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DETERMINISTIC CHAOS IN AN EXPERIMENTAL ECONOMIC SYSTEM

John D. Sterman

July, 1988 WP #2040-88

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Deterministic Chaos in an Experimental Economic System

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Abstract

An experiment with a simulated macroeconomic system demonstrates that the decision-making processes of agents can produce deterministic chaos. Subjects managed capital investment in a simple multiplier-accelerator economy. Performance, however, was systematically suboptimal. A model of the subjects' decision rule is proposed and related to prior studies of dynamic decision making. Econometric estimates show the model is an excellent representation of the actual decisions. The estimated rules are then simulated to evaluate the stability of the subjects' decision processes. While the majority of the estimated rules are stable, approximately 40% yield a variety of dynamics including limit cycles, period multiples, and chaos. Analysis of the parameter space reveals a complex bifurcation structure. Implications for models of human systems and experimental studies of economic dynamics are explored.

1. Introduction: coupling nonlinear dynamics and experimental economics

Recent work in the physical sciences has shown deterministic chaos to be a common mode of behavior in dynamic systems, thus stimulating the search for chaos and other highly nonlinear phenomena in human systems. Indeed, there has been a near explosion of models and empirical studies which seek to show the relevance of nonlinear dynamics and chaos in social and economic settings. Several journals have devoted special issues to chaos and models of nonlinear, disequilibrium dynamics in human systems.[1] This robust literature can be divided roughly into (1) theoretical models of nonlinear dynamics, and (2) empirical studies which seek evidence of chaos in economic data.

Theoretical studies include the work of Day (1982), Dana and Malgrange (1984), Grandmont (1985), Rasmussen and Mosekilde (1988), Stutzer (1980), and many others (see note 1). A survey of these studies suggests the following generalizations. First, many standard models, both micro- and macroeconomic, when modified to include realistic nonlinearities, can be shown to contain regimes of chaos and other nonlinear phenomena such as mode-locking, period multiples, and quasiperiodicity. Benhabib and Day's (1981) studies of such simple models lead them to "expect the possibility of erratic [chaotic] behavior for a wide variety of dynamic economic models involving rational decision-making with feedback." Subsequent work has borne out this conjecture – the possibility of chaos does not seem to depend on particular behavioral assumptions but on more fundamental properties. However, most models of nonlinear dynamics have been purely theoretical, and have not involved econometric estimation of the parameters. The few exceptions have found the estimated parameters lie well outside the chaotic regime, e.g. Candela and Gardini (1986) and Dana and Malgrange (1984). As Day and Shafer (1985, 293) note,

> "Whether or not we can construct empirical models that offer convincing explanation of *real world* macro activity [in terms of deterministic chaos] is an open question: most economists would probably agree that we are as yet quite far from a definitive answer to it. What we know now...is that among the empirical phenomena that we can hope to explain...are stochasticlike fluctuations in economic data. Moreover, we need not expect that exotic assumptions or bizarre model structures will be required" (emphasis in original).

The relative simplicity and theoretical focus of the models is entirely appropriate to the early exploration of new concepts and analytical tools. However, the theoretical work to date leaves unanswered questions about the relevance of these models. Can chaos arise from the behavior of actual agents? Do the chaotic regimes in the models lie in the realistic region of parameter space, or are they mathematical curiosities unrealized in actual economic systems?

The empirical literature has sought to answer these questions by searching for evidence of chaos in economic data. This literature is notable for the clever adaptation of techniques originally applied in physical settings, such as the Takens (1985) method for recovering low-dimensional attractors from a single time series, the Wolf, Swift, Swinney, and Vastano (1985) technique for estimation of Lyapunov exponents from experimental time series, and the Grassberger-Procaccia (1983) correlation dimension. The results are tantalizingly suggestive but inconclusive. Brock (1986) demonstrates a method to test the hypothesis of chaos in economic data against explicit alternative hypotheses, but finds "that there is not enough information available in U.S. real GNP, real gross private domestic investment, and Wolfer's sunspot series...to reject the null hypothesis that...[these series were] generated by an AR(2) process." Chen (1988) and Barnett and Chen (1988), however, find evidence of low dimensional strange attractors in some but not all measures of U.S. monetary aggregates. Their conclusions are tempered, however, by uncertainties such as the sensitivity of the methods used to the number of data points, the number of points per orbit, the (unknown) magnitude and statistical character of process noise, and the (unknown) magnitude and character of measurement error [Ramsey and Yuan (1987)]. Brock (1986, 192) concludes "It is *not* enough when you are working with short data sets to report low dimension and positive Lyapunov exponents to make the case for deterministic chaos in your data" (emphasis in original). Ramsey, Sayers, and Rothman (1988) have identified significant biases in calculations of correlation dimension caused by small sample size and conclude "that while there is abundant evidence for the presence of nonlinear stochastic processes, there is virtually no evidence at the moment for the presence of simple chaotic attractors of the type that have been discovered in the physical sciences."

The prevalence of chaos in the models but low power of aggregate statistical tests motivates a complementary approach based on laboratory experiments with simulated economic systems. The pioneering work of Smith (1982, 1986), Plott (1986), and others has demonstrated that many economic theories can be successfully tested in the laboratory. This paper applies these techniques to the investigation of chaos in economic systems. I report the results of an experiment with a simulated macroeconomic system, specifically a multiplier-accelerator model. In the experiment, subjects play the role of managers of the capital-producing sector of an economy. Each time period they must make a capital investment decision. The task of the agents is to manage a complex dynamic system in disequilibrium, a system with time lags, multiple feedbacks, and nonlinearities. I show that the behavior of the subjects is systematically suboptimal, suggesting the use of a common heuristic for decision making. A model of the subjects' decision rule is proposed. The model is well grounded in the literature of economics, psychology, and behavioral decision theory. Econometric estimation shows the decision rule explains the agents' behavior well. Next the estimated decision rules are simulated, and it is shown that approximately 40% of the agents produce unstable behavior, including chaos. The parameter space of the system is mapped and shown to contain a complex bifurcation structure. Thus experimental evidence is adduced that the actual decision processes of agents in a common economic context can produce chaos. Limitations of the method, implications, and suggestions for future research are discussed.

2. The Model

The experiment is based on a simple model of the capital investment accelerator and is fully described in Sterman (1988a). The model creates a two-sector economy with a capital producing and goods producing sector. The focus is the capital investment accelerator. Goodwin (1951, 4) notes that the traditional acceleration principle assumes

> ...that actual, realized capital stock is maintained at the desired relation with output. We know in reality that it is seldom so, there being now too much and now too little capital stock. For this there are two good reasons. The rate of investment is limited by the capacity of the investment goods

> industry....At the other extreme there is an even more inescapable and effective limit. Machines, once made, cannot be unmade, so that negative investment is limited to attrition from wear....Therefore capital stock cannot be increased fast enough in the upswing, nor decreased fast enough in the downswing, so that at one time we have shortages and rationing of orders and at the other excess capacity with idle plants and machines.

A single factor of production (capital plant and equipment) is considered. The model includes, however, an explicit representation of the capital acquisition delay (construction lag) and the capacity of the investment goods sector. As a result, orders for and acquisition of capital are not necessarily equal, and at any moment there will typically be a supply line of capital under construction. For simplicity, the demand for capital of the goods-producing sector is exogenous, and there is no representation of the consumption multiplier.

The model allows for variable utilization of the capital stock. Thus production P is the lesser of desired production P* or production capacity C. Capacity is proportional to the capital stock, with capital/output ratio κ:

$$P_t = \mathrm{MIN}(P^*_t, C_t) \tag{1}$$

$$C_t = K_t/\kappa. \tag{2}$$

The capital stock of the capital sector is augmented by acquisitions A and diminished by depreciation D. The average lifetime of capital is given by τ:

$$K_{t+1} = K_t + (A_t - D_t) \tag{3}$$

$$D_t = K_t/\tau. \tag{4}$$

The acquisition of capital by both the capital and goods sectors (A and AG) depends on the supply line of unfilled orders each has accumulated (the backlogs B and BG) and the fraction of the backlog delivered that period ϕ (the suffix 'G' denotes a variable of the goods-producing sector):

$$A_t = B_t \cdot \phi_t \tag{4}$$

$$AG_t = BG_t \cdot \phi_t \tag{5}$$

$$\phi_t = P_t/P^*_t \tag{7}$$

$$P^*_t = B_t + BG_t. \tag{8}$$

Each period both the capital and goods sectors acquire the full supply line of unfilled orders unless the capital sector is unable to produce the required amount. If capacity is insufficient so that $P^*>C$, $\phi<1$ and shipments to each sector fall in proportion to the shortfall. Note that the formulation for ϕ implies that $A+AG=P$ at all times, ensuring that output is conserved. The explicit representation of the construction supply line and the constraint on production mean that the lag in acquiring capital may be variable. It is easily shown that the average capital acquisition lag $\Lambda=1/\phi$. Normally, $\phi=1$ and $\Lambda=1$ period. If capacity is inadequate, however, ϕ falls and Λ lengthens as the backlogs of unfilled orders grow relative to output.[2]

The supply lines of unfilled orders for each sector B and BG are augmented by orders for capital placed by each sector and emptied when those orders are delivered:

$$B_{t+1} = B_t + (O_t - A_t) \tag{8}$$

$$BG_{t+1} = BG_t + (OG_t - AG_t). \tag{10}$$

Orders placed by the goods sector are an exogenous input to which the subjects of the experiment must respond by ordering an appropriate amount of capital for their own use:

$$OG_t = \text{exogenous} \tag{11}$$

$$O_t = \text{determined by subject.} \tag{12}$$

Equations (1)-(12) thus define a third-order nonlinear difference equation system. The system has the interesting property that the nonlinear capacity utilization function of eq. (1) divides the system into two distinct regimes, $\phi=1$ and $\phi<1$. Furthermore, the equilibrium point $P^* = C$ lies exactly at the boundary. Considering each region in turn reveals interesting properties of the open loop system. When $P^*=P<C$, $\phi=1$, and the system is linear:

$$\begin{bmatrix} K_{t+1} \\ B_{t+1} \\ BG_{t+1} \end{bmatrix} = \begin{bmatrix} 1-1/\tau & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} K_t \\ B_t \\ BG_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} O_t \\ OG_t \end{bmatrix} \tag{13}$$

Excess capacity implies each sector receives the quantity ordered after one period. Note that the system in this regime is always stable. The capital stock is controllable via orders O (it depreciates with lifetime τ towards an equilibrium of $\tau \cdot O$). BG is not controllable – when there is excess capacity, the goods-producing and capital-producing sectors are decoupled.

