general, the effect of a variable change on the value of risky debt is somewhat offset by the change in the value of the implied option. F_v^{II} is greater than D_v^{II} because the value of the implied put option also decreases with firm value.

Concavity of the line is shown by (8.g). This is to be expected for a debt-like security.

C.2 <u>A Discussion of Line Value</u>

Figure 3 shows a typical plot of line commitment value as a function of firm value. The most striking feature of the figure is the possibility of a negative value. The reason for this should be clear from the portfolio characterization of line value; the value of the stream of fixed payments, D^{II}, is worth less to the bank than the short position in the implied put option, O^{II}. In fact, for a given set of parameters, the point at which the value of the line changes from negative to positive represents the firm value at which the line was originally issued. If issued in Region II, the line will be fairly priced when its value is zero, where the value of the option to borrow exactly equals the value of the payments for this option. If issued in Region I, the line will be fairly priced when its value equals the amount of the borrowing.[15]

For a given firm value at which the line is issued, Equation (5) can be solved for a range of fixed verses variable costs. Depending upon the demands of the firm, the relative size of these two variables is discretionary. Since the riskless rate is determined elsewhere, setting the variable rate is equivalent to setting the risk premium on borrowing. The relative size of the fixed cost and the risk premium determines the pattern of borrowing on the line. In effect, the bank changes the security issued to the firm by changing the relative costs. A relatively small risk premium, and hence a relatively large fixed cost, implies that the firm

