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A. BREAKDOWN OF OXYGEN, NITROGEN, AND AIR

Breakdown fields of oxygen, nitrogen, and air have been obtained, using a one-quarter-inch gap O.F.H.C. copper cavity ($\Lambda = 0.202$ cm). The results for oxygen, nitrogen, and pure air are shown in Figure II-1, where the
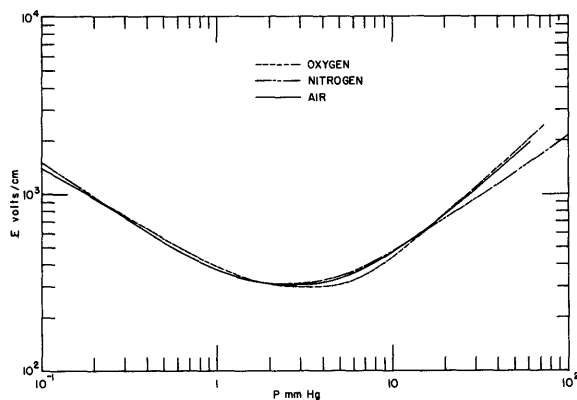

Fig. II-1 Breakdown fields of oxygen, nitrogen, and air.

experimental points (all lying within 2 per cent of the curves) have been omitted for the sake of clarity. In the pressure range 0.2 − 2.0 mm Hg, the nitrogen and air curves coincide, and below 0.2 mm Hg, the oxygen and air curves coincide.

Since the probabilities of excitation, P_x, are not known for the gases, detailed interpretation is not possible. However, assuming the P_x to be not very different, as seems likely from these results, the air should behave mostly like nitrogen, and a little like oxygen. This is seen to be the case, except at pressures above 16 mm Hg ($E/p < 40$ volts/cm mm Hg).

It is found that successive discharges lower the breakdown field of air; an equilibrium is obtained after several discharges in the same sample. Curves obtained in this way are rather like those of nitrogen, in that, at high pressure, the field is markedly lowered below that of pure air; however, the curves are, for the most part, a little lower than those of nitrogen. A breakdown calculated from data by Herlin and Brown (1) gives a curve of the same shape as, but about eight per cent lower than that for the contaminated air. This contamination is believed due to the formation of NO and NO_2. Ephraim (2) discusses their formation and adsorption on materials, especially on copper and cuprous oxide. By such processes, the oxides may

be retained in the cavity for appreciable lengths of time. Their lower ionization potential might then account for the difference in breakdown fields.

The conclusion to be drawn from this study is that the breakdown of pure air can be accomplished only by measuring a single breakdown; the air in the cavity is then contaminated and must be changed.

B. BREAKDOWN WITH PULSED FIELDS

The single electron picture of breakdown predicts that when an electric field is applied to a cavity, the energy of the electrons will increase until ionization occurs. At this point, the density of the electrons will increase and breakdown occurs. The density build-up is very rapid (10^{-9} seconds). Preliminary experiments reveal no change in pulsed breakdown for changes in the initial number of electrons or for repetition rates. This indicates that the important phenomenon is the build-up of the energy.

Elementary breakdown theory predicts that the time rate of change of total energy should equal the power put into the electrons minus the recoil and diffusion power losses. The power input is

$$\overrightarrow{eE_f} \cdot \overrightarrow{V} = \frac{eE_f^2}{m\nu_c} \quad ,$$

where

$$E_f = E\left(\frac{\nu_c^2}{\nu_c^2 + \omega^2}\right)^{1/2} \quad ,$$

is the peak effective electric field, ν_c is the collision frequency, and ω the angular frequency of the field. Hence

$$\frac{du}{dt} = \left(\frac{eE_f^2}{m\nu_c}\right) - \frac{2m}{M}\nu_c u - \nu_d u \quad .$$

The steady-state or c-w breakdown condition is

$$\frac{du}{dt} = 0 = \frac{eE_{of}^2}{m\nu_c} - \frac{2m}{M}\nu_c u_o - \nu_d u_o \quad ,$$

where E_{of} is the effective c-w breakdown field, and u_o some energy above the ionization energy.

