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NEW EVIDENCE ON THE NATURE OF SIZE
RELATED ANOMALIES IN STOCK PRICES

by

Terry A. Marsh***

Philip Brown*

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#1379-82

November 1982

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ABSTRACT

This paper is concerned with the size-related anomalies in stock returns reported by Banz (1981) and Reinganum (1981). They showed that small firms have tended to yield returns greater than those predicted by the traditional CAPM. We find that the size effect is linear in the logarithm of size, but reject the hypothesis that the ex ante excess return attributable to size is stable through time. We briefly analyze the Seemingly Unrelated Regression Model (SURM) and a two-step procedure as two alternative estimators of the size effect. Due to the instability of the effect, we find that the estimates are sensitive to the time period studied.

1. INTRODUCTION

There is evidence that excess returns¹ can be earned over time by ranking securities on certain variables. Basu (1977) and Reinganum (1981) report that excess returns on common stocks are a monotone decreasing function of the ranks of their earnings-price (E/P) ratios. Banz (1981) shows that rankings based on firm size can be used to earn excess returns, and Reinganum (1981, p. 20) shows further that "after controlling 'abnormal' returns for market value effects, one could not detect an independent E/P effect."

The size effect has manifested itself beyond academic circles. For example, in the well-publicized Institutional Investor (1980, p. 29) "Is Beta Dead?" article, Richard Michaud of Bache is quoted as follows: "Instead of using the beta model, I would probably like to use one other factor such as capitalization. It's related to return." The American National Bank and Trust Company of Chicago went a step further. It set up a "passive management...Market Expansion Fund" of small firm stocks. The class of "small firm growth stocks," considered in the Wall Street Week (1980) program, provides another illustration. Indeed, the implications of the anomaly for "small firm growth stocks" are especially interesting. As small firm stocks they allegedly earn positive excess returns; while the typical growth stock has a low, not high, E/P ratio, and (presumably) negative, not positive excess returns.

Banz and Reinganum both conclude that their evidence is consistent with misspecification in the capital asset pricing model (CAPM) benchmark used to assess excess returns rather than with market inefficiency [see also Ball (1978)]. However, in common with the examples just cited, they

implicitly assume that the expected size effect, or the distribution of the size effect, is constant over the periods examined. For example, Banz (1981, p. 15) presents mean monthly returns on "arbitrage" portfolios constructed to be long in small firms and short in large firms over the period 1931-1975 and the nine non-overlapping five-year subperiods. He finds a significant overall size effect -- the portfolios earn positive average excess returns over time with a reported t-statistic of 2.99 over the total period. But only two of the nine subperiods have t-statistics greater than 2.0, and in two subperiods the "arbitrage" portfolio strategy yields negative excess returns. Banz (1981, p. 16) attributes the reversals to a lack of diversification caused by portfolio concentration in the stocks of very small firms with high residual risk. This interpretation requires that the underlying size effect is stable over the entire period and that the reversals are due to unusual drawings from a stationary distribution. Without this assumption the t-statistics lose meaning, and the apparent CAPM misspecification is itself misspecified.

We investigate and reject the stability of excess returns obtained by ranking firms according to market value of equity. In some years the distribution of ex ante excess returns for small firms has a positive expected value and the distribution for large firms has a negative expected value, while in other years the effect is reversed. The issue of the stability of size-related excess returns is important to their measurement. But just as importantly, potential explanations of the size anomaly can be evaluated more readily if the anomaly is itself described properly.

Insofar as measurement is concerned, we use a Seemingly Unrelated Regression Model (SURM) and a two-step generalized least squares (GLS) procedure to measure size effects and find that they are statistically

insignifioant over the period January 1967 to June 1979. Over the subperiod January 1967 to December 1975, we find a "positive" but insignificant size effect, that is, small firms have negative excess returns and large firms have positive excess returns. However, Banz (1981) reports a negative size effect over the period 1966-1975 with a t-statistic of 1.55. In addition, over the 1973-1979 subperiod, we find a significantly negative size effect. These seemingly contradictory results are not surprising in light of our finding of instability of the size effect.

If size effects are not constant, some explanations of their existence can be ruled out, others need modifying, and still others suggest themselves. If small firm stocks are expected to earn positive excess returns because of differential commission costs in trading those stocks, or because they tend to provide less diversification service than large firms, then the expected return premium should be positive in every period. Roll's (1981) explanation of size effects in terms of non-trading induced biases in estimated beta coefficients is not consistent with our results. As we discuss in Section 5, it is unclear whether a modified version of the explanation which allowed instability in the pattern of non-trading across firms would be consistent with the results.

There appears to be a seasonal affecting the returns of both small and large firms, which is consistent with the "January effect" reported by Keim (1981). However, even after taking account of a January effect, the instability in the size effect remains. Although excess returns are unstable since 1926, the phenomenon is most pronounced since the late 1950's.

In Section 2, we discuss the alternative estimation procedures and the data in detail. We provide evidence that the anomaly may be treated as linear across size rankings but, as pointed out in Section 3, with an unstable slope. Section 4 evaluates the alternative estimation procedures and presents evidence on the magnitude and size of the anomaly over those periods in which stationarity appears to be a reasonable assumption. Some potential explanations of the size anomaly are considered in Section 5. A brief summary is given in Section 6.

2. METHODOLOGY AND SAMPLE

2.1 The CAPM and the Size Anomaly

The well-known market model written in terms of risk premia is:

$$(R_{it} - R_{ft}) = \alpha_i + \beta_{iM}(R_{Mt} - R_{ft}) + \tilde{e}_{it} \quad i = 1, \dots, N; t = 1, \dots, T \quad (1)$$

where R_{it} = the rate of return on asset i in period t ;

R_{Mt} = the rate of return on the market portfolio of assets in period t ;

\tilde{e}_{it} = a security specific disturbance term;

R_{ft} = the riskless rate in period t ;

$\beta_{iM} \equiv \text{Cov}(R_{it}, R_{Mt}) / \text{Var}(R_{Mt})$;

α_i = an arbitrary constant;

N = total number of assets;

and where E , Cov and Var are expectation, covariance and variance operators, respectively. Equation (1) is justified by the linearity properties of a multivariate normal (or multivariate student-t) joint distribution of asset returns [Fama (1976, Chapter 2)] or by the direct assumption $E(\tilde{e}_{it} R_{Mt}) = 0$, $E(\tilde{e}_{it}, \tilde{e}_{is}) = 0$, $s \neq t$.

The Sharpe-Lintner² CAPM implies that α_1 is an "excess return", i.e., $\alpha_1 = 0$. If there is a size anomaly relative to the Sharpe-Lintner CAPM, α_1 will be related to size for at least some asset i . The CAPM as a null hypothesis obviously provides no clue as to the relation between excess returns and size.

Banz's work (1979, 1981) implies two possible functional forms³ based on Merton's (1977) consumer services model and the information model of Klein and Bawa (1977). Banz argues that the former model implies a negative excess return for very large firms whereas the latter implies positive excess returns for very small firms. His empirical work implies that the appropriate form is a nonlinear declining function of equity (market) value. Reinganum (1981) reports a similar result. However, the distribution of market value of equity across the securities they studied is extremely positively skewed⁴ which may at least in part account for the reported non-linearities. Further, Reinganum does not always adjust for risk differences across size-ranked portfolios.

Rather than form a prior specification for the relation between α_1 and the size of firm i , we use the CRSP daily (beta) excess return file⁵ to estimate the cross-sectional pattern of average excess returns for ten portfolios ranked on size.

