

VII. ATOMIC BEAMS

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A. MEASUREMENT OF THE VELOCITY OF LIGHT

A proposed measurement of the velocity of light in terms of two primary standards was described in the Quarterly Progress Report of January 15, 1956. In this connection, a series of preparatory experiments was successfully completed, and actual resonance measurements were obtained from the microwave cavity used in transmission at 9300 mc. This investigation was designed to give experimental answers to some of the fundamental problems on which the high-precision measurement depends. These problems, which are not accessible to theoretical investigation alone, are now outlined.

1. Manufacturing Problems of the Cavity

A cylinder of moderate precision, 4 inches in diameter and 12 inches in length, was optically lapped and polished out of pyrex stock. The cylinder, which was good to about 4×10^{-4} inch at reception, seems to have deteriorated to about 12×10^{-4} inch on account of structural stresses, leaving it oval in shape.

2. Silver-Plating Problems of the Cavity (Glass or Pyrex)

A thick chemically-deposited silver layer with only slight buffing turned out to be the most adequate of various silver-plating procedures that were investigated. The silver was reduced and precipitated in standard fashion out of an ammoniacal silver-salt solution. Various electroplating procedures, including alternate copper and silver plating, failed to give a satisfactory, thick, bright layer on top of a thin chemical or evaporated film.

3. Iris Feeding and Receiving of Microwave Radiation

Two brass end pistons, one of which had two iris holes, 0.125 inch in diameter and about 0.050 inch in length, were lapped and polished on the end plates. A bright silver plating was obtained by electrodeposition. Rectangular X-band tubing was soldered directly to the end plate at the irises that were oriented to feed the TE_{01N} mode properly. The lapping of the end plates was achieved by careful use of successively finer sheets of emery polishing paper, constantly lubricated with xylene (for lack of naphtha). The paper was layed on a machinist's flat plate, the last paper being 4-0. Hand buffing with a commercial brass-polishing agent (Noxon) terminated the procedure. The surfaces were optically flat to within 5 visible fringes (deviations from sphericity were

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about 5×10^{-5} inch). Attempts at optical grinding and polishing did not prove successful. The surfaces finally obtained were quite scratch-free. The silver was electroplated out of a "bright" silver-plating solution, great care being taken to obtain an even deposit.

The irises were centered on the maximum of the circular E component determined by the Bessel function $J_1(3.83 r/a)$, that is, at about 0.48 of the cylinder radius a , the two irises being at a 90° angle from one another in order to suppress observation of the TE_{11N} mode, the TE_{01N} mode being the one in which we are interested.

4. Clearance between Piston and Cavity

Clearance between piston and cavity is permissible in the TE_{01N} mode and it is desirable for simplification of the problems of mechanical translation and centering. It was calculated that a radial clearance of 0.030 inch would give a very small power loss and very slight shift in natural frequency. Experiments with the present equipment showed that this section of waveguide beyond cutoff is fully effective as long as more than 0.5 inch of overlap between cavity and sidewall is provided.

5. Preliminary Experiments

To provide a rough check on the Q of the cavity and the insertion loss caused by the iris holes, a 9300-mc signal was fed into the cavity from a TS-120 klystron signal generator and the output signal was detected by a spectrum analyzer. The insertion loss was about 50 db, and the Q was approximately 10^4 . It is expected that improvement in the parallelism of the end plates will increase the Q somewhat.

The preliminary experiments were carried out on a converted lathe with an out-of-round pyrex cylinder, as discussed above. Two needs are immediately apparent. First, the M.I.T. ruling engine must be used to provide a much more delicate mechanical movement, and second, a fused quartz cavity must be used to provide the required precision in diameter.

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B. THEORY OF ATOMIC HYPERFINE STRUCTURE

1. Second-Order Corrections to the Octopole Interaction

An earlier study (1) showed that second-order hyperfine structure, involving the product of magnetic dipole and electric quadripole matrix elements, could give an apparent first-order octopole effect. In order to get a value for the actual magnetic octopole interaction, these second-order perturbations must be calculated explicitly. This calculation was carried out, taking into account only the nearest doublet state of

the atom. At the same time we also considered the infinity of excited states for the valence electron from the ground state up to the ionization energy and we showed that the additional contribution of these states was negligible (less than 1 per cent). We have now calculated roughly all other terms and we find that the corrections to the first term – the doublet state – are all negligible.