In the pulsed case under study, the energy, u, must build up to u_o in a time equal to the pulse width, τ. In the only case for which data

have been taken, $p \Lambda \geqq 2$ so that the diffusion losses are much less than the recoil losses. Considering ν_c constant and integrating

$$E_f^2 - E_{of}^2 = E_f^2 \, e^{-2\frac{m}{M}\nu_c \tau} \quad .$$

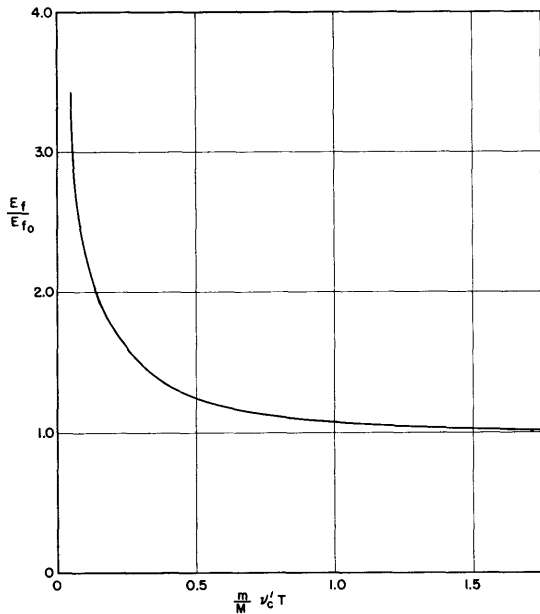This function is plotted in Figure II-2. The collision frequency is not a

Fig. II-2 Effective breakdown fields for pulsed and c-w cases for single electron theory.

constant for air and this simple theory neglects inelastic collisions. However, as can be seen from Figure II-3, there is functional agreement.
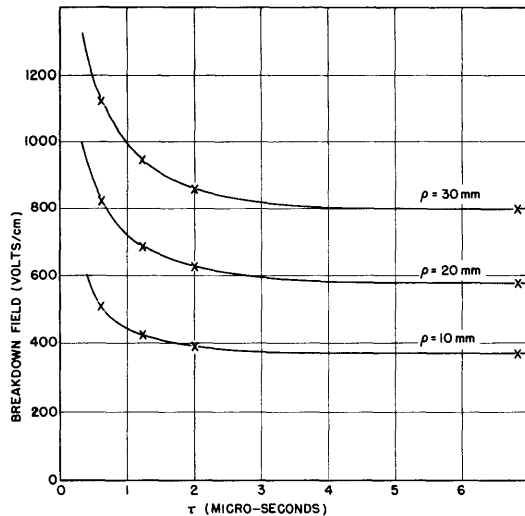

Fig. II-3 Experimental curves for breakdown in air as a function of pulse width.

The next step is to use helium, where the theory is more applicable.

C. MEASUREMENT IN HELIUM GAS PURIFIED WITH A LIQUID HELIUM TRAP

An oxygen-free copper 10-cm resonant cavity which can be outgassed and which permits the application of a d-c field to the discharge region has been constructed and is shown in Figure II-4. The cavity is essentially
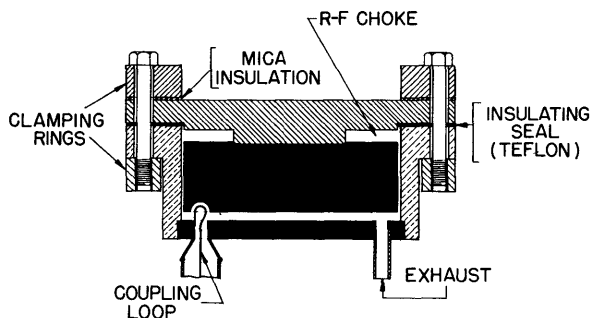

Fig. II-4 Parallel-plate resonant cavity.

a TM_{010} cylincrical mode cavity with a d-c break around the diameter. This break is backed up by an r-f choke. An insulating gasket made of sheet Teflon serves also to position the inner cavity assembly and as a vacuum seal. The flanges are clamped together on the gasket with clamping rings and bolts. This permitted baking the cavity at 290°C without developing a gasket leak or damaging the Teflon.

