

**USER COST IN OIL PRODUCTION**

by

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### ABSTRACT

The assumption of an initial fixed mineral stock is superfluous and wrong. User cost (resource rent) in mineral production is the present value of expected increases in development cost. It can be measured as the difference between in-ground market value and development cost, or estimated approximately from current development cost. For private or national-income accounting, mineral reserves should be treated as a renewable inventory. Adjustment for change in inventory may increase or decrease the income of a mineral producer, but an increase is more likely.



## USER COST IN OIL PRODUCTION

### INTRODUCTION

To reckon the income generated in mineral production, whether in a single unit or a whole nation, one must subtract out the value of the ore used up. In estimating this value, commonly called user cost (or resource rent), an initial fixed "non-renewable" mineral stock is usually assumed. In equilibrium the value must rise at the appropriate discount rate, or else it pays to arbitrage.<sup>1</sup> Prices (net of extraction cost) must rise at that rate. The higher the discount rate, the less the present value of the asset, and the faster the optimal depletion rate [Hotelling, 1931][Dasgupta & Heal 1979]. Hence the emphasis many writers place on the importance of the "social" discount rate, which allegedly is much lower than the market rate, in determining a nation's optimal depletion rate.

However, the assumption of an initial fixed stock is superfluous, and wrong. Only a fraction of the mineral in the earth's crust, or in any given field, will ever be used. The size of the fraction will depend on costs and prices, including those of substitutes. To define the initial fixed stock as "the economic portion" of what is in the earth, and then derive a price-output profile from it, is circular reasoning. Prices need not rise over time. In fact, decreases are usual, increases rare. [Adelman 1990]

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<sup>1</sup> If the expected rate of price increase exceeds [is less than] the discount rate, it pays owners to hold ore off the market [offer more ore], until the current price rises [declines] to where the two rates are equal.

What we observe in the real world are not one-time stocks immaculately created to be consumed, but inventories of "proved reserves," constantly renewed by investment in finding and development. Over time, the investment needed per unit-added is forced up by diminishing returns, and forced down by increasing knowledge. So far, knowledge has prevailed. It need not always do so. There is no general underlying price trend; it varies among minerals, in the short and long run.

Removing a wrong assumption does not make the Hotelling theory wrong. It makes it useful, by focussing on the true measure of mineral scarcity: the present value of a mineral reserve to be extracted. To estimate reserve values concentrates the minds of scientists, engineers, and investors. Economists need to observe the market in assets values, which is a market in ideas, good and bad.

**An opposing view** [Nordhaus 1973, pp. 530-538] gives a very good statement of the case that resource prices and values are not a valid indicator of real mineral scarcity.

"[A] full set of futures and insurance markets is not available....In the cosmic framework of the ultimate exhaustion of fossil fuels...[sales of in-ground reserves] cover a very short span." (534-535) [Consequently, private discount rates are too high, and resources undervalued.] "Too high an interest rate casts a long shadow over the future...resources are consumed too quickly." [Decisions are myopic, planning horizons too short.] (535) [Therefore private markets are] "an unreliable means of pricing and allocating exhaustible appropriable natural resources." (537)

However, if we drop the assumption of a fixed stock, none of these strictures apply. A high interest rate may speed up or slow down the depletion rate, and the effect will not be strong because the force works both ways. [Adelman 1990] If mineral reserves are a type of inventory, in which investment is governed by the

usual incentives and errors, they are no worse valued than any other asset. And both buyers and sellers have a strong inducement to use "a carefully constructed econometric and engineering model of the economy". (537)

**Purpose** This note is confined to crude oil. We first suggest how the in-ground value and the user cost are related to development investment. We test these concepts with a data set for the U.S.A., then apply them briefly elsewhere.

### INVESTMENT REQUIREMENTS AND IN-GROUND VALUES

**Investment in oil production** consists of two activities: discovery of new pools, and their development by drilling and equipping wells. The investment creates the inventory of developed barrels of "proved reserves", which are essentially a forecast of cumulative production through existing installations. At any given moment, if there is a market in reserves, we can write [Bradley 1989]:

$$VR - K(R) = UR \quad [1]$$

i.e., in-ground market value of a developed reserve, less development investment required, equals user cost. In equilibrium, user cost equals discovery cost, i.e., it just pays to find an additional barrel. A closer look shows that development cost and user cost are positively correlated.

Discovery-cum-development of new reservoirs, and development of known reservoirs, are competing investment outlets, alternative methods of creating reserves. They approach equality at the margin. If the operator chooses to develop a known pool more intensively, increased development cost is the penalty for using instead of holding the reserve, and it should equal the value of an undeveloped barrel, i. e.,

user cost.

To develop a reservoir, or a tranche thereof:

$Q$  = initial new production, barrels per year

$a$  = exponential decline rate of  $Q$ , percent per year

$$R = \text{current reserves} = Q \int_0^T e^{-at} dt = Q/a$$

$$a = Q/R \quad [2]$$

$K$  = capital expenditures =  $kaQ$

$k$  = an empirical constant per barrel, reflecting better or poorer geology.

The value of an undeveloped barrel, in infinite time:

$$VR = PQ/(a+i) - kaQ \quad [3]$$

$$V = Pa/(a+i) - ka^2 \quad [3a]$$

The decision variable is  $\underline{Q}$ , hence  $\underline{a}$ . All others are exogenous. It can be proved

[Adelman 1990] that value is maximized when

$$2ka^3 + 4ka^2 + 2kai^2 - Pi = 0 \quad [4]$$

To calculate the development cost of higher production rates, we make two extreme opposing assumptions, which bracket all possible cases:

(1) No resource limitation. Assume that the long run supply curve is horizontal. Production and reserves increase in the same proportion. That is,  $dR/R = dQ/Q$ , and  $dR/dQ = R/Q$ . Then  $\underline{a}$  remains constant, because from [2]:

$$da/dQ = \frac{R - Q(dR/dQ)}{R^2} = \frac{R - Q(R/Q)}{R^2} = 0$$



Investment per additional annual barrel:

$$dK/dQ = ka + kQ da/dQ = ka = K/Q \quad [5]$$

Investment per additional barrel in ground:  $dK/dR=K/R = ka^2$

(2) The reserve inventory R is fixed. Assume that additional investment can only accelerate output, not increase the total. Then  $a$  must increase proportionately with Q:

$$\text{If } dR/dQ = 0$$

$$da/dQ = \frac{R-Q(dR/dQ)}{R^2} = 1/R = a/Q$$

$$dK/dQ = d(kaQ)/dQ = ka + kQ(a/Q) = 2ka \quad [6]$$

Therefore in the limiting case of zero reserve additions from new investment, the investment per additional unit of capacity is twice what it would be in the opposed limiting case of new reserve creation. At the limit, the user cost, of shifting output from future to present, is equal to, and additive to, the cost of creating the new capacity. This follows from the quadratic cost function, obviously oversimplified but not untrue.

If the reserve is fixed, the cost of creating a new barrel in the ground is by assumption infinite. We treat the cost of creating an increment of capacity as a surrogate for the cost of obtaining the same result from creating a new barrel of reserves.

If the ratio of user cost per barrel to development cost per barrel fluctuates between zero and unity, then by Equation [1] the ratio of total in-ground value to

development cost fluctuates between unity and 2. We turn now to the data.

