XIV. SEMICONDUCTOR NOISE

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RESEARCH OBJECTIVES

Our investigation of the physical origins of 1/f noise in germanium has been brought to a conclusion, at least for the present, on the basis of the research completed last year by A. L. McWhorter and the work of L. Bess.

Our general interest in the circuit effects of semiconductor noise continues. In particular, the following topics are under study:

1. The amplitude probability distribution of 1/f noise. This work is a by-product of a more general problem concerning the effect of broadband filtering on non-gaussian noise.

2. Investigation of the properties of semiconductor diodes at low temperatures, with a view toward obtaining superior noise performance at high frequencies.

3. Determination of the dynamic factors, if there are any, that govern the performance of point-contact diodes at high frequencies. This work actually supplements topic 2, since it is of interest to know whether or not the dc volt-ampere characteristics of a crystal are actually of major significance in its operation as a mixer.

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A. THE EFFECT OF LOW-FREQUENCY FILTERING ON A NONGAUSSIAN AMPLITUDE PROBABILITY DISTRIBUTION

In making measurements of the first-order probability distribution of 1/f semiconductor noise in order to determine whether or not this distribution is gaussian, consideration must be given to the effect of rejecting a significant portion of the noise power at very low frequencies. This rejection is inevitable because of limitations in the length of the period of measurement.

We have adopted an approach to this problem which consists of looking at a simplified nongaussian microscopic model and calculating the effects of network transfer characteristics on the probability distribution of the model. Since we are interested in the approach to a gaussian distribution, attention is centered on skewness and excess (1) as measures of deviation from a gaussian distribution.

Let us consider a series of unit steps with leading edges Poisson-distributed to represent a noise model. These pulses are passed through the network H(s) (see Fig. XIV-1). The cumulants of the distribution of $e_o(t)$ are calculated from Middleton's equation (2)

$$\kappa_m = \bar{n} \int_{-\infty}^{\infty} [e_1(t)]^m \, dt$$

where κ_m is the mth cumulant of distribution of $e_o(t)$; $\bar{n}$ is the average number of pulses
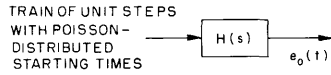
Fig. XIV-1. Preliminary model.

starting per second; and $e_1(t)$ is the response of $H(s)$ to a single unit step at $t = 0$.

Now (see ref. 1)

$$\gamma_1 = \text{coefficient of skewness} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\kappa_3}{\kappa_2^{3/2}}$$

and

$$\gamma_2 = \text{coefficient of excess} = \frac{\mu_4 - 3\mu_2^2}{\mu_2^2} = \frac{\kappa_4}{\kappa_2^2}$$

where μ_n is the nth central moment of the distribution.

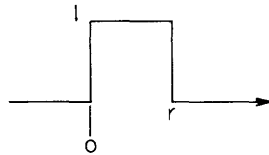Consider a simple case: $H(s) = \dfrac{s}{s + \frac{1}{\tau}}$; so that $e_1(t) = e^{-t/\tau}$. Therefore

$$\kappa_m = \bar{n} \int_{-\infty}^{\infty} (e^{-t/\tau})^m \, dt = \bar{n} \, \frac{\tau}{m}$$

$$\gamma_1 = \frac{1}{\frac{3}{2\sqrt{2}} (\bar{n}\tau)^{1/2}}$$

and

$$\gamma_2 = \frac{1}{\bar{n}\tau}$$

The physical significance of this model can be seen by comparing these values to those for unmodified Poisson-distributed pulses of the form

for which $\kappa_m = \bar{n}r$ and $\gamma_1 = (\bar{n}r)^{-1/2}$; $\gamma_2 = 1/\bar{n}r$. (This is a Poisson amplitude distribution with density of pulses $\bar{n}$ and length of pulses r.)