When capacity is inadequate, however, $P^*>P=C$, $\phi<1$, and the system is nonlinear. Linearizing the system around the operating point $(\overline{K}, \overline{B}, \overline{BG})$ and defining $\overline{P^*} = \overline{B}+\overline{BG}$ and $\overline{\phi} = (\overline{K}/\kappa)/\overline{P^*}$ yields

$$\begin{bmatrix} K_{t+1} \\ B_{t+1} \\ BG_{t+1} \end{bmatrix} = \begin{bmatrix} 1-\frac{1}{\tau}+\frac{\overline{B}}{\kappa\overline{P^*}} & \overline{\phi}\frac{\overline{BG}}{\overline{P^*}} & -\overline{\phi}\frac{\overline{B}}{\overline{P^*}} \\ -\frac{\overline{B}}{\kappa\overline{P^*}} & 1-\overline{\phi}\frac{\overline{BG}}{\overline{P^*}} & \overline{\phi}\frac{\overline{B}}{\overline{P^*}} \\ -\frac{\overline{BG}}{\kappa\overline{P^*}} & \overline{\phi}\frac{\overline{BG}}{\overline{P^*}} & 1-\overline{\phi}\frac{\overline{B}}{\overline{P^*}} \end{bmatrix} \begin{bmatrix} K_t \\ B_t \\ BG_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} O_t \\ OG_t \end{bmatrix} \tag{14}$$

When capacity is inadequate the goods and capital sectors are highly coupled. The eigenvalues of the open loop system for $\phi<1$ are readily found to be

$$\lambda = \begin{cases} 1 \\ 1-\frac{1}{\tau}+\frac{\overline{B}}{\kappa\overline{P^*}} \\ 1-\overline{\phi} \end{cases} \tag{15}$$

All three eigenvalues are real, indicating that the open loop system is not oscillatory. Note also that the system always has an eigenvalue of unity, indicating neutral stability and the importance of the higher order terms excluded from the linearization. The role of nonlinearity is also highlighted by the other eigenvalues, both of which are functions of the operating point around which the system is linearized. While $1-\overline{\phi} \leq 1$ for all $\overline{\phi}$, the term $1-(1/\tau)+(\overline{B}/\kappa\overline{P^*})$ may easily exceed unity, in which case the open loop system is unstable.

Thus the simplicity of the system belies considerable complexity of the dynamics. Of course, the closed loop properties of the system will depend on the decision rule for orders O. Nevertheless, the sharp differences between the two regimes and strong effects of the nonlinearities suggest effective management of the system will be difficult.

3. Experimental Protocol

The methodological foundations of experimental economics are discussed in the seminal work of Smith (1982) and Plott (1986). The experimental protocol used here is described in Sterman and Meadows (1985) and Sterman (1987, 1988a). A continuous time version of the model is developed and analyzed in Sterman (1985). For the experiment it has been converted to discrete time. Simulation and formal analysis confirm that the conversion to discrete time does not alter the essential dynamics of the system [Rasmussen, Mosekilde and Sterman (1985), Sterman (1988b)]. The experiment is implemented on IBM PC-type microcomputers. A 'game board' is displayed on the screen and provides the subjects with perfect information. Color graphics and animation highlight the flows of orders, production, and shipments to increase the transparency of the structure (figure 1).[3] No overt time pressure was imposed. The parameters ($\kappa=1$ and $\tau=10$) were chosen to minimize the computational burden imposed on the subjects, while remaining close to the original values.

The subject population (N=49) consisted of MIT undergraduate, master's and doctoral students in management and engineering, many with extensive exposure to economics and control theory; scientists and economists from various institutions in the US, Europe, and the Soviet Union; and business executives experienced in capital investment decisions including several corporate presidents and CEOs.

Subjects are responsible for only one decision – how much capital to order. The goal of the subjects in making these decisions is to minimize total costs. The cost function or score S is defined as the average absolute deviation between desired production P* and production capacity C over the T periods of the experiment:

$$S = \left(\frac{1}{T}\right)\sum_{t=0}^{T} \left|P^*_t - C_t\right|. \tag{16}$$

The cost function indicates how well subjects balance demand and supply. Subjects are penalized equally for both excess demand and excess supply. Absolute value rather than quadratic or asymmetric costs provide an incentive to reach equilibrium while minimizing the complexity of the subjects' decision-making task. A caveat: monetary rewards were not used, in violation of Smith's (1982) protocol for experimental microeconomics. While many economists argue that significant performance-based rewards are necessary to establish external validity, a number of experiments in preference reversal [Grether and Plott (1979), Slovic and Lichtenstein (1983)] suggest performance is not materially affected by reward levels. Similar experiments have found weak or even negative effects of incentives on performance, though the further study is needed [Hogarth and Reder (1987)]. Other experiments in dynamic decision making suggest the results are robust with respect to significant variations in the experimental environment. Sterman (1988c) describes an experiment with a simulated production-distribution system in which financial rewards were used. Like the model here, the experimental system contained multiple feedbacks, nonlinearities, and time lags. Yet despite large differences in the experimental cover story, information set, incentives, time pressure, and complexity of the underlying system, the results strongly reinforce those of the present experiment and support the same decision rule tested here, suggesting the relative insensitivity of the subjects to incentives and the dynamic structure of the system. Other studies of dynamic decision making which generally support the results here include Brehmer (1987) and MacKinnnon and Wearing (1985).

4. Results

The trials reported below were run for 36 periods. All were initialized in equilibrium with orders of 450 units each period from the goods sector and capital stock of 500 units. Depreciation is therefore 50 units per period, requiring the capital sector to order 50 units each period to compensate. By eq. (8) desired production then equals 450 + 50, exactly equal to capacity, and yielding an initial cost of zero. Orders for capital from the goods sector OG, the only exogenous input to the system, remain constant at 450 for the first two periods. In the third period OG rises from 450 to 500, and remains at 500 thereafter (figure 2). The step input is not announced to the subjects in advance.

Several trials representative of the sample are plotted in figure 3; table 1 summarizes the sample. Trial 16 is typical. The subject reacts aggressively to the increase in demand by ordering 150 units in period 2. The increase in orders further boosts desired production, leading the subject to order still more. Because capacity is inadequate to meet the higher level of demand, unfilled orders accumulate in the backlog, boosting desired production to a peak of 1590 units in period 6. The fraction of demand satisfied ϕ drops to as low as 52%, slowing the growth of capacity and frustrating the subject's attempt to satisfy demand. Faced with high and rising demand, the subject's orders reach 500 in the fifth period. Between periods 7 and 8 capacity overtakes demand. Desired production falls precipitously as the unfilled orders are finally produced and delivered. A huge margin of excess capacity opens up. The subject slashes orders after period 5, but too late. Orders placed previously continue to arrive, boosting capacity to a peak of over 1600 units. Orders drop to zero, and capacity then declines through discards for the next 12 periods. Significantly, the subject allows capacity to undershoot its equilibrium value, initiating a second cycle of similar amplitude and duration.

The other trials are much the same. While specifics vary the pattern of behavior is remarkably similar. As shown in table 1, the vast majority of subjects generated significant oscillations. Only 4 subjects (8%) achieved equilibrium before the end of the trial.

5. Proposed decision rule and estimation results

The qualitative similarity of the results suggests the subjects, though not behaving optimally, used heuristics with common features. The decision rule proposed here was used in the original simulation model upon which the experiment is based [Sterman (1985)] and is a variant of rules long used in models of corporate and economic systems [Samuelson (1939), Metzler (1941), Holt et al. (1960), Forrester (1961), Low (1980)]. The rule determines orders for capital O as a function of information locally available to an individual firm. Such information includes the current desired rate of production P*, current production capacity C, the rate of capital discards D, the supply line SL of orders for capacity which the firm has placed but not yet received, and the capital acquisition lag Λ:

$$O_t = f(P^*_t, C_t, D_t, SL_t, \Lambda_t). \tag{17}$$

Specifically, capital orders O are given by replacement of discards modified by an adjustment for the adequacy of the capital stock AC and an adjustment for the adequacy of the supply line ASL. Accounting for the nonnegativity constraint on gross investment and allowing for an additive disturbance ε yields:

$$O_t = \mathrm{MAX}(0, D_t + AC_t + ASL_t + \varepsilon_t). \tag{18}$$

Each of the three terms represents a separate motivation for investment. To maintain the existing capital stock at its current value, the firm must order enough to replace discards. The firm is assumed to adjust orders above or below discards in response to two additional pressures. The adjustment for capital AC represents the response to discrepancies between the desired and actual capital stock. The adjustment for supply line ASL represents the response to the quantity of capital in the supply line, that is, capital which has been ordered but not yet received.

The adjustment for capital is assumed to be proportional to the gap between desired capital stock DK and the actual stock. Desired capital is determined from the desired rate of production P* and the capital/output ratio κ:

$$AC_t = \alpha_K(DK_t - K_t) \tag{19}$$

$$DK_t = \kappa \cdot P^*_t \tag{20}$$

The adjustment for capital creates a straightforward negative feedback loop. When desired production exceeds capacity orders for capital will rise above discards until the gap is closed. An excess of capital similarly causes orders to fall below replacement until the capital stock falls to meet the desired level. Note, however, that due to the capital acquisition lag this negative loop contains a significant phase lag element, introducing the possibility of oscillatory behavior. The aggressiveness of the firm's response is determined by the adjustment parameter α_K.

The adjustment for the supply line is formulated analogously. Orders are adjusted in proportion to the discrepancy between the desired supply line DSL and the actual supply line:

$$ASL_t = \alpha_{SL}(DSL_t - SL_t). \tag{21}$$

In general the supply line of unfilled orders may include several stages of the capital acquisition process such as orders in planning, orders in the backlog of the supplier, and orders under construction. In the experiment these are aggregated into the backlog of unfilled orders B, thus SL=B. The desired supply line is given by

$$DSL_t = D_t \cdot \Lambda_t. \tag{22}$$

To ensure an appropriate rate of capital acquisition a firm must maintain a supply line proportional to the capital acquisition delay. If the acquisition delay rises, firms must plan for and order new capital farther ahead, increasing the desired supply line. The desired supply line is based on the capital discard rate – a quantity readily anticipated and subject to little uncertainty. To illustrate the logic of the supply line adjustment, imagine an increase in desired capital. Orders will rise due to the gap between desired and actual capital stock. The supply line will fill. If orders in the supply line were ignored ($\alpha_{SL}=0$), the firm would place orders through the capital stock adjustment, promptly forget that these units had been ordered, and order them again. The supply line adjustment creates a second negative feedback loop which reduces orders for new capacity if the firm finds itself overcommitted to projects in the construction pipeline, and boosts orders if there are too few. It also compensates for changes in the construction delay, helping ensure the firm receives the capital it requires to meet desired production. α_{SL} reflects the firm's or subject's sensitivity to the supply line.