Before helium was released into the vacuum system, the cavity was baked for a week at 280°C, the cavity having had an extended history of outgassing. In order to obtain the degree of purity of helium required for the experiment, a trap cooled by liquid helium was employed through the cooperation of the Cryogenics Group, R.L.E. In this way, any impurities initially present in the helium bottles (estimated to be of the order of 10^{-5} for spectroscopically pure helium) are condensed, as well as any small amounts of impurities evolved from the metal during a discharge which could accumulate to form an appreciable concentration. In the actual experiment, the concentration of impurities was estimated to be not greater than one part in 10^6.

The measurement of D/μ, converted to the average energy of electrons as described in the last Progress Report, (April 15, 1949), has been made under these conditions of purity in helium and is shown in Figure II-5. Measurements by Townsend (3), who employed a d-c method not seriously impaired by impurities, are included for lower E/p in helium.

Townsend's first ionization coefficient α/p can be calculated from the experiment by the relation

$$\frac{\alpha}{p} = \eta \frac{E}{p} = \frac{D/\mu}{(p\Lambda)(E\Lambda)} \quad .$$

A curve so obtained is given in Figure II-6. Enclosed as well is a curve
obtained by Townsend (4) based on the multiplication of small d-c currents
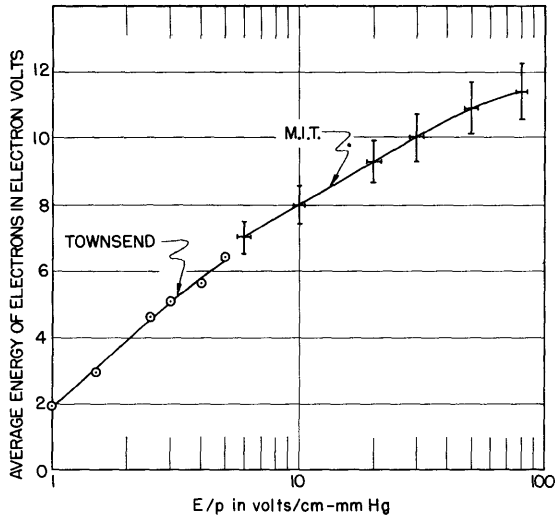in the gas in the standard manner.

Fig. II-5 Average energy of electrons
in pure helium.

It will be noted that an essentially d-c gas parameter, α/p, has been
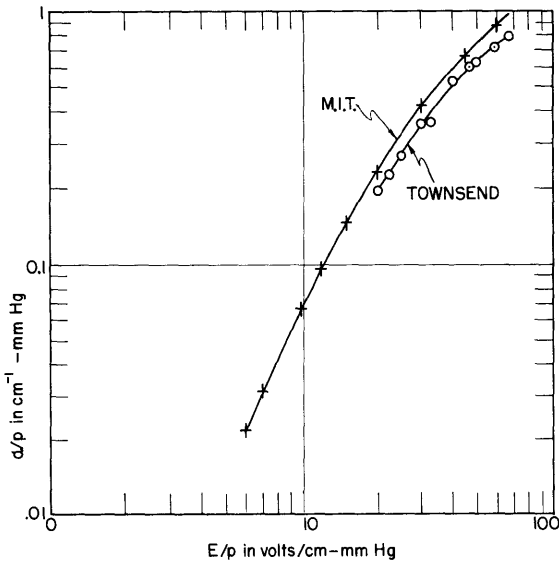measured by means of a microwave experiment. In addition, the range of

Fig. II-6 First Townsend coefficient
for pure helium.

α/p has been extended to the lower E/p region where d-c measurements were
seriously handicapped by low ionization as well as a more serious purity
problem, controlled here by the liquid helium trap.

