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Equations of State for Collisionless Guide-Field Reconnection

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Direct *in situ* observation of magnetic reconnection in the Earth's magnetotail as well as kinetic numerical studies have recently shown that the electron pressure in a collisionless reconnection region is strongly anisotropic. This anisotropy is mainly caused by the trapping of electrons in parallel electric fields. We present new equations of state for the parallel and perpendicular pressures for magnetized electrons. This model—derived here and tested against a kinetic simulation—allows a fluid description in a collisionless regime where parallel electric fields and the dynamics of both passing and trapped electrons are essential.

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Magnetic reconnection is believed to play a central role in some of the most violent plasma phenomena observed, including solar flares, coronal mass ejections, magnetic storms in the Earth's magnetosphere, and sawtooth crashes in tokamaks. It entails the change of connectivity by magnetic field lines of plasma elements. Despite early theoretical predictions that reconnection is slow in highly conducting plasmas, fast magnetic reconnection has been directly observed in fusion research confinement devices [1] and by spacecraft in Earth's magnetotail and the far solar wind [2,3].

In guide-field reconnection, the parallel electric field is proportional to the reconnection rate. To support such an electric field, and hence reconnection, the electron pressure is important for the electron momentum balance. Standard fluid models and simulation schemes often rely on isothermal or adiabatic equations of state for a fluid closure [4–6]. Meanwhile, measurements taken by the Wind spacecraft in a reconnecting current sheet in the Earth's magnetotail show that the electron phase space density is highly anisotropic, with $T_{\parallel} \gg T_{\perp}$ [2] (here subscripts $\parallel$ and $\perp$ denote the directions parallel and perpendicular to the magnetic field, respectively). Similar pressure anisotropy is observed in fully kinetic reconnection simulations [7]. Recently, Egedal *et al.* introduced a mechanism that accounts for the electron pressure anisotropy [8]. Extending those results, we present here, and confirm using the results of a kinetic simulation, equations of state for $p_{\parallel}$ and $p_{\perp}$. This fluid closure includes the anisotropic electron pressure caused by particle trapping, and it provides a new framework for modeling the electrons in fluid codes and interpreting space data. The model predicts a strong, nonlinear dependence of the parallel pressure on density variations, typically yielding an order of magnitude gain in $p_{\parallel}$ compared to the commonly used isothermal scaling ($p = nT$).

Our starting point is a model for the gyro-averaged electron distribution function $f(v_{\parallel}, v_{\perp})$ derived in Ref. [8], which is generalized below. (Note that our model does not include gyrophase-dependent terms, which are

sizable very near the $\mathbf{x}$ line [9], but otherwise negligible.) The model is based on the dynamics of magnetized electrons in the limit where the electron thermal speed is much larger than the Alfvén speed, $v_{th,e} \gg v_A$. In this limit, magnetic field variations and the parallel electric field trap particles, and the trapped particles bounce many times while they convect through the current sheet. We consider a reconnection region embedded in a uniform current sheet, as sketched in Fig. 1(a). As indicated, sufficiently far from the $\mathbf{x}$ line it is assumed that the magnetic field strength is uniform, $B = B_{\infty}$, and that $\mathbf{E} \cdot \mathbf{B} = 0$.

From Liouville's theorem ($df/dt = 0$ along particle trajectories), it follows that the phase space density $f(\mathbf{x}, \mathbf{v})$ for a point $(\mathbf{x}, \mathbf{v})$ inside the diffusion region is identical to $f_{\infty}(\mathbf{x}_{\infty}, \mathbf{v}_{\infty})$, where $(\mathbf{x}_{\infty}, \mathbf{v}_{\infty})$ is the phase space point in the ambient plasma. The electron distribution in the ambient plasma, f_{∞} , is assumed gyrotropic (generalizing Ref. [8], which takes a fully isotropic distribution for simplicity) so that $f(\mathbf{x}, \mathbf{v}) = f_{\infty}(\mathcal{E}_{\parallel\infty}, \mathcal{E}_{\perp\infty})$. Thus, to obtain $f(\mathbf{x}, \mathbf{v})$ we need only characterize the kinetic energies $\mathcal{E}_{\parallel\infty}$ and $\mathcal{E}_{\perp\infty}$ that the electron had before it entered the reconnection region.

