VI. MICROWAVE ELECTRONICS

Prof. L. J. Chu
Prof. H. A. Haus
L. D. Smullin
A. Bers
D. L. Bobroff
T. J. Connor
C. Fried
A. J. Lichtenberg
C. Morgenroth

A. A KINETIC POWER THEOREM FOR THE TRANSVERSE FIELD TUBE

The kinetic power theorem for an electron beam that interacts solely with longitudinal electric fields was formulated by L. J. Chu (1). It has served to explore the conditions under which growing waves can be excited by interaction of a beam with an rf structure and has established an analogy between longitudinal beam amplifiers and linear passive networks.

A similar theorem has been proved for Pierce's model of a transverse field tube (2) in which both longitudinal and transverse rf fields interact with the beam. The following assumptions have been made:

1. Small-signal theory is applicable.

2. The transverse dimensions of the beam are so small that the modulation quantities are constant through a cross section of the filament (or strip beam). This assumption implies that all modulation quantities are functions of one single distance variable.

3. The undisturbed flow of the electrons is rectilinear and follows an axis (or plane) of symmetry of the magnetic focusing field. All time average accelerations are in the direction of the electron flow.

These assumptions are consistent with Pierce's model of a transverse field tube (2). We allow for a variation in the magnitude of the magnetic focusing field as a function of the distance z along the flow lines of the undisturbed electrons. (See Fig. VI-1.)

An identity that is analogous to the conventional complex Poynting theorem can be derived from the linearized Maxwell equations.

$$-\nabla \cdot \hat{E} \times \hat{H}^* = \hat{E} \cdot \hat{J}^* + j\omega \left[\mu \hat{H} \cdot \hat{H}^* - \epsilon \hat{E} \cdot \hat{E}^*\right] \tag{1}$$

The circumflex indicates small-signal, complex vector quantities. The left-hand side of Eq. 1 is the divergence of the complex power flow density associated with the sinusoidally time-varying fields of frequency ω. The real part of this quantity is determined entirely by Re $[\hat{E} \cdot \hat{J}^*]$, a quantity associated with the modulation of the electron beam.

Integration of Eq. 1 over a volume of length Δz which contains a section of the electron beam (Fig. VI-2) gives

$$-\oint \hat{E} \times \hat{H}^* \cdot d\bar{S} = \hat{E} \cdot \int \hat{J}^* \, da \, \Delta z = \hat{E} \cdot \hat{I}^* \Delta z \tag{2}$$

since the stored energy in the volume that includes the filament (or strip) beam can be neglected.

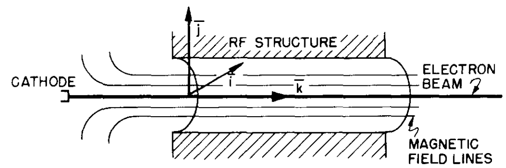

Fig. VI-1
Schematic of a transverse field tube.

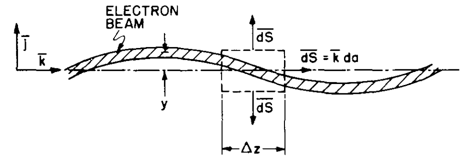

Fig. VI-2
Volume of integration.

The longitudinal component of the alternating current is $I_z = \rho_o v_z + \rho_1 u_o$, where ρ_o is the time-average charge density per unit length, u_o is the time average velocity, and ρ_1 and v_z are the corresponding time varying components. In Pierce's model of a transverse field tube (2) the transverse components of the alternating current are given by $I_x = j\omega\rho_o x$ and $I_y = j\omega\rho_o y$, where x and y are the transverse deviations of the filament beam from its undisturbed position. Indeed, Pierce's determinantal equation (Eq. 13.23 of ref. 2) can be obtained from Eq. 2 by identifying the term $1/2 \oint \hat{E} \times \hat{H}^* \cdot d\bar{S}$, which is the electromagnetic power of the beam delivered within the length Δz, with the change of the electromagnetic power flow ΔP on the structure

$$\frac{1}{2} \oint E \times H^* \cdot d\bar{S} = \Delta P \tag{3}$$

From reference 3 we have in Pierce's notation:

$$\Delta P = \frac{1}{2} \frac{|\Gamma|^2 |V|^2}{K} \frac{\left(\Gamma^2 - \Gamma_1^2\right)^*}{\Gamma_1^* \Gamma^{*2}} \Delta z \tag{4}$$

We also have in the absence of an x-component of the rf field

$$\hat{E} \cdot \hat{I}^* = E_y I_y^* + E_z I_z^* \tag{5}$$

The use of Eqs. 2, 4, and 5 in conjunction with Eqs. 13.5, 13.21, 13.22, and 2.15 of reference 2 gives the complex conjugate of the determinantal equation (13.23) within the approximations involved.