D. TRANSIENT DISCHARGE CHARACTERISTICS

The study of electron-ion recombination has been extended to the

monatomic gases He, Ne and A and to the diatomic gases H_2, N_2 and O_2.

The value of the recombination coefficient, α, measured in He at T = 300°K and p = 21 and 29 mm Hg is

$$\alpha = 1.7 \times 10^{-8} \text{ cc/ion-sec.}$$

The large ambipolar diffusion loss at lower pressures prevents measurement of the recombination over a wide pressure range.

Neon has been studied extensively because recombination is the dominant removal process over a wide pressure and temperature range and because the commercially prepared gas samples are sufficiently pure to give reproducible results with many different samples of neon. The accuracy of the data for neon is illustrated by Figure II-7.
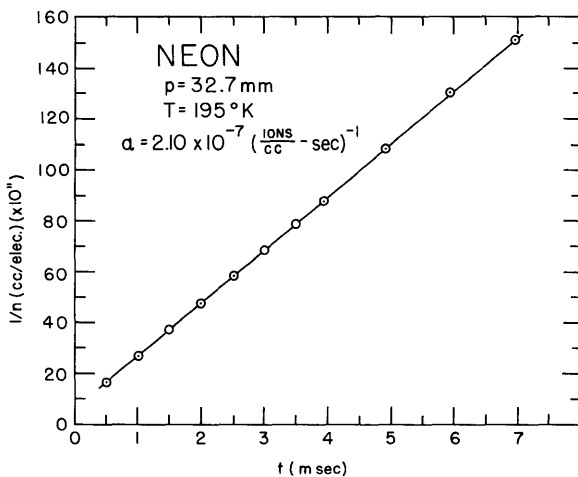

Fig. II-7 Recombination in neon at T = 195°K.

The variation of the recombination coefficient with pressure at various temperatures has been studied. Since the electrons and ions are in thermal equilibrium with the gas, the gas temperature is a measure of the electron and ion energy. The results of the measurements are shown in Figures II-8 and II-9. The recombination is independent of pressure and temperature except at T = 77°K, where a strong pressure dependence sets in. However, the residual value of α (the extrapolated value of α at p = 0) is the same at 77°K as at 195°K and 300°K. Empirically, it is found that the recombination coefficient at 77°K is given by

$$\alpha = \alpha_r + Ae^{bp}$$

where

$$\alpha_r = 1.94 \times 10^{-7} \text{ (cc/ion-sec)}$$
$$A = 0.13 \times 10^{-7} \text{ (cc/ion-sec)}$$

$$b = 0.16 \ (\text{mm Hg})^{-1} \ .$$

α_r is called the residual part of the recombination coefficient since it does not seem to depend on temperature or pressure. No explanation of the exponential behavior of the pressure dependent part
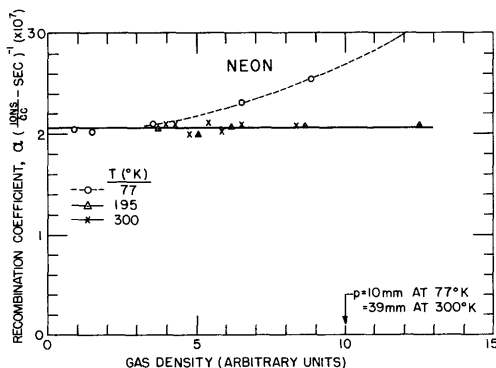
$$\alpha(p) = Ae^{bp}$$

is available at present.

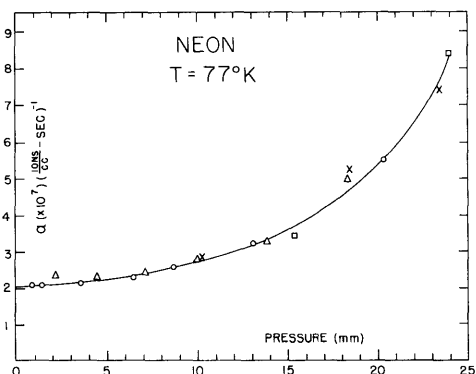Fig. II–8 Recombination in neon as a function of gas density and temperature.

