II. MICROWAVE GASEOUS DISCHARGES

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RESEARCH OBJECTIVES

The aim of this group continues to be to study some of the fundamental properties of gas discharge physics by microwave techniques, to study microwave oscillation within a gas discharge plasma, and to attempt to understand the details of the interaction between high-frequency electric fields and ionized gases. More and more emphasis is being placed on the high-density plasmas and plasmas in magnetic fields. We have made considerable progress in the understanding of the mechanisms of plasma oscillations, and, also, in the understanding of the interaction between high-frequency fields and discharges in a region where the electron density is higher than the plasma resonance condition in our high-frequency probing fields. We have been studying a new type of breakdown in high-frequency electric fields with superimposed magnetic fields at very low pressures. We hope that these studies will lead us to methods for producing high-density plasmas, so that we may study the fundamental properties of these types of plasma in the near future.

S. C. Brown

A. PLASMA OSCILLATIONS

Previous discussions of plasma oscillations in the Quarterly Progress Reports have been concerned with a uniform plasma. Recent work is directed toward studying the effects of boundary sheaths.

A particular aspect of the problem, long known and little understood, is the way in which the plasma maintains a Maxwellian distribution in discharges at pressures and densities so low that the mean free path for collisions with the gas or other electrons and ions is large compared with the diameter of the tube. Under these conditions, the high-energy tail of the distribution should be depleted. But probe measurements, carried out by many workers, have shown no depletion.

The only reasonable explanation has been that plasma oscillations in the sheath allow a much more efficient energy-exchange mechanism than electron-electron interactions or collisions. Recently Gabor (1) observed these oscillations. We have worked out a theory to explain these mechanisms, and the theoretical results will be compared with Gabor's experimental data.

Gabor was able to measure the dc and ac electric fields in the sheath, as well as the frequency. Figure II-1 is a plot (2) of his experimental results. E_o represents the dc field, and E_1 the ac field. Over most of the region a reasonable approximation can be made that both fields increase linearly with distance, from the sheath-plasma boundary toward the wall. It will be shown that, with this assumption, most of the features of his experimental results can be reasonably explained.

Basically, it has to be shown that the plasma electrons can supply energy to the ac
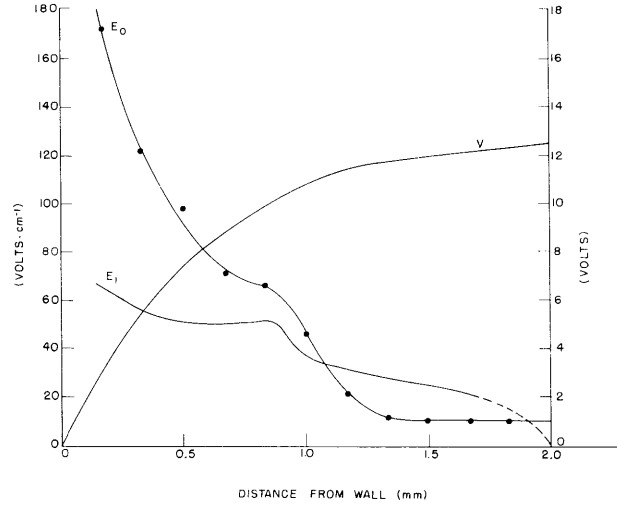
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Fig. II-1.   The dc and ac fields in the sheath.

electric field.   The frequency of oscillation is determined by balancing the stored kinetic energy of the electrons and the stored electric energy of the fields.   Since the electric field is also directed toward the wall and the curl is zero, the magnetic field can be taken to be zero.   Energy conservation requires that

$$\int \int \int \left[\vec{J}_p(\vec{r}) + j\omega\epsilon_o \vec{E}_1(\vec{r}) \right] \cdot \vec{E}_1 \, d^3r = 0 \tag{1}$$

The modulated electron current J_p, developed by the ac field as the electrons move in and out of the sheath, is computed in the following way by using a small-signal approach.