Let us consider a network including both low-frequency and high-frequency filtering:

$$H(s) = \frac{1}{\tau_0} \frac{s}{\left(s + \frac{1}{\tau_0}\right)\left(s + \frac{1}{\tau}\right)}$$
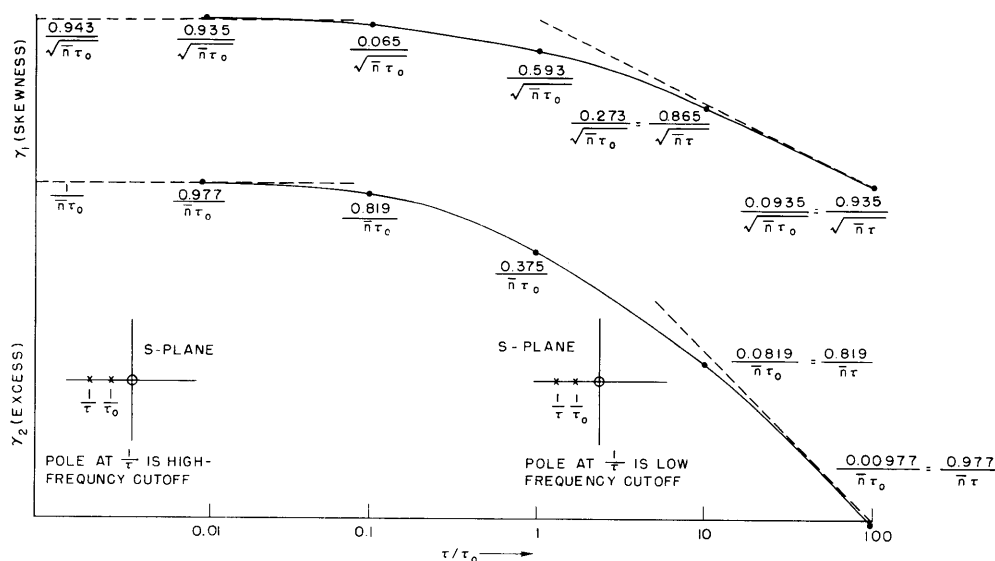
Fig. XIV-2. Skewness and excess for $H(s) = \dfrac{1}{\tau_o} \dfrac{s}{\left(s + \dfrac{1}{\tau_o}\right)\left(s + \dfrac{1}{\tau}\right)}$.

Let us see to what extent the probability distribution is modified as the low-frequency cutoff is varied:

$$\gamma_1 = \cfrac{1}{\dfrac{3}{2\sqrt{2}}\,(\bar{n}\tau_o)^{1/2}} \cfrac{1}{\cfrac{\left(\dfrac{2\tau}{\tau_o} + 1\right)\left(\dfrac{\tau}{\tau_o} + 2\right)}{2\left(\dfrac{\tau}{\tau_o} + 1\right)^{3/2}}} \quad ; \quad \gamma_2 = \cfrac{1}{\bar{n}\tau_o} \cfrac{1}{\cfrac{\left(\dfrac{\tau}{\tau_o} + 3\right)\left(\dfrac{3\tau}{\tau_o} + 1\right)}{3\left(\dfrac{\tau}{\tau_o} + 1\right)}}$$

The results of varying τ are shown in Fig. XIV-2.

We see that the important frequency in the approach to gaussian distribution is the lower-frequency cutoff, and that as the system limits the low-frequency energy, the distribution departs from gaussian. A physical interpretation can be given by considering the Central Limit Theorem, which states, roughly, that the superposition of increasingly large numbers of independent events leads one toward a gaussian distribution. By limiting the low-frequency energy of a nonoscillatory system we shorten its memory and so decrease the number of events that it superimposes.

Several other network configurations have also been investigated. The results seem to support the general conclusion that if the memory of a system excited by a nongaussian input is decreased, the probability distribution departs from gaussian.

In relation to experimental methods of measuring the probability distributions of semiconductor noise, we conclude that eliminating the low-frequency energy leads to measurements that are farther from gaussian than the actual noise distribution, unless this distribution is gaussian itself. If the semiconductor noise is gaussian, the measurements will merely confirm it.

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References

1. H. Cramér, Mathematical Methods of Statistics (Princeton University Press, 1946), p. 183.

2. D. Middleton, On the theory of random noise. Phenomenological models. I., J. Appl. Phys. 22, 1143 (1951).