Trapped and passing electrons are treated separately, as in Ref. [10], for example. A particle becomes trapped only if its initial $|v_{\parallel}|$ at the boundary is small. Thus, for most trapped electrons, the initial kinetic energy is $\mathcal{E}_{k\infty} \sim \mathcal{E}_{\perp\infty} \sim \mu B_{\infty}$. On the other hand, for passing electrons entering and leaving the reconnection region in a single shot along a magnetic field line, the perpendicular and parallel kinetic energies in the ambient plasma are given by $\mathcal{E}_{\perp\infty} = \mu B_{\infty}$ and $\mathcal{E}_{\parallel\infty} = \mathcal{E}_k - \mu B_{\infty} - e\Phi_{\parallel}$. Here, the acceleration potential, $\Phi_{\parallel}$, is defined as

$$\Phi_{\parallel}(\mathbf{x}) = \int_{\mathbf{x}}^{\infty} \mathbf{E} \cdot d\mathbf{l}, \quad (1)$$

where the integration is carried out from the point $\mathbf{x}$ along the magnetic field to the outer region where $\mathbf{E} \cdot \mathbf{B} = 0$. Combining the trapped and passing contributions, we obtain

FIG. 1 (color). (a) Schematic diagram of field lines in a reconnecting current sheet; separator is indicated by solid line. (b) Typical trapped (blue) and passing (red) electron orbits in the domain covered by simulation. (c) Contours of f with trapped and passing regions shaded. Our equations of state for (d) $p_{\parallel}(n)$ and (e) $p_{\perp}(n)$ plotted for various values of B .

$$f(\mathbf{x}, \mathbf{v}) = \begin{cases} f_{\infty}(\mathcal{E}_{\parallel\infty}, \mu B_{\infty}), & \text{passing} \\ f_{\infty}(0, \mu B_{\infty}), & \text{trapped} \end{cases} \quad (2)$$

The trapped-passing boundary is given by $\mathcal{E}_{\parallel\infty} = 0$, and there are trapped particles when $e\Phi_{\parallel} > 0$ or $B/B_{\infty} < 1$, typical of reconnecting current sheets where the in-plane components of $\mathbf{B}$ vanish.

Typical orbits for passing and trapped thermal electrons going through a point $\mathbf{x}$ are superimposed in Fig. 1(b) on the in-plane projection of magnetic field lines. The passing particle (red) has sufficient parallel energy to move directly through the reconnection region, but the trapped particle (blue) bounces several times inside the potential well produced by both magnetic trapping and $\Phi_{\parallel}$. In Fig. 1(c), contours of $f(v_{\parallel}, v_{\perp})$ at $\mathbf{x}$ [in Fig. 1(b)] are displayed for the case where f_{∞} is a Maxwellian. The trapped contribution (blue) to f is flattened by its independence of $v_{\parallel}$, while the passing portion (red) remains a Maxwellian modified by $\Phi_{\parallel}$. The location of the trapped-passing boundary at low perpendicular energies is determined by $e\Phi_{\parallel}$ ($v_{\parallel} \approx \sqrt{2e\Phi_{\parallel}/m}$), and at energies much greater than $e\Phi_{\parallel}$, the boundary approaches the loss cone associated with magnetic trapping ($v_{\parallel} \approx v_{\perp}\sqrt{1 - B/B_{\infty}}$).