We solve the equation of motion of the electron in the combined ac and dc field. This yields the position in the sheath

$$z = z(t - t_o, \ t_o, \ v_z, \ E_o, \ E_1) \tag{2}$$

where t_o is the time at which the electrons entered the sheath field, v_z is the initial velocity at t_o, and E_o and E_1 are the dc and ac fields.   We expand the actual transit time in the fields.

$$t - t_o = \tau_o(z, v_z) + \tau_1(z, v_z) e^{j\omega t} \tag{3}$$

into dc and ac parts, substitute in Eq. 2, and determine τ_o and τ_1.   The unperturbed transit time is τ_o.   The current of particles with velocity v_z is determined by writing from continuity

$$J(z, t, v_z) = J(0, t_o, v_z) \, dt_o/dt = J_o(v_z) \left(1 - j\omega\tau_1 \, e^{j\omega t}\right) \tag{4}$$

The direct current is conserved and the modulated component is identified as

$$J_1(v_z) = -J_o(v_z)j\omega\tau_1 \tag{5}$$

$\vec{J}_1(v_z) \cdot \vec{E}_1$ is integrated over the volume of the field and then averaged over the velocity distribution, to give the total power dissipated, supplied, or stored by the plasma, exclusive of collisions. Without going into detail, we can determine the effective admittance of the plasma entering the sheath. This is shown in Fig. II-2. The abscissa is $(\omega T/2)/2\pi$, where ω is the frequency of the oscillations and $T/2$ is the average time spent in the sheath as a result of the motion in the dc field. Since the field is linear, the potential is parabolic (harmonic oscillator) and all electrons spend the same average time in the sheath independent of velocity — namely, half a period of the harmonic oscillator frequency, which is determined from the slope of the dc field by writing

$$T = 2\pi\left(\frac{e}{m} \frac{dE}{dx}\right)^{-1/2} \tag{6}$$

It is only in a region in which both g and b are negative $(\omega T/4\pi < 0.64)$ that oscillations can exist. In order for the electrons to supply energy to the field, g must be
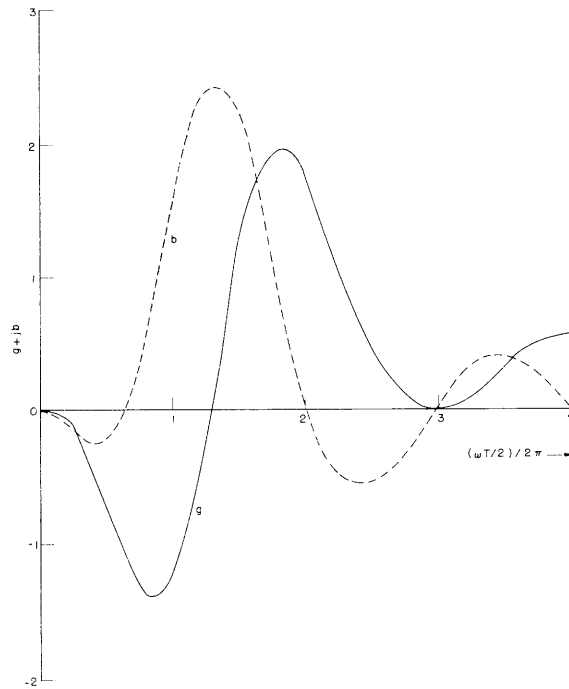

Fig. II-2. The admittance of the plasma in a reflecting sheath.

negative and b must be negative so that it represents stored kinetic energy rather than potential energy, and thus balances the electric energy stored in the ac field. The stored electric energy, in turn, depends on the thickness of the sheath. The requirement can be shown to be

$$(\omega T/4\pi)^3 b = -\frac{\omega^3}{12\pi} \left(\omega d \left(\frac{\pi m}{2kT_e} \right)^{1/2} \right)^3 \frac{\omega^2}{\omega_p^2} \tag{7}$$

where T_e is the temperature of the plasma electrons, d is the thickness of the sheath, and ω_p is the plasma frequency outside the sheath. Solving this graphically by using Gabor's data ($T_e = 2.6 \times 10^4$ °K, $n = 10^9$ cm^{-3}) yields $\omega T/2 = 0.4$ or a little less than one-half cycle of the rf field. This is consistent with the determination of T from Eq. 6 and the observed frequency of oscillation.