### RESERVE VALUES: ANALYSIS OF SALES OF PROPERTIES

The market value  $MV$  of an oil/gas property is a function of the amount of oil reserve  $O$ , unit value  $b$ , and of the amount of gas  $G$  and unit value  $c$ .

$$MV = b O + c G + e \quad [7]$$

#### [TABLE I HERE]

Table I shows regression estimates for recent years. There are some obvious problems. The samples are small. They cover only voluntary disclosures. The independent variables are measured with error. Non-reserve assets and liabilities and other special factors may greatly influence the value of a transaction. The sizes of the properties sold vary enormously. A regression showing heteroscedasticity (asterisk) was replaced by a weighted regression. [Kmenta 1984, p. 287]

We also computed all "pure oil" or "pure gas" cases, considering everything above 90 percent as "pure". For example, in 1979, the average wellhead oil price was \$12.64, and average wellhead gas price (a defective figure due to price regulations, but all we have) was \$1.178. [MER]) Shell Oil Co. bought the South Belridge field, credited with 364 BCF of gas and 365 MMB of oil. The current value of the oil was therefore 92 percent of total current value, and the oil consideration was then estimated as \$3.36 billion (92 percent of \$3.65 billion), which came to \$9.20 per barrel. This looks extremely high, but there is balm in hindsight. During 1979-1989 inclusive, cumulative production from the field was 494 million barrels, or 35 percent more

**TABLE I. REGRESSION ESTIMATES OF OIL AND GAS RESERVE VALUES  
1979-1988**

YEAR	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988
Sample	7*	10	5	5	12*	20*	15	26	46*	37*
<u>Oil Value</u>										
\$/BRL	7.62	14.34	5.02	5.83	4.42	5.95	6.85	5.72	4.35	5.32
t-stat	8.0	22.0	8.3	20.5	1.6	10.8	19.3	6.5	3.8	5.8
"Pure Oil"										
Number	2	2	1	1	1	3	3	3	12	8
Avg Val	8.32	7.07	5.34	8.10	3.39	5.64	6.60	6.30	5.06	6.06
<u>Gas Value</u>										
\$/MCF	0.26	1.96	1.53	0.89	0.68	0.99	0.32	0.81	0.81	0.74
t-stat	1.6	8.6	1.7	9.6	1.1	6.9	2.9	20.3	10.0	9.4
"Pure Gas"										
Number	--	2	--	--	4	2	2	9	11	7
Avg Val	--	0.57	--	--	1.19	0.88	1.5	0.65	0.79	0.69

Source: Scotia-OGJ Data Base, using "adjusted price." Method: ordinary least squares regressions. Asterisk \* indicates heteroscedasticity, in which case we use the corrected (weighted) regression results. "Pure oil" and "pure gas" explained in text.



than the total estimated reserve bought. It produced in 1989 four times as much as in 1978, and at the end of the year its current reserves were figured at 357 million [OGJ 1-29-79 p. 133, and 1-29-90, p. 74], which is probably too low. Obviously Shell won big when it bet on improved technology.

Turning to Table I: the 1980 oil and gas values are hardly credible, 2-3 times as high as the two "pure" cases. The 1985 gas value is hard to credit. In the next three years, however, the samples are larger and the results more stable. The 1986 decline was relatively mild, but continued even stronger into 1987, as old purchase/sale commitments ran out and the industry faced the new price levels. The 1988 revival reflected the price recovery of late 1986 and 1987.

Following a procedure similar to our earlier paper, the Scotia group have classified acquisitions as predominantly oil or predominantly gas. We use them below (page 11) to calculate a market estimate of expected price changes, to be compared with actual change.

One piece of information is missing: current production in the properties sold. Papers published over 30 years ago ([PPH 1962], [PEH 1987]) showed that the higher the ratio of production to reserves ( $Q/R$ ), the higher the ratio of value to wellhead price ( $V/P$ ). A glance at equation [3a] shows that for a developed reserve, with development cost zero by definition, reserve value and wellhead price are connected by the formula:

$$V/P = a / (a + i) \quad [8]$$

where  $\underline{V}$  is the per-barrel value,  $\underline{P}$  is the price net of current costs and taxes,  $\underline{a} = Q/R$ ,

and  $i$  is the discount rate. There is a good fit [Adelman 1990], but a bias: buyers consistently pay more than they "should." We suggest that in return they are getting an option on more intensive development. The Shell purchase was a strong example.

These results have an important implication for the theory of mineral depletion. Given the fixed-stock assumption, the value of a unit in-ground should equal the spot price net of extraction cost.[Dasgupta & Heal 1979, p. 158] [Miller & Upton 1985] If this assumption held, then the price would appreciate at the appropriate interest rate, and the waiting time to production (i.e., depletion rate) would not affect the value of the reserve. But in fact, as these results show, it clearly does. And where oil in the ground is worth roughly one-half of its net wellhead price, coal reserves are usually less than one percent. [For some recent examples, see Appendix I] This cannot be reconciled with a "tilt in the competitive price path which is an inescapable feature of an exhaustible resource" [Dasgupta & Heal, p. 159]

### INTER-TEMPORAL CHANGES IN VALUES AND COSTS

User cost is the penalty for developing more intensively at any given time instead of waiting for a later date and higher prices. But if development cost, or discovery cost, or both, are expected to increase in the future, the present value of an existing barrel will also increase. This increases user cost and with it the incentive for additional investment in discovery.

For long-term perspective, we have tabulated the oil reserve valuations issued by the John S. Herold Company beginning 1946. They calculate the present value of the proved reserves of a large number of companies, as they are expected to be

depleted over time, and discounted at what the estimators consider an appropriate rate.

We make only a limited use of these data. The Herold reserve estimate and valuation for a specific company are not substitutes for estimates made by a professional team who expose their data and reasoning, and sign their names. A market transaction is based on these professional appraisals, and incorporates a "peer review", because the final number has been accepted by both sides who are wagering money on the accuracy of the work.

We use no specific Herold numbers, but we do calculate annual averages to measure time trend. This is defensible because, first, the Herold valuations are themselves subject to a market process; the nearer they come to what investors consider reasonable, i. e. would pay or demand, the more successful they are. The survival of the company for this long a period indicates that they have been useful, and used. Herold valuations are frequently quoted in the financial press, a more demanding use than ours.

Second, we can test the reasonableness of the Herold annual averages by comparing them with our regression results, with an industry rule of thumb, and with altogether independent estimates of oil development cost. The details are in Table II.

**[TABLE II HERE]**

In Table II, Column (1) shows the average gross wellhead price of crude oil. Column (2) is an estimate of the price net of operating expenses, royalties, State

TABLE II. WELLHEAD PRICE, COST, AND RESERVE VALUES  
 USA 1946-1987  
 (Current Dollars per Barrel)