The decision rule in equations (18-22) is intendedly quite simple. Orders are determined on the basis of information locally available to the firm itself. Information an individual firm is unlikely or unable to have, such as the value of the equilibrium capital stock or the cost minimizing solution to the nonlinear optimal control problem, is not used. The firm's forecasting process is rather simple: capacity is built to meet current demand. The rule includes appropriate nonlinearity to ensure robust results: orders remain nonnegative even if there is a large surplus of capital. The rule also expresses the corrections to the order rate as linear functions of the discrepancies between desired and actual quantities. Undoubtedly the ordering rules of firms are more complex, and other work such as Senge (1980) considers various subtleties. One can think of the linear part of the rule as the first term of the Taylor series expansion of the more complex underlying investment rule. A large literature in psychology documents the ability of linear decision rules to provide excellent models of decision-making, even when interactions are known to exist [Dawes (1982), Camerer (1981)].

It is useful to interpret the rule in terms of the cognitive processes of the agents. The ordering rule can be interpreted as an example of the anchoring and adjustment heuristic [Tversky and Kahnemen (1974)]. In anchoring and adjustment, a subject attempting to determine an unknown quantity first anchors on a known reference point and then adjusts for the effects of other cues which may be less salient or whose effects are obscure. For example, a firm may estimate next year's sales by anchoring on current sales and adjusting for factors such as macroeconomic expectations, anticipated competitor pricing, etc. Studies have shown anchoring and adjustment to be a widespread heuristic. Indeed, anchoring is so common that many people use it inappropriately. Numerous studies have documented situations in which the adjustments are insufficient or in which judgments are inadvertently anchored to meaningless cues [Hogarth (1987)]. In the experimental context, the capital discard rate forms an easily anticipated and interpreted point of departure for the determination of orders. Replacement of discards will keep the capital stock of the firm constant at its current level (assuming the capital acquisition delay remains constant). Adjustments are then made in response to the adequacy of the capital stock and supply line. No assumption is made that these adjustments are in any way optimal or that firms actually calculate the order rate as given in the equations. Rather, pressures arising from the factory floor, from the backlog of unfilled orders and disgruntled customers, and from commitments to projects in the construction pipeline cause the firm to adjust its investment rate above or below the level which would maintain the status quo. For agents in the experiment the interpretation is parallel: replacing discards to maintain the status quo is a natural anchor. Adjustments based on the adequacy of the capital stock and supply line are then made. Again, there is no presumption that subjects explicitly calculate the adjustments using the formulae in the equations [see e.g. Einhorn, Kleinmuntz, and Kleinmuntz (1979)].

The adjustment parameters α_K and α_{SL} reflect the firm's or subject's response to disequilibrium: large values indicate an aggressive effort to bring capacity and the supply line in line with their desired levels; small values indicate a higher tolerance for, or negligence of, discrepancies between desired and actual stocks. For both the real firm and the subjects, the hypothesis that decisions are made via a heuristic such as the proposed rule is motivated by the observation that the complexity of determining the optimal rate of investment overwhelms the abilities of the managers and the time available to make decisions [Simon (1982)].

To test the rule only the two adjustment parameters α_K and α_{SL} need be estimated. All other data required to determine orders are presented directly to the subjects. Maximum likelihood estimates of the parameters for each trial were found by grid search of the parameter space, subject to the constraints $\alpha_K, \alpha_{SL} \geq 0$. Assuming the errors ε are Gaussian white noise then the maximum likelihood estimates of such nonlinear functions are given by the parameters which minimize the sum of squared errors. Such estimates are consistent and asymptotically efficient, and the usual measures of significance such as the t-test are asymptotically valid [Judge et al. (1980)].

Estimates for 49 trials together with t-statistics are given in table 2. The model's ability to explain the ordering decisions of the subjects is excellent. R^2 varies between 33% and 99+%, with an overall R^2 for the pooled sample of 85%.[4] All but two of the estimates of α_K are highly significant. The supply line adjustment parameter is significant in 22 trials, and not significantly different from zero in 27. Of course, zero is a legitimate value for α_{SL}, and the estimate of α_{SL} for 23 subjects is zero. The estimates of α_{SL} range from 0 to 4.4 while the mean 95% confidence band for the zero estimates is .17, less than 4% of the range of α_{SL}, indicating that the 23 zero estimates of α_{SL} are quite tight.

6. Simulation of the estimated decision rules

The estimation results indicate that the model is a good representation of the subjects' decision-making. Sterman (1988a) analyzes the estimated parameters and identifies several 'misperceptions of feedback' which are responsible for the subjects' poor performance.

One of these is the tendency for subjects to give insufficient attention to the supply line, as indicated by the large number of small estimates for α_{SL}. By ignoring the supply line subjects continue ordering even after the construction pipeline contains sufficient units to correct any stock discrepancy. Such overordering is a major source of instability in the closed loop system. The present concern, however, is the relationship between the estimated parameters and the regimes of behavior in the model. Even though the subjects do not behave optimally in disequilibrium, one might expect that their decision rules would ultimately return the system to a low cost equilibrium. Simulation of the estimated rules shows this is not the case.

The rightmost column of Table 2 indicates the mode of behavior produced by simulation of the decision rule with the estimated parameters. The parameters estimated for thirty subjects (61%) are stable. Most of these produce overdamped behavior of the capital stock in response to the step input. Seven parameter sets produce limit cycles of period 1 and one produces period 5. The parameters which characterize eleven subjects (22%) produce chaos. Inspection of table 2 shows that the subjects whose parameters are stable performed best in the task while those whose parameters produce periodic behavior or chaos generally had the highest costs. 1-way ANOVA confirmed the relationship: the costs achieved by the subjects strongly depend on the mode produced by simulation of the estimated decision rule ($p<.01$ when the modes were coded as stable, periodic, or chaotic).

Figure 4 shows time domain and phase portraits for simulations of several sets of estimated parameters. In all cases the orbits are roughly egg-shaped, with clockwise flow. To explicate the dynamics, consider figure 4a, showing the period 1 limit cycle produced by the parameters of subject 18 (.62, .43; $R^2 = .86$). In equilibrium desired production must equal capacity; the locus of such points is given by the 45° line. Below the line there is excess capacity and the system is linear and stable; above it there is insufficient capacity and the system is highly nonlinear and unstable. Given the steady input of orders from the goods sector of 500 units, the equilibrium point for the system as a whole lies at (555.55, 555.55). At t=1 capacity is insufficient. Shipments lag new orders, so the backlogs of the goods sector and capital sector grow. Rising desired production induces additional orders from the capital sector, causing rapid increase in desired production. Capacity, held down by the inability of the capital sector itself to fill all orders and the consequent rationing of output, lags behind. As capital stock grows, however, new orders placed by the capital sector slow, and the backlog is shipped at an increasing pace. Desired production peaks at t=6. Capacity now grows rapidly as the capital sector is increasingly able to fill orders. Between t=7 and t=8 capacity overtakes desired production, which falls rapidly since capacity is now large enough to deliver the entire supply line in one period. As desired production plummets, capacity reaches its peak and capital sector orders to fall to zero (t=9). Desired production is then sustained only by the exogenous demand of the goods sector (500 units/period). Depreciation causes capital stock to decline slowly, until at t=15 capital stock has fallen to a level low enough to cause the capital sector to place new orders. However, these new orders cause desired production to rise, and by the next period capacity is once again insufficient to satisfy demand, initiating the next cycle. The dynamics are the same for the period multiples and chaotic solutions, except that the trajectory does not close after one orbit.

The trajectories of chaotic systems are sensitive to initial conditions. The routes of nearby points through phase space diverge exponentially until the initial difference in the positions balloons out to fill the entire attractor. The time average rate of exponential divergence of neighboring points is given by the largest Lyapunov exponent L_+ [Wolf et al. (1985)] which may be defined as

$$L_+ = \lim_{t\to\infty} \frac{1}{t} \log_2 |\mathbf{x}| \tag{23}$$

where the separation vector **x** connects neighboring points in phase space. A positive exponent means nearby points diverge and indicates chaos; a negative exponent denotes convergence of nearby points. Because the Lyapunov exponents describe the long-term average behavior of nearby trajectories, any finite segment of the behavior may diverge from that average, including temporary reversals of sign. Nevertheless, a rough estimate of L_+ is given by the slope of

$$\widehat{L}_+(t)t = \log_2|x| \tag{24}$$

over long intervals. To calculate $\widehat{L}_+(t)t$ for each of the estimated decision rules, production capacity was perturbed by one part in a trillion in the 10,000th period. The separation vector $|x|$ was measured in the two-dimensional space defined by production capacity and desired production. Since desired capacity is the sum of the two backlogs, this output space reflects all three state variables in the system. Figure 5 shows the evolution of $\widehat{L}_+(t)t$ for the parameters of subject 16 after the perturbation. The phase plot for these parameters is shown in figure 4. The average slope of $\widehat{L}_+(t)t$ is clearly positive, indicating the system is chaotic. The magnitude of the Lyapunov exponent is approximately .1 bits/period. The values of $\widehat{L}_+$ for the subjects whose decision rules are chaotic range from about .01 to .1 bits/period, with an average of about .04 bits/period.

The magnitudes of the exponents determine the rate at which information about the state of the system, and hence the ability to predict its trajectory, is lost. The large measurement errors in economic systems [Morgenstern (1963)] dictate severe limits on predictability in chaotic systems. Thus if the states of the model economy were known with the not unrealistic measurement error of about 12% (3 bits of precision), the average Lyapunov exponent of .04 implies the uncertainty in the trajectory would grow to fill the entire attractor after only about 75 periods, corresponding in the experimental system to roughly 5 orbits. Additional precision buys little in additional predictability: cutting measurement error by a factor of two would delay the complete loss of predictability by less than 2 orbits. Of course these calculations assume no external noise, perfect specification of the model, and perfect estimates of the parameters, and thus represent an upper bound on the prediction horizon.