Thus, at the same time that individual electrons can make an encounter with the sheath, gaining or losing a few volts of energy and thus allowing the tail of the distribution to build up, the time-average motion of the electrons allows a net transfer of energy to the sheath fields. This process is dissipated by radiation that arises because the oscillations are not coherent over large regions of the sheath, and by collisions in the sheath.

E. Gordon

References

1. D. Gabor, E. A. Ash, and E. D. Dracott, Nature 176, 916-919 (Nov. 1955).
2. Figure II-1 is drawn from Fig. 47 of a Ph.D. thesis submitted to the University of London, Nov. 1955, by E. D. Dracott.

B. NONUNIFORM ELECTRIC FIELDS

One complicating feature of microwave breakdown is the effect of the nonuniformity of the breakdown field. The effect is usually (1, 2) computed by approximating the breakdown equation with a hypergeometric equation in order to obtain an effective diffusion length Λ_e. The effective diffusion length is a measure of the effect of the nonuniformity of the field on microwave breakdown. This procedure is often mathematically complex.

It is somewhat simpler, and at times more accurate, to obtain the effective diffusion length Λ_e directly from the breakdown equation by using a variational principle. The method is as follows. The breakdown equation is rewritten in the form of a Sturm-Liouville problem.

$$\frac{d}{dz}\left[p(z)\frac{d\phi}{dz}\right] + q(z)\,\phi + \lambda r(z)\,\phi = 0 \tag{1}$$

where the function ϕ is defined in the range $a \leqslant z \leqslant b$. When ϕ is made to satisfy certain boundary conditions, the constant λ becomes a characteristic value of the equation and can be obtained (3) from the variational principle

$$[\lambda] = \frac{\int_0^b \left[p(z)\left(\frac{d\phi}{dz}\right)^2 - q(z)\,\phi^2\right] dz}{\int_0^b r(z)\,\phi^2\,dz} \tag{2}$$

As an example, we shall calculate the effect of the nonuniformity of the field on breakdown in a spherical cavity. MacDonald and Brown (1) have shown that the equation for the breakdown field in its lowest mode can be written as

$$\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) + k^2\left[1 - \left(\frac{r}{a}\right)^2\right]^{\beta} r^2\,\psi = 0 \tag{3}$$

where ψ is the electron-diffusion current-density potential, a is the radius of the cavity, β is a parameter that depends on the gas and on the electron energy, and k is related to the effective diffusion length by $k = 1/\Lambda_e$. Equation 3 is to be solved, subject to the boundary conditions

$$\psi(r)\Big|_{r=a} = 0 \qquad \frac{d\psi}{dr}\Big|_{r=0} = 0 \tag{4}$$

Using the variational principle, we have

$$[k^2] = \frac{\int_0^a \left(\frac{d\psi}{dr}\right)^2 r^2\,dr}{\int_0^a \left[1 - \left(\frac{r}{a}\right)^2\right]^{\beta} [\psi(r)]^2\,r^2\,dr} \tag{5}$$