2.2 Data

Our primary sample consists of the 566 firms studied by Reinganum (1981) in which a size-related anomaly is reported. The sample is, in turn, a subset of 577 companies analyzed by Latane and Jones (1977).⁶ The reason for using this sample is that it has proven informative about the size anomaly, and Reinganum has generously supplied us with his data. Since all 566 firms were required to have complete quarterly data from June

1967 to December 1975, a survivorship bias is possible. Reinganum extended the series for eight quarters from the fourth quarter of 1975, and we have further extended it to the fourth quarter of 1979.

Of the 566 firms in existence at December 1975, 535 survived through December 1977 (Reinganum's sample) and 496 through December 1979. Since our primary focus is on firm size, Table 1 tabulates the reasons for incomplete data as at December 31, 1979 for each size decile where size is defined as market value of equity at December 1975. Assuming a combination of two listed firms is more likely to result in the exchange's delisting of the stock of the smaller than of the larger, it is not surprising that 45 of the 62 mergers and acquisitions resulted in the disappearance of firms smaller than the median firm size. When we consider firms below the median size, there is no obvious relation between size and the incompleteness of data (except possibly that due to bankruptcy).

A feature of Reinganum's data set is that 98 of the 566 stocks, including all of the smallest 31, are not in the CRSP monthly price relative file because they were not traded on the NYSE.⁷ All but three of the 98 had returns on the CRSP Daily Returns file which were used to construct monthly returns. Because the latter file contains both AMEX and NYSE listed companies, it is likely that Reinganum's data set is more informative about a size-related anomaly than a set confined to the larger firm stocks traded on the NYSE.⁸

Table 2 reports Spearman's rank order correlation coefficients between size (as measured by market value of equity) at June 1967, size at December 1975, leverage at December 1975 (with equity measured at both book and market values) and total assets in December 1966 and December 1975. Total assets and book value of debt and equity are taken from the Compustat

annual tape, and market value of equity from CRSP monthly price files (checked against the Compustat tape).

Table 2 shows a high degree of stability in market value of equity rankings over the period June 1967 to December 1975, the strong correlation between total assets and market value of equity, and the stability in asset ranking from 1966 to 1975. The high degree of stability and correlation among the alternative size measures suggest that our results are not likely to be sensitive to the particular size variable used, nor to the use of a constant size measure through time. The other results in Table 2 are discussed in Section 5 below.

2.3 Linearity of the Size Effect

We turn now to the form of the relation between excess returns and size. To avoid the possible survivorship bias until 1976, we rank securities on market value of equity at December 1975 and estimate the average daily excess return for each security from January 6, 1976 (the second trading day in 1976) to December 29, 1978, using the CRSP (beta) daily excess returns tape. Table 3 contains a summary of the sign and magnitude, relative to standard error, of the securities' excess returns across size deciles. While it is not at all clear how t-statistics are to be interpreted in the context of CRSP's population and its excess return estimation procedure,⁹ one conclusion is reasonably certain: the anomaly does not appear to be powered by small stocks alone. All of the largest 56 firms studied had negative excess returns, on average, from January 1976 to December 1978.¹⁰

To test whether these results are sample specific, the experiment was repeated for the population of all stocks for which market values of equity were available as at the first day of the CRSP daily returns tape (July 2,

1962) and for which any excess returns were available. The results are also presented in Table 3. Although it appears that the monotone effect in the Reinganum sample is not sample specific, the effect is clearly attenuated over the longer period covered by this larger sample. For example, although 10.8% of the t-statistics in Reinganum's sample exceed 2 in absolute value, there are only 5.2% in this category for the expanded sample.

Both the apparently monotonic effect across size deciles and the attenuation over the longer sample period are also shown by the values of the average daily excess return in Table 3. Again, the main difference between the results appears to be that the absolute values of the average excess returns for both the smallest and largest firms are greater in the sub-period. These results foreshadow Section 3 which shows that although the excess returns are linear in log size, the slope of the linear relation changes sign through time. Consequently, the effect measured over a long period during which the slope changes sign will be attenuated relative to a sub-period in which the sign is constant.

Finally, the linearity is perhaps most apparent when illustrated graphically, here using ten portfolios formed from a cross-sectional ranking of the sample securities in ascending order of size which we will use in most of the later results. Figure 1 shows scatter plots of average daily excess returns for Reinganum's sample¹¹ and (a) portfolio size calculated as the mean size of all firms in each size decile, (b) the natural logarithm of portfolio size, and (c) size decile. The plots imply that although the relation between excess return and untransformed firm (here portfolio) size may be non-linear, an approximately linear relation exists when either log size or size decile is used. Further the results

are very similar for log size and size decile, indicating that mere size ranking may contain as much information about the anomaly as size per se. In either case, we proceed with our analysis of monthly data on the assumption of a linear relation between excess return and log size.

2.4 Test Structure

If we assume linearity of the excess return α in log size, (1) may be generalized to include a (constant)¹² size measure:

$$(\tilde{R}_{1t} - R_{Ft}) = \gamma_{01} + \beta_{1M} (\tilde{R}_{Mt} - R_{Ft}) + \gamma_{11} \bar{S}_1 + \tilde{\epsilon}_{1t} \quad (2)$$

$$i = 1, \dots, N; t = 1, \dots, T$$

where \bar{S}_1 = a measure of size for asset 1 applicable to the period (1, T) and γ_{01}, γ_{11} are scalar parameters.

\bar{S}_1 will be assumed constant for any asset 1 over any time period (1, T). Since the relation between excess returns and \bar{S}_1 is cross sectional, it is not possible to estimate a separate γ_{01} and γ_{11} for each asset. The cross sectional information can be incorporated in two ways. First, (1) can be considered separately for each asset 1, and an estimate $\hat{\alpha}_1$ can be obtained by (say) OLS. Then in a second step a common¹³ γ_0 and γ_1 can be estimated from the cross sectional regression:

$$\hat{\alpha}_1 = \gamma_0 + \gamma_1 \bar{S}_1 + \tilde{\eta}_1 \quad i = 1, \dots, N \quad (3)$$

Alternately, since \bar{S}_1 varies across equations, a common γ_0 and γ_1 may be estimated directly using the Seemingly Unrelated Regression Model (SURM) as follows:

$$\begin{bmatrix} \tilde{R}_1^* \\ \tilde{R}_2^* \\ \vdots \\ \vdots \\ \vdots \\ \tilde{R}_N^* \end{bmatrix} = \begin{bmatrix} \underline{1}_T & \tilde{R}_M^* & \underline{0}_T & \dots & \underline{0}_T & \underline{S}_1 \\ \underline{1}_T & \underline{0}_T & \tilde{R}_M^* & & \underline{0}_T & \underline{S}_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \underline{1}_T & \underline{0}_T & \underline{0}_T & & \tilde{R}_M^* & \underline{S}_N \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \beta_{1M} \\ \beta_{2M} \\ \vdots \\ \vdots \\ \gamma_1 \end{bmatrix} + \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \vdots \\ \vdots \\ \vdots \\ \tilde{e}_N \end{bmatrix}, \quad (4)$$

where \tilde{R}_1^* , \tilde{R}_M^* , $\underline{0}_T$ are $(T \times 1)$ time series vectors of $(\tilde{R}_{1t} - \tilde{R}_{Ft})$, $(\tilde{R}_{Mt} - \tilde{R}_{Ft})$, \tilde{e}_{1t} ; and $\underline{1}_T$, $\underline{0}_T$, \underline{S}_1 are $(T \times 1)$ vectors of ones, zeros and (constant) size of asset 1. The \tilde{e}_1 are assumed non-autocorrelated, but (in general) cross correlated, with $E(\tilde{e}_1 \tilde{e}_j') = \sigma_{1j} I_T$ (a $T \times T$ diagonal matrix).