7. Mapping the parameter space

Additional effects of chaos on predictability arise due to uncertainty in the estimated parameters. Figure 6 locates the modes of behavior of the estimated decision rules in parameter space. The estimated parameters are clustered in the region $0 \leq \alpha_K \leq 1$, $0 \leq \alpha_{SL} \leq 1$, with the few outside this region falling approximately along the line $\alpha_{SL}=\alpha_K$. Consistent with intuition, the stable decision rules are confined to the region where α_K is small and α_{SL} is large, while the chaotic decision rules generally involved aggressive stock adjustment and weak supply line adjustment. More aggressive attempts to correct the discrepancy between the desired and actual capital stock are destabilizing: by ordering more aggressively the subject induces a larger increase in total demand, thus exacerbating disequilibrium and encouraging still larger orders in future periods. Conversely, more aggressive response to the supply line is stabilizing by constraining orders as the supply line fills. More formally, α_K determines the gain of the oscillatory negative feedback loop while α_{SL} controls the first-order, stabilizing supply line loop. To test this hypothesis the estimated parameters were regressed on the log of the cost function S. Costs provide a rough measure of instability since high costs indicate large excursions from equilibrium (standard errors in parentheses, trial 1 deleted as an outlier):

$$\ln(S) = \underset{(.13)}{5.3} + \underset{(.33)}{1.7}\alpha_K - \underset{(.30)}{1.1}\alpha_{SL}, \quad R^2=.43, \quad F=16.8 \quad (N=48). \tag{25}$$

The results are highly significant and confirm the overall relationship between the parameters and stability. But is the parameter space as smooth as these results suggest?

Figure 7 maps the parameter space for $0 \leq \alpha_K \leq 1$ and $0 \leq \alpha_{SL} \leq 1$ in steps of .005, representing over 40,000 simulations.[5] This region includes 86% of the estimated parameter sets, clearly showing that the fluctuating steady state solutions, including the chaotic solutions, lie in the managerially meaningful region of parameter space. The resulting bifurcation structure is surprisingly structured. First, the boundary for the bifurcation from fixed point to cyclic attractors appears to be a straight line, with stable solutions satisfying

$$\alpha_{SL} > \underset{(.0085)}{2.29}\alpha_K - \underset{(.0046)}{.706}, \quad R^2 = .99946 \quad (N=42). \tag{26}$$

Rasmussen, Mosekilde, and Sterman (1985) show that the transition from local stability to instability at the equilibrium point involves a Hopf bifurcation. In the region where capacity is inadequate ($\phi<1$) the linearized closed-loop system is oscillatory, but for small values of α_K or large values of α_{SL}, the eigenvalues lie inside the unit circle, producing damped behavior and a stable fixed-point attractor. As the parameters become less stable (larger α_K or smaller α_{SL}) the eigenvalues cross the unit circle and the system produces expanding oscillations. These fluctuations are ultimately bounded by the nonlinearities, particularly the nonnegativity constraint on orders and the flexible utilization of capacity in equation (1).

Inside the region of fluctuating steady state solutions, several features are apparent. Note first the striations of periodic behavior which cut across the space at a somewhat shallower angle than the stability/instability boundary. The bands are thicker near the transition to stability and thinner away from it. Second, note the several large regions in which the periodic solutions are very sparse.

Figure 8 magnifies the region $.55\leq\alpha_K\leq.65$, $.50\leq\alpha_{SL}\leq.60$ by a factor of ten in each direction. Both the chaotic regions and the bands of periodic behavior are now seen to contain irregularly distributed islands of other periodicities. Further magnification (not shown) reveals still more such islands. Such irregularity is characteristic of the fractal boundaries common in the bifurcation maps of many systems.

Simple though the model is, it is capable of generating a wide array of complex behaviors. The complexity of the parameter space shows that even small errors in estimates of the parameters may have large qualitative effects on the mode of behavior produced by the system. Indeed, the simulations here do not exhaust the possibilities. All the simulations described here involved no external forcing (goods sector orders for capital were constant). Larsen, Mosekilde, and Sterman (1988) have shown that sinusoidal forcing in goods sector orders (mimicking the effects of the business cycle or other cyclical modes in the economy) causes mode-locking, quasi-periodic solutions, and a devil's staircase to emerge.

8. Discussion

It is common in the social sciences to assume that decision-making behavior and thus the dynamics of human systems are, if not optimal, then at least stable. These results show that formal rules which characterize actual managerial decision making can produce an extraordinary range of disequilibrium dynamics, including chaos.

Such complexity suggests strong lessons for modelers of economic and social dynamics. The experiment shows that the regimes of fluctuating steady-state behavior, including chaos, lie squarely in the middle of the realistic region of parameter space. In consequence modelers can ignore nonlinear dynamics only at their peril. Models of economic and social dynamics should portray the processes by which disequilibrium conditions are created and dissipated. They should not assume that the economy is in or near equilibrium at all times nor that adjustment processes are stable. Models should be formulated so that they are robust in extreme conditions, since it is the nonlinearities necessarily introduced by robust formulations that crucially determine the modes of the system [see Day (1984)].

At the same time a number of questions regarding the the generalization of the results to the real world and the practical significance of chaos must be asked. Chaos is a steady-state phenomenon which manifests over very long time frames, but many policy-oriented models are concerned with transient dynamics and nearly all with time horizons much shorter than those used in the analysis of chaotic dynamics. For example the simulations here were run for 10,000 periods or more. Over such extended time horizons the parameters of the system cannot be considered static but will themselves evolve with learning, evolutionary pressures, and exogenous changes in the environment. There is evidence [Sterman (1988a), Bakken (1988)] that subjects begin to learn within just a few cycles, modifying the parameters of their ordering function. It appears that in the present experiment learning slowly moves the subjects away from the chaotic region towards the region of stability. However, the existence of chaos may itself hamper learning. Even though deterministic cause-effect relations exist in chaotic systems, it is impossible to predict the effects of small changes in initial conditions or parameters. Does such 'randomness' slow the discovery of cause and effect by agents in the economy and thus hinder learning or evolution towards efficiency? Indeed, does learning alter the parameters of decision rules so that systems evolve towards or away from the chaotic regime? Some argue that chaos may be adaptive. In a world whose 'fitness space' contains many local optima, a decision rule that produces chaos, by constantly exploring new pathways, may help a system evolve faster than a stable, incremental decision making strategy [Prigogine and Sanglier (1987), Allen (1988)].

Chaos places an upper bound on prediction, but is that bound a binding constraint in social systems? Real social systems are bombarded by broadband noise, and it is well known that such random shocks severely degrade predictability. Does the magnitude of stochastic shocks swamp the uncertainty in trajectories caused by chaos? How does the existence of chaotic regimes in a model influence its response to policies, and the predictability of that response? Particularly troubling here is the potential for fractal basin boundaries in both initial condition and parameter space. Policy interventions often imply changes in the parameters of a decision rule or model. How can policy analysis be conducted if the "policy space" contains fractal basin boundaries? In such systems parameter changes on the margin may produce unpredictable qualitative changes in behavior, as illustrated by the fractal distribution of modes shown in figures 7-8. Therefore learning and experience may not transfer to circumstances which differ only slightly. Learning often involves a hill-climbing procedure of incremental movement towards a profit-maximizing peak in parameter space. How well can agents negotiate that space when the landscape not only has many local optima but is fractal as well? The development of principles for policy design in such systems is a major area for future research. The practical significance of chaos and other nonlinear phenomena in policy-oriented models of social and economic behavior remains clouded while these questions are unanswered. Some of these questions may be resolved by further application of the experimental techniques demonstrated here.

9. Conclusions

The discovery of nonlinear phenomena such as deterministic chaos in the physical world naturally motivates the search for similar behavior in the world of human behavior. Yet the social scientist faces difficulties in that search which do not plague the physicist, at least not to the same degree. Aggregate data sufficient for strong empirical tests simply do not exist for many of the most important social systems. Social systems are not easily isolated from the environment. The huge temporal and spatial scales of these systems, vast number of actors, costs and ethical concerns make controlled experiments on the systems themselves difficult at best. Finally, the laws of human behavior are not as stable as the laws of physics. Electrons do not learn, innovate, collude, or redesign the circuits in which they flow.

Laboratory experiments appear to provide a fruitful alternative. Since experiments on actual firms and national economies are infeasible, simulation models of these systems must be used to explore the decision making heuristics of the agents. Such experiments create 'microworlds' in which the subjects face physical and institutional structures, information, and incentives which mimic (albeit in a simplified fashion) those of the real world. It appears to be possible to quantify the decision making heuristics used by agents in such experiments and explain their performance well. Simulation then provides insight into the dynamic properties of the experimental systems.

These results demonstrate that chaos can be produced by the decision making processes of real people. The experiment presented subjects with a straightforward task in a common and important economic setting. The subjects' behavior is modeled with a high degree of accuracy by a simple decision rule consistent with empirical knowledge developed in psychology and long used in economic models. Simulation of the rules produces chaos for a significant minority of subjects. Chaos may well be a common mode of behavior in social and economic systems, despite the lack of sufficient information to detect it in aggregate data.

Notes

1 Journal of Economic Theory, 40 (1) 1986, Journal of Economic Behavior and Organization, 8 (3), September 1987; System Dynamics Review, 4, 1988, European Journal of Operational Research, 35, 1988. See also Goodwin, Kruger, and Vercelli (1984) and Prigogine and Sanglier (1987).

2. Conservation of output requires $P = A + AG$. But $A + AG = B \cdot \phi + BG \cdot \phi = \phi(B + BG) = (P/P^*)(B + BG) = P$. By Little's law, the average residence time of items in a backlog is the ratio of the backlog to the outflow, here given by $\Lambda = (B + BG)/(A + AG)$. By eq. 4-5, $(B + BG)/(A + AG) = (B + BG)/(B \cdot \phi + BG \cdot \phi) = 1/\phi$.

3. Disks suitable for IBM PC's and compatibles, or for the Apple Macintosh, are available from the author.

4. Note that the function $O=f(\cdot)$ does not contain an estimated regression constant. Thus the correspondence of the estimated and actual capital orders, not just their variation around mean values, provides an important measure of the model's explanatory power. Since the residuals e need not satisfy $\Sigma e_t = 0$, the conventional R^2 is not appropriate. The alternative $R^2 = 1 - \Sigma e_t^2 / \Sigma O_t^2$ is used (Judge et al. 1980). This R^2 can be interpreted as the fraction of the variation in capital orders around zero explained by the model.

