# X. COMMUNICATION RESEARCH


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## A. INFORMATION THEORY

### 1. Introduction

Information theory indicates that the performance of existing communication systems could be greatly improved by suitably processing the signal before transmission. In practice, two major problems confront us when we attempt to realize any substantial improvement. The first problem involves the identification of the data to be transmitted and the measurement of the corresponding amount of information. The second problem, often referred to as coding, concerns the representation of the data in a form most suitable for transmission through the available communication channel.

The main difficulty connected with the first problem results from the fact that the communication system to be designed may be but a link in a larger system in which the ultimate receiver, at least, is a human brain. This is the case, for instance, in telephony and television. Since our knowledge of the human part of the communication system is still very poor, we are forced to employ a cut-and-try approach to the problem together with a measuring technique of the null type. Thus we assume that the signal to be transmitted belongs to some particular subclass of time function, and then we try to transmit it through a channel of limited capacity by making use of the characteristics of the assumed subclass. If the linear receiver can extract the desired information from the output of the channel of limited capacity as well as from the original signal, we can state that the signal belongs to the assumed subclass and that the average rate at which information needs to be transmitted does not exceed the capacity of the channel employed. This approach is being used in connection with speech and picture transmission.

In more conventional terms, the goal of this research program is the reduction of the required frequency band; in fact, reducing the bandwidth of a channel results, for a given signal-to-noise ratio, in a proportionate reduction of the channel capacity. It should be stressed that our present primary goal is the determination of upper bounds to the required channel capacities rather than the design of complete transmission systems; no attention is being given, for the moment, to the problems presented by the space separation of transmitter and receiver, nor to those involving equipment complexity.

In the case of speech communication, the program amounts to ways and means of improving the performance of vocoders and similar devices. Better techniques are being investigated for the analysis and synthesis of frequency spectra. We are also completing a device that should constitute an appreciable improvement over the speech compression system of the chopper type that was developed at the University of Illinois (G. Fairbanks et al.: Methods for Time or Frequency Compression-Expansion of Speech, Convention Record of the I. R. E., 1953 National Convention, p. 120). In the new device, the chopping is synchronous with the pitch, so that the natural characteristics of the voice should be preserved even for large compression ratios.

The work on picture transmission has been resumed along new lines. Previous work in this Laboratory, as well as elsewhere, has indicated that quantization of the light intensity is not a promising approach. It appears, instead, that the derivative (possible in two dimensions) of the light intensity can be quantized to a relatively small number of levels without introducing appreciable picture degradation. The present program calls for the determination of the minimum number of levels and for the measurement of their probability distribution. There are reasons to believe that such results will indicate the possibility of reducing the channel bandwidth by an appreciable amount.

Work is in progress on the second problem mentioned above: coding. The fundamental theorem of information theory states that if the data to be transmitted are properly coded in blocks of increasing size (this operation involves a correspondingly increasing delay), it is possible to transmit information at any rate not exceeding the channel capacity with a vanishingly small probability of error. We present in the next section a new proof to this theorem that is more satisfactory, in some respects, than the one previously available. It is hoped, in addition, that the new approach will yield useful upper bounds to the probability of error for finite coding delay.

The derivation of a general coding procedure that realizes, in practice, the behavior predicted by the new theorem is the second objective of the theoretical part of the research program. Such a coding procedure would be of particular practical importance in channels disturbed by nonadditive noise in which the probability of error cannot be decreased at will by increasing the transmitter power. Progress in this area is expected to be rather slow because of the extreme difficulty of the problem.

R. M. Fano

### 2. A New Proof of Shannon's Theorem for Noisy Channels

A new proof of Shannon's theorem for noisy channels has been obtained. It yields the further result that the equivocation, as well as the probability of error, vanishes. The main ideas in the proof are summarized below.

Let us consider a communication channel defined by a transmitter alphabet $x_1, x_2, \ldots, x_m$, a receiver alphabet $y_1, y_2, \ldots, y_{m'}$. Shannon's theorem states that if we consider sequences of n symbols it is possible to select from all such sequences of x-symbols a number N such that if only these sequences are allowed to be transmitted, the probability that the receiver will make an error in identifying the transmitted sequences will approach zero as n approaches infinity, as long as N satisfies the inequality $\log_2 N < nC$. The constant C represents the channel capacity per symbol; thus this inequality states that the rate of transmission of information must be smaller than the channel capacity.