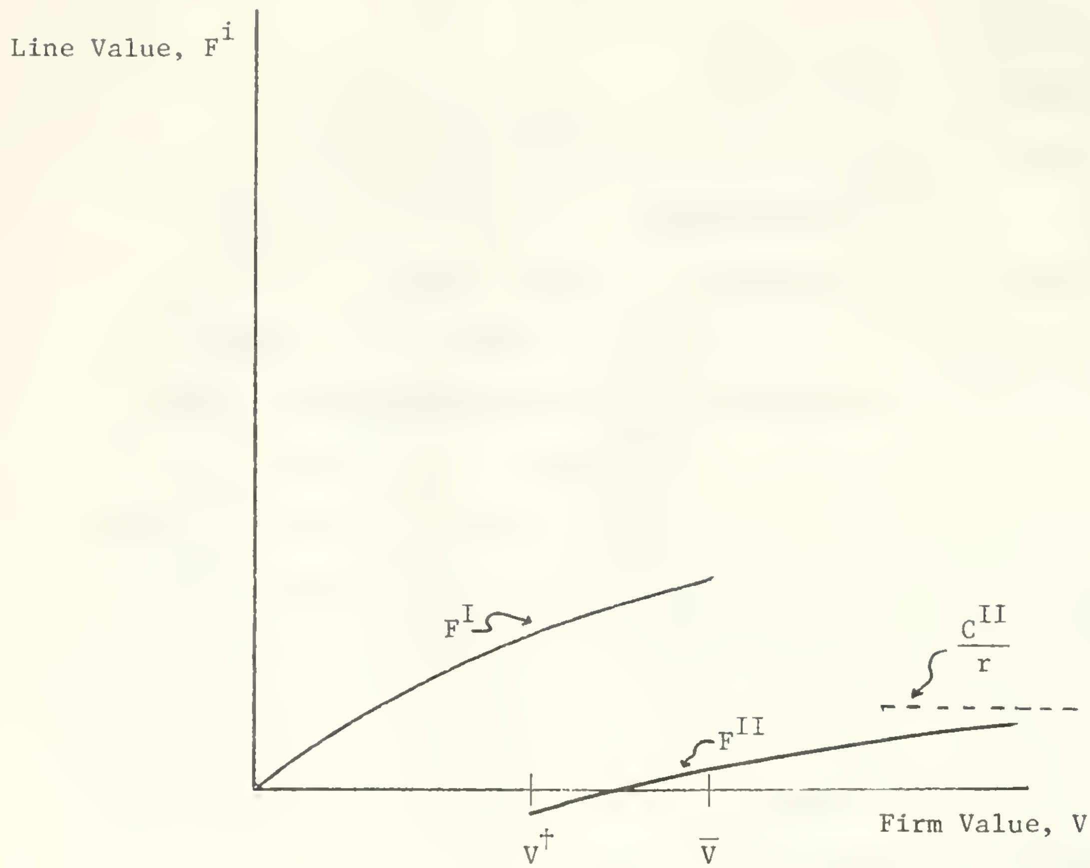

Figure 3- Line value as a function of firm value for a Fixed Firm Payout Policy. The firm repays line borrowing, \$L, at $\overline{V}$ so line value drops by this amount. † Negative line value after borrowing is repaid, at V, reflects that the value of the option for the firm to borrow is more valuable than the fixed cost of the line. The value of the line asmytotically approaches the value of riskless debt with the same coupon C^{II}.

will borrow on the line at all but very high firm values. At the extreme of a zero risk premium, the bank has only issued regular risky debt. The options have no value since they will never be exercised; the firm will always borrow. For the line to be correctly priced in this case, the fixed cost would equal the risk premium on regular debt.

C.3 Elasticity Coefficient

One variable of interest is the risk of the line relative to the risk of the firm for different levels of firm values. It is easy to show from the nature of the processes assumed that $\frac{\sigma^F}{\sigma} = \frac{VF_V}{F} \equiv \Theta$.[16] Θ is defined to be the elasticity coefficient of the line value with respect to firm value. It is also easy to show that Θ defines the relative beta of the line and the firm.

In Region I, Θ equals one for V=0 and decreases as firm value increases. This is common for most securities which are concave in firm value. Region II exhibits a different behavior. At $V^{\dagger}$, Θ is greater than one and then decreases as firm value increases.[17] The size of the elasticity coefficient is greatly dependent on the relative weights of the fixed debt, D^{II}, and the implied option which comprise the line at $V^{\dagger}$. (Of course Θ also depends on the riskiness of D^{II} and the put. Both risks are declining in firm value. However, in Region II the riskiness of D^{II} is very small even at $V^{\dagger}$.)

The value of the implied option, O^i, in either region is:

$$O^i = D^i - F^i$$

which implies

$$O_V^I = [j-1][1-P(a+1, ad^I)] \geq 0$$

$$O_V^{II} = [J-1]P(a, ad^{II}) \leq 0$$

For j,J>1, the value of the implied call is increasing in firm value and the value of the implied put is decreasing. This is expected since as the firm value moves away from $\overline{V}$ in Region I and $V^{\dagger}$ in Region II, the respective options become further "out of the money."

It is also of interest that the put is a convex function of firm value:[18]

$$O^{II}_{VV}=\frac{2\phi L/\sigma}{V^2}[J-1][P(a,ad^{II})-P(a+1,ad^{II})] \geq 0$$

D. Model 2-Fixed Investment Policy

In the FFPP Model no payments to equityholders are allowed at any point during the life of the line commitment. The FIP Model allows the firm to pay a dividend equal to the line borrowing when the firm draws down the line. Conversely, the firm must issue new equity when repaying line borrowing. The value of equity under this line, g, is shown in Figure 4. Because investment policy is fixed, firm value is continuous across the boundary; the firm is just changing its capital structure.

The boundary conditions for equity under this model in Region I are:

$$g^{I}(0,\tau)=0 \quad (9.a) \qquad g^{I}(V,0)=\text{Max}[V-L,0] \quad (9.b)$$

The boundary conditions for Region II are:

$$g^{II}(V,0)=V \quad (9.c) \qquad g^{II}/V \leq 1 \quad (9.d)$$

and the shared boundary conditions are:

$$g^{I}(\overline{V},\tau)=g^{II}(\overline{V},\tau)-L \quad (9.e) \qquad g^{I}_{V}(\overline{V},\tau)=g^{II}_{V}(\overline{V},\tau) \quad (9.f)$$

The major modification in this model is the replacing of (3.e) with (9.e) to reflect the dividend policy of the firm.