YEAR	WELLHEAD PRICE		AVG. RES. VALUE	STAND-ARD DEVIA-TION	SAMPLE SIZE	DEVELOPMENT COST		USER COST (VALUE LESS POST-TAX COST)	VALUE AS RATIO TO:			
	GROSS	NET				PRE-TAX	POST-TAX		DEVELOP-MENT COST		PRE-TAX	POST-TAX
									GROSS PRICE	NET PRICE		
1	2	3	4	5	6	7	8	9	10	11	12	
1946	1.41	0.91	0.30	0.04	16				0.21	0.33		
1947	1.93	1.24	0.43	0.06	7				0.22	0.35		
1948	2.60	1.67	0.75	0.19	25				0.29	0.45		
1949	2.54	1.64	0.73	0.09	19				0.29	0.45		
1950	2.51	1.62	0.70	0.05	21				0.28	0.43		
1951	2.53	1.63	0.73	0.14	16				0.29	0.45		
1952	2.53	1.63	0.68	0.07	23				0.27	0.42		
1953	2.68	1.73	0.75	0.19	41				0.28	0.43		
1954	2.78	1.79	0.69	0.07	38				0.25	0.39		
1955	2.77	1.78	0.89	0.22	88	0.80	0.71	0.18	0.32	0.50	1.11	1.25
1956	2.79	1.80	0.95	0.25	103	0.80	0.71	0.24	0.34	0.53	1.18	1.33
1957	3.09	1.99	0.94	0.18	118				0.30	0.47		
1958	3.01	1.94	0.91	0.18	117				0.30	0.47		
1959	2.90	1.87	0.88	0.21	124	0.50	0.45	0.44	0.30	0.47	1.77	1.98
1960	2.88	1.85	0.83	0.22	110	0.66	0.59	0.24	0.29	0.45	1.25	1.41
1961	2.89	1.86	0.83	0.19	110	0.58	0.52	0.31	0.29	0.46	1.43	1.61
1962	2.90	1.87	0.87	0.20	131	0.77	0.69	0.19	0.30	0.47	1.13	1.27
1963	2.89	1.86	0.83	0.20	79	0.74	0.66	0.17	0.29	0.45	1.12	1.26
1964	2.88	1.85	0.87	0.20	117	0.61	0.54	0.33	0.30	0.47	1.43	1.60
1965	2.85	1.84	0.83	0.21	103	0.53	0.48	0.35	0.29	0.45	1.55	1.75
1966	2.88	1.85	0.84	0.29	119	0.54	0.48	0.36	0.29	0.45	1.55	1.74
1967	2.92	1.88	0.82	0.22	111	0.56	0.50	0.32	0.28	0.44	1.46	1.64
1968	2.94	1.89	0.83	0.23	113	0.71	0.63	0.20	0.28	0.44	1.17	1.31
1969	3.09	1.99	0.85	0.23	123	0.85	0.75	0.10	0.28	0.43	1.00	1.13
1970	3.18	2.05	0.81	0.24	125	0.63	0.56	0.25	0.26	0.40	1.29	1.45
1971	3.39	2.18	0.90	0.36	96	0.68	0.61	0.29	0.27	0.41	1.32	1.48
1972	3.39	2.18	0.99	0.56	112	1.10	0.98	0.01	0.29	0.45	0.90	1.01
1973	3.89	2.51	1.41	0.59	119	0.89	0.79	0.62	0.36	0.56	1.58	1.78
1974	6.74	4.34	2.28	0.83	121	1.46	1.30	0.98	0.34	0.53	1.56	1.75
1975	7.56	4.87	2.39	0.84	130	2.99	2.66	-0.28	0.32	0.49	0.80	0.90
1976	8.19	5.27	2.74	0.03	126	4.17	3.71	-0.97	0.33	0.52	0.66	0.74
1977	8.57	5.52	3.03	1.71	137	3.85	3.43	-0.40	0.35	0.55	0.79	0.88
1978	9.00	5.80	3.39	1.29	145	2.76	2.45	0.94	0.38	0.58	1.23	1.38
1979	12.64	8.14	4.62	2.11	143	3.35	2.98	1.64	0.37	0.57	1.38	1.55



**TABLE II. WELLHEAD PRICE, COST, AND RESERVE VALUES**  
**USA 1946-1987**  
**(Current Dollars per Barrel)**

YEAR	WELLHEAD PRICE		AVG. RES. VALUE	STAND-ARD DEVIATION	SAMPLE SIZE	DEVELOPMENT COST		USER COST (VALUE LESS POST-TAX COST)	VALUE AS RATIO TO:			
	GROSS	NET				PRE-TAX	POST-TAX		DEVELOPMENT COST		PRE-TAX	POST-TAX
									GROSS PRICE	NET PRICE		
1	2	3	4	5	6	7	8	9	10	11	12	
1980	21.59	13.9	7.91	3.58	131	3.48	3.10	4.81	0.37	0.57	2.27	2.55
1981	31.77	17.51	9.72	3.90	132	6.42	5.71	4.01	0.31	0.56	1.51	1.70
1982	28.52	16.28	8.31	2.94	129	11.29	10.05	-1.74	0.29	0.51	0.74	0.83
1983	26.19	15.43	8.62	4.49	126	4.00	3.56	4.96	0.33	0.55	2.13	2.39
1984	25.88	16.67	8.80	2.58	130	3.60	3.20	5.60	0.34	0.53	2.45	2.75
1985	24.09	13.80	8.19	2.64	122	4.08	3.63	4.56	0.34	0.59	2.01	2.26
1986	12.68	8.15	5.88	2.16	129	4.96	4.42	1.46	0.46	0.72	1.19	1.33

Source: Price, DOE/EIA, Annual Energy Review 1986, tab. 60. For 1946-1948, DeGolyer & McNaughton, Twentieth Century Petroleum Statistics 1986, page 99 (ultimate source DOE). Net price is gross less operating costs, royalties, and taxes excluding Federal income taxes but including Windfall Profits Tax after 1979. From M. A. Adelman, Oil Production Costs in the USA, 1918-1985, forthcoming in J. R. Moroney, ed., Advances in the Economics of Energy & Resources (JAI Co., 1991). Average ratio of net to gross during 1955-1980 was .644, standard deviation .016. The ratio of net to gross was considerably less, and fluctuated, in later years, so individual years' values are used. Data ceases to be available after 1984. For 1985, we have used the 1981-84 average. For 1986-1987, we have used the previous average. The difference is almost entirely in the Windfall Profits Tax, which became inoperative in 1986, because of lower prices. Reserve values, calculated from J. S. Herold Company tabulations.

Notes: average values

- 1959 One outlier omitted: \$4.73/barrel
- 1961 Two outliers omitted: \$90, \$331/barrel
- 1970 One outlier omitted: \$959/barrel
- 1971 Two outliers omitted: \$7.02, \$7.75/barrel
- 1972 One outlier omitted: \$193/barrel
- 1975 One outlier omitted: \$59/barrel

Explanation: Post Tax Costs

The reduction in cost aims to capture the net advantage of drilling for oil instead of buying. This is the result of the tax advantage of charging off intangible drilling expenses.

Intangible drilling costs are "between 60 and 70 percent of the entire well cost." (Petroleum Production Handbook 1962, page 38-22, repeated at page 44-11 of Petroleum Engineering Handbook, 1987). However, a special API tabulation released in 1985 showed intangibles as 34 percent in 1984. This discrepancy is due to the fact that drilling and completion account for only about 60 percent of total development cost including lease equipment, pressure maintenance programs, etc. For the whole period, therefore, development outlay post tax is reckoned at 83 percent of pre-tax, by the formula;  $X - 1 - (.34)(1-.5) = .83$ . The net present value is 63 percent of the gross saving. This is calculated by assuming that cost would otherwise be uniformly charged off over 25 years, and discounting at 10 percent. This would be worth 0.367 of an immediate payment, i.e.  $1 - .367 = .633$ . Then  $0.17(.63) = 0.11$ , and development cost is reduced by 11 percent.

We have not made any adjustments to cost to allow for the effects of percentage depletion. That would be double counting, since the present value of percentage depletion is already reflected in the value of the developed reserve.

severance taxes and the Windfall Profits Tax, which was an excise per barrel, not a profits tax. For 1955-82, there are actual data on these deductions. Since the dispersion about the period mean is quite small, it seems safe to extend it forward and back.

Columns (3) and (4) show, respectively, the annual average and standard deviation of the Herold reserve valuations for the individual companies. We will use their relation to oil prices, and to oil development costs to demonstrate user costs following Equation [1].

### OIL RESERVE VALUES AND OIL PRICES

**The one-third rule** For many years, the industry has had a rule of thumb of in-ground value as one-third of the wellhead price, i. e. about one-half of the net price (ex-operating costs, royalties, and non-income taxes). Recall equation [8] above: if  $a (=Q/R)$  and  $i$  are roughly equal, as has been true since World War II, then  $V$  should be approximately half of net  $P$  and one-third of gross  $P$ .

The one-third rule seems to agree well with our estimates. Column (3) is generally mildly lower than one third of Column (1) before 1973, mildly higher afterward.

**Reserve asset value as predictor** In general, a long-lived asset rises in price when the market expects an increase in the prices of the goods in which the asset will be embedded through future production. Hence asset price changes are a leading indicator of product price changes.