The first group of excited states that we considered was the continuum of levels for the valence electron from energy zero to $+\infty$. Since the sum over levels up to zero energy was decreasing, we expect that it will continue to decrease for some distance above zero. An interesting possibility is that the sum may rise sharply for very large energies (large momenta). In that case we would use the closure approximation for the second-order perturbation theory

$$W^{(2)} = \sum_n \frac{(0|D|n)(n|Q|0)}{E_0 - E_n} \approx \frac{-1}{\bar{E}} (0|DQ|0) \quad (1)$$

where D is the dipole operator $\approx \vec{a} e \mu_N r^{-2}$ (with \vec{a} representing the Dirac velocity operator), Q is the quadripole operator $\approx e^2 R_N^2 r^{-3}$, and \bar{E} , the mean excitation energy, will be fixed by the following considerations.

For a valence $p_{3/2}$ orbit the matrix element $(0|DQ|0)$ would be infinite on account of the singularity at $r \rightarrow 0$ of the product of the D and Q operators. Thus the sum over positive energy states in Eq. 1 would also diverge. This divergence is halted by the physical limitation of the size of the nucleus, since the singular forms of the D and Q operators given above holds only for $r > R_N$. Hence, we shall cut off the radial integral at $r = R_N$, instead of going to $r = 0$. The cutting off at $r = R_N$ implies that the \sum_n which would have diverged, starts to converge at some high momentum related to R_N in the usual way

$$p_{\max} = \hbar/R_N$$

Thus the appropriate excitation energy will be (relativistically)

$$\bar{E} \approx \hbar c/R_N$$

Now we calculate the ratio of Eq. 1 to the actual first-order octopole interaction.

$$\frac{e^3 \mu_N R_N^2 \int_{R_N} \psi^* \vec{a} r^{-5} \psi dv}{\hbar c/R_N e \mu_N R_N^2 \int_0^\infty \psi^* \vec{a} r^{-4} \psi dv}$$

This ratio will be roughly $e^2/\hbar c \sim 1$ per cent, or actually smaller upon more careful analysis.

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The second class of excited states is the one in which an electron from a closed shell of the core is excited to any unoccupied level (either below or above zero energy). For the excitation to any single level, there will be several states corresponding to different couplings of the several angular momenta of the core (with a hole in it), the excited electron, and the valence electron(s). Within the limits of the very good approximation that the energy splitting of this multiplet is small compared with the energy of exciting the core electron into this new state, we are free to use any convenient scheme for coupling the angular momenta. Let us describe the states of this excited multiplet in terms of J' , the resultant angular momentum of the core-hole coupled to the excited core electron. We then have to calculate matrix elements from the initial core system of spin zero ($0|$ to the excited core system $|J'\rangle$). The dipole operator is a vector and thus it requires $J' = 1$, while the quadrupole operator requires $J' = 2$. Since the two values of J' do not coincide, there will be no $(0|D|n)(n|Q|0)$ product.

This result, of course, does not depend upon the particular representation that was chosen for coupling the angular momenta. It can be stated that if one used any other representation and then summed the DQ contributions over this complete set of states (with the same energy denominator for all), the total DQ term would vanish.

There is, however, one special set of excited states for which this result does not hold. If an electron from the core is excited into the valence shell, then the Pauli principle restricts the allowed angular momentum states and we cannot carry out this "sum over a complete set."

For a valence $p_{3/2}$ electron, or $p_{3/2}$ vacancy in the valence shell, this type of DQ correction relative to the one for the $p_{1/2}$ doublet state turns out to be (aside from the sign)

$$\frac{4}{5} \sum_{p'} \frac{\delta_p \langle p' | r^{-3} | p \rangle^2}{E_{p'} \langle p | r^{-3} | p \rangle^2}$$

where p refers to the valence p -state and p' refers to the p -states of the core. This expression may be approximated by

$$\frac{4}{5} \sum_{p'} \frac{\delta_{p'}}{E_{p'}}$$

These values of the fine structure and term energy for the filled p -shells of the atomic core can be obtained from X-ray data. The numerical values for this correction are: 1 per cent for chlorine, about 5 per cent for bromine, and perhaps 20 per cent for iodine. Since this is a percentage correction to the perturbation of the $2p_{1/2}$ term and the

importance of this $2p_{1/2}$ term decreases with increasing atomic weight, we can now feel quite confident about the entire problem of achieving the correct octopole interaction.