Condition (9.e) appears to violate the condition of Merton (1977) that the value of any contingent claim must be continuous everywhere. It is true that the total equity value takes a jump at the boundary. This

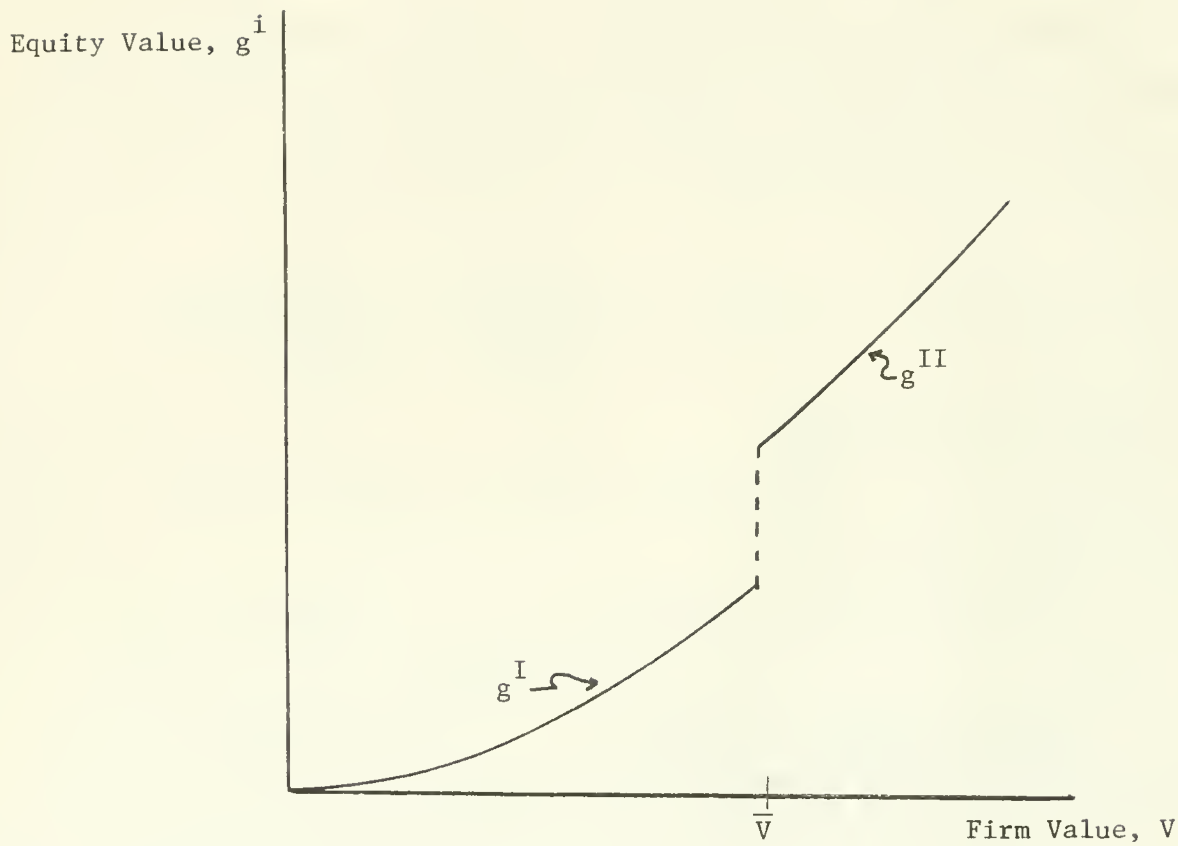

Figure 4- Equity value, g^i, as a function of firm value for a firm with a Fixed Investment Policy. The firm is borrowing along g^i. When repaying line borrowing of \$L, at $\overline{V}$, the firm is replacing line borrowing with equity of an equal value.

happens because new equity has been issued to replace bank borrowing. The key point is that the value of an equity share has not taken a jump as would be required for the possibility of dominance. This can be seen by considering the change in total equity value at the boundary as

$$dg = NdS + (S + dS)dN$$

where S is the share price and N is the number of shares outstanding. Share price is continuous while total equity jumps at the boundary, and since the new shares, dN, are fairly priced, no arbitrage possibilities exist.

The set of conditions (9) together with Assumption A8 imply, as they did in the FFPP Model, that the firm can do no outside borrowing.