We choose for $\psi(r)$ the trial function

$$\psi(r) = \left[1 - \left(\frac{r}{a}\right)^2\right]^{\alpha} \tag{6}$$

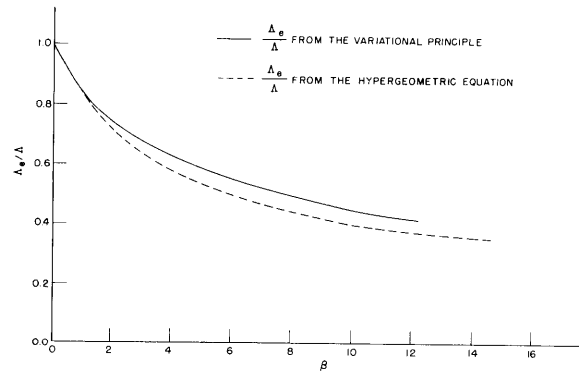where α is a variational parameter whose value depends on the particular β for which

Fig. II-3. Plot of Λ_e/Λ against β for spherical geometry.
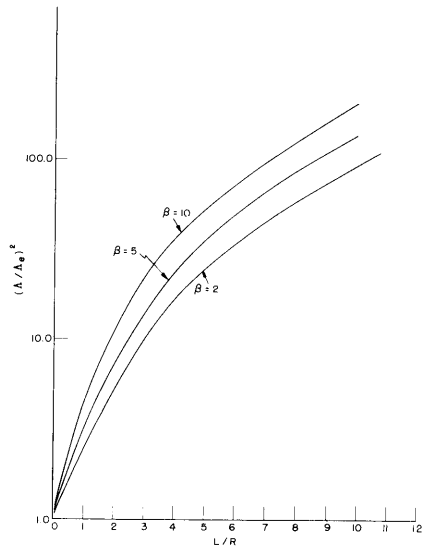

Fig. II-4. Plot of Λ/Λ_e against L/R for cylindrical geometry.
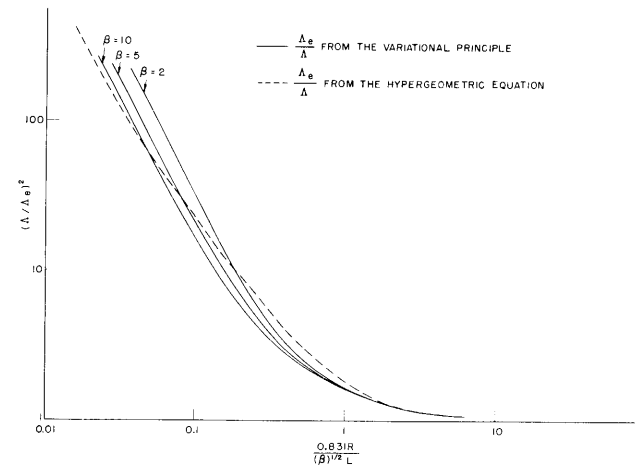

Fig. II-5. Comparison of effective diffusion lengths obtained from the variational principle and from an approximate hypergeometric equation for cylindrical geometry.

$[k^2]$ is to be computed. The result is

$$[k^2 a^2] = \frac{4a^2 \int_0^1 (1-t)^{2(a-1)} t^{3/2} \, dt}{\int_0^1 (1-t)^{\beta+2a} t^{1/2} \, dt}$$

$$= \frac{6a^2 \, \Gamma(2a - 1) \, \Gamma(\beta + 2a + 5/2)}{\Gamma(\beta + 2a + 1) \, \Gamma(2a + 3/2)} \qquad (7)$$

Equation 7 is minimized with respect to a, and the minimum value of $[ka]$ so obtained is used to calculate the effective diffusion length Λ_e. Figure II-3 is a plot of Λ_e/Λ (in which Λ is the geometrical diffusion length) calculated from Eq. 7 as a function of β. For comparison, values of Λ_e/Λ that were obtained from the solution of an approximate hypergeometric equation are also shown. It is seen that the curve obtained from the variational principle lies above the curve obtained from the hypergeometric equation. In view of the fact that $[ka]_{min} \geq (ka)_{exact}$, the variational solution is closer to the true one than the hypergeometric solution.