The force equation

$$\frac{d\bar{v}}{dt} = -\frac{e}{m} \left[\bar{E} + \bar{v} \times \bar{B}\right] \tag{6}$$

can be split into a time-independent and a sinusoidally-varying part. The magnetic forces on the beam are neglected except for those that are caused by the focusing field $\bar{B}_o$. It is assumed that the magnetic field has only a y- and a z-component. A proof similar to the one that follows can be carried out for other symmetries of the magnetic field. The alternating force exerted by the magnetic field has the components:

$$-\frac{e}{m}\left[v_y B_{oz} - u_o y \frac{\delta B_{oy}}{\delta y}\right] \quad \text{in the x-direction}$$

and (7)

$$-\frac{e}{m}\left[-v_x B_{oz}\right] \quad \text{in the y-direction}$$

The force component, $(e/m)\, u_o y\, (\delta B_{oy}/\delta y)$, is caused by the fact that the modulated beam passes through a magnetic field with a positive or negative y-component as it enters above or below the symmetry plane of the magnetic field.

The use of the small-signal part of Eq. 6, the application of the expressions of Eq. 7, and the divergence relation of the magnetic field lead eventually to:

$$\hat{E} \cdot \hat{I}^* = \frac{d}{dz}\left[U_x I_x^* + U_y I_y^* + U_z I_z^*\right] - j\omega\rho_o \frac{m}{e}\left[v_x^2 + v_y^2 + v_z^2 - \omega_c\left(y v_x^* + y^* v_x\right)\right] \quad (8)$$

where

$$U_x = -\frac{m}{e} u_o (v_x - \omega_c y), \qquad U_y = -\frac{m}{e} u_o v_y, \qquad U_z = -\frac{m}{e} u_o v_z,$$

$$I_x = -j\omega\rho_o x, \qquad I_y = j\omega\rho_o y, \qquad I_z = \rho_o v_z + \rho_1 u_o, \qquad \text{and } \omega_c = \frac{e}{m} B_{oz}.$$

Equations 2, 3, and 8 finally give for the rate of change of the real part of the complex power flow on the rf structure:

$$\frac{d}{dz}\left[\text{Re}(P)\right] = -\frac{d}{dz}\,\text{Re}\left[U_x I_x^* + U_y I_y^* + U_z I_z^*\right] \quad (9)$$

The quantity under the differentiation sign on the right-hand side of Eq. 9 can be interpreted as a kinetic power flow whose rate of decrease is identical with the rate of increase of the power produced on the rf structure. Because of the relation given in Eq. 9 one can represent any transverse field tube in terms of a passive linear network with at least eight pairs of terminals (Fig. VI-3). In the particular case of a lossfree rf structure, the matrix characterizing the structure must be that of a lossless (in general, nonreciprocal) network. Regions in which no electromagnetic power is extracted from the beam are lossless six-terminal pair transducers. Velocity jumps and

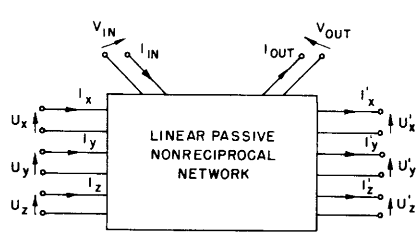

Fig. VI-3
Transverse field tube in network representation.

regions of changing magnetic focusing field are examples of such transducers.

As another example of the application of the kinetic power theorem, Eq. 9, we present the proof that only waves with a phase velocity slower than the beam velocity can grow or decay in a transverse field tube. Both the kinetic power in the beam and the electromagnetic power on the circuit grow in magnitude at the same rate, if only one growing wave is considered at a time. Conservation of the over-all power, Eq. 9, requires that the kinetic power be negative, since the electromagnetic power is positive in a conventional tube. For Pierce's two-dimensional transverse field tube, we have, in Pierce's notation,

$$\mathrm{Re}\left[U_x I_x^*\right] = 0$$

$$\mathrm{Re}\left[U_y I_y^*\right] = \frac{I_o}{2V_o} \frac{C\beta_e^2 \phi'^2 |V|^2}{\left|(j\beta_e - \Gamma)^2 + \beta_m^2\right|^2} \mathrm{Im}[\delta]$$

$$\mathrm{Re}\left[U_z I_z^*\right] = \frac{I_o}{2V_o} \frac{C\beta_e^2 \phi^2 |V|^2}{|j\beta_e - \Gamma|^4} \mathrm{Im}[\delta]$$

The sign of the kinetic power is determined entirely by the sign of $\mathrm{Im}[\delta]$, which has to be negative if growth should occur. An $\mathrm{Im}[\delta] < 0$ means that the phase velocity of the wave is smaller than the electron velocity u_o, which is a necessary (but not sufficient) condition for a growing wave.