D.1 Solution of the FIP Model

The solution for the FIP Model is very similar to that of the FFPP Model and is shown in Figure 5:

$$g^{I} = k[V - D^{I}(V,\infty)] \tag{10.1}$$

and

$$g^{II} = V - [1-K]\frac{C^{II}}{r} - KD^{II}(V,\infty) \tag{10.2}$$

which implies that the line value is

$$G^{I} = [1-k]V + kD^{I}(V,\infty)$$

and

$$G^{II} = [1-K]\frac{C^{II}}{r} + KD^{II}(V,\infty)$$

where

$$k \equiv \frac{h + HD_{v}^{II}(\bar{V},\infty)}{D_{v}^{II}(\bar{V},\infty) + h - hD_{v}^{I}(\bar{V},\infty)}$$

$$K \equiv \frac{1 - H + HD_{v}^{I}(\bar{V},\infty)}{D_{v}^{II}(\bar{V},\infty) + h - hD_{v}^{I}(\bar{V},\infty)}$$

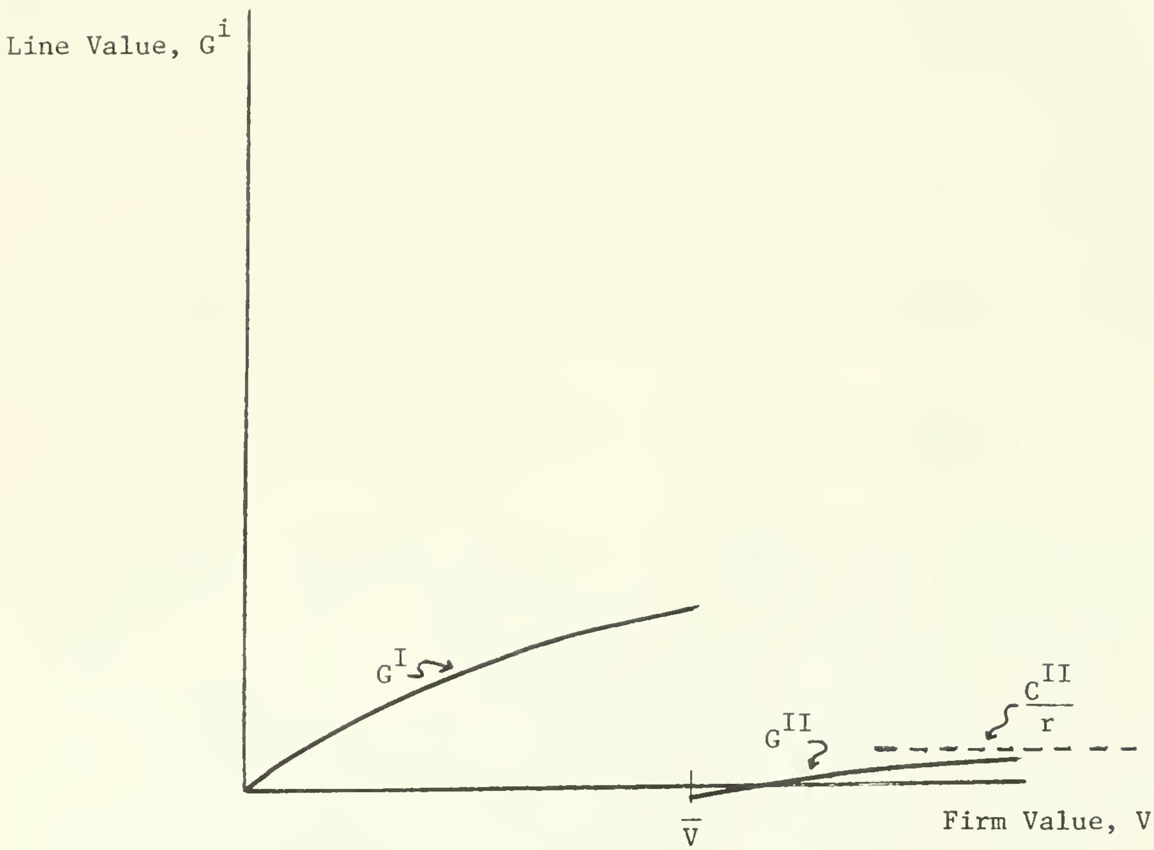

Figure 5- Line value as a function of firm value for a firm with a Fixed Investment Policy. The firm _is_ borrowing along G^I and repays the borrowing, \$L, at $\overline{V}$.