Similar variational calculations were also carried out for cylindrical geometry (2). In Fig. II-4 the ratio $[\Lambda/\Lambda_e]$ is plotted against L/R with β as a parameter, in which L is the length, and R is the radius of a cylindrical cavity in which breakdown is produced with the TM_{010} mode. To compare the variational results with those obtained from an approximate hypergeometric equation, the results are replotted in Fig. II-5, in which the parameter $(0.831 \, R/(\beta)^{1/2}L)$ is plotted along the abscissa. The approximate hypergeometric equation indicates that the ratio Λ/Λ_e is a function of that parameter only (dashed curve in Fig. II-5). The variational calculation seems to indicate that this is not so.

S. J. Buchsbaum

References

1. A. D. MacDonald and S. C. Brown, Can. J. Research A28, 168 (1952).

2. M. A. Herlin and S. C. Brown, Phys. Rev. 74, 1600 (1948).

3. P. M. Morse and H. Feshbach, Methods of Theoretical Physics (McGraw-Hill Book Company, Inc., New York, 1953), p. 721.

C. PULSED BREAKDOWN

A steady-state gas discharge breakdown occurs when the gain in electron density caused by ionization is just balanced by the loss of electrons from diffusion. When the electric fields are increased by a few per cent over those required for the breakdown,

the electron density builds up to a value large enough to be detected in a very short time (of the order of a few microseconds). This increase in electron density causes a frequency shift and a drop in the Q-value of the cavity, as a result of which the transmitted microwave power decreases. This decrease can also be observed by looking at the light with the help of a photomultiplier tube.

It was observed that as the field strengths are continuously increased over those required for breakdown, the light as a function of time during the total pulse interval changes shape, until at one instant it goes out, and then slowly reappears. The phenomenon was photographed at each stage. (See Figs. II-6, II-7, and II-8.) A four-way band switch was used in conjunction with a photomultiplier tube, a Tektronix oscilloscope, and an A/R oscilloscope to observe the light and the incident, transmitted, and reflected signals. When the production of electrons caused by ionization is greater than the loss of electrons by diffusion, the discharge builds up. The build-up of energy is very rapid, whereas the density builds up slowly at first, when the free diffusion is the controlling factor, and then more rapidly as the diffusion is inhibited by the space charge. At a particular density N_b, which depends on the pressure and the oscillating frequency of the field, power begins to be reflected from the cavity. At this stage, the light from the discharge is correlated with the transmitted power, as shown in Fig. II-9. Point A in the light pulse corresponds to point A in the transmitted power pulse. As the microwave field is increased, the time interval for achieving the density N_b decreases rapidly, the transmitted pulse changes shape, and the light pulse follows it. There is a time lag between the energy and the light. Therefore, when the transmitted power reaches its maximum value, the light is still traversing the decay curve. A stage is reached when the transmitted power goes below the maintaining value for the discharge. Thereafter, the energy increases again, while the light which was lagging behind achieves its minimum value and goes out. As far as the time scale is concerned, this minimum value of the light corresponds to a point on the rebuild-up curve of the transmitted power.