As Columns (9) and (10) show, reserve values in 1946-72 failed to reflect

higher prices to come. There is a clear shift in 1973. Higher percentages beginning that year were correct at least as to sign in predicting higher prices, through 1980.

The gradual weakening of prices in 1985, and collapse in 1986, are only partially reflected in value changes, and therefore columns (9) and (10) go to record heights. This reflects the expectation of a large price rise: false after 1985, true after 1986.

For a little more precision, we can re-write Equation (8) above to allow for an expected price increase at some constant annual rate  $g$ :

$$V/P = a / (a + i - g) \quad [9]$$

$$g = i - a (1 - (P/V)) \quad [10]$$

This measure is a residual, sensitive to errors in the discount rate, as well as in the estimated value. However, it seems to be of some help in interpreting fluctuations in recent years.<sup>2</sup> In Appendix II we show the gain or loss to postponement of an oil project, if the price is expected to change.

[TABLE III HERE]

Table III presents estimates of expected changes in wellhead price (net of extraction costs and taxes), in recent years, in percent per year. We have used the 12

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<sup>2</sup> [Verleger 1990] has analyzed a royalty trust issued by British Petroleum Company. Its current market value contains an implicit forecast of the future price of crude oil, given assumptions about inflation and interest rates. He concludes (page 12) that "investors do not expect increases in oil prices to exceed the rate of inflation in the near future." The price spike of July-August 1990 was due to an unexpected restraint on supply.

TABLE III. OIL AND GAS PRICES & RESERVE VALUES 1982-1990

OIL VALUES (\$/Brl)						
YEAR	NET PRICE	IN-GROUND VALUE	RATIO	Q/R=a	DIS-COUNT RATE	IMPLICIT PRICE FORECAST change/yr
1982	16.28	8.50	0.52	0.106	0.12	-0.004
1983	15.43	7.10	0.46	0.109	0.12	-0.032
1984	16.67	6.80	0.41	0.106	0.12	-0.068
1985	13.80	7.50	0.54	0.108	0.12	-0.007
1986	8.15	6.50	0.80	0.111	0.12	0.081
1987	9.927	5.10	0.51	0.105	0.12	-0.008
1988	8.10	5.50	0.68	0.105	0.12	0.049
1989	10.21	5.10	0.50	0.105	0.12	-0.015
1990	11.44	3.80	0.33	0.105	0.12	-0.136
:1Q						
GAS VALUES (c/mcf)						
YEAR	NET PRICE	IN-GROUND VALUE	RATIO	Q/R=a	DIS-COUNT RATE	IMPLICIT PRICE FORECAST change/yr
1982	NA	1.20	NA	0.099	0.12	NA
1983	NA	1.30	NA	0.090	0.12	NA
1984	NA	1.10	NA	0.099	0.12	NA
1985	1.62	1.20	0.72	0.095	0.12	0.053
1986	1.11	0.90	0.81	0.088	0.12	0.059
1987	0.97	0.75	0.78	0.092	0.12	0.058
1988	1.13	0.80	0.71	0.092	0.12	0.043
1989	1.16	0.80	0.69	0.092	0.12	0.038
1990	1.23	0.65	0.53	0.092	0.12	-0.016
:1Q						

Sources: Oil prices, Monthly Energy Review. Gas prices are spot, from National Gas Clearing House, reprinted monthly in Oil & Gas Journal; annual average. Reserve values are from Scotia/OGJ Data Base, classified as predominantly oil or predominantly gas, value rounded to nearest 10 cents. Q/R ratios, from Basic Petroleum Data Book. Discount rate assumed (usual published assumption in financial press.)

percent discount which is conventional in the investment community. The implied forecasts were surprisingly good for oil. Prices were expected to decline during 1982-85. The low prices of 1986 were not expected to last, and the anticipation of rising prices was borne out in 1987, when pessimism again took over and was justified the next year.

But for gas, the price forecasts were all too optimistic. Gas prices were expected to rise soon because a temporary surplus would disappear. Instead, the "gas bubble" became the "gas sausage". There is no sign of increasing gas prices, yet reserves have stabilized. This suggests that current prices induce enough investment to maintain reserves.

#### RESERVE VALUES AND OIL DEVELOPMENT COSTS

Returning to Table II, we now compare oil reserve values with development cost per barrel of newly-booked reserves. Since values are ex-tax, they should be compared with the post-tax development cost, column (7) of Table II. The sum of development drilling and non-drilling expenditures are divided by gross reserve-additions to give unit development cost.

To estimate finding cost per unit, corresponding to development cost per unit, would require knowledge of separate oil and gas finding expenditures. That is impossible because much or most finding cost is joint. Econometric analysis might give us marginal relations, but only with data on the amounts of oil and gas found year by year (not merely what has been developed into new proved reserves). No such numbers exist. The financial press frequently quotes "finding costs per barrel of

oil-equivalent." This equals the sum of exploration plus development expenditures, oil and gas together, divided by increments to proved reserves, oil plus gas equivalents. But not only does it omit newly-found undeveloped reserves, there is no stable or necessary relation of oil to gas in respect of price, value, or cost. "Finding cost per barrel of oil equivalent" amounts to adding apples to oranges, then dividing by pineapples plus bananas. It deserves no attention.

[FIGURE 1 HERE]

Following Equation [1], Figure 1, from Column 8 of Table II, shows oil user cost. It is a surrogate for finding cost. In long-run equilibrium, they should be equal.

[FIGURE 2 HERE]

The market value averaged 1.47 times development cost during 1955-1973, and all observations were in the predicted range between 1.0 and 2.0. During 1974-86, the mean was 1.62 but the dispersion much greater. This suggests that user cost/finding cost has been about one-half of development cost.

Before 1973, there was no upward trend in gross or net values, nor in development cost as measured here (and also in an independent ordinal measure [Adelman 1991]). It may seem surprising that there was no increase in scarcity, nor in finding cost, in view of the well-known decrease in average size of new field found. But just as average size of new fields discovered is a decreasing function of cumulative effort, knowledge is an increasing function. The more wells drilled, the more physical infrastructure, the better the technology, and the more is known about the geology of the proved and unproved areas, hence the more efficient the search