Fig. II–9 Recombination in neon at T = 77°K.

The lack of pressure dependence at various temperatures indicates that the residual part of the recombination involves a two-body radiative capture of the electron by the ion.

The measurements in argon are less satisfactory because we have been unable to obtain sufficiently pure gas samples to give adequately reproducible results. The value of the recombination coefficient in argon, obtained from an average of many different gas samples, appears to be

$$\alpha = 3 \times 10^{-7} \ \text{cc/ion-sec.}$$

The recombination coefficient for hydrogen is more than 100 times

that of helium, indicating that the vibration and rotation levels which exist in diatomic molecules greatly enhance the capture probability. The data for hydrogen are shown in Figure II-10. The recombination is found to be independent of pressure over the measured range.
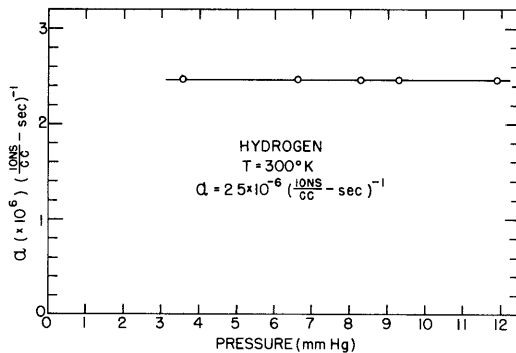

Fig. II-10   Electron-ion recombination in hydrogen at T = 300°K.

The data for nitrogen are shown in Figure II-11. It is found that a pressure dependence similar to that observed in neon at T = 77°K sets in at room temperature in nitrogen.
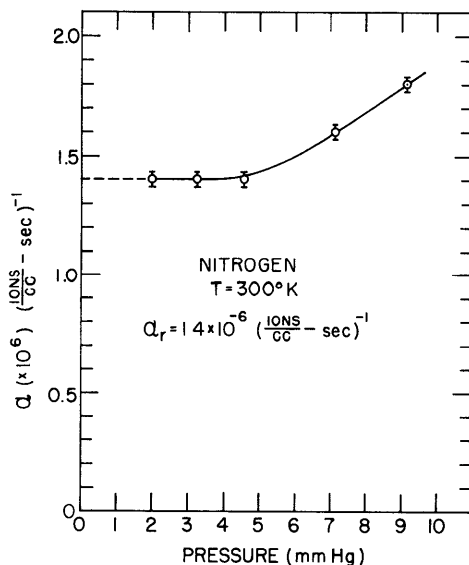
Fig. II-11   Recombination in nitrogen at T = 300°K.

The data for oxygen are given in Figure II-12. The pressure dependence is observed to be much stronger for O_2 than for N_2. Also it is found that the recombination coefficients decrease with increasing atomic number for diatomic gases while the recombination increased with increasing mass number for monatomic gases.

Existing quantum mechanical theories for recombination between electrons and ions predict coefficients many orders of magnitude smaller than

those observed in this experiment. The major contribution to recombination occurs, according to those theories, for capture of the free electron into ground state or low-lying excited levels. We have

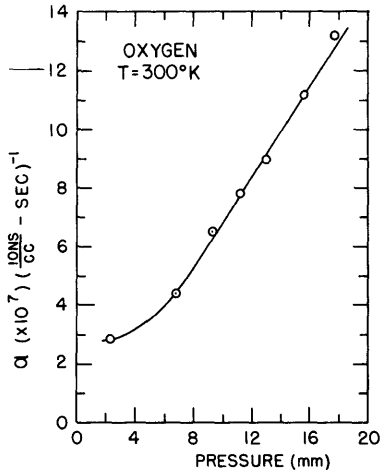Fig. II-12 Recombination in oxygen at T = 300°K.

developed a classical theory which calculates the recombination for electrons captured into the highest lying excited levels. Although developed independently, a number of the features of this theory had previously been calculated by Kramers and Eddington (5).