The behavior of the light in going out when it achieves its zero value and reappearing thereafter can be explained as follows. At the beginning of the pulse, the microwave field strength is large, which causes a high frequency of ionization and a large electron density during the build-up time. The electron density is concentrated around the center of the cavity. This high value of density results in a shift in the resonant frequency, a drop in the Q-value, and a detuning of the cavity. The transmitted power and the light from the discharge decay. The decay of power or energy, as compared with the decay of electron density, is very fast. During the time interval in which there has been hardly any appreciable change in the density the energy already reaches a lower value than that required for maintaining the discharge. This energy is insufficient to excite the gas atoms, and the light goes out. This fact is illustrated in the photographs,
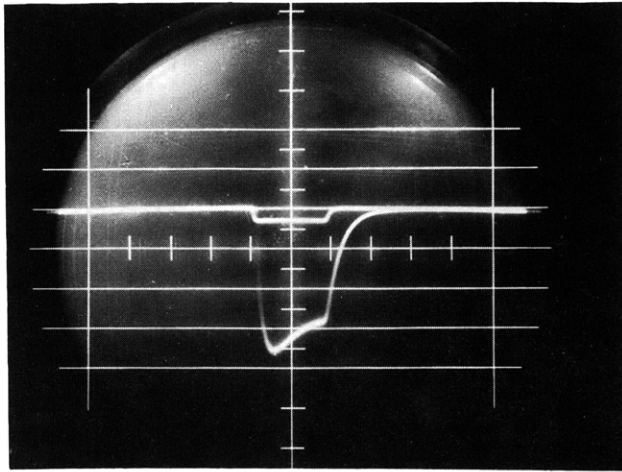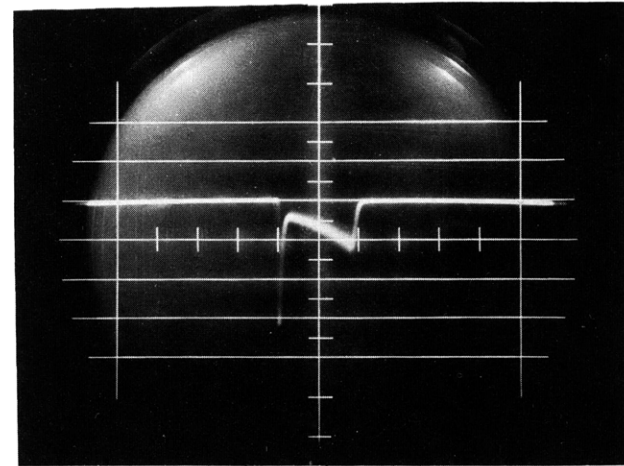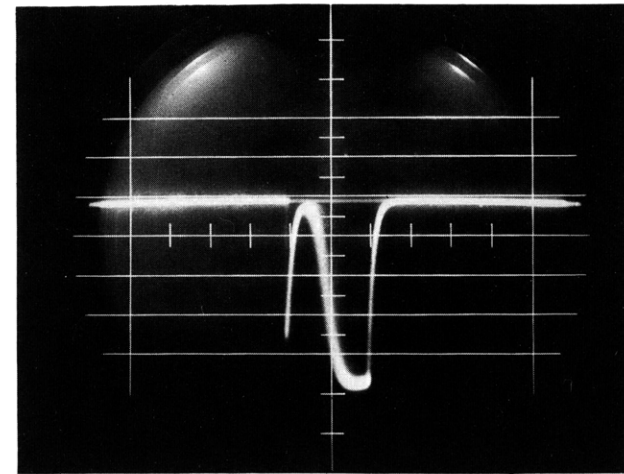
Fig. II-6. Incident power pulse and light pulse (with low overvoltage) as a function of time.
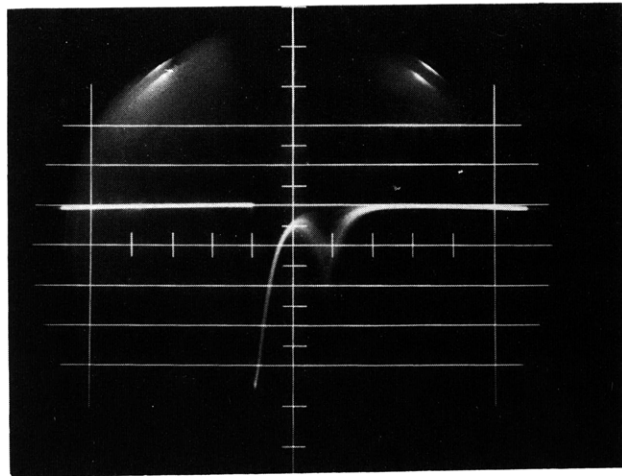

(a)

Fig. II-7. Light pulse as a function of time (high overvoltage).