Our model applies to thermal electrons magnetized by the guide magnetic field, which determine the fluid properties of the plasma. The features of f resulting

from trapped electron dynamics strongly influence the fluid quantities determined by taking moments of Eq. (2). Note that our distribution function is the zero-order term in an expansion $f = f^{(0)} + f^{(1)}(u/v_{\text{the}}) + f^{(2)}(u/v_{\text{the}})^2 + \dots$, where u is the mean flow speed. Even moments, such as the density, $n = \int f d^3v$, the parallel pressure, $p_{\parallel} = \int m v_{\parallel}^2 f d^3v$, and the perpendicular pressure, $p_{\perp} = (\frac{1}{2}) \times \int m v_{\perp}^2 f d^3v$, can be obtained by direct integration. Odd moments, such as the fluid flow $\mathbf{u}$, must be found using other methods. For example, the perpendicular fluid flow $\mathbf{u}_{\perp}$ can be obtained by considering momentum balance in given electric and magnetic fields, and the parallel flow $u_{\parallel}$ can then be inferred from the continuity equation.

To formulate a fluid description, we begin by directly computing $n = \int f d^3v$ using Eq. (2), which gives the density as a function of B and $\Phi_{\parallel}$. For example, when $\Phi_{\parallel}$ and B both contribute to trapping, the electron density for a Maxwellian f_{∞} is

$$\frac{n}{n_{\infty}} = \frac{2}{\sqrt{\pi}}(1-b)\sqrt{u} + e^u \Psi(\sqrt{u}) - b^{3/2} e^{u/b} \Psi\left(\sqrt{\frac{u}{b}}\right) \quad (3)$$

for $u \equiv e\Phi_{\parallel}/T_{\infty} > 0$ and $b \equiv 1 - B/B_{\infty} > 0$, and where Ψ is the complementary error function. Note that physically, $\Phi_{\parallel}$ varies on ion scales, and the electron density response to $\Phi_{\parallel}$ maintains quasineutrality. Next, the relationship $n(\Phi_{\parallel}, B)$ is inverted numerically to give $\Phi_{\parallel} = \Phi_{\parallel}(n, B)$. By substituting $\Phi_{\parallel}(n, B)$ back into Eq. (2), higher moments of f for the parallel and perpendicular pressures can also be recast as functions of only n and B . Despite its physical significance, the acceleration potential $\Phi_{\parallel}$ in our fluid model thus becomes an intermediate calculational tool for relating more convenient fluid and magnetic field quantities. The pressure moments close our collisionless fluid picture by yielding general equations of state for $p_{\parallel}(n, B)$ and $p_{\perp}(n, B)$, which can be tabulated numerically for inclusion in a fluid code.

Although the inversion $n(\Phi_{\parallel}, B) \rightarrow \Phi_{\parallel}(n, B)$ must be done numerically, analytical scalings in the limits of deeply trapped (large $e\Phi_{\parallel}$ and $B/B_{\infty} \leq 1$) or fully passing distributions illustrate the new features of our fluid model. The passing electrons behave as an isothermal population: they exhibit a Boltzmann-like response with $\Phi_{\parallel}$ replacing the usual electrostatic potential. But trapping by parallel electric fields and the magnetic geometry alter the trapped electron response. For constant $B = B_{\infty}$, our results reduce to those of the 1D analysis in Ref. [10], with a slowly varying $\Phi_{\parallel}(t)$ serving to trap particles. Including magnetic effects in the trapped regime, our perpendicular pressure is consistent with the conservation of μ , which forces the perpendicular temperature to be proportional to B , or $p_{\perp} \propto nB$. Additionally, the parallel pressure is $p_{\parallel} \propto n^3/B^2$. Thus, for trapped electrons, we recover the double-adiabatic scalings of Ref. [11] appropriate for well-

magnetized electrons with negligible parallel heat conductivity.