We shall calculate the orbit of electrons accelerated in the field of positive ions and compute the energy radiated in traversing the orbit. Electrons which radiate their initial kinetic energy or greater will be considered to be captured by the ion. A critical capture orbit can be calculated for an electron of initial velocity, v_1, to lose just its initial kinetic energy. A capture cross-section can be defined in terms of this orbit.

The rate of removal of electrons of velocity, v, by recombination is

$$\frac{\partial n_-}{\partial t} = -\alpha(v)n_- n_+$$

where $\alpha(v)$ is the recombination coefficient for monoenergetic electrons of velocity, v, and n is density. In terms of the capture cross-section

$$\alpha(v) = \pi r_x^2 v$$

where r_x is the radius of the capture cross-section.

An electron starting at "infinity" with a velocity v_1 pointing at a target distance r_x from an ion will be accelerated toward the ion and may have a closest distance of approach, r_a. If the electron's energy is small

compared to the potential at r_a from the ion, we have from conservation of momentum and energy, in unrationalized esu units:

$$r_x = \sqrt{\frac{2Z^{*}e^{2}r_a}{m}} \frac{1}{v_1}$$

where Z^{*} is the effective charge of the ion.

The rate of radiation of energy by an accelerated electron is

$$\frac{du}{dt} = \frac{2}{3} \frac{e^2 a^2}{c^3}$$

where e is the electron's charge, a is its acceleration, and c the velocity of light. The energy radiated along the orbit will be given by

$$u_{rad} = 2 \int_{\infty}^{r_a} (\frac{du}{dt}) dt \quad .$$

We express du/dt in terms of r by means of Coulomb's Law and dt in terms of r from the solution of the central force equations. The solution for u_{rad} may be expressed in terms of r_x and v_1 and is given, for electrons of small initial kinetic energy, by

$$u_{rad} = \frac{2\pi(Z^{*})^4 e^{10}}{m^4 v_1^5 c^3 r_x^5} = \frac{1}{2} m v_1^2$$

where we have set u_{rad} equal to the electron's initial kinetic energy along the critical capture orbit. The recombination coefficient for electrons of velocity, v, is thus

$$\alpha(v) = 8.64 \left\{ \frac{(Z^{*})^{8/5} e^4}{m^2 c^{6/5} v^{9/5}} \right\} \quad .$$

It is found that, according to this classical theory, the electron penetrates the electron shell of the ion and hence is accelerated in a field arising from a variable charge. Using the quantum mechanical model for the helium ion and the Fermi-Thomas model for the neon and argon ions it is found that at the closest distance of approach of the electron to the ion, the effective charge, Z^{*}, is

Ion	Z^{*}
He^+	~2
Ne^+	8 to 9
A^+	11 to 13 .

We shall use the value of Z^* at the closest distance of approach, r_a, to calculate the radiated energy. Since most of the energy is radiated when the electron is very near the ion this value of Z^* introduces only a small error into the calculation. Averaging the monoenergetic recombination coefficient, $\alpha(v)$, over the Maxwellian energy distribution of the electrons, we find that $\alpha(T)$, the observable recombination coefficient, is given by

$$\alpha(T) = 14.5 \left(\frac{m}{2kT}\right)^{0.9} \frac{(Z^*)^{8/5} e^4}{m^2 c^{6/5}}$$

where T is the electron temperature. The values of $\alpha(T)$ at 300°K are compared with measured values in the following table:

Recombination Coefficients at T = 300°K

Gas	$\alpha(T)$ Theory	α(measured)	Ratio of values
He	2.0×10^{-7}	1.7×10^{-8}	12
Ne	2.2×10^{-6}	2.05×10^{-7}	11
A	3.3×10^{-6}	3×10^{-7}	11

The theory predicts an order of magnitude greater recombination coefficient than is observed (as compared to previous theories which predict 10^4 times too small a coefficient). In addition it predicts the _relative_ values of α in going from He to Ne to A quite accurately. The better agreement between this theory and experiment suggests that recombination of electrons into high-lying excited states is quite important, in disagreement with the results of previous quantum mechanical theories.