5. Each simulation was 10,000 periods long. The first 8000 were discarded in assessing the steady state mode of behavior. 10 digit accuracy was used. A simulation was assumed to be chaotic if the trajectory did not close after 100 orbits.

References

Allen, P., 1988, Dynamic models of evolving systems, System Dynamics Review, 4.

Bakken, B., 1988, Learning system structure by exploring computer games, presented at the 1988 International System Dynamics Conference, San Diego, CA, July.

Barnett, W. and P. Chen, 1988, The aggregation theoretic monetary aggregates are chaotic and have strange attractors: an econometric application of mathematical chaos, in Barnett, Berndt, and White (eds.), Dynamic Econometric Modeling, Cambridge University Press, Cambridge.

Benhabib, J. and R. Day, 1981, Rational choice and erratic behavior, Review of Economic Studies, 48, 459-471.

Brehmer, B., 1987, Systems design and the psychology of complex systems, in Rasmussen, J. and P. Zunde (eds), Empirical Foundations of Information and Software Science III, Plenum, New York.

Brock, W., 1986, Distinguishing random and deterministic systems: abridged version, Journal of Economic Theory, 40, 168-195.

Camerer, C., 1981, General conditions for the success of bootstrapping models, Organizational Behavior and Human Performance, 27, 411-422.

Candela, G., and A. Gardini, 1986, Estimation of a non-linear discrete-time macro model, Journal of Economic Dynamics and control, 10, 249-254.

Chen, P., 1988, Empirical and theoretical evidence of economic chaos, System Dynamics Review, 4, 81-108.

Dana, R., and P. Malgrange, 1984, The dynamics of a discrete version of a growth cycle model, in Ancot, J. (ed.), Analysing the Structure of Econometric Models, Martinus Nijhoff, The Hague.

Dawes, R., 1982, The robust beauty of improper linear models in decision making, in Kahneman, Slovic, and Tversky (eds), Judgment under uncertainty: Heuristics and biases, Cambridge University Press, Cambridge.

Day, R., 1982, Irregular Growth Cycles, American Economic Review, 72, 406-414.

Day, R., 1984, Disequilibrium Economic Dynamics: A Post-Schumpeterian Contribution, Journal of Economic Behavior and Organization, 5, 57-76.

Day, R., and W. Shafer, 1985, Keynesian chaos, Journal of Macroeconomics, 7, 277-295.

Einhorn, H., D. Kleinmuntz, and B. Kleinmuntz, 1979, Linear regression and process-tracing models of judgment, Psychological Review, 86, 465-485.

Forrester, J. 1961, Industrial Dynamics, MIT Press, Cambridge.

Goodwin, R., 1951, The nonlinear accelerator and the persistence of business cycles, Econometrica, 19, 1-17.

Goodwin, R., M. Kruger, and A. Vercelli, 1984, Nonlinear models of fluctuating growth, Springer Verlag, Berlin.

Grandmont, J.-M., 1985, On endogenous competitive business cycles, Econometrica, 53, 995-1046.

Grassberger, P., and I. Procaccia, 1983, Measuring the strangeness of strange attractors, Physica, 9D, 189-208.

Grether, D., and C. Plott, 1979, Economic theory of choice and the preference reversal phenomenon, American Economic Review, 69, 623-638.

Hogarth, R., 1987, Judgment and Choice, 2nd ed., John Wiley, Chichester.

Hogarth, R., and M. Reder, 1987, Rational Choice: The contrast between economics and psychology, University of Chicago Press, Chicago.

Holt, C., F. Modigliani, J. Muth, H. Simon, 1960, Planning Production, Inventories, and Work Force, Prentice-Hall, Englewood Cliffs, N.J.

Judge et al., 1980, The Theory and Practice of Econometrics, Wiley, New York.

Larsen, E., E. Mosekilde, and J. Sterman, 1988, Entrainment between the economic long wave and other macroeconomic cycles, presented at the IIASA Conference on Instabilities in Long-Term Economic Growth, Novosibirsk, USSR.

Low, G., 1980, The multiplier-accelerator model of business cycles interpreted from a system dynamics perspective, in J. Randers (ed.), Elements of the System Dynamics Method, MIT Press, Cambridge.

MacKinnon, A., and A. Wearing, 1985, Systems analysis and dynamic decision making, Acta Psychologica, 58, 159-172.

Metzler, L., 1941, The nature and stability of inventory cycles, Review of Economic Statistics, 23, 113-129.

Morgenstern, O., 1963, On the Accuracy of Economic Observations, 2nd ed., Princeton University Press, Princeton.

Plott, C. R., 1986, Laboratory experiments in economics: The implications of posted price institutions, Science, 232 (9 May), 732-738.

Prigogine, I., and M. Sanglier, 1987, Laws of nature and human conduct, Task Force of Research Information and Study on Science, Brussels.

Ramsey, J., and H.-J. Yuan, 1987, Bias and error bars in dimension calculations and their evaluation in some simple models, New York University.

Ramsey, J., C. Sayers, and P. Rothman, 1988, The statistical properties of dimension calculations using small data sets: some economic applications, RR88-10, New York University Department of Economics.

Rasmussen and Mosekilde, 1988, Bifurcations and chaos in a generic management model, European Journal of Operational Research, 35, 80-88.

Rasmussen, S., E. Mosekilde and J. Sterman, 1985, Bifurcations and chaotic behavior in a simple model of the economic long wave, System Dynamics Review, 1, 92-110.

Samuelson, P. A., 1939, Interactions between the multiplier analysis and the principle of acceleration, The Review of Economic Statistics, 21, 75-78.

Senge, P., 1980, A system dynamics approach to investment function formulation and testing, Socio-Economic Planning Sciences, 14, 269-280.

Simon., H. A., 1982, Models of Bounded Rationality, The MIT Press, Cambridge.

Slovic, P. and S. Lichtenstein, 1983, Preference reversals: a broader perspective, American Economic Review, 73, 596-605.

Smith, V. L., 1982, Microeconomic systems as an experimental science, American Economic Review, 72 , 923-955.

Smith, V. L., 1986, Experimental methods in the political economy of exchange, Science, 234, (10 October), 167-173.

Sterman, J. D., 1985, A behavioral model of the economic long wave, Journal of Economic Behavior and Organization, 6, 17-53.

Sterman, J. D., 1987, Testing behavioral simulation models by direct experiment, Management Science, 33 , 1572-1592.

Sterman, J. D., 1988a, Misperceptions of feedback in dynamic decision making, Organizational Behavior and Human Decision Processes, forthcoming.

Sterman, J. D., 1988b, Deterministic chaos in models of human behavior: Methodological issues and experimental results, System Dynamics Review, 4, 148-178.

Sterman, 1988c, Modeling managerial behavior: Misperceptions of feedback in a dynamic decision making experiment, Management Science, forthcoming.

Sterman, J. D. and D. L. Meadows, 1985, STRATEGEM-2: a microcomputer simulation game of the Kondratiev Cycle, Simulation and Games, 16, 174-202.

Stutzer, M., 1980, Chaotic dynamics and bifurcation in a macro model, Journal of Economic Dynamics and Control, 2, 353-376.

Takens, F., 1985, Distinguishing deterministic and random systems, in Borenblatt, G, G. Iooss, and D. Joseph (eds.), Nonlinear Dynamics and Turbulence, Pitman, Boston.

Tversky, A. and D. Kahneman, 1974, Judgment under uncertainty: heuristics and biases, Science, 185 (27 September), 1124-1131.

Wolf, A., J. Swift, H. Swinney, and J. Vastano, 1985, Determining Lyapunov exponents from a time series, Physica, 16D, 285-317.

Figure 1. Computer screen showing experimental economy, initial configuration. The subject is about to enter new orders for capital sector.

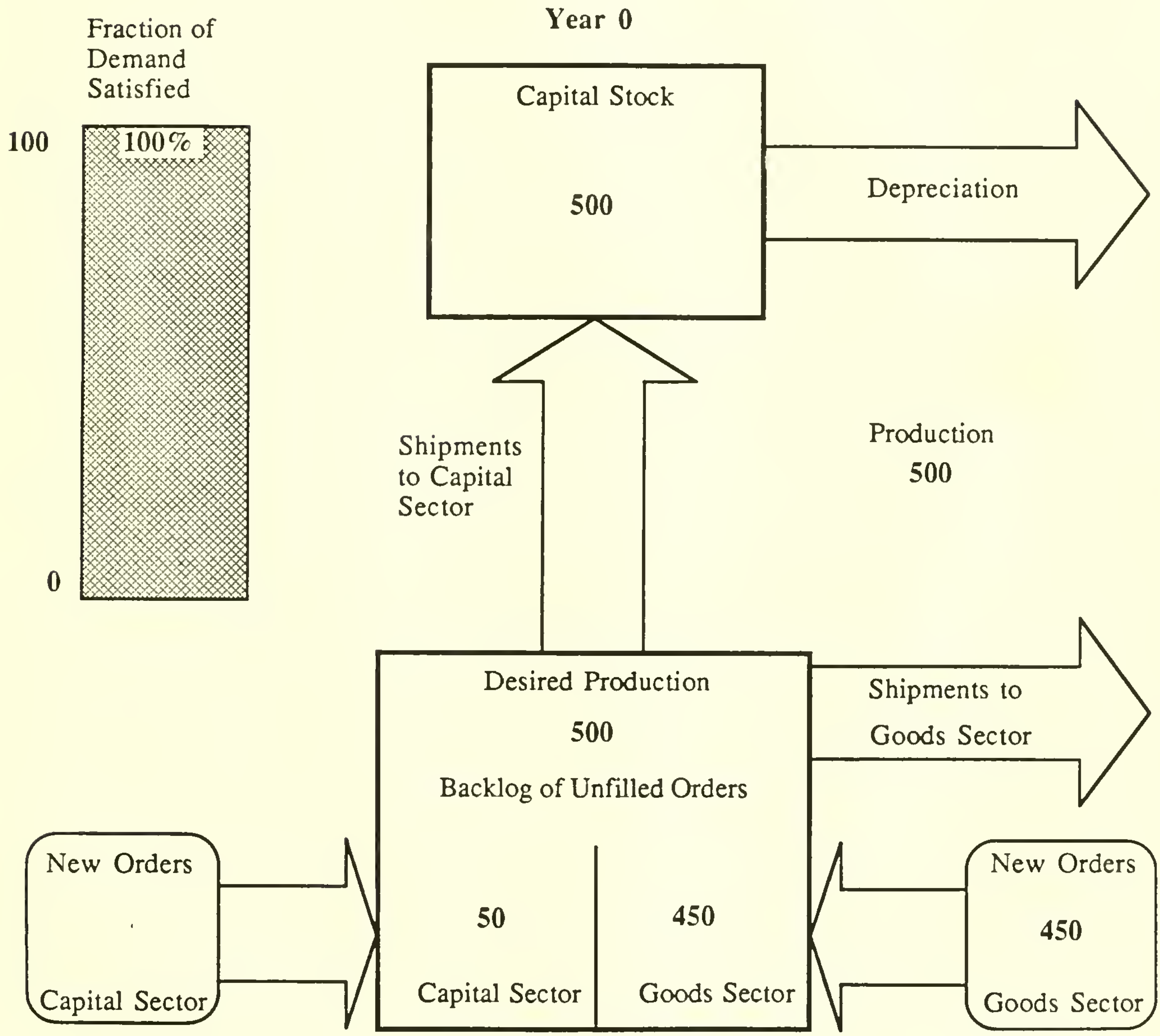


Figure 2. Exogenous orders of the goods sector. Each trial begins in equilibrium. In period 2 there is an unannounced step increase in new orders placed by the consumer goods sector from 450 to 500 units. Compare against subjects' behavior shown in figure 3.

