# II. MICROWAVE GASEOUS DISCHARGES*

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## A. ION CYCLOTRON RESONANCE

In order to obtain an additional diagnostic tool for the study of ionized plasmas, an experiment designed to measure the cyclotron resonance of molecular and atomic ions in a hydrogen plasma is in progress. In principle the experiment is a simple one. An ion in a crossed rf electric field and a static magnetic field will gain energy from the E-field if the magnitude of the B-field is so adjusted that the ion cyclotron frequency $\omega_b = (eB)/M$ is of the order of $\omega$, the frequency of the rf field. The rate of gain of energy is largest when $\omega = \omega_b$, and when the ion collision time is very short compared with the rf period. In practice, the experiment is complicated by the presence of electrons in the plasma, which, because of their much smaller mass, are particularly adept at extracting energy from the electric field. There are other limitations and requirements. Because the mass of even the lightest ion – the proton – is rather large, $\omega_b$ is limited to the megacycle region. This implies that in plasmas under study, where the electron plasma frequency is in the microwave region, the rf field must interact with a plasma whose plasma electron frequency, $\omega_{pe}$, is a few orders of magnitude larger than $\omega$. If the configuration of the E-field in the plasma is to be known (as it must for the experiments to be a diagnostic tool), two conditions must be satisfied. The E-field must be perpendicular to plasma gradients (1, 2) and the effective skin depth of the plasma must be large in comparison with plasma dimensions. These conditions are satisfied in the following arrangement (3). A plasma in the form of a long, narrow cylinder (radius, 1 cm) is placed coaxially in a coil that is driven by a 1-mc generator. A static magnetic field is applied axially. Particular care is used to shield out from the plasma any axial rf E-fields. The azimuthal induction field of the coil satisfies the conditions set above. From the known formula for the plasma conductivity we can easily calculate that, at least in the absence of the static magnetic field, the skin depth of the plasma, although smaller than the wavelength (300 meters) remains larger than the radius of the plasma so long as the inequality $n/p < 10^{15}$ holds, where $n$ is the plasma electron density per $cm^3$, and $p$ is the hydrogen gas pressure in mm Hg (in this formula $p$ must be larger than $1\mu$).

The effect of the ions on the interaction of the plasma with the rf field is obtained

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from the plasma conductivity tensor, $\overleftrightarrow{\sigma}$, obtained by Allis (4). The $\sigma_{\theta\theta}$ component of $\overleftrightarrow{\sigma}$ (the diagonal component), is given by

$$\sigma_{\theta\theta} = \frac{ne^2}{2m}\left[\left(\frac{1}{\nu_{m_e}+j\left(\omega+\omega_{b_e}\right)}+\frac{1}{\nu_{m_e}+j\left(\omega-\omega_{b_e}\right)}\right)+\frac{m}{M}\left(\frac{1}{\nu_{m_i}+j\left(\omega+\omega_{b_i}\right)}+\frac{1}{\nu_{m_i}+j\left(\omega-\omega_{b_i}\right)}\right)\right]$$

$$= \sigma_{\theta\theta_e} + \sigma_{\theta\theta_i} \tag{1}$$

where, for simplicity, we assumed only a single ion specie of mass M and where the subscript e refers to electrons and subscript i to ions. In the absence of the static magnetic field, the effect of the ions is negligible as a consequence of the smallness of the mass ratio m/M. At ion cyclotron resonance, however, $\left(\omega = \omega_{b_i}\right)$, the contribution of ions to $\sigma_{\theta\theta}$ can be made appreciable. For example, the ratio

$$R = \frac{\text{Re}\,\sigma_{\theta\theta_i}}{\text{Re}\,\sigma_{\theta\theta_e}} \approx \frac{m}{M}\,\frac{\omega_{b_e}^2}{\nu_{m_i}\,\nu_{m_e}}$$