FIGURE 1

USER COST (DEVELOPED VALUE LESS  
DEVELOPMENT COST), US CRUDE OIL

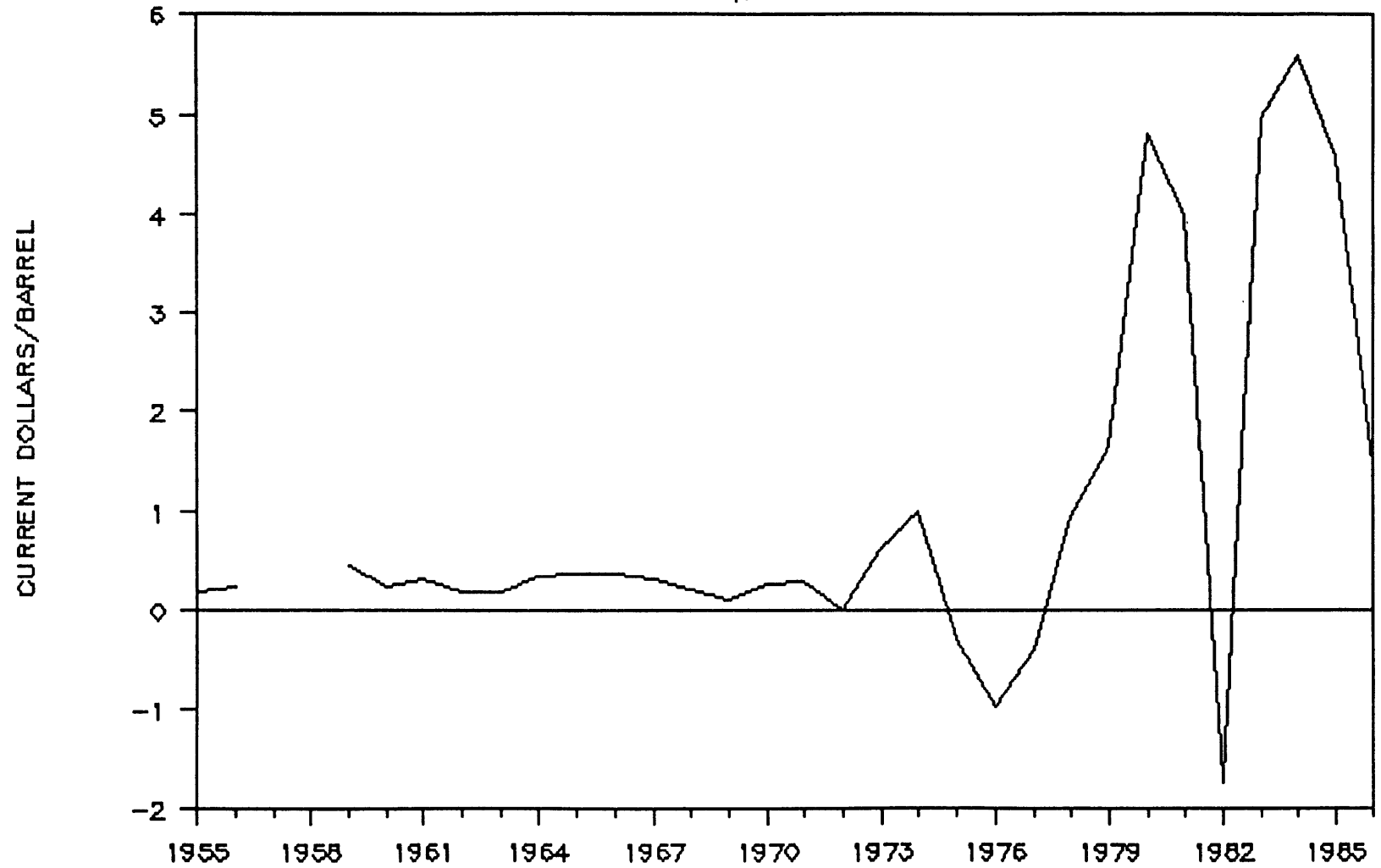
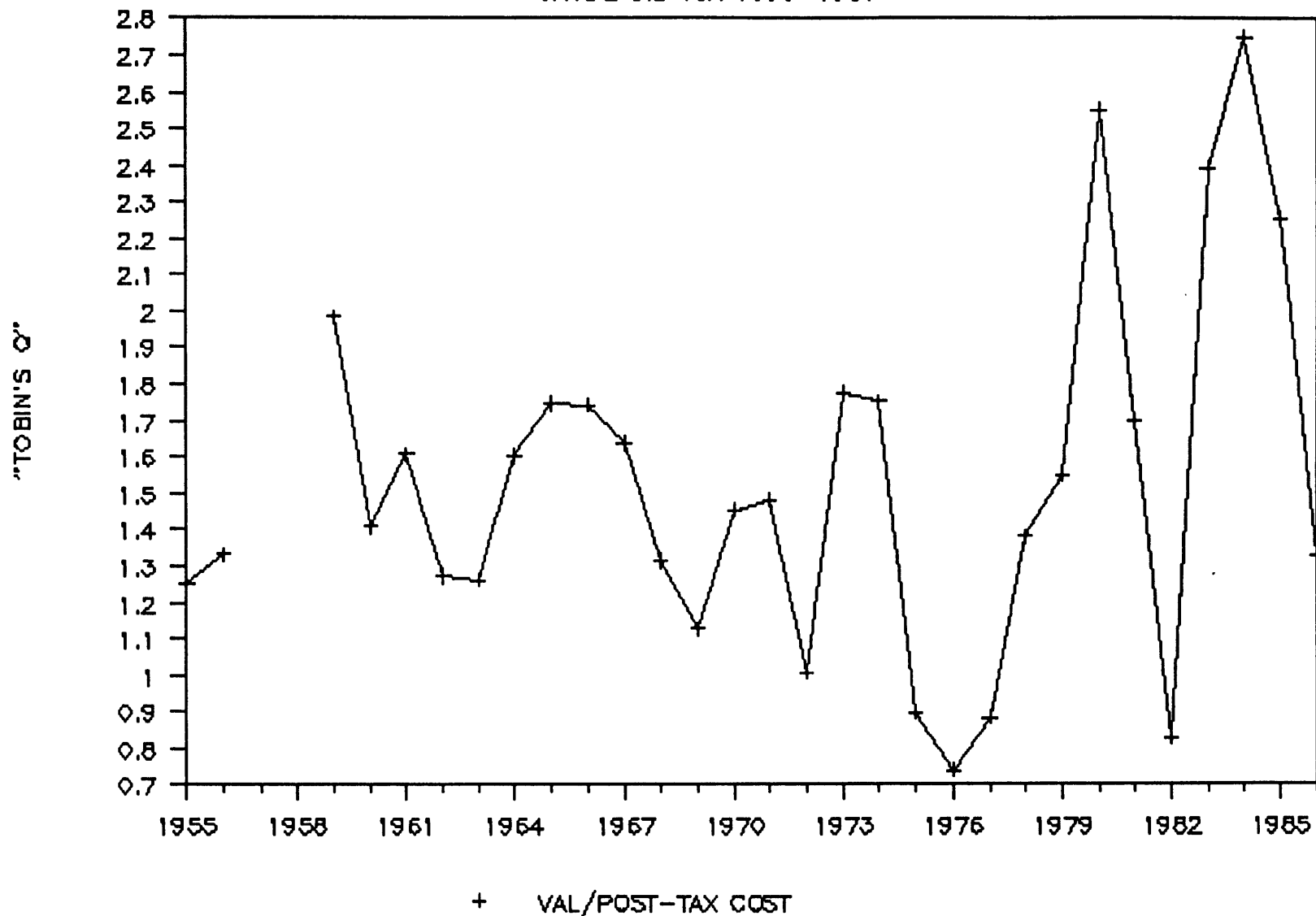


FIGURE 2

# VALUE RELATIVE TO DEVELOPMENT COST

CRUDE OIL USA 1955-1986





process. It is a perpetual tug-of-war. An important qualification: development cost and market value stayed approximately constant because the strain on the system increased only very slowly. When the effort was stepped up, efficiency suffered greatly, both because poorer prospects were drilled, and because of the general waste of a frantic boom.

The recent report [AAPG 1989] on reserves which might be developed under various price levels is in effect a forecast that the future will resemble the past. The process of wringing more oil out of old fields, which has dominated the industry since discoveries began dwindling in 1930, still has a long way to go.

#### AN APPLICATION: 1976 USER COST IN SAUDI ARABIA

For Saudi Arabia, 1973 development investment per daily barrel is given in one place at \$147 [WO 1973], in another at \$197. [PIW 2-4-74:5 for capital expenditures, PIW 5-14-73:6 for the increased capacity, which may be too low: see PIW 4-22-74:12] Over the next three years, drilling factor prices rose by 58 percent [IPAA], so for 1976 we estimate \$312 per daily barrel, or \$0.854 per annual barrel. Production was 2 percent of proved reserves [OGJ 1977]. By equation [2], the investment to develop one additional barrel of reserves is  $K/R = (K/Q)a = \$0.854 \times .02 = \$0.0170$ . Assuming fixed reserves, then by equation [6] the marginal development investment would be another 1.7 cents per barrel, and this is the maximum estimate of user cost.