(b)

Fig. II-8. (a) Transmitted power pulse as a function of time (high overvoltage). (b) Light pulse as a function of time (high overvoltage).

POWER PULSE   LIGHT PULSE

Fig. II-9. Comparison of transmitted
power pulse and light pulse
as a function of time.

in which it is seen that the transmitted power achieves a minimum value greater than zero, but the light pulse goes to its zero value.

In the meantime, the electrons that were concentrated at the center diffuse outward. This spreading out of the electrons results in a decrease of the electron density at the point where it had reached maximum, and the cavity begins to come in tune again. The Q-value of the cavity increases and we are able to put in more power. The transmitted power increases slowly (the light following it) until the equilibrium stage is reached and we have a maintaining discharge.

M. P. Madan

D. MAGNETIC FIELD STUDIES

To study the effect of a magnetic field on a highly ionized plasma, certain conditions must be met by the magnetic field with regard to homogeneity in order to obtain interpretable results. (See the Quarterly Progress Report, Oct. 15, 1956, p. 11.)

The axial magnetic field must be constant within one-half per cent over a distance of 6 cm, and outside this region it must increase to a value five times as large as in the homogeneous region. The inhomogeneous magnetic field will produce a retarding force on electrons traveling from the center in the Z-direction and decrease the loss of electrons at the ends of the magnetic field.

A method of designing a solenoid to produce such a field configuration was suggested by Professor L. J. Chu. A full-sized model was constructed with the use of his method and it performed satisfactorily. The method of design is outlined briefly. The magnetic field that is desired along the Z-axis is expanded in a series of functions that satisfy Laplace's equation. The series chosen for this application is:

$$B_z = \sum_{i=1}^{3} A_i J_0(u_i r) \cosh(u_i z)$$

where the A_i are fitted empirically to the desired shape.

Next, using $\nabla \cdot B = 0$, we determine B_r:

$$B_r = \sum_{i=1}^{3} B_i J_1(u_i r) \sinh(u_i z)$$

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Thus we know the field over all space, having determined it on the Z-axis.

To find the streamlines or lines of B in the vector field, we must have:

$$\nabla\phi \cdot \nabla\psi = 0$$

where ϕ is the magnetic vector potential and ψ is the streamline function.

If we now replace a line of B with an infinitely thin current sheath whose current per unit length is equal to the value of B at that point, we shall obtain the field desired. The value of B on the outside of the current sheath must be zero so that the B on the inside of the solenoid is equal to the current per unit length in the sheath. This was accomplished in the model by having a soft iron shell on the outside of the solenoid.

The approximation to the current sheath was made by using several layers of No. 14 wire. The results from the model indicate that the approximation was satisfactory.

<div align="right">R. G. Meyerand, Jr.</div>

E. HIGHLY IONIZED MICROWAVE PLASMA[*]

The investigation of the optimum conditions for the production of highly ionized hydrogen plasma, described in the Quarterly Progress Report of Oct. 15, 1956, page 11, has led to a study of the maintaining conditions of the discharge immediately prior to extinction. In order for a discharge to be maintained, we must have

$$\nu_i \geqslant \nu_L$$

where ν_i is the ionization frequency, and ν_L is the frequency with which electrons are lost from the discharge. The minimum value of the ionization frequency to maintain the discharge will be given by