which measures the relative energy loss to ions and electrons, can easily be made unity or larger by reducing the collision frequencies $\nu_{m_i}$ and $\nu_{m_e}$. In an active hydrogen plasma, $\nu_{m_e} \approx 6 \times 10^9$ p, where p is the pressure in mm Hg; $\nu_{m_i}$ for atomic ions is not well known (it is one of the objects of this experiment to measure the elastic collision cross-section of thermal protons), but $\nu_{m_i}$ for molecular hydrogen ions can be calculated from the known ambipolar diffusion coefficient to be approximately $2 \times 10^7$ p for thermal ions. We must have $\nu_{m_i} < \omega$ for the resonance effect to be measurable. This sets the upper limit on the pressure as approximately 0.1 mm Hg. At this pressure the ratio $\frac{M}{m}$ R is approximately $10^5$, and is more than sufficient to offset the smallness of the mass ratio m/M. The current given by $\sigma_{\theta\theta}$ is not the only current that will flow in the plasma. Other components of the tensor will also contribute. Calculations are now in progress to determine the total effect of ions and electrons on the reflected impedance of the coil by taking into account all elements of the tensor and proper boundary conditions.

The disadvantage of the experimental arrangement is known to be the rather small effect that the plasma will have on the effective inductance and resistance of the coil. For example, in the absence of the static magnetic field, for an infinite coil of radius a and a plasma of uniform electron density n and radius R, the change in the Q value of the coil is given by

$$\Delta\left(\frac{1}{Q}\right) = \frac{\text{average power dissipated}}{\omega \text{ energy stored}}$$

$$\approx \frac{\pi^2}{2} \frac{\omega_p^2}{\omega^2} \frac{\omega}{\nu_{m_e}} \left(\frac{R}{\lambda}\right)^2 \left(\frac{R}{a}\right)^2$$

$$\approx 10^{-18} \frac{n}{p}$$

where n is the density per $cm^3$, and p is the pressure in mm Hg and where it was assumed that $\nu_{m_e} > \omega$. For a plasma density of $10^{10}$ $cm^{-3}$ and p = 0.1 mm, the change in the Q is only 1 part in $10^7$. The change in inductance is smaller still, being given $\left(\frac{\Delta L}{L}\right) \approx \frac{\omega}{\nu_m} \Delta\left(\frac{1}{Q}\right)$. A twin-T circuit capable of detecting these small impedance changes is discussed in Section II-B.

S. J. Buchsbaum

References

1. S. J. Buchsbaum and S. C. Brown, Phys. Rev. 106, 196 (1957).
2. K-B. Persson, Phys. Rev. 106, 191 (1957).
3. K-B. Persson, private communication.
4. W. P. Allis, Handbuch der Physik, Vol. XXI, p. 413.

## B. ION CYCLOTRON RESONANCE MEASUREMENT SYSTEM

Measurement of ionized plasma properties in the ion cyclotron resonance experiment requires measurement of $\partial L/L$, the normalized change in measurement coil inductance, and $(\partial R)/(\omega L)$, the normalized change in resistance. A system using a twin-T null network (1) with substitution that is synchronous with periodic modulation of $\partial L$ and $\partial R$ has been developed.

The measurement solenoid and modulator coil is shown in Fig. II-1. The 1-mc coil (top) is shown with one end of its electrostatic shield removed to expose the interior winding. The radial vanes and shielded cable connections to the coil winding are evident. The radial vanes assist the electrostatic shield in removing the axial electric field. The plasma discharge tube fits inside the vanes. The lower portion of the photo shows the modulator coil that surrounds the 1-mc measurement coil. The function of the modulator coil is to vary (at 100 cps) the axial magnetic flux density B and, therefore, to modulate periodically the ion cyclotron frequency. It is by this modulation process that $\partial R$ and $\partial L$ are varied. They are reflected as very small periodic variations in the input impedance of the measurement coil.