We adjusted to the year 1976 because for that year the estimate can be compared with an independent observation: a market value for an undeveloped

barrel, i. e. user cost. The 1976 buyout agreement, between Arabian-American Oil Co. (Aramco) and the Saudi government, set a discovery fee for newly discovered (i. e. undeveloped) oil of 6 cents per barrel, as produced. We assume zero development time: upon discovery, depletion starts immediately, at the current Saudi 2 percent per year. Let  $U'$  be the value of the undeveloped barrel, and  $i$  the discount rate. Then the present value of the stream of fee payments of 6 cents per barrel would be:

$$U'R = (.06) Q \int_0^T e^{-(a+i)t} dt = (.06) Q/(a+i)$$

$$\text{Substituting } Q/a \text{ for } R: U' = (.06)a/(a+i) \quad [11]$$

Assuming the discount rate alternatively at 5 and 20 percent:

$$U' = 6 (.02/.07) = 1.71 \text{ cents per barrel}$$

$$U' = 6 (.02/.22) = 0.54 \text{ cents per barrel}$$

Summing up: the estimated 1976 user cost, based on development cost, and on an actual transaction, are both in the neighborhood of one cent per barrel. The range of estimates is large relatively but tiny absolutely. No allowance for error can have an important effect on the result.

It should be no surprise that the fee was only 6 cents, and in-ground value no more than 2 cents, at a time when the world market price was over \$12; market price and in-ground value in the United States were respectively \$8.19 [MER] and \$2.74. When marginal cost is very low, so is the optimal marginal revenue of a monopolist. Saudi reserves were superabundant. Beyond the very low production/reserve ratio in

the 15 developed fields, there were 22 other commercial fields identified in 1976, but not operating. [AAPG:B 1977]. In 1988, with 52 fields now identified, only the same 15 produced. [Saudi Aramco 1989] Despite oil prices many times as high, Saudi oil well completions fell 96 percent from 1973 to 1988. [WO 1974, 1989] Since Saudi Arabia was the only buyer, its marginal revenue set the value of new reserves. A higher discovery fee would have brought them more new-found oil, but for them it was not worth finding. (See also the discussion of Saudi Arabia below.)

This extraordinary gap in asset values is a symptom of the unbalanced world oil market, where low-cost reserves are kept out of production to maintain the price. Were the world oil monopoly to disappear, prices would fall to a fraction of their present levels. In-ground values and user cost in high-cost countries would decline sharply, and investment with them. But in low-cost areas there would be a burst of investment as the owners tried to save something from the wreck, and compensate for low prices by higher production. Reserves would expand, but production would expand much more, as the reserves/production ratio would rise toward the optimal, the industry rule of thumb being about 15. Development cost would rise, and so would the present value of a barrel in-ground. This would stimulate discovery, also at increasing cost.

At some point, rising values in the low-cost countries and falling values in the high-cost countries would come together, until a barrel in any country was worth the same as a barrel in any other. That would signal the end of the monopoly. It is not in view.

## NATIONAL ACCOUNTING

In recent years, work has been directed to estimating national income adjustments because of mineral production. On the usual assumptions, the question is well posed as: what accumulation of reproducible capital would just offset current production, which depletes some of the initial non-reproducible fixed stock? [Solow 1986, p. 144]

El Serafy [in Ahmad et al 1989] has devised a measure of user cost which is a special case of our Equation [3]. Consider a fully developed reservoir, with no production decline. Output  $Q$  and sales receipts  $PQ$  are constant over a finite time  $T$ , then shut off. The value of the reserve is the present value of the flow. The interest rate measures time preference of consumption, since there is no investment in the system and no risk. Now define the true income  $Y^*$  as a perpetual stream with the same present value.

$$Y^* \int_0^{\infty} e^{-it} dt = Y^*/i = PQ \int_0^T e^{-it} dt = PQ (1 - e^{-iT}) / i$$

$$Y^*/PQ = (1 - e^{-iT}) \quad [12]$$

The fraction  $Y^*/PQ$  is the ratio of true income to apparent income. The user cost component or "capital component" of the nominal stream  $PQ$  is:

$$(1 - (Y^*/PQ)) = e^{-iT}. \quad [13]$$

"The unit value of the total resource is the same as the current price."

Disregarding this (cf. above p. 7), and some other untenable theses<sup>3</sup>, we apply his concept (using the lower 5 percent of his two suggested discount rates) to the large Persian Gulf oil producers for 1989. Obviously Y\* comes so close to PQ that the user cost component is negligible.

TABLE IV. USER COST COMPONENT OF SALES RECEIPTS

Country	Reserves: Production Ratio	User Cost as Percent of	
		Sales Receipts Discounted at: 5 Percent	10 Percent
Abu Dhabi	200	0	0
Iran	83	1.6	0
Iraq	100	0.7	0
Kuwait	167	0	0
Saudi Arabia	143	0	0

Sources: Reserve:production ratios from Oil  
& Gas Journal, December 25, 1989.

User Cost factors from El Serafy, Table 3-2, p. 15

Percentages rounded to nearest tenth of one percent.

<sup>3</sup> (1) "[T]he oil market had long been an oligopsonistic market, dominated by powerful multinational conglomerates." (12) This implies prices below some long-run competitive level. But the multinational companies were sellors not buyers, whose interest was in higher not lower prices, both to increase profits, and to decrease domestic political threats from European coal miners and USA oil producers.

(2) OPEC allegedly did not restrict competition, because they had no formal market allocation until 1982. (Actually, there was one in 1980.) Thus the private oligopolists could affect prices without formal market division, but the sovereign oligopolists, even more concentrated than the private, cannot. He does not explain this double standard.

(3) "...[M]any analysts in the 1970s appeared to think that if free competition were to prevail, competitive equilibrium would indicate a price equal to the marginal cost of extraction..." (Page 12, emphasis in original.) He cites none of the "many," and it is doubtful that he can find even one.

Askari and Weitzman [Askari 1990](including Appendix I, by Martin L. Weitzman) also apply Solow's principle. They modify Equation [10] by allowing P to increase by g percent per year, as in Equation [8] above. Then we have:

$$Y^* = iPQ [1 - e^{-(i-g)T}] / (i-g) \quad [14]$$

As with El Serafy, there is no consideration of the decline rate  $\underline{a}$ , nor of any investment cost factor  $\underline{k}$ .

The Askari-Weitzman "base case" is where  $g = i$ , which is said to be "not a bad assumption empirically." Where  $g = i$ ,  $Y^*$  is indeterminate,  $0/0$ . Weitzman avoids this by using L'Hospital's rule, taking the first derivative with respect to g of the numerator and the first derivative of the denominator. At the limit ( $i = g$ ):

$$Y^* = iPQT, \text{ or } Y^*/PQ = iT = iR/Q \quad [15]$$

Table V applies Weitzman's formula to recent years. For example, in 1989 conditions, if the real rate of return is 3 percent (as against El Serafy's 5 percent), the true or permanent income is actually four times as large as conventional income.

**TABLE V. Saudi Arabia: Ratio of Conventional to True National Product**

YEAR	Reserves/ Production	ASSUMED REAL RATE OF RETURN (PCT/YEAR)	
		1%	3%
1981	46	2.17	0.72
1982	71	1.41	0.47
1983	102	0.98	0.33
1984	113	0.88	0.29
1989	143	0.71	0.24

N. B. Reserves and production, OGJ, last issue of year.  
Estimation formula, Weitzman, App. I, eq. (16).

In general, if  $i(R/Q)$  (the discount rate times the reserve/production ratio) exceeds unity, the conventional national product understates the theoretically correct value. For example, with  $i = .03$ , the critical value is  $R/Q = 33$ .

Askari (p. 16, ch. IV, p. 182) and Weitzman (p. 198) argue that a higher rate than 1 percent is a better choice. At a modest 3 percent, which they themselves use, their thesis is overturned. For in every large OPEC country the reserve/production ratio exceeds 33, and true NNP necessarily exceeds conventional NNP. Applying Equation [12], Saudi 1989 user cost is a large negative:  $1-(Y^*/P) = 1-1/(.24) = -3.2$ , or negative \$3.20 for every dollar of conventional income.