The properties of both estimators are best developed in terms of the error components model. Suppose we stack the OLS equations (1) from which the $\hat{\alpha}_1$ are computed as follows:

$$\begin{bmatrix} \tilde{R}_1^* \\ \tilde{R}_2^* \\ \vdots \\ \vdots \\ \vdots \\ \tilde{R}_N^* \end{bmatrix} = \begin{bmatrix} \underline{1}_T & \underline{0}_T & \dots & \underline{0}_T & \tilde{R}_M^* & \underline{0}_T & \dots & \underline{0}_T \\ \underline{0}_T & \underline{1}_T & \dots & \underline{0}_T & \underline{0}_T & \tilde{R}_M^* & \dots & \underline{0}_T \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \underline{0}_T & \vdots & \dots & \underline{1}_T & \underline{0}_T & \vdots & \dots & \tilde{R}_M^* \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \vdots \\ \alpha_N \\ \beta_{1M} \\ \vdots \\ \vdots \\ \beta_{NM} \end{bmatrix} + \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \vdots \\ \vdots \\ \vdots \\ \tilde{e}_N \end{bmatrix} \quad (5)$$

where the variables are defined and have properties as in (4).

The estimation of α_1 in (1) and (5) as constants implies that the error terms \tilde{e}_1 in (3) arise from estimation error in $\hat{\alpha}_1$ and not from some cross sectionally random (but time-invariant) component of a random α_1 . If α_1 were random with a time invariant component, the removal of fixed effects $(\gamma_0 + \gamma_1 \underline{S}_1)$ as in (4) would leave a random time-invariant element

in the error term $\tilde{\epsilon}_1$. In this case $\tilde{\epsilon}_1$ could be written as:

$$\tilde{\epsilon}_{1t} = \tilde{\mu}_1 + V_{1t}, \quad t=1, \dots, T$$

where $\tilde{\mu}_1$ is an assumed cross sectionally random but time-invariant element and V_{1t} is an (orthogonal) time-dependent error component. Then even with i.i.d. V_{1t} , the $\tilde{\epsilon}_{1t}$ would be autocorrelated, with $E(\tilde{\epsilon}_{1t} \tilde{\epsilon}_{1s}) = E(\tilde{\mu}_1 \tilde{\mu}_1)$ for $t \neq s$. Thus, by assuming non-autocorrelated disturbances $\tilde{\epsilon}_{1t}$ in the SURM model (4), cross-sectionally random μ_1 have been ruled out. Likewise, they are ruled out by the assumption $E(\tilde{\epsilon}_{1t} \tilde{\epsilon}_{1s}) = 0$, $t \neq s$ in (1) and (5). Only firm size is allowed as a cross-sectional fixed effect.¹⁴

Any common fixed effects will be impounded in γ_0 and any security-specific time-varying effects will be in $\tilde{\epsilon}_{1t}$ in (4). Subject to identification,¹⁵ firm-specific fixed effects other than size could be estimated by firm-specific γ_{0i} . Finally, a more general SURM model (4) can be estimated if there are (cross-sectionally) random effects other than size in the model [e.g., Baltagi (1980)].

Since the regressors are identical in each equation in (1) or (5), estimation of (5) by SURM is identical to equation by equation OLS estimation of (1). The covariance matrix of \hat{a}_1 in (5) can be written as $\lambda \Omega$, where $\lambda \equiv$ the (1, 1) element of $[(\mathbf{1}_T \mathbf{R}_M^*)' (\mathbf{1}_T \mathbf{R}_M^*)]^{-1}$, and $\Omega \equiv$ the covariance matrix of the disturbances $\tilde{\epsilon}_1$. Since the source of the $\tilde{\eta}_1$ in (3) is specified as estimation error of \hat{a}_1 , we use GLS regression in the second step (3) with a covariance matrix estimated from the residuals of the first step regressions in (1).

The results from this two-step procedure are virtually identical to those of the SURM (4), and we present only the latter. However, the two-step procedure is comparable with the SURM only if GLS (not OLS) regression is applied in the second step. We provide OLS second step results for com-

parison, and at least for this application¹⁶ the misspecification results in severely understated standard errors in the second step.

2.5 Preliminary Estimates

The two-step procedure (1) and (3) and the SURM (4) were applied to the time series of monthly returns for ten portfolios of all possible members of the sample of 566 companies from January 1967 to June 1979 (150 months). The ten portfolios were formed by ranking the stocks according to market value of equity at June 30, 1967, portfolio one containing the smallest stocks. The results for OLS for equation (3) and the SURM for (4) are given in Table 4 for different periods.¹⁷ The first is the full 150 month period January 1967 to June 1979. The Subperiods I section takes account of the potential survivorship bias in the Reinganum sample prior to 1976 by breaking the overall period at the end of 1975. Finally, since stationarity in (at least) the β_{1M} of (4) may be untenable over long periods, we break the total period into the three 50 month subperiods of Subperiods II. The SURM procedure detects a statistically significant negative size effect only for the period May 1975 to June 1979, (including the period January 1976 to June 1979). Over the earlier subperiod January 1967 to December 1975, the point estimates of the size effect γ_i are positive, though only a two-step procedure using OLS in the second step shows it to be significant.

The results of Table 4 raise two interesting questions. First, a comparison of the results with those reported by Banz (1981, Table 1) reveals that over the subperiod 1966-1975 which most closely corresponds to our subperiod January 1967 to December 1975, he reports a negative size effect with a t-statistic of -1.55. By contrast, our SURM procedure yields

positive, but insignificant, point estimates for the subperiod January 1967 to December 1975. The (overstated) t-statistic for OLS in this subperiod is actually significant. Second, though point estimates in the period January 1967 to December 1975 are positive, those for the approximate two subperiod breakdown January 1967 to February 1971 and March 1971 to April 1975 are both negative. This loose analysis serves only to underscore a general impression that the subperiods in Table 4 are "different". We now consider the possible instability of the size effect in more detail.

3. STATIONARITY OF THE EXCESS RETURNS

3.1 The Techniques and Estimates

Two techniques were used here to study the stationarity properties of the expected size effect premium or discount. One is the recursive residuals technique associated with Brown, Durbin and Evans (1975) and the other is a Kalman filtering technique. Since the results were similar, only those from the Kalman filtering technique are discussed in detail.¹⁸

The Kalman filter technique requires specification of the stochastic process generating the ex ante excess return series $\{\alpha_1\}_t$. The results here are generated under the assumption that α_1 follows a random walk, i.e.,

$$\tilde{\alpha}_{1t} = \alpha_{1,t-1} + \tilde{u}_{1t} \quad \tilde{u}_{1t} \text{ i.i.d. } N(0, \sigma_{u_1}^2) \quad (6)$$