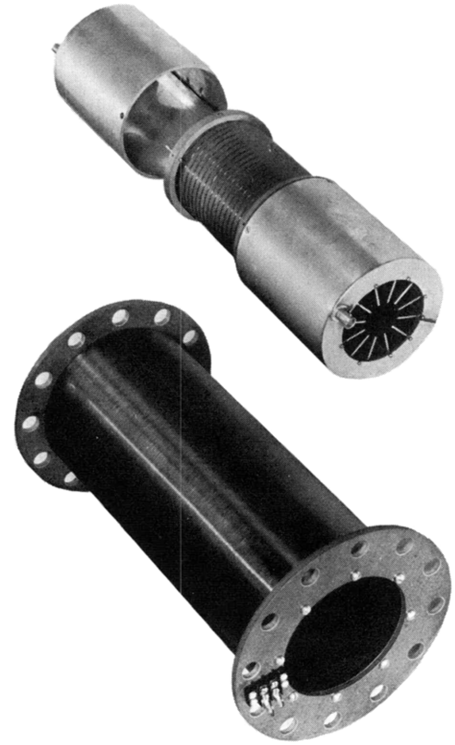

Fig. II-1. Measurement coil (top) and modulator coil (bottom).

A simplified block diagram of the complete measurement system is shown in Fig. II-2. The physical arrangement of various coils that are associated with the experiment is indicated at the top of the figure. Stated briefly, the twin-T null network is adjusted so that its null is at 1 mc. Periodic modulation at 100 cps in the plasma generates sidebands at the detector output. Three amplification and mixing operations are performed to recover the upper sideband and to measure its components in terms of conductance and susceptance. Some details of the detection process are shown in the diagram. Substitution measurements are accomplished by modulating the main balance capacitances in the twin-T so that a sideband equal in amplitude and opposite in phase to the signal is generated. With the proper substitutions, each integrator output is zero. An accurate estimate of the measurement coil impedance change at 100 cps is then obtainable from a knowledge of the substitution modulator characteristics. The accuracy of the measurement is not dependent upon the detector gain or phase characteristic.

A photograph of the complete electronics of the system is shown in Fig. II-3. The left portion of the equipment consists basically of the 1-mc generator and power amplifier, twin-T null circuit, and 100-cps substitution generator. The twin-T is the unit