$$h \equiv \frac{\frac{C^{II}}{r} - D^{II}(\bar{V},\infty)}{\bar{V} - D^{I}(\bar{V},\infty)}$$

$$H \equiv \frac{\bar{V} - L - \frac{C^{II}}{r}}{\bar{V} - D^{I}(\bar{V},\infty)} \quad .$$

$\bar{V}$ must satisfy the the relationship:

$$\frac{C^{I}}{r}[P(a,ad^{I})-P(a+1,ad^{I})] - \frac{C^{I}}{C^{II}} LP(a+1,ad^{II}) \left[\frac{P(a,ad^{I})-P(a+1,ad^{I})}{P(a,ad^{II})-P(a+1,ad^{II})}\right]$$

$$= \left[L + \frac{C^{II}}{r}\right][1-P(a+1,ad^{I})] - \frac{C^{I}}{r}[1-P(a,ad^{I})] \quad .$$

The solution for the boundary, $\bar{V}$, is different from that of the FIP Model. Simulations show that the $\bar{V}$ of the FFPP Model is greater than that of the FIP Model. However, the V^{+} in the FFPP Model is less than the $\bar{V}$ for the FIP Model. This last result is appealing. In the FIP Model the equityholders recieve a payment of \$L at the boundary as firm value decreases. In the FFPP Model they recieve nothing. Everything else equal, the sooner the \$L is recieved, the better off the equityholders.

The comparative statics of G are the same as for F and are shown in Equations (7).

D.2 Outside Borrowing in the FIP Model

An assumption for both models is that no borrowing from other sources is allowed. In the FFPP Model this assumption does not substantially change the analysis. Because of the payment of the loan amount to the equityholders in the FIP Model, the effect of outside borrowing is quite different.

What would be the nature of any debt issued in Region II to replace bank borrowing? It would have to be very short term (i.e., one period). If it were of a longer term then the firm would be giving up a portion of the implied put option from the line, since it has killed the option during the term of the outside debt. The debt would also be riskless, not only because of its short term and the nature of the stocastic process assumed for firm value, but more importantly, because the line commitment acts as a loan guarantee for any debt issued in Region II. Since the lender of Region II debt knows that the debt will be repaid when the firm value drops to $\overline{V}$, there is no risk of default.

The debt could also be made riskless for firm values less than $\overline{V}$ if the firm makes a side agreement with the non-bank lenders which states that the debt will be repaid with line borrowing at some $V^{*}>0$. This agreement would be costless to the firm, given that it is already paying the fixed cost to the bank, yet for a given amount of borrowing \$L, the riskless rate would be paid for the debt instead of the risky line rate. This V^{*} can be set by agreement such that it is reached only just before bankruptcy is imminent.

In effect, if outside borrowing is allowed in the FIP Model, the bank is only issuing a loan guarantee. The firm holds a put option sold by the bank. This indicates that the fixed fee, Φ, should be set at the same level as the risk premium on correctly priced regular risky debt. It does not matter at what level the risk premium on line borrowing, Δ, is set. The firm will never borrow from the bank.

When "rolling over" existing debt, the firm could still use the line as a loan guarantee; the restriction against additional outside borrowing is not sufficient. This provides the rationale for the materially adverse change clause. Firms are prevented from using the line as a guarantee below some firm value since the bank no longer has to lend them additional funds. Furthermore, banks do not ever specify the level of this cut-off value. This uncertainty makes it almost impossible to use the line as a guarantee.

The description of the FIP Model with outside borrowing is similar to that of a line commitment used to back-up commercial paper. The line is issued to insure that the firm can roll over the commercial paper at maturity. In this case, the line is not a loan guarantee in a strict sense, however. If the firm goes into default during the life of the commercial paper, \$L is not transferred from the bank to the paperholders. The line only insures that the paperholders recieve \$L at the maturity of their paper. Consequently, commercial paper is not a riskless security since it can have a maturity of up to nine months.