The equations of state are graphed in Fig. 1(d) and 1(e), where $p_{\parallel}(n)$ and $p_{\perp}(n)$ are plotted for various values of B and a Maxwellian f_{∞} . For low density, $p_{\parallel}$ and $p_{\perp}$ are simply proportional to n , but when the density is increased by trapping, the curves approach the double-adiabatic forms. While the equations of state are readily tabulated for accurate modeling, the following approximations represent a significant improvement over the isothermal approximation:

$$\begin{aligned}\tilde{p}_{\parallel} &= F(\alpha/2) \times \tilde{n} + F(\alpha^{-1}/2) \frac{\pi \tilde{n}^3}{6 \tilde{B}^2}, \\ \tilde{p}_{\perp} &= F(\alpha) \tilde{n} + F(\alpha^{-1}) \tilde{n} \tilde{B},\end{aligned}\quad (4)$$

where $\tilde{n} = n/n_{\infty}$, $\tilde{B} = B/B_{\infty}$, $\tilde{p}_{\parallel} = p_{\parallel}/p_{\infty}$, $\tilde{p}_{\perp} = p_{\perp}/p_{\infty}$, $\alpha = \tilde{n}^3/\tilde{B}^2$, and $F(x) = (1+x)^{-1}$.

We note that these pressure scalings are important when considering the generalized parallel Ohm's law that follows from the electron momentum equation. The gyro-tropic pressure can balance a parallel electric field

$$neE_{\parallel} = -\nabla_{\parallel} p_{\parallel} + (p_{\parallel} - p_{\perp}) \nabla_{\parallel} \ln B \quad (5)$$

where $\nabla_{\parallel} = (\mathbf{B}/B) \cdot \nabla$. The strong density dependence of the trapped electron pressure, $p_{\parallel} \propto n^3/B^2$, allows signifi-

cant parallel electric fields to develop: for a doubling of the density and a magnetic field strength reduced to half its boundary value, there is roughly a factor of 24 enhancement in $dp_{\parallel}/dn$ over the isothermal or adiabatic approximations. Furthermore, the pressure anisotropy provides an additional contribution.

We validate our fluid model by comparing it to the results of a kinetic, open boundary particle-in-cell (PIC) simulation of a reconnecting current sheet. The PIC simulation is translationally symmetric in the z -direction and has a total domain of 3072×3072 cells $= 569 \times 569 c/\omega_{pe}$. The initial state is a Harris sheet characterized by the following parameters: $m_i/m_e = 360$, $T_i/T_e = 2$, $B_{\text{guide}} = 0.5B_0$, $\omega_{pe}/\omega_{ce} = 2.0$, background density $= 0.30n_0$ (peak Harris density), and $v_{th,e}/c = 0.20$. Magnetic reconnection with a single x line evolves from a small perturbation, and we consider a time with approximately steady-state reconnection. The profiles of the magnetic field strength B , the electron density n , and the sheet current density j_z self-consistently produced by the simulation are plotted in Figs. 2(a)–2(c).

The PIC code tracks 2×10^9 electrons and thus allows the full electron distribution function to be constructed. The gyro-averaged distribution functions at four sample points are plotted in Figs. 2(d)–2(g). These simulation distributions are gyrotropic to a good approximation, and

FIG. 2 (color). PIC simulation results: (a) Out-of-plane sheet current density j_z , (b) magnetic field strength B with points used in Fig. 4 where $B/B_{\infty} = 0.65$ (white) and $B/B_{\infty} = 0.85$ (black), and (c) plasma density n . Dashed lines represent in-plane magnetic field lines. Simulation electron distribution functions with theoretical level lines superimposed along the cut $30d_e$ right of the x line at the locations indicated in (a) [(d) $y = 113d_e$, (e) $y = 123d_e$, (f) $y = 133d_e$, and (g) $y = 143d_e$].