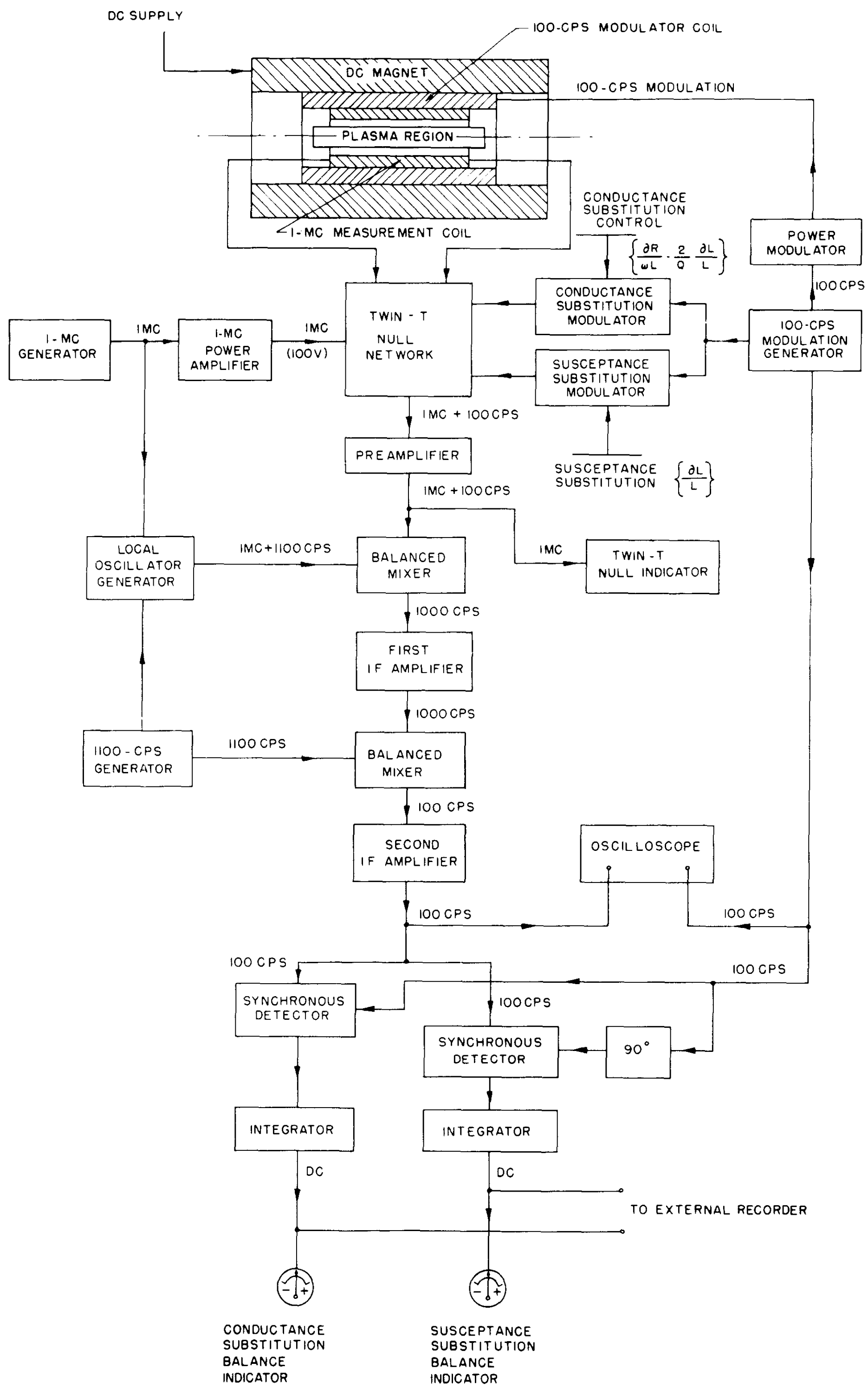


Fig. II-2. Ion cyclotron resonance measurement system.