C. L. Schwartz

References

1. C. L. Schwartz, Phys. Rev. 97, 380 (1955).

C. HYPERFINE STRUCTURE AND NUCLEAR MAGNETIC OCTOPOLE MOMENTS OF THE STABLE CHLORINE ISOTOPES

The atomic-beam magnetic-resonance method was used to remeasure (1) the zero field hyperfine intervals of the $^2P_{3/2}$ ground state in Cl^{35} and Cl^{37} , and the values for the nuclear magnetic octopole moments were calculated.

In order to obtain the high precision that is required in the measurements, the Ramsey technique of separated oscillating fields was used (2). Since the observed signal intensities in some transitions were small, the phase-modulation technique used by R. T. Daly (3) was employed, which effectively doubled the signal intensity. An automatic data-taking procedure displayed the Ramsey patterns on a Sanborn recorder.

The intervals were measured in magnetic fields ranging from 0.2 gauss to 0.7 gauss, extrapolated to zero field, and corrected for the perturbing effects of the neighboring $^2P_{1/2}$ metastable state. The magnetic dipole, electric quadripole, and magnetic octopole interaction constants, a, b, and c, respectively, and the intervals are listed in Table VII-1 and Table VII-2. The quoted uncertainties include the estimated experimental errors, as well as the errors in the theoretical corrections.

Table VII-1. Uncorrected Intervals and Interaction Constants

	Cl^{35}	Cl^{37}
Upper interval	670.015155 ± 0.000085 mc	555.305480 ± 0.000085 mc
Middle interval	355.219540 ± 0.000065 mc	298.126655 ± 0.000065 mc
Lower interval	150.173345 ± 0.000075 mc	127.440655 ± 0.000075 mc
a	205.046975 ± 0.000025 mc	170.686450 ± 0.000025 mc
b	54.874150 ± 0.000050 mc	43.246090 ± 0.000050 mc
c	9.30 ± 1.20 cps	5.35 ± 1.20 cps
	a^{35}/a^{37} 1.20130790 ± 0.0000015	
	b^{35}/b^{37} 1.2688812 ± 0.000013	
	c^{35}/c^{37} 1.75 ± 0.40	

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Table VII-2. Corrected Intervals and Interaction Constants

	Cl ³⁵	Cl ³⁷
Upper interval	670.013455 ± 0.000090 mc	555.304315 ± 0.000090 mc
Middle interval	355.221030 ± 0.000070 mc	298.127655 ± 0.000070 mc
Lower interval	150.173560 ± 0.000075 mc	127.440815 ± 0.000075 mc
a	205.046870 ± 0.000030 mc	170.686370 ± 0.000030 mc
b	54.872905 ± 0.000055 mc	43.245245 ± 0.000055 mc
c	-7.15 ± 1.20 cps	-5.55 ± 1.20 cps
	a ³⁵ /a ³⁷ 1.20130780 ± 0.00000020	
	b ³⁵ /b ³⁷ 1.2688773 ± 0.0000015	
	c ³⁵ /c ³⁷ 1.30 ± 0.33	

C. L. Schwartz calculated the electronic matrix elements that were necessary to extract the nuclear octopole moments from the measured octopole interaction constants. The values for the nuclear octopole moments are:

$$\Omega_{35} = (-0.0191 \pm 0.0032) \times 10^{-24} \text{ nuclear magneton cm}^2$$

$$\Omega_{37} = (-0.0148 \pm 0.0032) \times 10^{-24} \text{ nuclear magneton cm}^2$$

The dipole and quadripole ratios obtained from this experiment have been compared to other measurements of these quantities, which have been made by other techniques (4, 5). To the limit of the errors involved, the agreement is excellent.

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References

1. L. Davis, Jr., B. T. Feld, C. W. Zabel, and J. R. Zacharias, Phys. Rev. 76, 1076 (1949). See also, V. Jaccarino and J. G. King, Phys. Rev. 83, 471 (1951).
2. N. F. Ramsey and H. B. Silsbee, Phys. Rev. 84, 506 (1951).
3. R. T. Daly, Ph. D. Thesis, Department of Physics, M. I. T., 1954.
4. G. D. Watkins and R. V. Pound, Phys. Rev. 82, 343 (1951).
5. T. C. Wang, Phys. Rev. 99, 566 (1955).