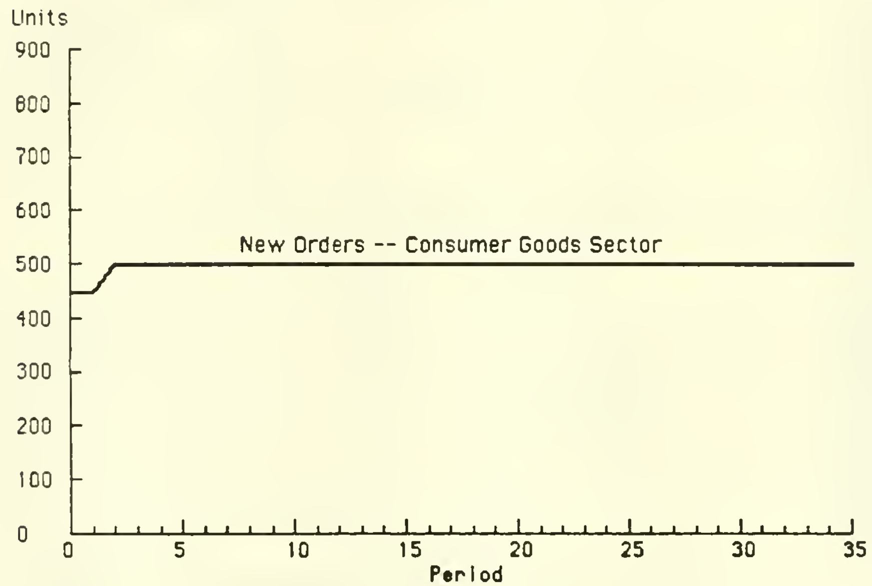


Figure 3. Typical experimental results. N.B.: vertical scales differ.

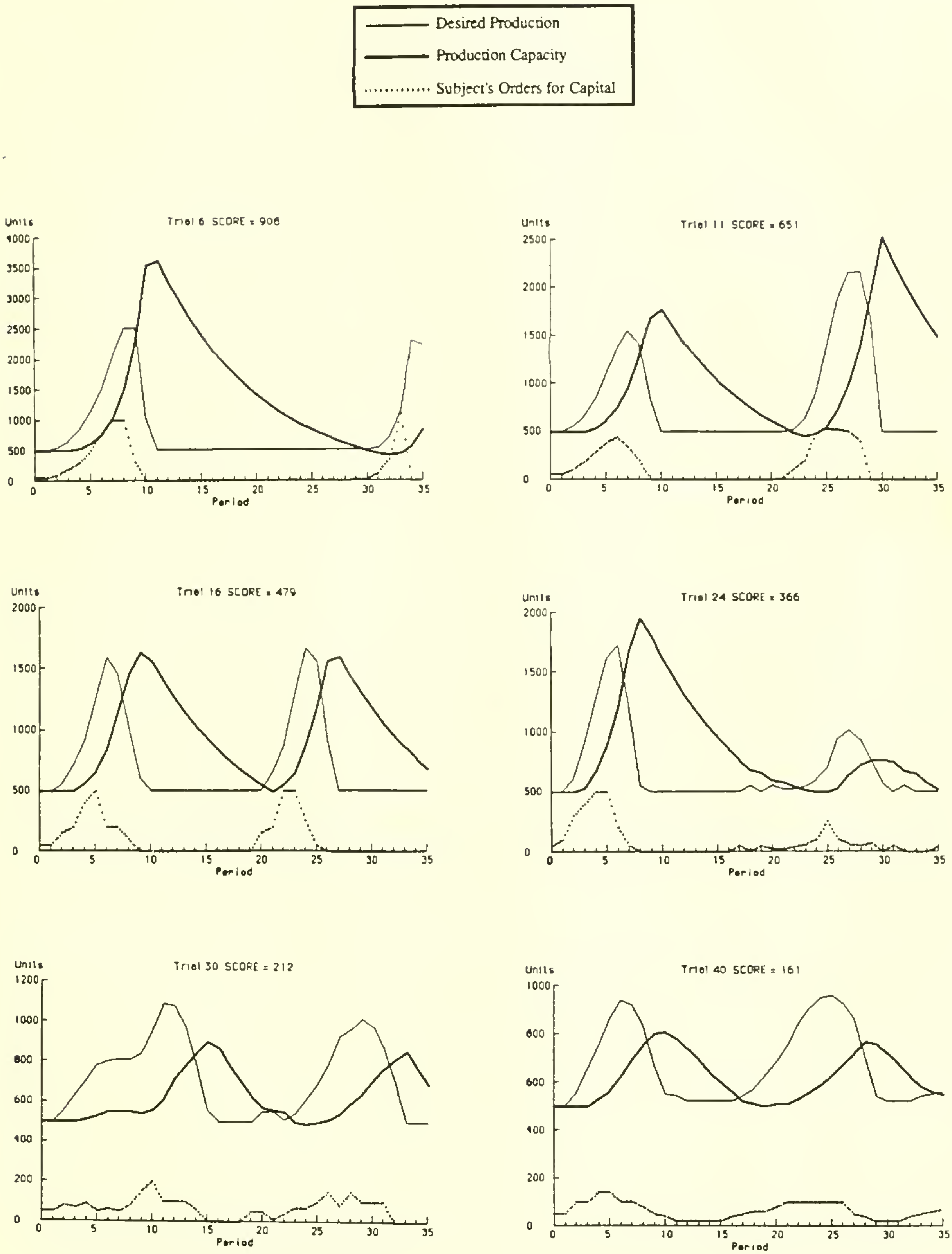


Table 1. Summary of experimental results.

	Experiment (Std. Dev. in parentheses) N=48*	Optimal
Costs (units)	432 (382)	19
Periodicity (periods)	45 (11)	NO CYCLE
1st Capacity Peak (units)	3935 (1346)	630
2nd Capacity Peak (units)	1139 (671)	NO 2nd PEAK
Peak Order Rate (units/period)	518 (501)	260
Minimum Order Rate (units/period)	4 (11)	0
Minimum Fraction of Demand Satisfied (ϕ) (%)	14 (13)	62

* Trial 1 excluded as an outlier (score 8229; capacity peak >27,000; maximum order rate of 6000 units).

Table 2. Estimated parameters and mode of behavior of simulated decision rules

Trial	Score	α_S	Std. Error*		α_{SL}	Std. Error*		R^2	Mode
1	8229	.88	(.04)	a	.24	(.03)	a	.90	C
2	1980	.56	(.10)	a	.26	(.13)	c	.39	P1
3	1335	1.00	(.004)	a	0.00	(.003)		.99+	C
4	1283	2.21	(.06)	a	1.51	(.06)	a	.86	C
5	965	1.35	(.01)	a	.59	(.01)	a	.99	P1
6	908	.38	(.09)	a	0.00	(.10)		.48	P1
7	890	1.28	(.02)	a	.72	(.02)	a	.97	C
8	786	.21	(.02)	a	.16	(.05)	a	.74	S
9	741	1.66	(.05)	a	1.83	(.07)	a	.76	C
10	715	.74	(.04)	a	.60	(.05)	a	.81	C
11	651	.69	(.02)	a	.33	(.03)	a	.94	C
12	638	.46	(.08)	a	.33	(.11)	a	.33	P5
13	634	3.73	(.08)	a	4.44	(.10)	a	.63	P1
14	554	.45	(.04)	a	0.00	(.06)		.87	P1
15	482	.32	(.05)	a	.20	(.10)	b	.57	S
16	479	2.49	(.04)	a	2.90	(.05)	a	.91	C
17	467	.87	(.05)	a	1.38	(.09)	a	.57	S
18	459	.62	(.04)	a	.43	(.05)	a	.86	P1
19	459	.13	(.02)	a	0.00	(.08)		.77	S
20	400	.14	(.04)	a	0.00	(.14)		.37	S
21	399	.23	(.02)	a	.28	(.06)	a	.83	S
22	384	.55	(.04)	a	0.00	(.05)		.90	C
23	373	.17	(.03)	a	.40	(.13)	a	.50	S
24	366	.56	(.06)	a	.11	(.08)		.76	C
25	311	.16	(.02)	a	0.00	(.08)		.80	S
26	267	.16	(.02)	a	0.00	(.09)		.78	S
27	265	.14	(.02)	a	0.00	(.08)		.88	S
28	241	.19	(.03)	a	0.00	(.10)		.73	S
29	229	.33	(.04)	a	0.00	(.09)		.82	P1
30	212	.22	(.02)	a	.15	(.08)	c	.85	S
31	212	.04	(.03)		.29	(.17)		.74	S
32	207	.16	(.02)	a	0.00	(.10)		.80	S
33	196	.09	(.02)	a	0.00	(.14)		.79	S
34	194	.29	(.06)	a	.25	(.13)	c	.45	S
35	189	.09	(.02)	a	0.00	(.11)		.90	S
36	183	.42	(.04)	a	0.00	(.06)		.92	C
37	164	.14	(.04)	a	0.00	(.13)		.74	S
38	164	.22	(.02)	a	0.00	(.06)		.94	S
39	161	.02	(.02)		0.00	(.19)		.87	S
40	161	.19	(.02)	a	.02	(.07)		.93	S
41	156	.24	(.04)	a	0.00	(.11)		.82	S
42	135	.32	(.04)	a	.69	(.14)	a	.74	S
43	120	.16	(.02)	a	0.00	(.09)		.93	S
44	118	.15	(.03)	a	0.00	(.12)		.90	S
45	108	.44	(.03)	a	.40	(.07)	a	.94	S
46	105	.08	(.02)	a	0.00	(.13)		.96	S
47	100	.16	(.03)	a	0.00	(.11)		.91	S
48	90	.60	(.02)	a	.85	(.05)	a	.96	S
49	77	.11	(.03)	a	.19	(.17)		.90	S
Mean:	591	.55			.40		Pooled R^2:	.85	

*t statistic for test of H_0: $\alpha = 0$ significant at a: .01; b: .05; c: .10 level. Mode: S=Stable, Pn=period n, C=Chaotic

Figure 4. Simulation of estimated decision rules. Note similarity to the experimental results (figure 3).