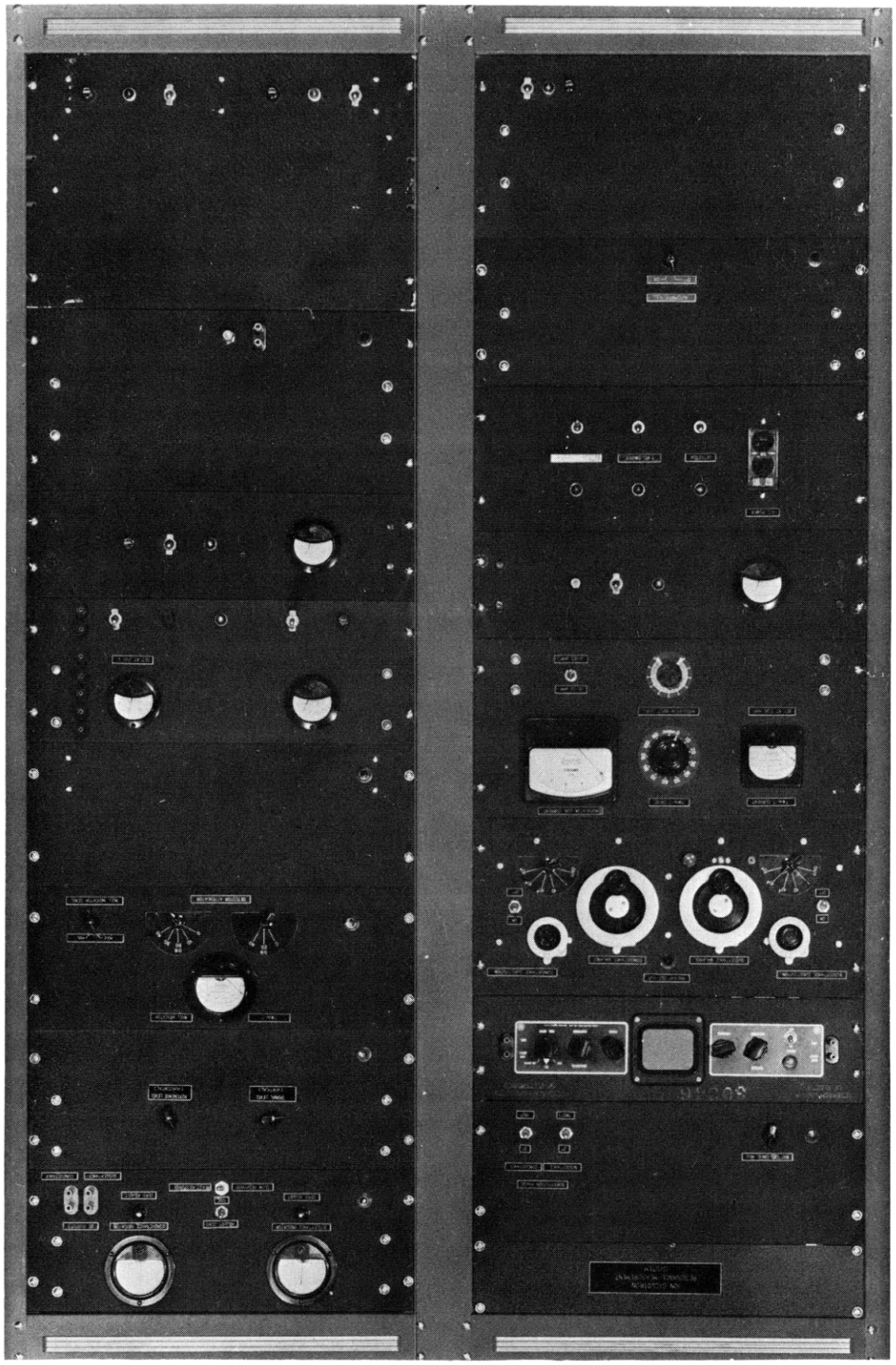

Fig. II-3. Ion cyclotron resonance measurement system.

with the two large main balance controls visible just below the oscilloscope. The substitution modulator controls are located at either side of the balance controls. The right side of the unit is devoted mainly to the detector units, power modulator unit, and their associated power supplies.

First measurements of detectability and measurement accuracy indicate that the residual noise level in normalized rms parts (with 3.5 seconds integration time) is about $10^{-9}$ in coil resistance. Normalized rms inductance change is about the same level.

Reasonably accurate estimates of relative loss and inductance changes require reasonable signal-to-noise ratios. Therefore, a normalized impedance change of about $10^{-8}$ represents the lower bound for accurate measurements.

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References

1. W. N. Tuttle, Bridge-T and parallel-T null circuits for measurements at radio frequencies, Proc. IRE 28, 23-29 (Jan. 1940).

## C. MEASUREMENTS OF THE COLLISION CROSS SECTION AND OF THE ENERGY LOSS OF SLOW ELECTRONS IN HYDROGEN

The work outlined in the Quarterly Progress Report of January 15, 1958, page 14, has been completed; the final results are shown in Fig. II-4.

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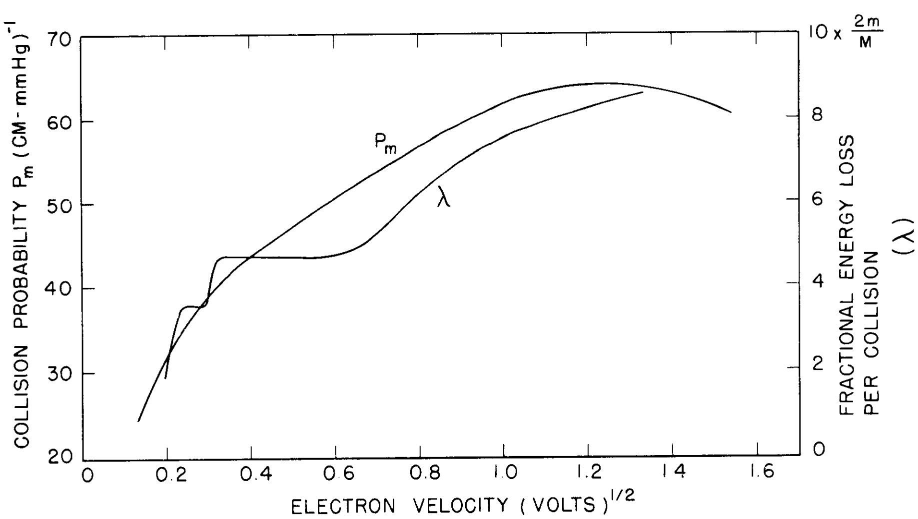


Fig. II-4. Collision probability and energy loss of slow electrons in hydrogen as a function of the electron velocity (m is the mass of the electron; M is the mass of the hydrogen molecule).