FIG. 3 (color). Comparison of fluid model and PIC simulation. Acceleration potential $\Phi_{\parallel}$ (a) predicted by model and (b) given by integral definition with the integration constant chosen so both match along the dashed lines. (c) Fluid model $p_{\perp}(n, B)$, (d) simulation $p_{\perp}$, (e) fluid model $p_{\parallel}(n, B)$, and (f) simulation $p_{\parallel}$.

FIG. 4. Comparison of $p_{\parallel}$ from fluid model to $p_{\parallel}$ from two PIC simulations with ion/electron mass ratios of (a) 360 and (b) 180. (c) and (d) Comparison of the analytical equations of state against PIC data from points marked in Fig. 2(b).

comparison with the superimposed level lines of our analytical solution for $f(n, B)$ shows that our model correctly predicts the broadening and flattening of the distribution. Note that while the original model in Ref. [8] used a uniform f_{∞} far from the x line [in the shaded boxes in Fig. 1(a)], due to computational constraints on the simulation size, the PIC code uses an open boundary condition that varies f at the edge to eliminate gradients in the density, fluid flow, and pressure tensor at the boundary. To approximate the PIC code's boundary values for f when comparing to our fluid model, we take f_{∞} a Maxwellian in the inflow region and, for passing electrons that originate in the outflow region, f_{∞} a bi-Maxwellian with $n = 0.33n_0$ and $T_{\parallel} = 2T_{\perp} = 2T_e$.

To compare our fluid model to the PIC simulation, we first determine $\Phi_{\parallel}(n, B)$ based on the profiles of B and n given by the simulation. The deduced $\Phi_{\parallel}(n, B)$ closely matches a direct evaluation of the defining integral (1). Because of the open boundary conditions, $\Phi_{\parallel}$ need not vanish at the simulation edge, so the integration constant is fixed by matching to $\Phi_{\parallel}(n, B)$ at the midway point of each field line [along the dashed lines in Figs. 3(a) and 3(b)]. In both cases, $e\Phi_{\parallel}$ reaches a maximum of $(4-5)T_e$, implying that the majority of electrons in regions of enhanced density are electrically trapped.

The parallel and perpendicular pressure profiles, calculated using $\Phi_{\parallel}(n, B)$, agree well with the pressures obtained directly from the PIC simulation [Figs. 3(c)–3(f)]. Except in a small region directly around the x line, the pressure is gyrotropic to within $\sim 5\%$. In the outflow region, the parallel pressure reaches nearly 5 times its boundary value, yet our fluid model differs from the PIC

simulation by less than $\sim 20\%$ throughout the simulation domain. Similarly, while the perpendicular pressure drops to half its boundary value, the numerical results and our model agree everywhere to within a few percent.

The parallel pressure from the PIC simulation and from our fluid model are plotted in Fig. 4(a) as functions of y along a cut $30d_e$ to the right of the x line [the same cut used in Figs. 2(d)–2(g)]. For comparison, we present in Fig. 4(b) similar plots based on data from another PIC simulation with a mass ratio of $m_i/m_e = 180$, but otherwise identical. Most notably, agreement between our fluid model and the PIC simulation improves for the more physical mass ratio as we approach the limit in which our model is derived.

In the inflow region, the boundary f_{∞} is Maxwellian, which allows us to compare our analytical equations of state directly to the PIC data. Good agreement is found in Figs. 4(c) and 4(d) between equations of state given by Eq. (4) and the simulation data from the points marked in Fig. 2(b).

Kinetic simulations of a reconnecting current sheet therefore verify that our collisionless equations of state, $p_{\perp} = p_{\perp}(n, B)$ and $p_{\parallel} = p_{\parallel}(n, B)$, correctly account for the main anisotropy of the electron pressure tensor. These equations of state are suitable for implementation in two-fluid codes and could be combined with a microscopic dissipation mechanism in the immediate vicinity of the x line (such as hyper resistivity) to investigate large-scale reconnection geometries. This fluid closure is also useful for modeling and interpreting measurements from low-collisionality plasmas in space when a fully kinetic treatment is intractable.

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