We return to this stochastic process assumption later, noting only at this point that diagnostic checks suggest it is sufficiently well specified. Conditional on the specified process, the technique estimates a time series of stochastic coefficients which has been filtered from the noise or error term as in signal extraction. It also tests whether the resulting series

is stochastic or just reflects sampling variation around a constant parameter. The results indicate that the assumption of a non-stochastic α_1 over the 150 month period is most seriously violated for the smallest and largest portfolios; that the excess returns $\{\hat{\alpha}_{1t}\}$ of the first (smaller firm) six portfolios are positively correlated¹⁹, as are those of the ninth and tenth (the largest firm) portfolios; that the excess returns of the first six portfolios are negatively correlated with those of the ninth and tenth; and that the excess returns of the seventh and eighth portfolios are not well specified by the random walk, but are in fact best described as a constant α_1 over the 150 month time period.²⁰ Previous writers [e.g. Black, Jensen and Scholes (1972)] have found that when portfolios are ranked on $\hat{\beta}_j$, those with a higher $\hat{\beta}_j$ tend to have higher standard errors and residual variances in (1). Since higher $\hat{\beta}_j$ firms tend to be smaller firms, and since the Kalman Filter estimate $\{\hat{\alpha}_{1t}\}$ "extracts" a component of the residual $\{\hat{\epsilon}_{1t}\}$ in (1), we likewise find higher standard errors of $\{\hat{\alpha}_{1t}\}$ for small firms. The above correlation estimates for $\{\hat{\alpha}_{1t}\}$ and $\{\hat{\alpha}_{jt}\}$ would also be consistent with high covariances between the residuals of adjacent Black, Jensen and Scholes (1972) portfolios.²¹

Figure 2 superimposes the time series of the estimated excess returns $\hat{\alpha}_{1t}$ for portfolios 1 (comprising the smallest firms), 5 and 10 over the 150 month period January 1967 to June 1979. The basic pattern shown by the smallest firm portfolio is repeated for each portfolio 2 through 6, although with different variance. Portfolio 10 (and to a lesser degree portfolio 9) whose returns are negatively correlated with Portfolios 1 and 5 displays a time series pattern which is somewhat of a mirror image of the others. The plots suggest that from say January 1969 to December 1973 there existed a relatively stable positive relation between excess return

and size, and from January 1974 to June 1979 there was a relatively stable negative relation between excess return and size.

3.2 Statistical Significance of the Nonstationarity

To establish that the expected excess returns attributed to size effects are nonstationary requires significance tests to be integrated with the preceding analysis. Two tests are conducted, both of which use the program VPAR written by Craig Ansley.

The first test is based on a statistic formed by taking the ratio of the variance of \tilde{u}_{it} in (6) to the variance of \tilde{e}_{it} in (1) for each (portfolio) $i=1, \dots, N$. For the large firm portfolio, this statistic is 2.23 which is significant in terms of the statistic's asymptotically normal distribution at the conventional 5% level.²² For the small firm portfolio the statistic is 2.01. Thus for portfolios at both ends of the size range a sampling theorist would reject at the 5% level the hypothesis that the excess return series $\{\hat{a}_i\}_t$ is merely variation around a constant α_i . The rejection is interesting for the small firm portfolio where the relatively large variance of the disturbance series $\{e_i\}_t$ (i.e., noise) increases the difficulty of extracting the variation α_t . Further, as one would expect, the $\{\alpha\}_t$ for the difference between the returns of the portfolios of the smallest and largest firms is not constant through time, with a test statistic of 2.20.²³

An equivalent test is given by the confidence intervals for $\{\hat{a}_{it}\}$. If, in a given subperiod, the lower confidence interval for $\{\hat{a}_{it}\}$ were above zero for (say) small firms and the upper interval below zero for large firms, a conclusion (based on a joint Bonferroni interval) would be that ex ante excess returns were significantly positive for small firms and significantly negative for large firms. Alternatively, the bands switching

from totally above zero to totally below zero would be sufficient (although not necessary) for a statistically significant switch in the sign of ex ante excess returns. For the portfolio of the largest firms which presents least difficulty in the signal extraction problem, the 95% confidence bands for $\{\hat{\alpha}_{it}\}$ drop from a region above zero prior to 1975 to a region below zero after that date.

4. THE SIZE EFFECT IN STATIONARY SUBPERIODS

Section 3 establishes statistically significant nonstationarity in excess returns earned by portfolios ranked on size over the period January 1967 to June 1979. However, in the subperiods January 1969 to December 1973 and January 1974 to June 1979, the excess returns seem reasonably stationary²⁴ (although the effect is reversed across the subperiods). As pointed out previously, it is only valid to estimate the regressions (1) and (3) or the SURM (4) over an interval during which the parameters, including the size-variable coefficient γ_1 , are constant. For the period January 1967 to December 1975, which Figure 2 shows as incorporating periods of both positive and negative size effects, OLS results showed a "significant" positive effect while the SURM showed no significant effect. Further, the effect in Figure 2 appears negative from 1967 to 1969, and a possible explanation for Banz's result of a negative effect over 1966-1975 is that the effect was negative in 1966 also. Finally from January 1976 to June 1979 during which a stationary negative size effect is shown in Figure 2, the SURM (and also OLS) showed a significant negative effect.

The results of the SURM estimation using monthly data for the two subperiods which have a stationary size effect (the "predetermined subperiods") are given in Table 4 along with the OLS results for

comparison. The SURM shows a strong positive size effect in the first period (the t-statistic for the slope coefficient γ_1 is 3.11) and the opposite effect in the latter period ($t(\gamma_1)$ is -3.17). Note that while the positive effect clearly shows up more strongly than in the Subperiods I-II of Table 4, so does the negative effect; this holds also for the OLS results. The economic significance of the phenomenon can be seen from the excess returns earned by small firms over large firms. From January 1969 to December 1973 small firms had ex ante negative excess returns of about 25% per annum,²⁵ while from January 1974 to June 1979 they had ex ante positive excess returns of more than 25% per annum.

The estimates for the Predetermined Subperiods in Table 4 were generated after using stationarity tests to "pick" the subperiods. Consequently, these "pre-test"²⁶ estimates are biased since the sample period was selected after searching through the data, and the Predetermined Subperiods section of Table 4 contains no further evidence of the instability of these effects. Also, the procedure in this section of Table 4 is not fully efficient. An efficient procedure would "pool" the information about parameters (other than γ_1) which are assumed to be constant. However, that procedure would pose formidable and unresolved problems of (econometric) identification and diagnostics.

5. POTENTIAL EXPLANATIONS OF THE STOCHASTIC "SIZE EFFECTS"

We note first that any explanation of size effects which implies a stable premium for small firms is at best incomplete.²⁷ The results also appear to be inconsistent with the simple version of Roll's (1981) explanation of size effects in terms of non-trading induced biases in estimated beta coefficients. It is certainly true that the Kalman filter

would pick up any autocorrelation in the market model residuals induced by non-trading, but the ex ante "excess" return attributable to (say) small firms is predicted to be of a constant sign and magnitude. Presumably Roll's explanation could be generalized to fit our results by allowing sufficient instability in the pattern of non-trading across firms, but there are still at least three factors which weigh against the explanation. First, the size effects show up both in daily excess returns computed using Scholes and Williams (1977) betas and in monthly data. Second, the results hold when the large firm portfolio is related to the value-weighted market index where the non-trading problem should be minimal. Finally, the recursive residuals (not reported) do not in general exhibit the substantial autocorrelation which would occur if the estimated beta were unstable because of this hypothesized instability in non-trading.²⁸

We now discuss some potential explanations that are consistent with a stochastic \tilde{a}_1 . First, can the zero-beta factor of the Black (1972) model explain the apparent excess return relative to the Sharpe-Lintner model? Suppose (1) is replaced by the unrestricted market model

$$\tilde{R}_{1t} = \alpha_1' + \beta_{1M}' \tilde{R}_{Mt} + \tilde{e}_{1t} \quad i=1, \dots, N; \quad t = 1, \dots, T \quad (7)$$

