XII. NOISE IN ELECTRON DEVICES*

Prof. R. B. Adler
Prof. H. A. Haus
Prof. J. B. Wiesner
Dr. J. K. D. Verma
J. G. Ingersoll

A. THE CHARACTERISTIC NOISE MATRIX FOR EQUILIBRIUM NETWORKS AND MASER AMPLIFIERS

In the Quarterly Progress Report of April 15, 1956, pages 87-88, and in published work (1) it was pointed out that a 2 × 2 matrix can be ascribed to a linear two terminal-pair network containing noise sources. This "characteristic noise matrix" expressed in terms of the rms open-circuit noise voltages E_1 and E_2 and the impedance matrix $\vec{Z}$ has the form

$$\vec{N} = -\frac{1}{2}(\vec{Z} + \vec{Z}^{\dagger})^{-1} \overline{\vec{E}\vec{E}^{\dagger}} \qquad (1)$$

where $\vec{E}$ is the column matrix

$$\vec{E} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

and the bar over $\vec{E}\vec{E}^{\dagger}$ indicates an ensemble average.

Alternatively, a characteristic noise matrix can be defined in terms of the scattering matrix $\vec{S}$ of the same network and the noise waves β_1 and β_2 that would be fed by the network into matched loads

$$\vec{N}' = (\vec{S}\vec{S}^{\dagger} - \vec{1})^{-1} \overline{\vec{\beta}\vec{\beta}^{\dagger}} \qquad (2)$$

where

$$\vec{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

and $\vec{N}'$ has the same eigenvalues as $\vec{N}$ defined in Eq. 1. The characteristic noise matrix $\vec{N}$ pertaining to a passive two terminal-pair network at an equilibrium temperature T has the simple form

$$\vec{N} = -\vec{1}kT\Delta f = \vec{N}' \qquad (3)$$

Therefore, we might be led to suggest that the coincidence of the eigenvalues and the diagonal form of the $\vec{N}$ matrix are characteristic of equilibrium networks. But,

*This research was supported in part by Purchase Order DDL-B187 with Lincoln Laboratory, which is supported by the Department of the Army, the Department of the Navy, and the Department of the Air Force under Contract AF19(122)-458 with M.I.T.

conventionally, equilibrium is defined only for passive networks for which, as we see from Eq. 3, the eigenvalues are negative. Thus, coincidence of the eigenvalues and the diagonal form of $\vec{N}$ can be interpreted as representing an equilibrium in the conventional sense only if the eigenvalues are negative.

Recent suggestions that the maser may be considered as consisting of a medium with a negative conductance at a negative temperature provide an interesting interpretation of the diagonal characteristic noise matrix with positive, coincident eigenvalues.

Indeed, it is easy to show from Wittke's (2) work that for the maser consisting of a lossless waveguide filled with a medium with inverted energy-level populations, we have

$$\vec{N} = (\vec{S}\vec{S}^\dagger - \vec{1})^{-1}\,\overline{\vec{\beta}\vec{\beta}^\dagger} = \vec{1}kT\Delta f \tag{4}$$

The temperature T is so defined that it provides the proper occupation N_2 of the higher-energy states and the occupation N_1 of the lower-energy states, according to an inverted Boltzmann distribution

$$\frac{N_2}{N_1} = \exp(-\hbar\omega/kT) \tag{5}$$

with $T < 0$. In Eq. 4 the assumption is made that $\hbar\omega \ll |kT|$.

A corresponding relatively simple expression is found for the maser that contains lossy material. We shall derive this expression in detail in a manner that brings out clearly the significance of the characteristic noise matrix as a measure of "equilibrium" (in a broader sense). Consider, first, a microwave transmission cavity or waveguide with lossy walls at temperature T. The characteristic noise matrix of the cavity when considered as a two terminal-pair network is given by Eq. 3. It is clear that the same result would be obtained with a cavity with lossless walls and filled with a medium of conductivity σ so as to give the same loss as the cavity in the preceding example. This statement can also be proved by direct computation if we assign to every differential rectangular box of volume dv, at the position $\bar{r}$ of the material, a mean-square short-circuit current-density fluctuation $\overline{J_x^2}$ in the x-direction, according to the Nyquist formula

$$\overline{J_x^2} = 4\sigma kT\Delta f\,\delta(\bar{r}) \tag{6}$$

and corresponding expressions for $\overline{J_y^2}$ and $\overline{J_z^2}$. Here $\delta(\bar{r})$ is the three-dimensional Dirac δ-function. The correlation between current densities in two different directions is zero, e.g., $\overline{J_xJ_y^*} = 0$.

Equation 6 is particularly convenient for generalization to maser analysis. According to Wittke's paper, a negative conductivity ($\sigma_m < 0$) can be assigned to a volume element occupied by maser material.

The short-circuit noise current $\overline{i^2}$ produced by the resulting equivalent negative conductance $G_m (< 0)$ of the lossless maser fulfills the Nyquist formula

$$\overline{i^2} = 4G_m kT_m \Delta f \tag{7}$$

where $T_m < 0$, and is defined by Eq. 5, with $T = T_m$. Equation 7 suggests that a Nyquist generator can be assigned to every volume element dv of maser material at a point $\bar{r}$ according to Eq. 6. Thus

$$\overline{J_x^2} = 4\sigma_m kT_m \Delta f \delta(\bar{r}) \tag{8}$$

and so forth, for the other current components.