D. STABILIZATION OF KLYSTRONS BY PHASE LOCKING TO LOW-FREQUENCY QUARTZ CRYSTAL OSCILLATORS

A stabilization system for an S-band klystron, which, essentially, transfers the stability of a 5-mc quartz crystal oscillator to the klystron, has been developed for use with the cesium clock. A stable S-band oscillator had already been constructed by J. G. Yates (1) for use with the clock; but the problem of dividing down from 3000 mc to the low-frequency region, in order to provide a 1-mc stable frequency standard, is sufficiently difficult that other types of driving oscillators for the clock were investigated, in particular, a quartz crystal oscillator followed by some kind of multiplier chain. The required stability of the X-band signal as determined by the cesium beam apparatus is of the order of 1 part in 10^8 for a tenth of a second. This figure is easily achieved by using present-day quartz oscillator techniques (2); therefore the essence of the problem is building a circuit to transfer the stability of the quartz crystal to a signal at X-band. Conventional frequency multipliers (class C, for example) would probably work, but the problems of increased power consumption and circuit complexity with increasing frequency suggest that techniques that use a little more finesse might produce better results. One technique of this sort was suggested by M. Peter and M. W. P. Strandberg (3); it involves using a klystron which is phase-locked to a harmonic of the crystal oscillator. There are many possible ways of achieving this phase lock; the work described here is concerned with the exploitation of only one of these techniques, that of one-step high-ratio frequency multiplication and sensitive superheterodyne detection of the error signal. The principal characteristics of this system — simplicity (only six tubes), high multiplication ratio (75), excellent stability (1 part in 10^{12} or better) and large control range — offer considerable advantages over other techniques that have been previously reported in the literature of the subject. It should be emphasized at this point that, although specific frequencies and multiplication ratios

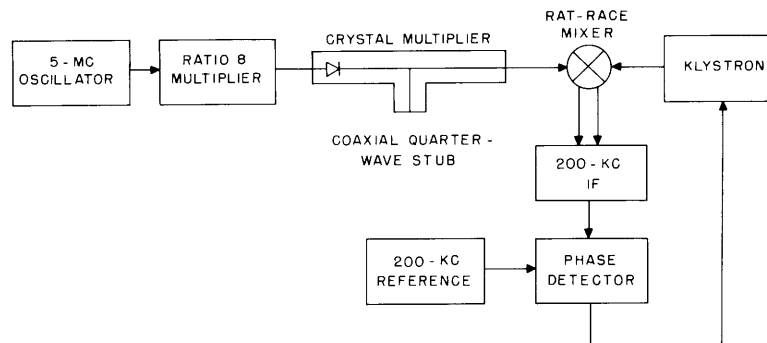


Fig. VII-1. Block diagram of the system.

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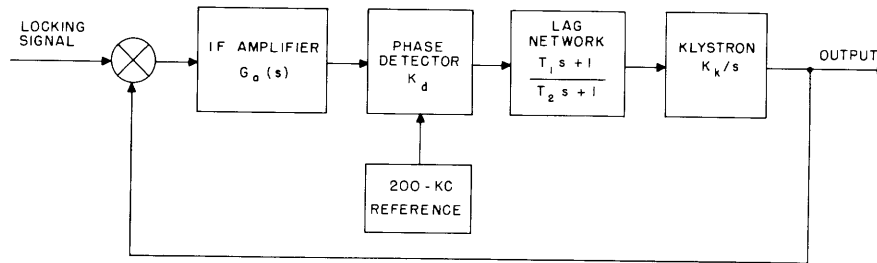


Fig. VII-2. Block diagram in servo form.

are discussed below, the technique has general applicability at any frequency at which voltage-tunable oscillators can be used.

A block diagram of the system is shown in Fig. VII-1. An oscillator operating at 5 mc was chosen as a source of the stabilization signal because of the availability of stable quartz crystals in this range. The oscillator frequency is multiplied by 600, first by 8 in a conventional class C multiplier, then by 75 in a crystal multiplier. The 3000-mc signal is mixed with the klystron signal and the difference is detected in a rat-race balanced mixer and amplified in a 200-kc amplifier. The phase difference between the 600th harmonic of the quartz crystal frequency and the klystron signal, which has been preserved in the mixing and amplifying processes, is converted to a dc voltage in the phase detector; then it is fed to the repeller of the klystron.