If the Sharpe-Lintner CAPM holds, $\alpha_{1t}' = (1 - \beta_{1M}') R_{Ft}$; if the Black model holds, $\alpha_{1t}' = (1 - \beta_{1M}') E(\tilde{R}_{Zt})$ where $E(\tilde{R}_{Zt})$ is the expected return on an asset whose return is orthogonal to that of the market index. If the Black model holds but instead the Sharpe-Lintner model is imposed on (8), there will be an artificially induced "excess return" of $(1 - \beta_{1M}')(E(\tilde{R}_{Zt}) - R_{Ft})$. If it were the case that small firms had an average β_{1M} substantially above one and vice versa for the large firms, the artificial excess return can only change sign across size groups if $(E(\tilde{R}_Z) - R_F)$ changes sign through

time. But $E(\tilde{R}_Z)$ must be greater than or equal to R_{Ft} if the ex ante mean-variance efficient portfolio boundary is concave. In the Fama and Macbeth (1973) study, the estimate of $(E(\tilde{R}_Z) - R_F)$ was (insignificantly) negative for only one of the six subperiods (January 1961 to June 1968) which they examined.

The sampling error implied by such a zero-beta factor explanation is enormous. In the period January 1974 to June 1979 small firms earned positive excess returns of 2.06% per month or 28% per annum (and hence, by this explanation, R_F exceeded \bar{R}_Z where \bar{R}_Z is the point estimate of $E(\tilde{R}_{Zt})$). The estimated β_{1M} for this period are 1.15 and 1.10 for small and large firm portfolios respectively. For the zero-beta explanation to account for the 2.06% per month actual difference for five years, R_F must have exceeded the true $E(\tilde{R}_Z)$ by 39.09% per month or 5142% per annum. For the subperiod January 1969 to December 1973 the implied excess of $E(\tilde{R}_Z)$ over R_F is 43% per annum (actual β_{1M} are 1.48 and 0.94).

As a further check, we repeated the recursive residuals analysis in Section 3 where α_1 was replaced by the α'_1 in (8). The time series behavior of α'_1 appears very similar to that of α_1 , which implies that the nonstationarity of the excess expected return is not very sensitive to the particular version of the CAPM used. Thus, even though it seems possible that the observed size effect might somehow be related to the Black, Jensen and Scholes (1972) finding of positive (negative) alphas for low (high) beta stocks, the connection is far from clear.

Another possibility is that the CAPM holds with respect to stocks, but that the stock betas are not constant through time because of an option effect of debt [see, for example, Galai and Masulis (1976)]. Since size is strongly negatively related to debt-equity ratios,²⁹ several specifications

of the beta non-constancy will generate nonstationary expected "excess" returns. However, an allowance for a stochastic $\tilde{\beta}_{1M}$ for each size portfolio does not seem to remove the apparent size effect in the expected excess return α_1 . We experimented with several different models of $\tilde{\beta}_{1M,t}$, but none had much of an impact on $\hat{\alpha}_1$. The robustness is consistent with Gibbons (1980, p. 79), who concludes that apparent parameter nonstationarity "does not seem to be 'causing' the rejection of the CAPM restriction."

Alternatively, the selection of the market proxy might account for our results [Roll (1977)]. Suppose the Sharpe-Lintner model holds when applied to the value-weighted market portfolio of all assets M^* , and the proxy we use, denoted M , is also on the efficient frontier but it differs from M^* . Then, given strict convexity of the efficient set, the value of " R_F " implied by the CAPM based on M , that is, $\alpha_1/(1 - \beta_{1M})$, will differ from the observed R_F . It can be shown [Roll (1977, p. 143)] that the ex post regression fitted using proxy M will be:

$$\bar{R}_1 - R_F = (\bar{R}_M - R_F) \beta_{2M^*} (1 - \beta_{1M}) + (\bar{R}_M - R_F) \beta_{1M} \quad (8)$$

where β_{2M^*} is the beta generated by the regression of the returns on the asset which is zero-beta with respect to M , on the true market M^* . The first term on the right hand side constitutes a "spurious intercept" α_1 in (1). If β_{1M} exceeds one for small firms and is less than one for large firms, any ex post variability of this spurious intercept will again be in opposite directions for small and large firms. The spurious intercept will be small if the common element β_{2M^*} is small. Unfortunately β_{2M^*} is not observable, so the importance of the market proxy error in (9) is difficult to evaluate. But the order of magnitude of this proxy error required to explain the excess returns is the same as that of the zero-beta error

considered above, i.e., implausibly high. Additionally, our results imply more than ex post variability of the spurious intercept in (9).³⁰

A final alternative is that the stochastic expected excess return is symptomatic of an underlying misspecification in the CAPM. Since the misspecification explanation is a catch-all, we consider a few concrete examples. First, suppose the traditional CAPM omits a state variable from the intertemporal capital asset pricing model (ICAPM) of Merton (1973), denoted $\tilde{X}(t)$ as a function of time. Through time the state variable follows (say) a mean reverting (elastic) random walk with central tendency μ . The omitted term in the CAPM might be of the form $\theta [E_t(\tilde{X}(t)) - \mu]$ where θ is related to size ranking. If $[E_t(\tilde{X}(t)) - \mu] \neq 0$, this omitted term contributes to a nonstationary expected excess return which is related to size.

Our conclusions with respect to the nonstationarity of the size effects do not appear sensitive to the exact form of the stochastic process.³¹ For example, we have evidence of a lag twelve factor in the diagnostics from the random walk fit for the small firm portfolio. The seasonal factor appears to be robust with respect to choice of the market index, and consistent with the evidence of a "January effect" reported by Keim (1981). If a dummy variable for January is included in our Kalman filter, it is significant for both small and large firms and both value-weighted and

equally-weighted market indices.³² In all cases, significant instability in the size effect remains after allowing for the January dummy variable.³³

We repeated the stochastic estimation procedures using all firms on the NYSE from 1926 to 1978, and all AMEX firms from 1963 to 1978. The results reported here are confirmed, but the nonstationary size effect takes on the magnitude shown here only since the ^{early 1960's} ~~late 1950's~~. It is possible to speculate on the importance for the size effect of variables which are associated with this later period, for example, expected and unexpected inflation effects. On the other hand, the data set of NYSE-AMEX returns constitutes only one sequence of realizations over the post-1950's period in which there may be "relatively unique" realizations of a number of highly collinear variables. If so, the data set may simply not provide enough degrees of freedom to pinpoint the source of the size effect.

6. SUMMARY

We believe that there are three new results here concerning size-related anomalies in stock returns. First, we have shown that the relation between excess returns and firm size can be regarded as linear in the log of size. The transform is important because of skewness in the distribution of firm size. Second, we have shown that the ex ante excess returns attributable to size are not constant through time. Third, we have shown that different estimation methodologies can lead to different conclusions about the size effects.

FOOTNOTES

¹By "excess return" we mean the extent to which the rate of return deviates from that predicted by the traditional capital asset pricing model (CAPM).

²In Section 5, we examine whether our results are affected if the Black (1972) CAPM is considered rather than the Sharpe-Lintner version.

³Banz (1979, 1981) does not work with (1) directly since he uses the Black (1972) version of the CAPM.

⁴See Banz (1979, p. 12) and Figure 1 below.