This odd result should not be taken too seriously, since it derives from the particular assumption that  $g=1$ . The important point is that the ratio  $Y^*/PQ$  and the user cost  $(1-Y^*/PQ)$  may be positive or negative, much or little, depending on the

parameters.

El Serafy, Askari, and Weitzman disregard investment in oil, a procedure which follows logically from the original premise of a fixed stock, which is "running out."<sup>4</sup> But even on that premise, one might ask: why do these asset holders keep such excessive in-ground inventory that it has, at the margin, no present value, and makes the national product less than it could be? Far from having long horizons and low rates of time preference: all but Kuwait have overspent their incomes, which was one reason for the Iraqi aggression in 1980 and 1990.

Be that as it may, the large OPEC producers have negligible (or negative) user cost. The massive gap between price and operating-cum-development cost remains unexplained. The calculations of El Serafy and Askari and Weitzman support the hypothesis that these producers are holding production off the market in order to maintain the price; i.e., are acting as a cartel.<sup>5</sup>

**Indonesia** The valuation by [Repetto et al 1989] of 1984 Indonesian oil reserves at \$24 per barrel in-ground is the market price, net of operating cost, undiscounted.

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<sup>4</sup> Both El Serafy and Askari err in supposing that production can proceed at a constant rate then abruptly cease. The decline rate stands at the center of every reservoir engineering calculation. Moreover, the rate of extraction is limited by sharply rising marginal costs. [Adelman 1990] However, this correction would not basically change the problem.

<sup>5</sup> Like El Serafy, Askari believes (p. 28) that before 1970 "the majors [multinational oil companies] kept oil prices at what may be considered artificially low levels." He does not explain why they should act against their own interests. Askari's policy prescription of a high savings rate makes good sense, if we disregard the "depletable nature of oil" as an un-fact, and instead accept that monopoly prices are inherently unstable and may drop sharply.



They must assume, with El Serafy that: "The unit value of the total resource is the same as the current price."

We now attempt a measure for 1989. Up to this point, we have only considered private values of reserves. In the United States, because oil profits are relatively lightly taxed, this does not involve any serious distortion. Elsewhere, it does. Table VI makes the adjustment.

**TABLE VI: INDONESIA  
1990 PUBLIC & PRIVATE VALUES**

A. Oil: \$/barrel				
	New Oil		Old Oil	
	Wellhead Price	In-ground Value	Wellhead Price	In-ground Value
Private	10	5	5	1
Public	8	4*	13	6*
Total	18	9*	18	7*

\*Inferred: assumes same ratio of value to price on public as on private new (low-cost) oil.

Source: Daniel Johnston, "How Indonesian Production Sharing Contracts Work," *Offshore*, July 1990, pp. 42-44

B. Natural Gas: \$/mcf				
	New Gas		Old Gas	
	Wellhead Price	In-ground Value	Wellhead Price	In-ground Value
Private	0.94	0.47	0.49	0.09
Public	0.76	0.37*	1.23	0.56*
Total	1.7	0.84	1.7	0.65

Source: Prices from *World Gas Intelligence*, 6-90-13. Rest of table assumes same tax treatment as oil, in panel A.

We take as average values \$8 for oil and 75 cents for gas, halfway between the new lightly taxed hydrocarbon and the old heavily taxed and higher-cost. Then we can calculate the total or social addition or subtraction during the year 1989:

**TABLE VII: Indonesia:  
Change in Values of Oil & Gas Reserves 1989**

	Increment to Reserves	Unit Value	Change in Value (in million \$)
Oil	-50 million barrels	\$8.00	-\$400
Gas	<u>+3.43 trillion cu ft</u>	<u>\$0.75</u>	<u>+\$2916</u>
Total	--	--	+\$2515

Source: Oil & Gas Journal, end-of-year issues for 1988 and 1989.

Indonesia gained, on balance, \$2.5 billions in 1989. Almost everywhere in the world outside North America, proved reserves have increased; hence true NNP exceeds conventional NNP.

**TABLE VIII: Net Change in Oil Reserves and Values,  
end-year 1988 to end-year 1989**

Area	Increment (MB)	Unit Value (\$)	Value (\$B)
USA	-640	6.25	-4.0
Canada	-653	5.00	-3.3
Other Market Economies, exOPEC	+11,021	4.00	+44.1

Sources: Reserves changes from International Oil & Gas Exploration & Development Activities, fourth quarter 1989 (Department of Energy, 1990). Unit values from First Boston Corporation, Oil & Gas Exploration and Production, OL 0675, March 21, 1990. NB. these values are private; complete or social values are larger, as calculated in the Indonesian example above.

**Producer country policies** If we discard the fixed-stuck assumption, we can see how overestimation of in-ground values and user costs has been costly to many countries. They demanded too much for oil exploration/ development rights, hence lost revenue they might have received. Moreover, by supposing a fixed stock of mineral wealth, they were careless of contract obligations because they overlooked the role of investment in creating new wealth.

The higher the discount rate, the lower the present value of an existing barrel. I have suggested [Adelman 1986] that in oil investment small Less-Developed-Countries [LDCs] have much higher discount rates than do private multinational companies. [Cf. Vaish 1990] If so, a given oil development is more valuable to a private company than to an LDC, and we would expect to see a drift toward allowing companies some oil development rights in countries which had previously nationalized their oil. The movement is now perceptible in nearly all

OPEC countries (as well as the Soviet Union and China). [PIW 7-16-90:Special Supplement] It is going very slowly, and may not make much difference soon, but is worth study.

### Conclusions

- 1 There is no fixed mineral stock, only an uncertain flow into mineral inventories, "proved reserves".
- 2 Mineral reserves are risky assets, like all other reproducible wealth. Capital markets do as good or bad a job of valuing them as they do other assets.
- 3 User cost or resource rent, the market value of an undeveloped reserve unit in-ground, is a valid measure of mineral scarcity.
- 4 This value is rarely observed directly. But it equals the market value of a developed reserve less the development investment, or the present value of changes in investment required per unit of reserves. At the margin the two are equal.
- 5 Adjustments to Net National Product, to allow for the depletion of mineral assets, are like any other adjustments for inventory change. They may in theory be large or small, positive or negative. Oil inventory changes appear to be relatively small, and mostly positive. A fortiori, this is true for coal; and probably for most minerals.
- 6 User cost explains none of the price-development cost gap for Persian Gulf crude oils. Monopoly is the only tenable explanation offered thus far.

## REFERENCES

(Periodicals cited by name, month-day-year: page)

- [AAPG:B] American Association of Petroleum Geologists Bulletin, monthly. Volume 61 (1977), page 1798.
- [AAPG 1989] American Association of Petroleum Geologists, Report of the Committee on the Resource Base (Tulsa n. d.)
- [Adelman 1986] M. A. Adelman, "Discount Rates for Oil Producing Nations", Resources & Energy, vol. 8 (1986), pp. 309-329
- [Adelman 1990] -----, "Mineral Depletion, with special reference to Petroleum", Review of Economics & Statistics, vol. 72 (February 1990), pp. 1-10
- [Adelman 1991] -----, "Finding and Development Costs in the USA 1945-86", forthcoming in Advances in the Economics of Energy and Resources, ed. John R. Moroney (1991)
- [Ahmad et al 1989] Yusuf J. Ahmad, Salah El Serafy, Ernst Lutz, Environmental Accounting for Sustainable Development, a UNEP-World Bank Symposium (Washington: World Bank 1989)
- [Askari 1983] Hossein Askari, "Management of External Surpluses in the Gulf Countries", American-Arab Affairs, No. 7, Winter 1983-84, pp. 100-106
- [Bradley 1989] Paul G. Bradley, Remarks to the National Energy Board Workshop on Benefit-Cost Analysis and Export Impact Assessment (Calgary, 27 November 1989)
- [Dasgupta & Heal] Partha Dasgupta and G. M. Heal, Economic Theory and Exhaustible Resources (Cambridge Economic Handbooks) (Cambridge University Press 1979)
- [DOE:AER] Department of Energy, Annual Energy Review
- [Griffin 1985] James M. Griffin, "OPEC Behavior: a Test of Alternative Hypotheses," American Economic Review, vol. 80, pp. 954-63 (1985).
- [Hotelling 1931] Harold Hotelling, "The Economics of Exhaustible Resources", Journal of Political Economy, vol. 39 (1931) 137-175
- [IPAA] Independent Petroleum Association of America, Report of the Cost Study Committee, twice yearly