The existence of lossless beam transducers in the case of transverse as well as longitudinal motion of the beam, allows (at least theoretically) a noise minimization and computations for the minimum noise figure of a transverse field tube in a manner analogous to the one used for longitudinal beam amplifiers. Collimators are nonlinear devices and are excluded from these schemes. Such a noise formalism will be discussed in the next report.

H. A. Haus

References

1. L. J. Chu, Paper delivered at the conference of the Professional Group on Electron Devices, Orono, Maine, June 1951.
2. J. R. Pierce, Traveling Wave Tubes (D. Van Nostrand Company, Inc., New York, 1950).
3. W. Kleen, Einfuehrung in die Mikrowellen-Elektronik (S. Hirzel, Zuerich, 1952), Eq. 3.23.

B. INTERCEPTION CURRENT NOISE MEASUREMENTS

When a thin electron beam is desired, the most obvious way of producing it is to use cathodes of smaller diameter; but this method introduces the problem of nonuniform cathode coating (edge effects). With these facts in mind, it was decided to investigate the effects of using a reasonably large cathode, with a very close electrode for intercepting most of the beam. The investigation is concerned mainly with the noise resulting from this interception current as a function of cathode to cathode-electrode spacing and voltage (see Fig. VI-4).

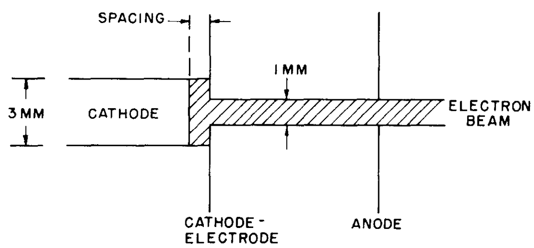

Fig. VI-4
Interception electrode.

Two different spacings have been tried, 0.005 inch and 0.011 inch. Because of the large noise standing-wave ratios obtained, a comparison of the effects of the different spacings, and of the effects of changing various parameters, based on the noise standing-wave average, might not be meaningful, since higher order modes prevent the accurate determination of the minimum. Therefore, as a first approximation of the noisiness of a beam under various conditions, the value of the first maximum of the standing wave was inspected.

The trend seems to show that the larger spacing produces a noisier beam than does the closer spacing. For example, with a beam voltage of 1500 volts, a cathode to cathode-electrode voltage of +5 volts, and an axial magnetic field of 600 gauss, the 0.005-inch spacing has its first maximum 11 db below shot noise, whereas the 0.011-inch spacing has its first maximum 5.7 db below shot noise.

This effect appears logical, since a closer spacing "shades" more of the cathode and prevents the random motion of the outer electrons into the main stream. Thus, it is also concluded that a stronger magnetic field and a higher cathode-electrode voltage produce like effects, that is, a less noisy beam, since they both tend to prevent sidewise motion of the electrons.

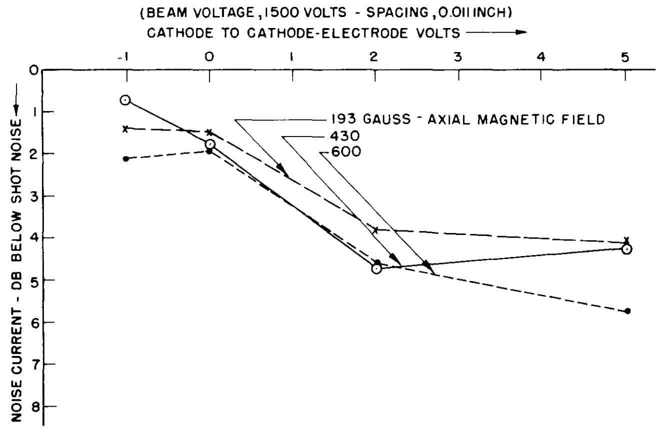

Fig. VI-5

Maximum of noise standing wave vs. cathode-electrode voltage for various values of axial magnetic field.

These effects are seen in Fig. VI-5. The data were taken on the gun with 0.011-inch cathode to cathode-electrode spacing.

The experiment is being continued. The present aims are to verify, duplicate, and extend these results.

T. J. Connor