The ratio 75 multiplier is a passive device; about 0.5 watt of power at 40 mc is fed into a 1N34 crystal and a 3000-mc signal of about 10^{-10} watt is fed to the mixer. Very broad filtering – either a short length of S-band waveguide or quarter-wave coaxial stub – is used to attenuate the first few harmonics of 40 mc that are many orders of magnitude larger than the desired signal. It is interesting to note that, because of the presence of a small amount of 5-mc phase modulation in the 40-mc multiplier output, harmonics appear at 5-mc intervals throughout S-band rather than at 40-mc intervals as might be expected (4). By suitably tuning the 40-mc tuned circuit and the S-band multiplier output, any one of these harmonics, or a group of them, can be maximized. Thus the multiplier and the locked oscillator can be used as a stable S-band signal generator.

Since the phase detector contains no tubes, it can be floated on the output of the reflector voltage power supply. Thus, dc amplifiers or floating power supplies are not required.

The i-f amplifier is three-stage, resistance-coupled, with 100 db gain. The 90-kc passband is completely determined by the one tuned circuit in the amplifier, the input coil. The advantage of this design is obvious, once the feedback stability of the loop is investigated.

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The block diagram of the stabilization loop, drawn in servo form, is shown in Fig. VII-2. Note that the feedback-control information is contained in the relative phase of the signal from the klystron output to the phase detector. There is no gain constant associated with the mixer or i-f amplifier because the amplitude of phase information cannot be changed except by frequency multiplication or division. However, it is obvious that phase shifts in the i-f amplifier do change the "phase" of the phase information; the function of $G_a(s)$ represents this effect.

Although the actual gain of the i-f amplifier does not enter directly into the loop-gain calculation, the magnitude of the output voltage of the amplifier certainly has direct bearing on the phase-detector gain constant K_d (volts/radian). The phase-detector output is actually a function of the amplitude characteristics of the i-f input tuned circuit, but the effect is nonlinear, and fortunately it is second-order for practical i-f bandwidths; accordingly, it will be neglected.

If the klystron has a frequency sensitivity of K_f cps/volt, then, because phase deviation and frequency deviation are directly related by the modulation frequency, the phase sensitivity can be written $2\pi K_f / j\omega_m = K_k / s$ (radians/volt). The klystron has, then, as far as the phase information is concerned, the properties of a perfect integrator; it is assumed to have no other frequency dependence.

In the particular loop that is being considered here, the klystron is a 2K41 with $K_f = 175$ kc/volt; hence $K_k = 10^6$ radians/volt. Since the phase-detector constant K_o

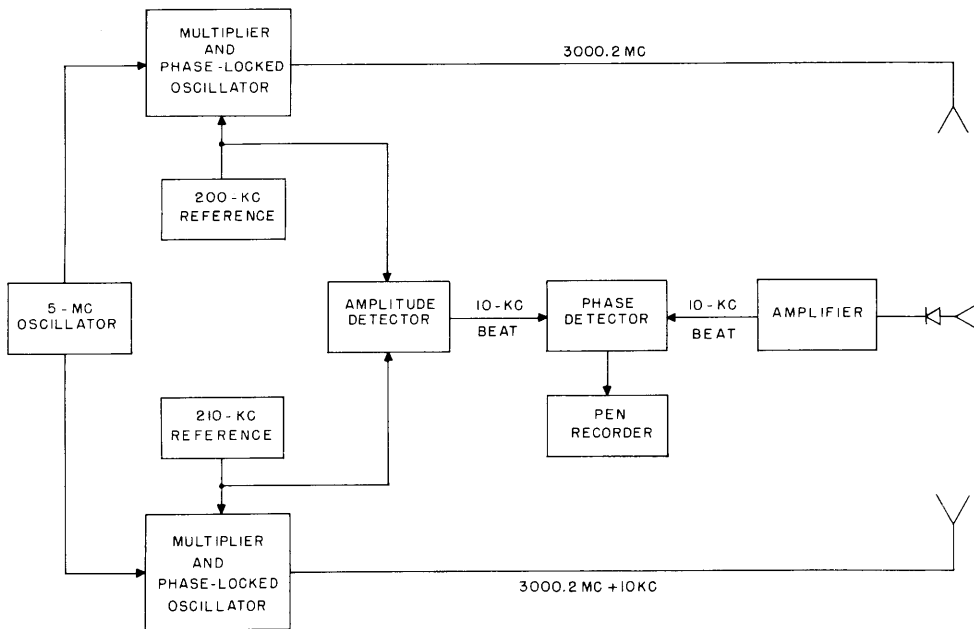


Fig. VII-3. Block diagram of stability-measuring system.