E. AN ERROR IN A PAPER BY LANDAU (6) ON COULOMB INTERACTIONS IN A PLASMA

Landau (6) attempts to show that the Coulomb interactions of electrons and ions in a plasma can be represented as the divergence of a flow vector in momentum space. To obtain this result, he expands the probability w of scattering through a small angle in powers of that angle without properly taking into account the singularity of w at zero deflection. Fortunately this expansion is avoided by reversing the integration by parts. Landau's Eq. (1) is

$$\int [n(p)n'(p') - n(p + \Delta)n'(p' - \Delta)] \, w(p - p', \Delta) d\tau' d\tau_\Delta \quad . \quad (1)$$

The distributions n, n' are continuous and can be expanded in powers of Δ

$$n(p + \Delta) = n(p) + \frac{\partial n}{\partial p_i} \Delta_i + \frac{1}{2} \frac{\partial^2 n}{\partial p_i \partial p_k} \Delta_i \Delta_k$$

$$n'(p' - \Delta) = n'(p') - \frac{\partial n'}{\partial p_i'} \Delta_i + \frac{1}{2} \frac{\partial^2 n'}{\partial p_i' \partial p_k'} \Delta_i \Delta_k \ .$$

Substituting in Eq. (1), the zero order terms cancel and the first order terms vanish on integration, leaving

$$\int \left[\frac{\partial n}{\partial p_i} \frac{\partial n'}{\partial p_k'} - n' \frac{\partial^2 n}{\partial p_i \partial p_k} + \frac{\partial n}{\partial p_i} \frac{\partial n'}{\partial p_k'} - n \frac{\partial^2 n'}{\partial p_i' \partial p_k'} \right] \pi \frac{\Delta_i \Delta_k}{2} \, d\tau' d\tau_\Delta. \quad (2)$$

This expression is correct. The last two terms are now integrated by parts, and use made of the relation

$$\frac{\partial w}{\partial p_k'} = - \frac{\partial w}{\partial p_k}$$

giving

$$\int \left[\left(\frac{\partial n}{\partial p_i} \frac{\partial n'}{\partial p_k'} - n' \frac{\partial^2 n}{\partial p_i \partial p_k} \right) w + \left(n' \frac{\partial n}{\partial p_i} - n \frac{\partial n'}{\partial p_i'} \right) \frac{\partial w}{\partial p_k} \right] \frac{\Delta_i \Delta_k}{2} \, d\tau' d\tau_\Delta. \quad (3)$$

This is Landau's result on p. 156 except for the sign before the second parenthesis. Unfortunately, this expression does not reduce to a divergence.

There is a further error in Landau's method, which has been followed in going from Eq. (2) to Eq. (3) in order to point out the error in sign. The vector $\vec{p} + \vec{p}'/2 = \vec{g}$ represents the motion of the center of gravity and the conservation laws require $\vec{p}$, $\vec{p}'$, $\vec{p} + \vec{\Delta}$, and $\vec{p}' - \vec{\Delta}$ to terminate on a sphere about $\vec{g}$. The integration by parts was done holding $\vec{p}$ and $\vec{\Delta}$ constant. This obviously restricts $\vec{p}'$. To be correct, the integration should be at constant scattering angle, in which case the quantity $\Delta_i \Delta_k$ occurs with w in the derivative $\partial(w\Delta_i\Delta_k)\partial p_k$.

Unless the second half of Eq.(3) can be shown to be small, these errors will affect the results of Cahn (7) in two recent papers on the velocity distribution in a plasma.

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(6) E. Landau, Physik Zeits. Sowjetunion 10, 154 (1936).

(7) J. H. Cahn, Phys. Rev. 75, pp. 293 and 838 (1949).