The basic idea in the new proof may be explained, approximately, as follows. Let us select a sequence $X_1$ of x-symbols and associate to it a set $S_1$ of sequences Y of y-symbols in such a way that the sum of the corresponding conditional probabilities for the specified $X_1$ satisfies the inequality

$$\sum_{S_1} P(Y/X_1) > 1 - e$$

where e is a small constant that will be interpreted later as the probability of error. Next, let us select in succession other sequences $X_i$ of x-symbols, together with the corresponding sets $S_i$ of Y sequences, in such a way that the sets $S_i$ do not overlap. The proof amounts, roughly speaking, to showing that if after selecting N sequences of x-symbols no additional sequences of x-symbols can be found that will meet the stated conditions, then N is sufficiently large to prove Shannon's theorem. More precisely, the probability of error e can be made to approach zero as n approaches infinity, while keeping the ratio $(\log_2 N)/n$ constant and as close as desired to the channel capacity.

The basic result about the magnitude of N consists of the inequality

$$N\, 2^{-n(C-\epsilon_1-\epsilon_2)} > (e-\alpha)\, 2^{-2n\epsilon_2} \left[\left(1 - \delta_2 - \frac{\delta_1}{\alpha}\right) - \frac{N}{2^{n(H(x)-\epsilon_2)}}\right] \tag{1}$$

where $\alpha$ may be taken as any positive constant smaller than e; $\delta_1$ and $\delta_2$, $\epsilon_1$ and $\epsilon_2$, are small constants that can be made to vanish simultaneously when n approaches infinity; and H(x) represents the entropy of the x-alphabet appearing in the usual computation of the channel capacity. It can be shown further from this inequality that for a given $\left[(\log_2 N)/n < C\right]$ the probability of error vanishes sufficiently fast with increasing n so that not only the ratio of the equivocation to the information transmitted vanishes but also the equivocation itself. This last statement constitutes a new result more powerful than the original statement of Shannon's theorem. There are good hopes, also, that inequality 1 will yield a useful upper bound to the probability of error for finite values of n.

A. Feinstein

## B. SECOND-ORDER CORRELATION

The second-order autocorrelation function of a random or periodic function $f_1(t)$ is defined as

$$\phi_{111}(\tau_1, \tau_2) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t) f_1(t + \tau_1) f_1(t + \tau_1 + \tau_2)\, dt$$

Work is being carried on toward the experimental evaluation of this function. The block diagram of the experimental setup is shown in Fig. X-1.

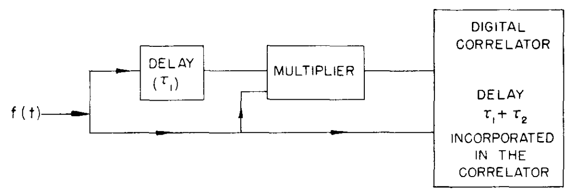


Fig. X-1

Setup for the experimental evaluation of second-order correlation functions.

At present the chief source of difficulty in the measurements is the multiplier. A duplicate of the multiplier designed by Mr. J. Miller, of M. I. T., has been constructed and is being tested. It is desired to have satisfactory frequency characteristics from 0-50 kc/sec with a minimum drift in the multiplier.

A. Bose, Y. W. Lee

## C. WIENER'S METHOD OF NONLINEAR SYSTEM CHARACTERIZATION

Work is being started on a method suggested by Dr. Norbert Wiener for the characterization of nonlinear systems. The method is confined to those nonlinear systems in which the remote past becomes less and less relevant to the behavior of the system as we push it back in time.

The object is to define for such a nonlinear system an operator that can be experimentally evaluated for the given system and that once evaluated, permits synthesis of the system. The probe for the investigation of the system and the evaluation of the operator is, as suggested by Dr. Wiener, gaussian noise. The method of characterization consists of expanding the output of the system in a series of Hermite polynomials of the Laguerre coefficients of the gaussian noise input.

A. Bose, Y. W. Lee