- [Kmenta 1984] J. Kmenta, Elements of Econometrics (New York: Macmillan Publishing Co., 1984)
- [MER] Department of Energy, Monthly Energy Review
- [Miller & Upton 1985] Merton H. Miller and Charles W. Upton, "A Test of the Hotelling Valuation Principle", Journal of Political Economy, 93 (Feb. 1985), 1-25
- [Nordhaus 1973] William D. Nordhaus, "The Allocation of Energy Resources", Brookings Papers on Economic Activity 1973, no. 3
- [OGJ:WVO] Oil & Gas Journal, weekly, annual supplement "World Wide Oil", last issue of year
- [PEH 1987] Howard B. Bradley, ed., Petroleum Engineering Handbook (Richardson, TX: Society of Petroleum Engineers, 1987), p. 41-5
- [PIW 1989] Petroleum Intelligence Weekly
- [PPH] Thomas Frick, ed., Petroleum Production Handbook (New York: McGraw-Hill Book Co. 1962) p. 20
- [Repetto et al 1989] Robert Repetto, William Magrath, Michael Wells, Christine Beer, Fabrizio Rossini, Wasting Assets: Natural Resources in the National Income Accounts (Washington: World Resources Institute, 1989)
- [Saudi Aramco 1988] Saudi Arabian Oil Company, Annual Report 1988, page 5
- [Solow 1986] Robert M. Solow, "On the Intergenerational Allocation of Natural Resources," Scandinavian Journal of Economics, vol. 88, no. 1, pp. 141-149, 1986
- [Vaish 1990] Pankaj Vaish, "Appropriate Discount Rates for Crude Oil Cashflows of the OPEC Members: Saudi Arabia, Kuwait and Venezuela," unpublished M. S. thesis, Sloan School of MIT, 1990
- [Verleger 1990] Philip K. Verleger Jr., "Reading the Tea Leaves in a Tempestuous Tea Pot", in CRA Petroleum Economics Monthly, vol. 7, no. 5, May 1990.
- [Weitzman 1976] Martin Weitzman, "On the Welfare Significance of National Product in a Dynamic Economy", Quarterly Journal of Economics, vol. 91, no. 1 (February)

[World Bank 1988] World Bank, World Development Report (Washington: 1988)

[WO 1973] World Oil, monthly, April 1973, p. 15

[WO] ———, "International Oil", August issue

[WSJ] Wall St. Journal

### APPENDIX I: RECENT COAL RESERVE SALES

A. Payment of \$715 million, for 55 percent of the assets of Peabody Coal Co., reported as producing 9 percent of coal produced in the US, with reserves at about 100 years at current rates.[WSJ 3-30-90:A10] Using 1989 coal production from [MER January 1990, p. 69]] and 1988 minemouth prices from [AER 1988, p. 191], one may calculate as follows:

$$\text{Value per ton in-ground} = \frac{\$715 \text{ m}}{975 \times .09 \times 100 \times .55} = \$.148/\text{ton}$$

$$\text{Value/minemouth price} = \$.148/\$22 = .0067$$

B. Payment of \$115 million for "more than 390 million tons of salable coal reserves". Revenues in 1989 were \$380 million on sales of 14.1 million tons. [NYT 5-26-90:31] Then value per ton in ground was \$.295, value/price was .011, and reserves were 27.7 times annual output.

These data are unfortunately subject to rather gross errors. But no allowance for them could increase the ratios by 30 times to be near equality with the ratios for oil.

### APPENDIX II: GAIN OR LOSS TO POSTPONEMENT

Restating Equation [3] as a quadratic:

$$a = \sqrt{P_i/2ka} - 1 \quad [A1]$$

To restate net present value as a function of time, we take the depletion rate as predetermined. The net present value of the project initiated in any later year  $t$  is:

$$\begin{aligned} \text{VR}(t) &= [(PQ e^{gt} / (a+i-g)) - kaQ] e^{-it} \\ &= (PQ/(a+i-g)) e^{(g-i)t} - kaQ e^{-it} \end{aligned} \quad [A2]$$

In a fully developed deposit, or in a very low-cost pool, with almost no investment, the negative term approaches zero. Then aside from the unlikely case of the price increase  $g$  exceeding the interest rate  $i$ , the higher is  $t$ , the less is  $VR$ . Postponement is unprofitable.

At the other extreme is a project which would barely repay the cost of capital, i.e.,  $PQ/(a+i-g) = kaQ$ , and  $VR(0) = 0$ .

$$VR(t) = PQ/(a+i-g) [e^{-(g-i)t} - e^{-it}] \quad [A3]$$

For any positive value of  $t$ , the bracketed expression of Equation [8] is positive,  $VR(t)$  exceeds zero, and it pays to postpone. Waiting raises the expected revenues, but not the needed investment. Therefore a "dog" of a project is always worth postponing, and has only option value.

To find the optimal time to postponement, we rewrite [A2] and differentiate with respect to  $t$ :

$$VR(t) = [PQ/(a+i-g) e^{gt} - kaQ] e^{-it}$$

$$= PQ/(a+i-g) e^{(g-i)t} - kaQ e^{-it}$$

$$d(VR(t))/dt = [PQ/(a+i-g)] (g-i) e^{(g-i)t} + kaQ (ie^{-it}) = 0$$

Divide by  $-Q(i-g)e^{-it}$ :

$$-P/(a+i-g) \frac{(g-i)}{(i-g)} e^{(g-i)t}/e^{-it} = kai e^{-it} / i-g (e^{-it})$$

Canceling:  $P/(a+i-g) e^{gt} = kai/i-g$

$$e^{gt} = \frac{k}{P} \frac{ai(a+i-g)}{i-g}$$

$$t = [(\ln k - \ln P) + \ln a + \ln i + \ln(a+i-g) - \ln(i-g)] / g \quad [A4]$$

The lower is the cost factor  $k$ , the less the postponement. Good projects should not wait. Negative  $t$  means project should have commenced before year 0. Saudi  $t$  runs to negative decades.

As for the derivative, only the last two terms in [A4] are terms in  $g$ . Hence:



$$dt/dg = \frac{g \cdot 1(-1) - \ln(a+i-g)(1)}{g^2} - \frac{g \cdot 1(-1) - \ln(i-g)(1)}{g^2}$$

$$dt/dg = \frac{\frac{g}{(i-g)} - \frac{g}{(a+i-g)} + \ln(i-g) - \ln(a+i-g)}{g^2}$$

Note that  $g/(i-g)$  must always exceed  $g/(a+i-g)$ , while  $\ln(i-g)$  must always exceed  $\ln(a+i-g)$ , and since both are negative the algebraic sum must be negative. Hence the first two terms sum to a positive, the second pair to a negative. But as  $g$  increases, the positive terms increase, the negative terms decrease in absolute values, so all terms increase algebraically. Hence while  $dt/dg$  may possibly be negative at small values of  $g$ , it must be positive as  $g$  increases.

The equation works empirically: if we take any likely values, and calculate values from [A4],  $t$  varies directly with  $g$ , and first differences are always positive.