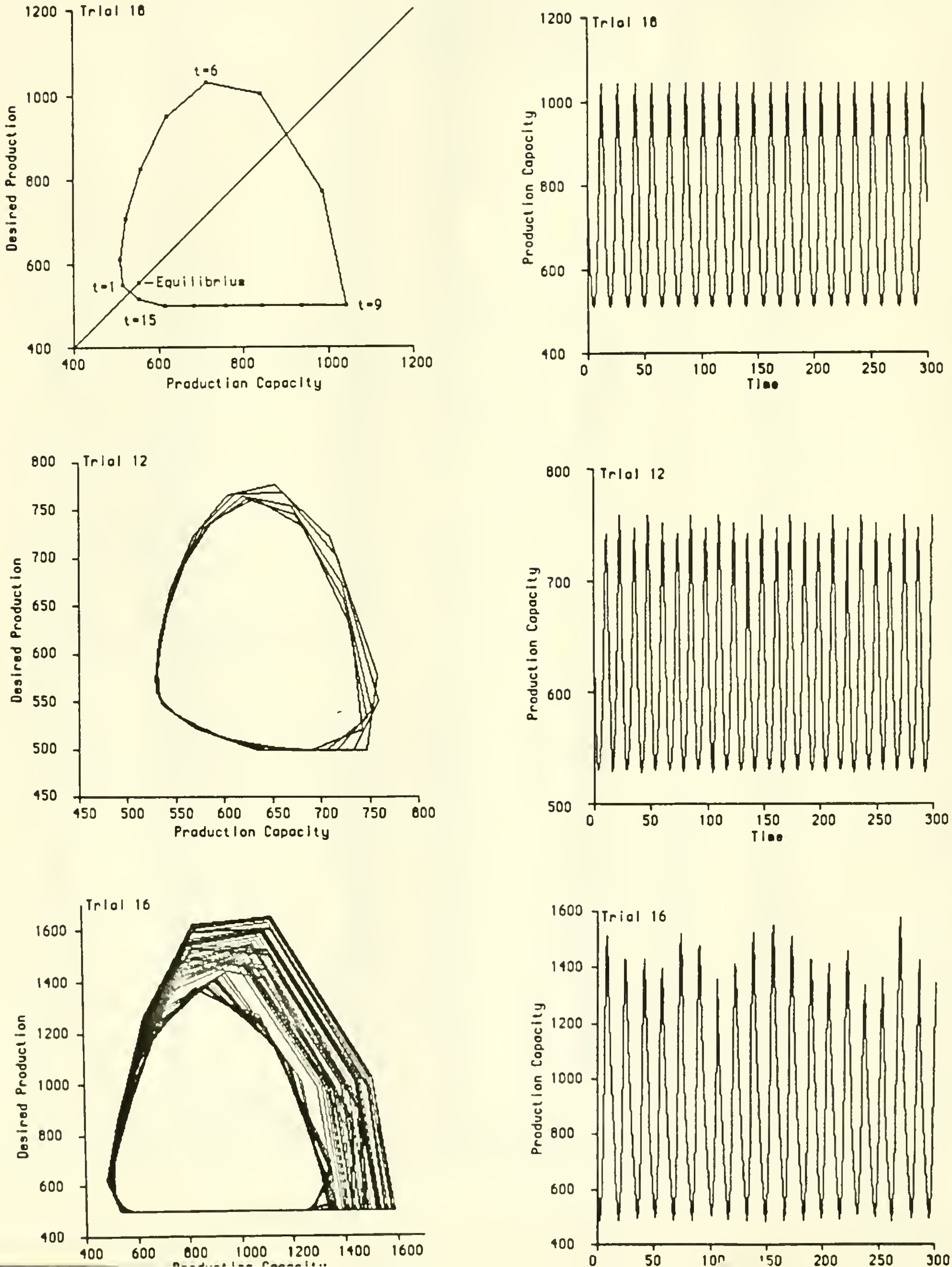


Figure 5. Evolution of $\log_2|\mathbf{x}|$, the distance between two neighboring trajectories, for simulation with parameters of subject 16 (2.49, 2.90) after perturbation of capacity by factor of 10^{-12} in period 10,000. The largest Lyapunov exponent, given approximately by the average slope, is positive, indicating that the two trajectories diverge exponentially and the system is chaotic.

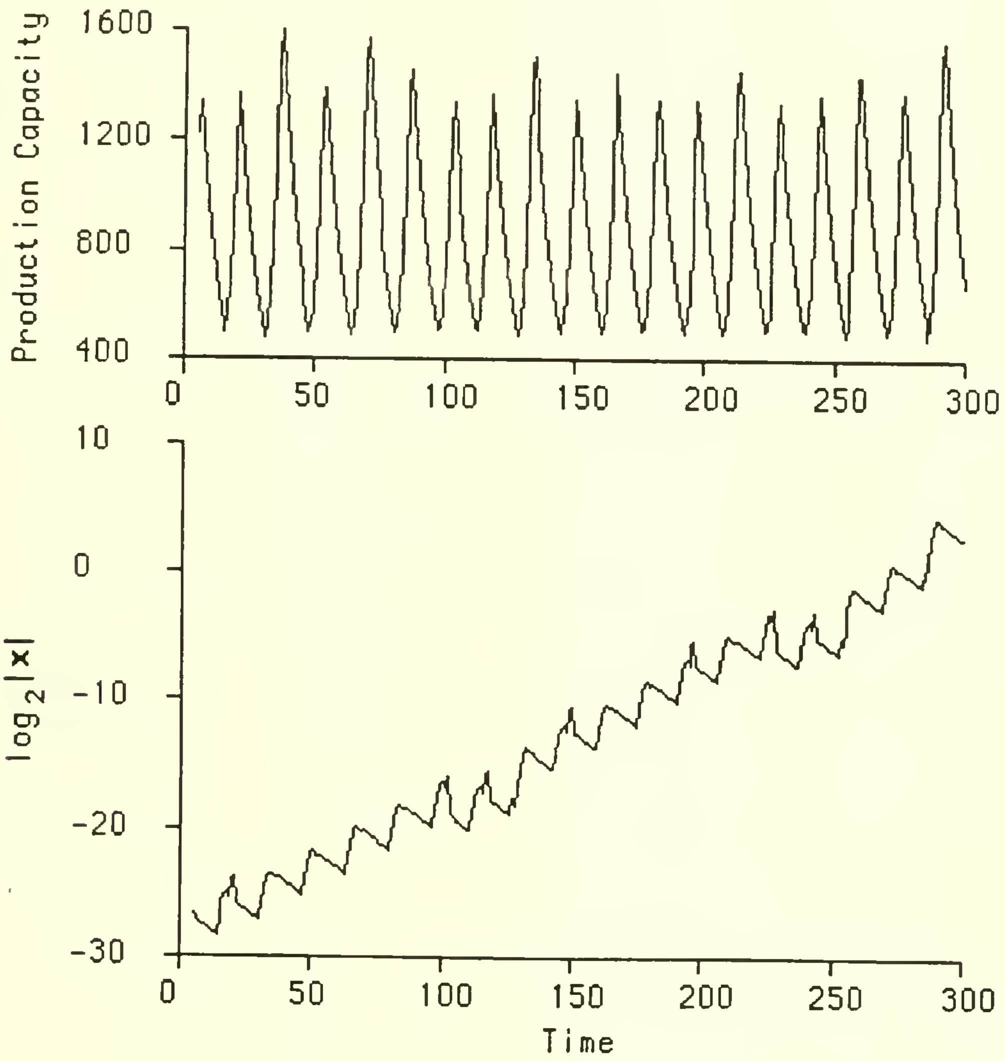
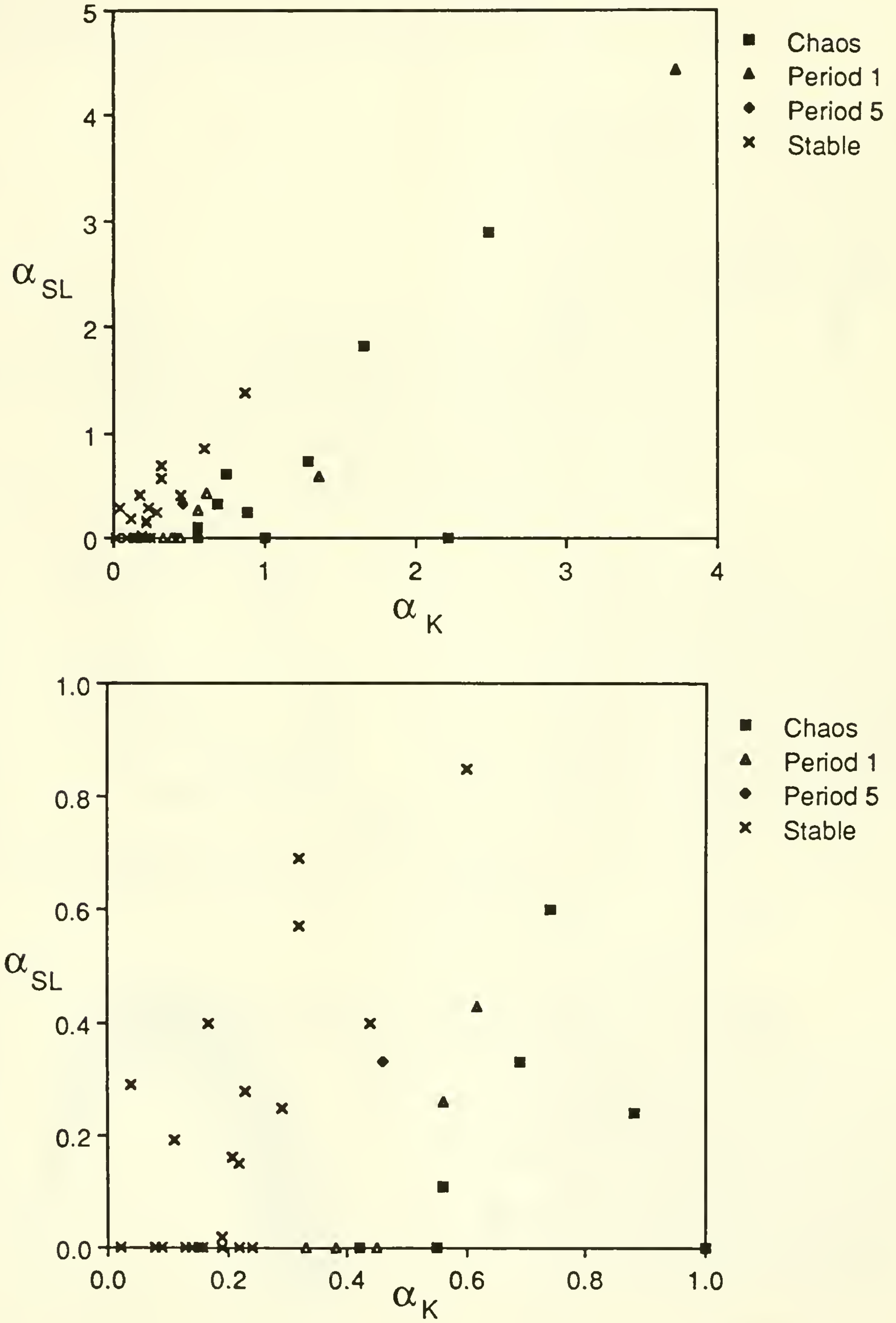