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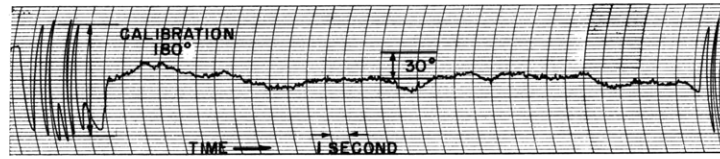


Fig. VII-4. Pen recordings of phase difference between two S-band oscillators.

is 3 volts/radian, the gain at $\omega_m = 1$ radian is approximately 3×10^6 , and extreme care must be taken to minimize phase shifts other than the 90° shift that is inherent in the integrator. For this reason, the i-f amplifier was designed to have effectively only one frequency-determining element, the input resonant circuit, which introduces a 45° phase shift at about 45 kc off center frequency. To insure adequate stability, the loop gain should be reduced to less than unity at this frequency. As the integrator at 45 kc has a gain slightly greater than 30, a lag network was added at the phase-detector output to introduce another 30-db loss at this frequency. The network break points were adjusted so that no added phase shift was introduced at 45 kc.

In order to determine accurately the stability that was achieved, we had to build two complete phase-locked oscillator systems and compare their outputs. Preliminary measurements indicated that the stability of the beat-note between the two systems was almost completely determined by the 5-mc and 200-kc oscillators. For the measurement of the true performance of the equipment, we decided to measure only the transfer stability (the ability of the equipment to transfer the stability of a 5-mc crystal to S-band) eliminating instabilities introduced by the 5-mc oscillator. The measurement apparatus was also designed to be insensitive to instabilities of the 200-kc oscillator on the basis that by crystal control (dividing down from 5 mc, for example) any instabilities introduced here could, if desired, be made negligibly small. A block diagram of the measuring system is shown in Fig. VII-3. The beat between the two S-band signals is detected and amplified; similarly, the beat between the two nominally 200-kc oscillators. These two 10-kc beat signals are fed into a phase detector. The detector output is fed to a pen recorder.

Typical pen recordings are shown in Fig. VII-4. Within the 100-cycle bandwidth of the recorder, two frequency components are predominant, a low-frequency drift of about 30° per minute, and 60 cycles with a peak deviation of 2° . Note that no special precautions were taken with the power supplies to get these results. The klystron filaments were supplied with 60-cycle power, and the repeller and anode voltages were obtained from a TVN11 klystron power supply. The maximum low-frequency drift amounts to a frequency change of a few parts in 10^{-11} , averaged over a second, and less than one part in 10^{-12} , averaged over a minute, a figure that substantiates the earlier observation that the 5-mc oscillator is the primary stability-determining element in

the system as a whole.

The advantages of using a phase-locked oscillator as a stable microwave source are fairly obvious from the results quoted above. The stability of a low-frequency quartz crystal oscillator can be transferred to a microwave oscillator by means of simple circuitry essentially without stability deterioration. The ability of the loop to counteract large drifts in the klystron frequency, either from supply voltage changes or mechanical changes in the tube, is determined almost wholly by the maximum available voltage from the phase detector. Thus, in the case of the loop discussed here, the maximum-phase detector output voltage is approximately ± 5 volts; therefore open-loop drifts of the S-band klystron of about ± 1 mc can be handled without loss of lock. Examination of the figures quoted above for loop-gain and maximum detector output voltage indicates why the system operates successfully on ordinary klystron regulated power supplies. The system can handle power-supply drifts of the order of a volt without any deterioration in performance. After suitable warm-up, most regulated supplies will have a drift of this order. The gain of the loop at 60 cycles is approximately 30,000; hence any hum modulation introduced by the power supply will be drastically reduced by the feedback.

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References

1. J. G. Yates, Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., January 15, 1954, p. 59.
2. P. G. Sulzer, Proc. IRE 43, 701 (June 1955).
3. M. Peter and M. W. P. Strandberg, Proc. IRE 43, 869 (July 1955).
4. D. D. McRae, S. M. Thesis (to be submitted to the Department of Electrical Engineering, M.I.T., June 1956).