⁵For construction details, see Center for Research in Security Prices (1980). Briefly, all available securities are ranked into ten portfolios according to their Scholes-Williams (1977) betas calculated on daily returns. A security's day by day excess return is then calculated as the raw return less the equally weighted average return for that day for the portfolio into which the security is ranked. Note that this method is potentially biased against finding any relation between excess return and a variable such as size that is correlated with beta since the mean daily return for each portfolio is taken out of these excess returns.

⁶The companies had 35 quarters of complete data for earnings, dividends, and prices from June 1967 through December 1975 on a quarterly Compustat tape. All companies had fiscal years ending on December 31. For the analysis in this paper 566 of the 577 companies were used. The numbers differ because ten companies were not contained on the CRSP daily master and return tapes and because Reinganum was unable to find the earnings announcements for one multinational company, Unilever Ltd.

⁷However 563 of the 566 are in the CRSP daily excess returns file, although 9 of the 563 have no excess return data since January 6, 1976.

⁸In fact, AMEX/NYSE firms are "large" relative to the size (market value of equity) that many appear to have in mind when discussing small firms. If one really wished to focus on transaction/information costs, OTC stocks would seem more appropriate all else equal. [See for example, "Drexel Burnham Alde Seeks Bargains Among Small Firms That Others Shun," The Wall Street Journal, Tuesday, June 12, 1979, p.6].

⁹Note for example the overall ratio of negative to non-negative averages in Table 3. As a check, the excess returns for all securities on the excess returns tape for this period were also calculated; there were 952635 negative and 826958 non-negative averages, and the average for all securities for this period was close to zero (0.00008).

¹⁰Since many earnings announcements would be made in the three months after December 1975, we also examined excess returns from April 1976 on with similar results.

¹¹ The size and return variables for each portfolio are equally weighted averages of the component securities. The results for the expanded sample are virtually identical to those in Figure 1.

¹² It is important to understand that (2) incorporates only one particular size effect variable, albeit the one which has been the focus of attention in the studies mentioned earlier. It is possible that size rankings, given in (2) by the average size over time, could be "priced", as could temporal deviations of firm size around this mean.

¹³ A common γ_0 or γ_1 must be imposed to allow estimation, although in general both parameters need not be common across all assets. If a common γ_1 is imposed, equality of two γ_0 is necessary to avoid singularity in the regressor matrix, and at least one more γ_{0i} must be restricted to allow any degrees of freedom in a cross sectional regression such as (3).

¹⁴ Although this specification is formally necessary to achieve consistency between the two-step GLS and SURM estimation, it does not seem overly restrictive since the CAPM as a null hypothesis also rules out any cross sectionally random but time-invariant component of α_1 .

¹⁵ As discussed in footnote 13.

¹⁶ In a related application, Fama (1976, p. 336) notes that the parameters obtained in Fama and MacBeth (1973), by cross sectional OLS regression of portfolio returns on (estimated) portfolio betas, have Gauss-Markov minimum variance properties only if the cross-sectional error terms are i.i.d..

¹⁷ Estimation of (4) is by the full information maximum likelihood (FIML) procedure of TSP using the Gauss method [see Hall and Hall (1979)].

¹⁸ All recursive residual results are available on request. The Kalman filter results are obtained using programs written by Craig F. Ansley which are described in Ansley (1979) (1980).

¹⁹ Under the assumption that α_1 follows a random walk, the "correlations" of the $\{\hat{\alpha}_{1t}\}$ are not defined. However, the stationary first differences of the $\{\hat{\alpha}_{1t}\}$ for the first six size portfolios are also highly mutually correlated ($.96 \geq \rho \geq .85$, where $\rho \equiv$ the correlation coefficient), and strongly negatively correlated with those of portfolio 10 ($-.57 \geq \rho \geq -.74$). For portfolio 9, the $\{\hat{\alpha}_{1t}\}$ are estimated as close to constant, and the first differences are not significantly correlated with those of the other portfolios.

²⁰ Note that given the positive correlation amongst the returns of the first six size-ranked portfolios, there must be some negative correlation between one or more of them and the remaining four portfolios, or between the returns on the remaining four (since excess returns must sum cross-sectionally to zero). The cross-sectional constraint does not imply, however, the particular cross-sectional correlation pattern observed here.

²¹ Note that the estimate obtained by averaging $\{\alpha_{it}\}$ across the portfolios $i=1, \dots, 10$ is analogous to the R_{zt}^* in Black, Jensen and Scholes (1972, p. 109). They report a first order autocorrelation of 0.414 for R_{zt}^* over the period April 1957 to December 1965. Since the average of our α_{it} across portfolios would include the intermediate size portfolios, their finding is fully consistent with our random walk assumption.

²² At least for the case of a random walk slope coefficient, simulation results by Garbade (1977) suggest that the asymptotic (chi-square) likelihood ratio test is a conservative approximation under the null hypothesis of constancy of the coefficient, and an increasingly accurate approximation as the instability under the alternative hypothesis increases, even with a sample size of thirty-one. Pagan (1980) establishes the consistency and asymptotic normality of the variance ratio (in a standardized form) but his results hold only when the stochastic coefficient version of (1) is identified.

²³ These results were derived using the value-weighted market index as a measure of R_{Mt} in (1). When the equally-weighted index is used, the statistics are 1.70 and 0.37 (10^{-5}) for large and small firm portfolios respectively and 1.8 for the difference. When a dummy variable for January is included (see pp. 21-22), the statistics are 1.93, 1.84, and 2.27 respectively (for the equally-weighted index). The large difference in the test statistic for the small firm portfolio and equally-weighted index when the January dummy is included confirms our diagnostics indicating misspecification when the seasonal dummy is omitted.

²⁴ Though constancy over these two subperiods might not hold exactly, the induced bias (squared) might be less than the increased variance of the estimators which would result from (pre-test) estimation of a finer partitioning of the overall period.

²⁵ Since the period January 1969 to December 1973 shows such a strong positive size effect, we examined the daily excess returns data to see if this effect is replicated there, and confirmed the apparent switching in the size effect found using monthly data.

²⁶ An example might assist those unfamiliar with pre-test estimators. Some introductory econometrics textbooks contain examples of tests for changes in parameters between pre- and post-war periods. If the null hypothesis of no change in parameters is rejected, the parameters are estimated separately over the two periods. The latter parameter estimators are pre-test estimators. See also Leamer (1978).

²⁷ Of course, some of these explanations might be implausible on other grounds also. For example, commission costs, diversification costs etc. could be reduced by financial intermediation.

²⁸ However, since this recursive residuals test does not detect the stochastic nature of α , it may have low power in detecting an unstable beta.

²⁹ Table 2 gives the rank order correlation coefficients between market value of equity (size) and the debt-total asset ratio, a simple transform of the debt-equity ratio. A more complete analysis of the relation is available on request. Christie and Hertz (1981) have also examined the relation between size and debt-equity ratios.

³⁰ However, one logical possibility is that ex ante shifts in the relation between the true index and (say) the NYSE index induce (ex ante) shifts in β_{ZM} .

³¹ In Monte Carlo experiments (with a sample size as small as 35), Cooley and Prescott (1973) also found that the random walk intercept model is robust, in terms of mean square forecast error, to specifications of the intercept's stochastic process. The alternatives considered were a model with a small probability of a large change in the intercept in each period, and a model with a constant change in the intercept each period.