## D. MICROWAVE MEASUREMENTS OF ELECTRON TEMPERATURES IN THE PRESENCE OF A STATIC MAGNETIC FIELD

It is well known that electrons (and ions) in a crossed microwave E-field and a static B-field exhibit a cyclotron resonance when the microwave frequency $\omega$ is equal to the cyclotron frequency $\omega_b$[= (eB)/m]. It will be shown that a resonance also occurs when the microwave E-field is parallel to the B-field if the plasma temperature is finite (so that the Larmor radius of the average electron is not zero) and if the plasma finds itself in a region where there exists an appreciable gradient of the E-field in a direction perpendicular to the B-field. In a properly designed experiment, the magnitude of the resonance is directly proportional to the square of the Larmor radius and hence it can be used as a direct determination of the electron (or ion) temperature.

Simple Analysis

The motion of an average electron in parallel E- and B-fields along the direction of the fields (assumed to be along the z-axis) is given by

$$m\frac{dv_z}{dt} + m\nu_m v_z = e[E\ r(t)]\ e^{j\omega t} \tag{1}$$

where we assumed that the microwave field is axially symmetric and that it is a function of the radius; $\nu_m$ is the collision frequency for momentum transfer (assumed to be independent of electron velocity). Expanding E about $r_o$, the position of the guiding center,

$$E(r) = E(r_o) + \frac{dE}{dr_o}(r-r_o) + \dots \tag{2}$$

and setting

$$r - r_o \approx r_b \cos(\omega_b t + \phi) \tag{3}$$

where $r_b$ is the Larmor radius and $\omega_b$ is the cyclotron frequency, the solution to Eq. 1 is obtained:

$$v = \frac{e}{m}\left[\frac{E(r_o)}{j\omega + \nu_m} + \frac{r_b}{2}\frac{dE}{dr_o}\left(\frac{\exp[j(\omega_b t+\phi)]}{\nu_m + j(\omega+\omega_b)} + \frac{\exp[-j(\omega_b t+\phi)]}{\nu_m + j(\omega-\omega_b)}\right)\right]\exp(j\omega t + \nu_m) \tag{4}$$

In microwave diagnostics we are interested in the time average of $J^* \cdot E/2$; Re $\overline{J^* \cdot E/2}$ is the microwave power per unit volume dissipated in the plasma and is, therefore, related to the change in the loaded Q value of the cavity while Im $\overline{J^* \cdot E/2}$ is the average energy density stored in the plasma and, therefore, is related to the shift in the resonant frequency of the cavity. Since J = nev, where n is electron density (we neglect the contribution of the ions), we have

$$\overline{J^* \cdot E} = \frac{ne^2}{m}\left\{\frac{E^2(r_o)(\nu_m - j\omega)}{\omega^2 + \nu_m^2} + \left(\frac{r_b}{2}\frac{dE}{dr_o}\right)^2\left[\frac{\nu_m - j(\omega-\omega_b)}{\nu_m^2 + (\omega-\omega_b)^2} + \frac{\nu_m - j(\omega+\omega_b)}{\nu_m^2 + (\omega+\omega_b)^2}\right]\right\} \tag{5}$$

The change in the Q value of the cavity is then given by

$$\Delta\left(\frac{1}{Q}\right) = \frac{\omega_p^2}{\omega^2}\left\{\frac{\nu_m/\omega}{1+(\nu_m/\omega)^2}\frac{\int_p E^2\,dv}{\int_c E^2\,dv} + \left(\frac{r_b}{2}\right)^2\frac{\nu_m/\omega}{(\nu_m/\omega)^2 + [(\omega-\omega_b)/\omega]^2}\frac{\int_p [(dE)/(dr)]^2\,dv}{\int_c E^2\,dv}\right\} \tag{6}$$