E. The Systematic Effects of Aging

It is difficult to determine how changing maturity affects the value or pattern of use of the line commitment without a proper model. Not only does the value of the line change for a fixed boundary, but the boundary can also be changing with time. It is possible that these two effects offset each other.

"Normally," the risk premium on line borrowing should be an increasing function of time.[19] Therefore, for a given borrowing rate, the value of the straight debt, and hence the value of the line when borrowing, should increase as maturity increases. However, the change in the straight debt causes the boundary to occur at lower firm values causing the line value to decrease due to the increased value of the implied option. The sum of these two effects is indeterminant. There is also the additional factor of the fixed debt in both regions. As maturity nears, the value of the fixed debt decreases since it has zero maturity value, $D_{\tau}^{II} \geq 0$. Offsetting this, the value of the implied option is decreasing, $0_{\tau}^{II} \geq 0$. The effect of these variables on both line value and the boundary is also uncertain for changing maturity.

IV. Conclusion

Any study of revolving credit agreements should explore the reasons for their dramatic increase in popularity during the past decade. Obviously these agreements provide an unusual payout structure, as is evidenced by their unique option characterization.

The demand for revolving credit agreements could be explained by the increasingly volitile interest rates of the last several years. However, it is probably not the volitile borrowing rates which directly explain the demand; bank term loans are also variable rate securities and could meet this demand. As noted above, revolving credit agreements are similar to callable bonds. For a given firm value during a period of volitile interest rates, a callable bond is likely to reach its call price at some time. This leads to large and frequent transactions costs due to refinancing. The variable rate revolving credit agreement is less likely to be called. Even if the line is repaid there is little, if any transaction cost due to repayment. Consequently, even if both securities are correctly priced at issue, the probability of higher transaction costs argues against callable bonds.[20]

On the supply side, lenders must be able to meet the fluctuating and unpredictable demand inherent in loans from revolving credit agreements.

The depth of the Federal funds market has probably made banks the institutional framework best suited to meet this demand. Whenever a bank can't make a revolving credit loan from internally generated monies, it can go to the Federal funds market.

The principal insight of the present paper is the option nature of line commitments in general, and revolving credit agreements in particular. The two models used to determine the properties of line commitments represent polar cases based on a fundamental firm accounting identity. Their differing assumptions -a fixed investment or fixed dividend policy- highlight the rationale behind several line covenants, especially the materially adverse change clause. In reality, a revolving credit agreement contains elements of each model since neither policy is strictly forced on a firm.

Two important issues were only partially developed in this paper: the effect of changing maturity and the high incidence of renegotiation. The former could be examined with the use of numerical methods to solve Equations (2) with the proper time variables. The latter, renegotiation of terms, seems to be a common occurrence. The reasons for frequent renegotiation are not clear from the current analysis but it is possible that a realistic treatment of changing maturity would help to explain them.

FOOTNOTES

1. A typical line commitment includes a variable rate loan on which the interest rate per period is the prime rate that period plus a given premium, either a fixed amount or a percentage of prime. A less common type of line involves a fixed rate loan on which the interest rate is fixed regardless of the prevailing market rate at the time borrowing is undertaken. As Bartter and Rendleman (1979) show, analysis of this latter type of line borrowing differs substantially from that taken here.

2. Most revolving credit agreements which require compensating balances stipulate that a minimum balance is to be kept at all times. In this case compensating balances are not a method of paying interest on demand deposits but represent a cost of the line. Black (1975) discusses this issue. Bartter and Rendleman (1978) consider the trade-off between fees and compensating balances in pricing a line. The use of compensating balances is not efficient for pricing because of the deadweight loss due to reserve requirements. For this reason compensating balance requirements are declining as a feature of revolving credit agreements.

3. In reality, there are three interrelated choice variables: 1) μ, the line fee, 2) δ, the explicit risk premium, and 3) α, the level of compensating balances. However, these three variables, as shown by Equation (1'), reduce to only two relevant variables: the fixed and variable payments.

   It is shown in Section IIIC that the bank, based on the firm's demands, has a certain amount of latitude in setting the relative size of the fixed and variable costs.