$$\nu_i = \nu_L$$

The ionization frequency as a function of electron energy, for various pressures of hydrogen gas, is shown in Fig. II-10. If we assume a function relating the frequency of electron loss with electron energy (Fig. II-10), the condition to maintain the discharge with the minimum ionization frequency at a given electron energy is determined by the intersection of the ν_i and ν_L curves for a particular electron energy. From these intersections, the function relating the gas pressure with electron energy can be derived. This function is shown in Fig. II-11. The maximum electron energy is achieved at the condition of cyclotron resonance when $\omega = \omega_b$. For $\omega \neq \omega_b$, two lower energies will
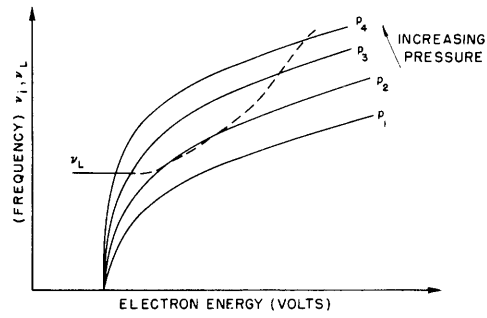
Fig. II-10. Ionization frequency (ν_i) as a function of electron energy for hydrogen at four pressures and postulated electron loss frequency (ν_L) in the system as a function of electron energy.
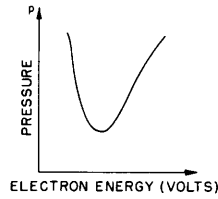

Fig. II-11. The minimum gas pressure required to maintain a discharge as a function of electron energy.
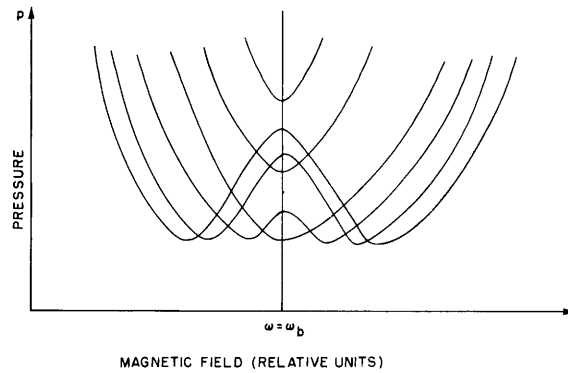

Fig. II-12. Theoretical curves showing the dependence of the minimum gas pressure on the magnetic field strength.
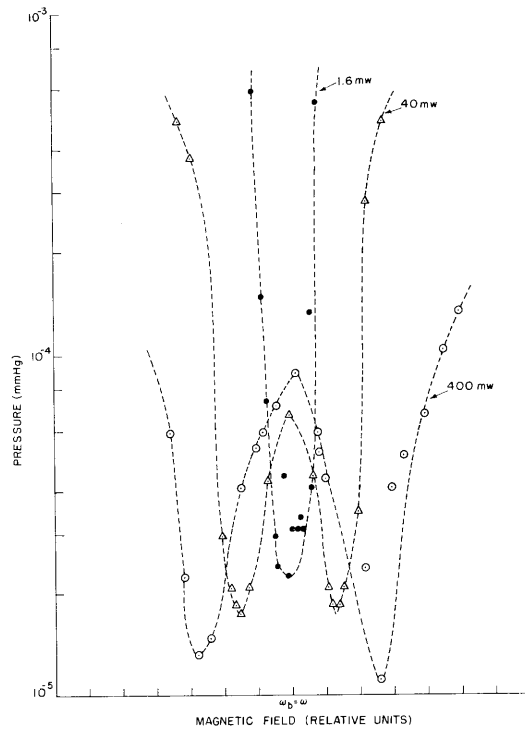
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Fig. II-13.   Experimental curves showing the dependence of the minimum gas pressure on the magnetic field strength.

exist, corresponding to $\omega > \omega_b$ and $\omega < \omega_b$. The gas pressure, plotted as a function of ω_b (showing variations of the incident power), can be derived. This function, which is shown in Fig. II-12, provides a further explanation of the widening of the breakdown curves that were previously observed and reported.

The maintaining conditions of the discharge for $\nu_i = \nu_L$ were studied experimentally, and a series of curves was obtained for varying incident power. The results of these studies are shown graphically in Fig. II-13. Comparison with the simple theoretical curves indicates a good correlation between the two functions.

D. O. Akhurst, S. J. Buchsbaum, E. Gordon

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