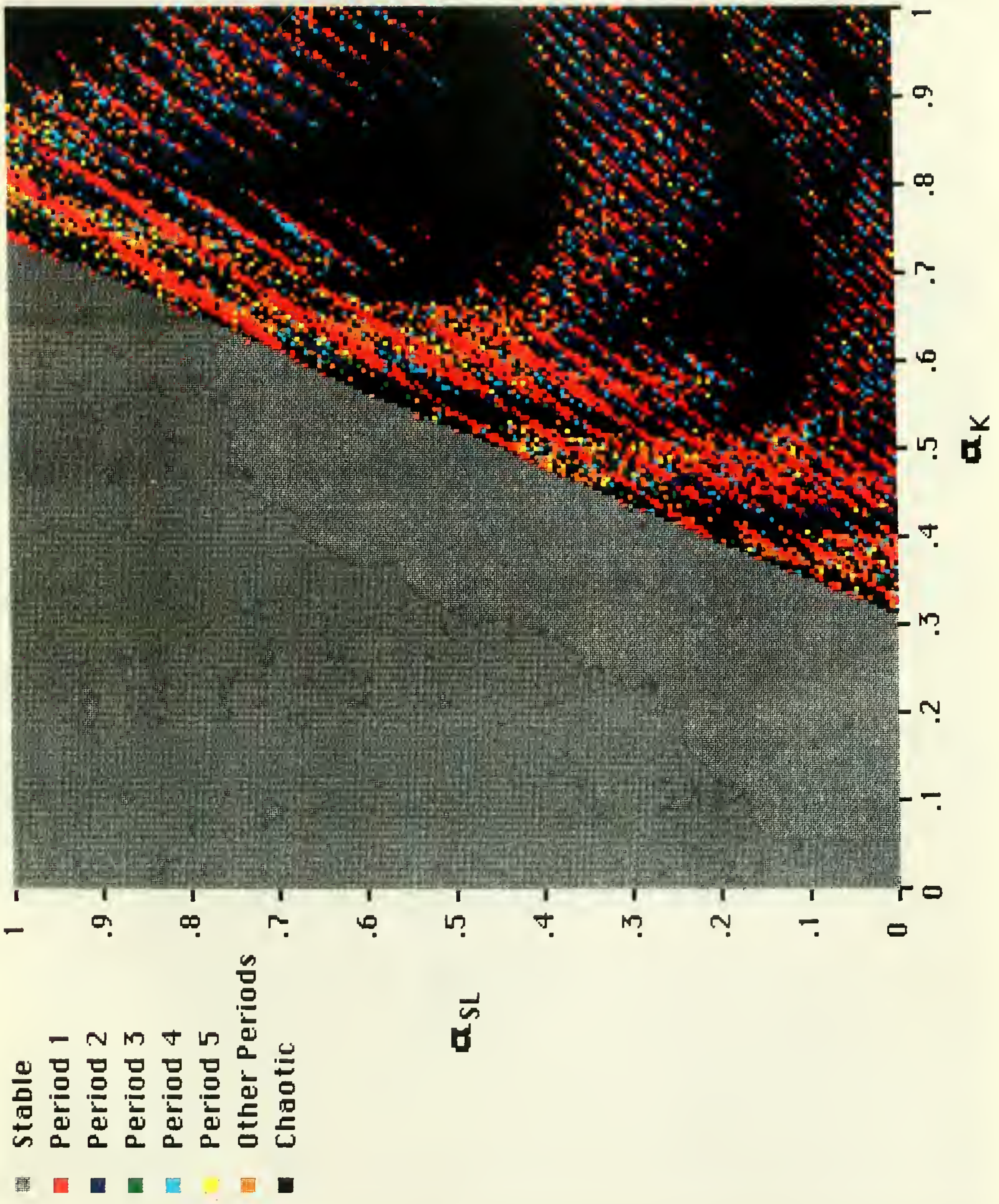
Figure 6. Modes of behavior produced by simulation of the estimated parameters. Lower graph magnifies area bounded by $0 \le \alpha_K \le 1$, $0 \le \alpha_{SL} \le 1$.



(Captions for figures 7 & 8; figures on following pages)

Figure 7. Map of parameter space for $0 \leq \alpha_K \leq 1$ and $0 \leq \alpha_{SL} \leq 1$. Simulations were performed in steps of .005, representing $201^2 = 40{,}401$ simulations.

Figure 8. Map of the region $.55 \leq \alpha_K \leq .65$, $.50 \leq \alpha_{SL} \leq .60$ in increments of .0005. Magnifies figure 7 by a linear factor of 10 in each direction. Note the islands of higher periodicities irregularly distributed within the bands of periodic behavior.


Stable
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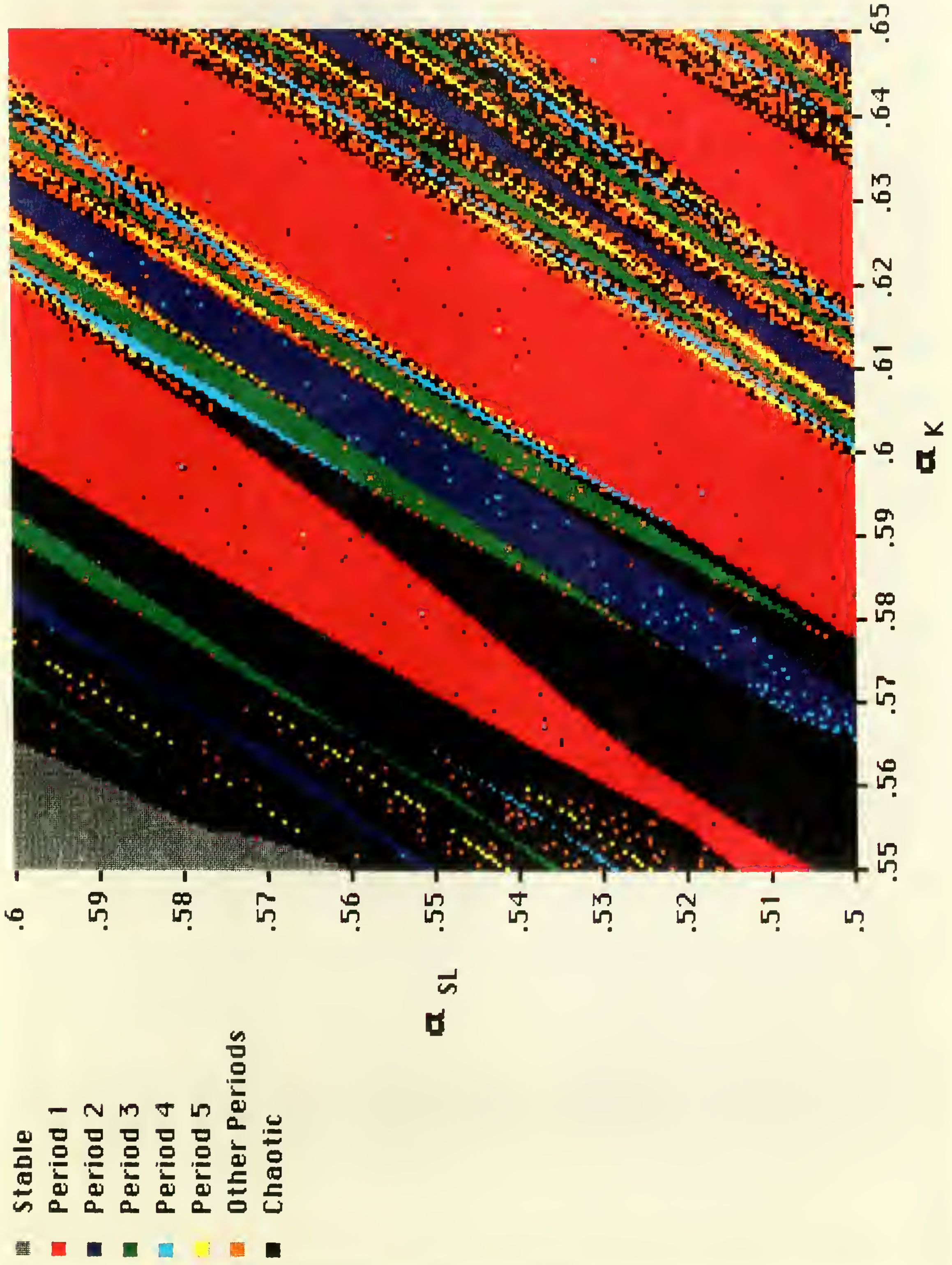
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Date Due

AP 7 '89	JAN 2 1994	
	JUN 1 5 1996	
MY 01 '89		
OC 19 '89		
NOV 15 1990		
JAN 03 1992		
Mar 1 92		
MAY 25 1992		
MAR. 07 1993		

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