4. Of course there may be partial markdowns in the short-term lines used for working capital purposes. A reason for partial markdowns due to market imperfections will be suggested later.

5. An alternative to the level of the penalty set in the last section is for the bank to make ρ very high. This would ensure that the firm keeps compensating balances most of the time. However, in some instances, such as very low firm values, even "borrowing" the compensating balances and paying the high penalty would be optimal for the firm. For a large penalty there would be three regions: $\$N = -CB$, $\$N = L - CB$, and $\$N = L$. Having three borrowing regions, instead of two, would unnecessarily complicate the analysis.

6. As with private placements, it is to be expected that the required return on bank borrowing is higher than for a public offering. This higher rate offsets to some extent the advantage of lower transactions costs.

7. A detailed discussion of these "boilerplates" is given in Smith and Warner (1979).

8. Assumptions A1 through A7 are common in the option pricing literature, see Merton (1973). Assumption A8 is discussed in detail for each of the models.

9. It is implicit in both models that the firm operates in a competitive market; by expanding the scale of its assets, the firm is investing in zero NPV projects. Under this assumption, the firm, by selling assets at $V^{\dagger}$, maximizes equity

value. Neither model restricts the issuing of new equity at any time. However, if the firm issues new equity to invest in zero NPV projects, the bank gains, since the line is "safer", at the expense of existing shareholders.

10. See Black and Cox (1976) for an example of the effect of other borrowing. Merton (1973) has an option model which allows for continuous dividends which are proportional to firm value.

11. See Hildebrand (1962) for the solution given in Ingersoll (1976).

12. $P(a,x) = \frac{1}{\Gamma(a)} \int_0^x e^{-s} s^a ds$. $\Gamma(\ )$ is the gamma function. (P) takes on values between 0 and 1 inclusive. Davis (1966) contains the necessary relationships for the manipulation of the incomplete gamma function.

13. In Section IIIA, it was noted that $\bar{V}$ was defined to be that point where the value of the borrowing portion of the line, DL, was equal to \$L. Alternatively the boundary can be defined as the firm value (or two values, $\bar{V}$ and $V^{\dagger}$, in this case) where $D^{II}(\bar{V},\infty) - 0^{I} = D^{II}(V^{\dagger},\infty) - 0^{II}$. The value of the right-hand side decreases more quickly as σ increases.

14. The value of the risk premium is $\Delta = \frac{\alpha}{1-\alpha} r + \frac{\alpha}{1-\alpha} (\delta - \mu)$ from A10 and Equation (1'). For δ and μ both fixed, the present value of the risk premium, Δ/r, decreases as r increases.

15. The solution for line commitment value assumes that all parameters of the contract, except for firm value, are fixed. If firm value increases by "enough", the possibility of ever using line borrowing would become remote. The firm would like to renegotiate the line at this point. The bank, if it renegotiated a new, fairly priced line, would be giving up the value of the fixed costs. In effect, if the bank did renegotiate, the firm orginally paid too little for the implied options. It is not at all clear why the bank would forego the value of the fixed costs by renegotiating a new line agreement. It is possible, since lines are commonly renegotiated, the firm pays a higher fixed cost for the right to renegotiate if firm value increases substantially.

16. A derivation of this relationship is in Merton (1974).

17. Merton (1973) also describes a security, the "down and out" option, which is concave and has an elasticity with respect to firm value greater than one. Ingersoll (1976) shows that callable, convertible discount bonds exhibit the opposite behavior for large firm values: convex with elasticity less than one.

18. We should expect from Merton (1973) that a put is convex in firm value.

19. For discount bonds, Merton (1974) <u>does not</u> find that the risk premium is "normally" an increasing function of time. When the present value of the face value, discounted at the riskless rate, Be^{-rt}, is greater than the value of the firm, the risk premium is decreasing in τ. I am basing my "normally" increasing function of τ on the fact that only "bad things can happen to DL or it will be called. This is only speculation and may be wrong since I do not have a model which allows me to derive it.

20. Small firms should have a greater demand for revolving credit than large firms. Small firms seem to have a higher variance and therefore should call more frequently. If small firms demand some form of callable debt, revolving credit agreements would provide the cheapest transactions cost.

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