³² The January dummy variable coefficient, its t-statistic, and the test statistic for instability in the size effect with the January dummy, and the test statistic for instability in the size effect without the January dummy, respectively, are: -0.11, -3.12, 2.31, 2.23 for the large-firm portfolio and value-weighted market index; -0.037, -4.85, 1.93, 1.70 for the large firm portfolio and equally-weighted market index; 0.100, 7.60, 2.47, 2.01 for the small-firm portfolio and value-weighted index; 0.053, 5.96, 1.84 and 0.37 (10^{-5}) for the small-firm portfolio and equally-weighted index; 0.111, 7.46, 2.48, and 2.20 for the difference between small and large firm portfolios and value-weighted index; and 0.090, 6.76, 2.27 and 1.80 for the difference between small and large portfolios and the equally-weighted index.

³³ Indeed, in the case of the small-firm portfolio and the equally-weighted market index, the instability in the size effect becomes statistically significant only after the January effect is included.

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Table 1

Reasons for incomplete data, over the period January 1976 through December 1979, for 70 of the 566 stocks in the Reinganum sample: Classified by market value of equity as of December 1975.

Size Decile	Merger or Acquisition	Merger & Fiscal Year Change	Fiscal Year Change	Bankruptcy	Liquidation	Total
0-1 ^a	5	1	-	2	-	8
.2	10	-	1	1	-	12
.3	9	-	-	-	-	9
.4	14	-	1	-	-	15
.5	6	-	1	-	2	9
.6	7	-	-	-	-	7
.7	6	-	-	-	-	6
.8	3	-	-	-	-	3
.9	1	-	-	-	-	1
.9-1.0	-	-	-	-	-	-
TOTAL	61	1	3	3	2	70

^a Ranked from smallest to largest.

Table. 2

Spearman's rank order coefficients of correlation between firm size, assets, and leverage: 566 stocks studied by Reinganum.

	Market Value of Equity June 1967	Market Value of Equity December 1975	Book Leverage December 1975	Market Leverage December 1975	Total Assets Book Value December 1975
Market Value of Equity December 1975	.96				
Book Leverage December 1975	-.06	-.08			
Market Leverage December 1975	-.40	-.47	.80		
Total Assets Book Value December 1975	.89	.89	.24	-.07	
Total Assets Book Value December 1966	.80	.81	.19	-.03	.94

Book Leverage = (Assets - Common Equity)/Assets, at Book Values

Market Leverage = Book Debt/(Book Debt + Market Value of Equity)

Table 3

Classification for each size decile,^{a/} of: (i) The stocks in the Reinganum sample^{b/} over the period January 1976 to December 1978, and (ii) The population of all stocks over the period July 1962 to December 1978 for which market values of equity were available on the first day of the CRSP daily returns tape^{c/} and for which any excess returns were available, according to the sign and magnitude of the t-statistics for their average daily returns.^{d/}

REINGANUM SAMPLE					ALL STOCKS						
January 1976 - December 1978					July 1962 - December 1978						
Size Decile	$ t < 2$		$ t \geq 2$		Average Daily Excess Return ^{e/}	$ t < 2$		$ t \geq 2$		Average Daily Excess Return ^{e/}	t-statistic ^{f/}
	t<0	t \geq 0	t<0	t \geq 0		t<0	t \geq 0	t<0	t \geq 0		
0.0-0.1	10	37	0	1	0.79	43	154	3	2	0.58	7.95
0.1-0.2	22	32	1	1	0.25	60	140	1	2	0.34	6.05
0.2-0.3	25	32	0	0	0.11	69	130	3	1	0.19	3.78
0.3-0.4	22	30	1	1	0.03	81	117	3	2	0.10	2.39
0.4-0.5	33	23	0	1	-0.10	81	117	2	3	0.08	2.12
0.5-0.6	40	16	1	0	-0.32	104	94	3	2	-0.05	-1.69
0.6-0.7	37	17	2	0	-0.30	130	66	6	1	-0.11	-3.88
0.7-0.8	45	5	5	1	-0.55	137	51	15	0	-0.16	-6.52
0.8-0.9	37	3	17	0	-0.75	160	32	11	0	-0.22	-10.75
0.9-1.0	28	0	28	0	-0.85	149	8	46	0	-0.30	-18.75
TOTAL	299	195	55	5	-0.20	1014	909	93	13	-0.0018	-0.16

^{a/}"Size" is measured by the market value of common stock at June 30, 1967.

^{b/}CRSP daily excess return data are unavailable for 12 stocks.

^{c/}The first day on the tape was July 2, 1962.

^{d/}The CRSP daily (beta) excess returns are derived by ranking all securities on the CRSP daily returns tape into ten portfolios on the basis of their Scholes-Williams (1977) betas, and then computing the daily excess return of any security as the difference between its daily raw return and the (equally weighted) average daily return of the portfolio of which it is a member.

^{e/}Multiplied by 10^3 .

^{f/}Computed for the expanded sample on decile mean excess return assuming independently and identically distributed returns for all companies and all days in each decile. The computed t-statistics are biased to the extent that returns are cross-sectionally correlated.

Table 4

Results of the regression of monthly excess returns on the logarithm of average portfolio size^{a/} for ten portfolios formed by dividing the size-ranked stocks in the Reinganum sample^{b/} into deciles, over (i) the period January 1967 to June 1969 and given subperiods, and (ii) the subperiods January 1961 to December 1973 and January 1974 to June 1979 over which preliminary tests showed the size effect γ_1 to be constant.

$$\text{Model for OLS: } (\tilde{R}_{it} - R_{Ft}) = \tilde{\alpha}_i + \beta_{iM}(\tilde{R}_{Mt} - R_{Ft}) + \tilde{\epsilon}_{it} \quad (1)$$

$$\tilde{\alpha}_i = \gamma_0 + \gamma_1 \tilde{s}_i + \tilde{\eta}_i \quad i = 1, \dots, 10 \quad (3)$$

where

\tilde{R}_{Mt} = value weighted market index

\tilde{s}_i = log size of portfolio i

Model for SURM:

$$(\tilde{R}_{it} - R_{Ft}) = \gamma_0 + \beta_{iM}(\tilde{R}_{Mt} - R_{Ft}) + \gamma_1 \tilde{s}_i + \tilde{\epsilon}_{it} \quad (4)$$

$$i = 1, \dots, N; \quad t = 1, \dots, T$$

Period	Model	$\hat{\gamma}_0^{c/}$	$t(\hat{\gamma}_0)$	$\hat{\gamma}_1^{c/}$	$t(\hat{\gamma}_1)$	Annualized Small Firm Excess Return (%) ^{d/}
<u>TOTAL</u>						
1/67-6/79	OLS	10.72	5.20	-0.52	-3.03	4.26
	SURM	13.15	1.36	-0.67	-1.01	5.51
<u>SUBPERIODS I</u>						
1/67-12/75	OLS	-7.54	-2.45	0.84	3.28	-6.96
	SURM	1.79	0.15	0.18	0.23	-1.47
1/76-6/79	OLS	59.69	13.01	-4.03	-10.96	37.36
	SURM	45.98	2.99	-3.18	-3.00	28.55
<u>SUBPERIODS II</u>						
1/67-2/71	OLS	2.22	0.52	0.16	0.44	-1.28
	SURM	5.20	0.38	-0.00	-0.00	0.13
3/71-4/75	OLS	-19.81	-3.74	1.77	4.00	-15.12
	SURM	12.37	0.66	-0.51	-0.40	4.12
5/75-6/79	OLS	48.20	12.12	-3.39	-10.15	30.66
	SURM	39.18	2.87	-2.74	-2.92	24.24
<u>PREDETERMINED^{e/} SUBPERIODS</u>						
1/69-12/73	OLS	-46.03	-11.02	3.58	10.22	-32.68
	SURM	-32.87	-2.70	2.62	3.11	-23.09
1/74-6/79	OLS	48.42	20.70	-3.28	-16.73	29.60
	SURM	45.81	3.22	-3.09	-3.17	27.66

^{a/}"Size" is measured by the market value of common stock at June 30, 1967.