Turning now to the maser with loss, we note that the conductivities σ (attributed to loss) and σ_m (attributed to the gain mechanism) appear in parallel and therefore give the net conductivity

$$\sigma_t = \sigma_m + \sigma \tag{9}$$

The noise currents are also uncorrelated and add up to

$$\overline{J_x^2} = 4\Delta f \delta(\bar{r}) \left[\sigma_m T_m + \sigma T\right] \tag{10}$$

and so forth, for the other current components. But, Eqs. 9 and 10 show that the maser with loss may be considered as consisting of a material with conductivity σ_t and temperature T_t defined by

$$\sigma_t T_t = \sigma_m T_m + \sigma T \tag{11}$$

The characteristic noise matrix then follows directly as

$$\overrightarrow{N} = -kT_t \Delta f \overrightarrow{1} \tag{12}$$

If the maser has gain, $\sigma_t < 0$, and therefore $T_t < 0$. Thus, $\overrightarrow{N}$ has two coincident positive eigenvalues. The optimum noise figure is given directly by the positive eigenvalues $kT_t\Delta f$ of $\overrightarrow{N}$

$$F_e - 1 = -\left(1 - \frac{1}{G_e}\right)\frac{kT_t\Delta f}{kT\Delta f} = -\left(1 - \frac{1}{G_e}\right)\frac{T_t}{T} \tag{13}$$

where T is the temperature of the circuit and the lossy part of the maser (walls), and the subscript e refers to the generalizations of F and G in terms of "exchangeable power" (3).

If we use Wittke's notation, we have

$$\frac{\sigma_m}{\sigma_t} = -\frac{a}{a - a_g}; \qquad \frac{\sigma}{\sigma_t} = \frac{a_g}{a - a_g}$$

$$\frac{N_1}{N_2} = \exp(+\hbar\omega/kT_m) \approx 1 + \hbar\omega/kT_m; \qquad F_e = F; \qquad G_e = G$$

and thus we obtain

$$-kT_m \approx \frac{\hbar\omega}{1 - \frac{N_1}{N_2}} \tag{14}$$

Introducing expression 14 into Eq. 13 and using Eqs. 9 and 10, we obtain

$$F - 1 = \left(1 - \frac{1}{G}\right)\left[\frac{h\nu}{kT}\,\frac{1 + \frac{a_g}{a - a_g}}{1 - \frac{N_1}{N_2}} + \frac{a_g}{a - a_g}\right]$$

which is in agreement with Wittke's expression.

Thus we conclude that a maser is a device with simple equilibrium properties. This makes the analysis of maser noise performance through the characteristic noise matrix particularly attractive.

R. B. Adler, H. A. Haus

References

1. H. A. Haus and R. B. Adler, Invariants of linear noisy networks, IRE Convention Record, Part 2, 1956, pp. 53-67.
2. J. P. Wittke, Molecular amplification and generation of microwaves, Proc. IRE 45, 291-316 (1957).
3. H. A. Haus and R. B. Adler, An extension of the noise figure definition, Proc. IRE 45, 690-691 (1957).

B. LOW-NOISE 30-MC AMPLIFIER

Construction of a low-noise 30-mc amplifier has been completed. This amplifier uses Western Electric 416B planar triodes in a cascode circuit. It is well known that the cascode combination (grounded cathode-grounded grid) gives the best results of any of the nine possible combinations of two triodes with respect to noise figure, gain, and stability. The 416B tube was designed for operation at 4000 mc, but recently it has also been used at lower frequencies; for example, at 60 mc for low-signal operation. With an optimum value of 1620 ohms for source resistance, the theoretical noise figure of the amplifier is 1.06, or 0.26 db. The corresponding gain and bandwidth are 32 db and 6.2 mc, respectively.

This "preamplifier" is followed by a "postamplifier" that uses 417A tubes in cascode and a 6AK5 tube in the final stage. The measured noise figure of the postamplifier is of the order of 1.4 db. It can be easily seen that connecting the two amplifiers in cascade does not appreciably affect the noise figure of the preamplifier. The noise figure of the whole amplifier is given by the relation

$$F_{12} = F_1 + \frac{F_2 - 1}{G} \tag{1}$$

where F_{12} is the noise figure of the whole amplifier, F_1 is the noise figure of the preamplifier, F_2 is the noise figure of the postamplifier, and G is the available gain of the first cascode. In this relation, G is of the order of 1000, and hence the noise contribution of the second term on the right-hand side in Eq. 1 is negligibly small.

The measured noise figure of the amplifier as a whole is of the order of 0.6 db. The noise figure as measured by existing commercial equipment, for example, by RadaNode of Kay Electric Company, is approximate, since the accuracy of these instruments is poor in the lower ranges. Hence, the radiometer method (1) is being used. The amplifier will be used to measure noise in semiconductors.

J. K. D. Verma

References

1. G. E. Valley, Jr. and H. Wallman, Vacuum Tube Amplifiers, Radiation Laboratory Series Vol. 18 (McGraw-Hill Book Company, Inc., New York, 1st ed., 1948), p. 718.

C. CHARACTERIZATION OF PROBABILITY DISTRIBUTIONS FOR EXCESS PHYSICAL NOISE

This work was completed and is being published as Technical Report 276, which is based on a thesis that was submitted to the Department of Electrical Engineering, M.I.T., Sept. 1956, in partial fulfillment of the requirements for the degree of Doctor of Science.

J. Hilibrand