^{b/}The 563 stocks of the Reinganum sample of 566 stocks which are on the CRSP daily (buta) excess returns file, and on which Table 3 is based, are used as the sample in this table.

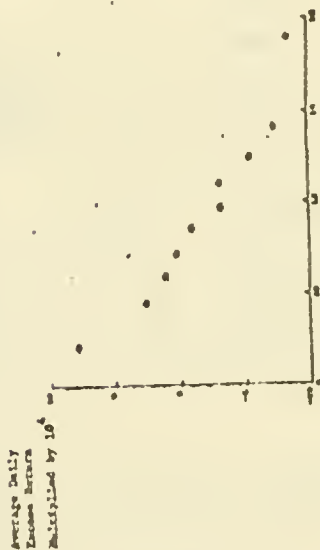
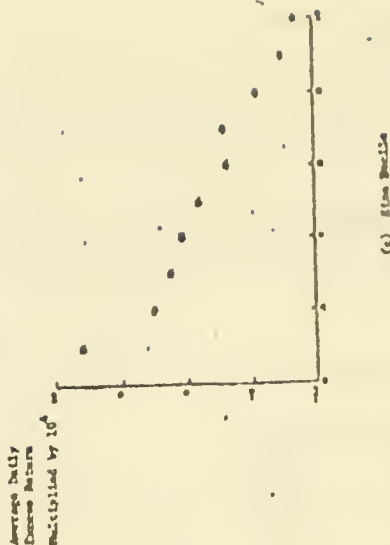
^{c/}Multipplied by 10^3 .

^{d/}Annualized small firm excess return = $(1 + \gamma_1(\bar{s}_1 - \bar{s}_{10}))^{12} - 1$, where \bar{s}_i = the log size of portfolio i .

^{e/}These subperiods are found by Kalman filtering techniques to have approximately constant effects. The t-statistics should be interpreted with this in mind.

Figure 1

Scatter plots of the relation between average daily excess returns and:
(a) Portfolio size (in thousands of dollars); (b) Natural log of portfolio size; and (c) Size deciles, over the period January 1967 to December 1978.



^a Daily excess returns are obtained from the Jett daily (Jett) excess returns file. The Jett daily excess returns are derived by ranking all securities on the Jett daily returns tape into ten portfolios on the basis of their Scholes-Williams (1977) betas, and then computing the daily excess return of any security as the difference between its daily raw return and the (equally weighted) average daily return on the portfolio of which it is a member.

^b The firms in Brinson's sample have been allocated to ten portfolios according to their size which is measured by the market value of common stock at June 1967. The size of each portfolio is an equally weighted average of the size of its component stocks.

Figure 2

Time series of risk adjusted excess returns (\hat{a}_{it}) for portfolios 1, 5, and 10
estimated as random walks for the period January 1967 to June 1979

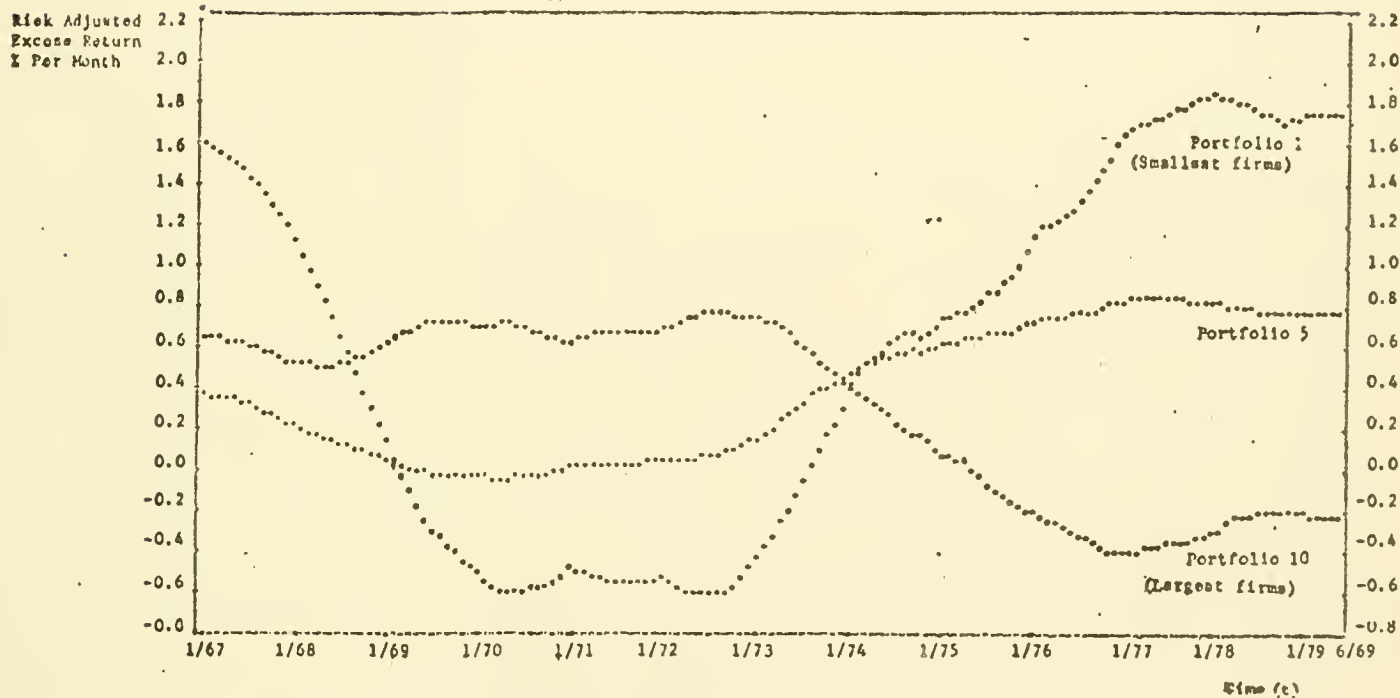
$$(R_{it} - R_{Pt}) = a_{it} + \beta_{iM}(R_{Mt} - R_{Pt}) + \epsilon_{it} \quad (1)$$

$$a_{it} = a_{i,t-1} + u_{it} \quad (6)$$

$$E(\epsilon_{it}^2) = \sigma_\epsilon^2$$

$$E(u_{it}^2) = \sigma_u^2$$

$$E(\epsilon_{it}, u_{it'}) = 0 \text{ for all } t, t' = 1, \dots, T, \quad i = 1, 5, 10$$



R_{it} = month t rate of return on equi portfolio i , $i=1,5,10$

R_{Mt} = month t rate of return on value-weighted market index

R_{Pt} = month t rate of return on a Treasury Bill maturing at the end of month t

The model estimates of a_{it} are obtained by using a Kalman filter algorithm to generate a smoothed trajectory $(\hat{a}_{it})_t$ in (1) which, together with an estimate of β_{iM} , can be used to generate the trajectory $(\hat{a}_{it})_t$. The optimally smoothed (filtered) estimate of $(\hat{a}_{it})_t$ is a linear function of the "total" noise term $(\epsilon_{it} + u_{it} - u_{it-1})_t$ which arises when (1) is first differenced and $(a_{it})_t$ is described by (6).

BASEMENT

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