where subscripts p and c indicate integrations over the volume of the plasma and cavity, and where, for simplicity, we assumed a uniform plasma and neglected the nonresonant term. The first term of Eq. 6 is the one commonly used in microwave diagnostics in the absence of a B-field. The second term exhibits a resonance at $\omega = \omega_b$. Its magnitude is proportional to $r_b^2$ and therefore to plasma temperature. The change in the resonant frequency of the cavity is given by

$$\frac{\Delta f}{f} = \frac{1}{2}\frac{\omega_p^2}{\omega^2}\left[\frac{1}{1+\nu_m^2/\omega^2}\frac{\int_p E^2\,dv}{\int_c E^2\,dv} + \frac{[(\omega-\omega_b)/\omega]}{(\nu_m/\omega)^2 + [(\omega-\omega_b)/\omega]^2}\left(\frac{r_b}{2}\right)^2\frac{\int_p [(dE)/(dr)]^2\,dv}{\int_c E^2\,dv}\right] \tag{7}$$

Since in most plasmas the Larmor radius is very small [$r_b \approx 0.02$ cm for T = 100 ev, $\omega_b = 2 \times 10^{10}\ \text{sec}^{-1}$] we must examine whether or not it is possible to observe the resonance effect experimentally. Consider the ratios

$$R_1 = \frac{\Delta(1/Q)_{\omega=\omega_b}}{\Delta(1/Q)_{\omega_b=0}} = a\left(\frac{r_b}{2}\right)^2\frac{1+(\nu_m/\omega)^2}{(\nu_m/\omega)^2} = a\left(\frac{r_b}{2}\right)^2\left(\frac{\omega}{\nu_m}\right)^2 \qquad \text{for } \frac{\nu_m}{\omega} \ll 1 \tag{8}$$

and

$$R_2 = \frac{(\Delta f/f)_{\omega-\omega_b=\nu_m}}{(\Delta f/f)_{\omega_b=0}} = a\left(\frac{r_b}{2}\right)^2\frac{1}{2}\frac{1+(\nu_m/\omega)^2}{\nu_m/\omega} = \frac{a}{2}\left(\frac{r_b}{2}\right)^2\left(\frac{\omega}{\nu_m}\right) \qquad \text{for } \frac{\nu_m}{\omega} \ll 1 \tag{9}$$

where

$$a = \frac{\int_p [(dE)/(dr)]^2\,dv}{\int_p E^2\,dv} \tag{10}$$

We see that at low pressures ($\nu_m \ll \omega$), $\Delta(1/Q)$ is the more sensitive of the two effects.

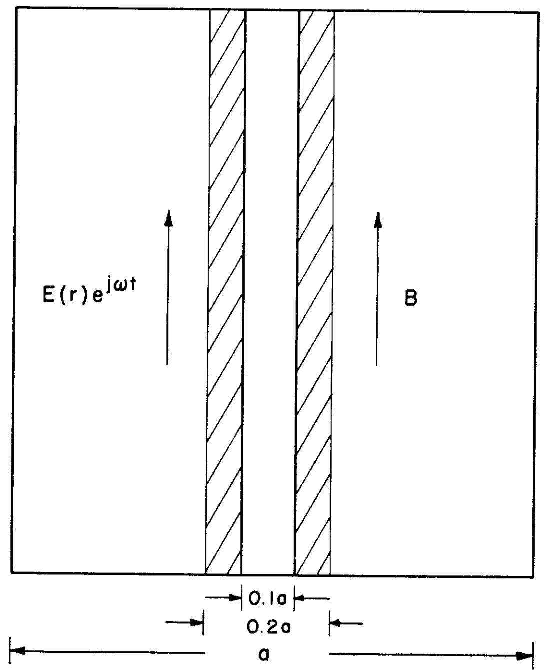


Fig. II-5. A $TM_{010}$ cavity for microwave measurement of plasma temperature.

To make $R_1$ as large as possible, it is necessary to make $\alpha$ as large as possible; that is, the plasma must be placed only in that part of the field where the gradient is large but the field is small.

A possible field configuration is that of the $TM_{010}$ mode of a coaxial cavity whose center conductor is thin. It was calculated that if the radius of the inner conductor is one-tenth the radius of the outer conductor (see Fig. II-5) and if the plasma is allowed to surround the inner conductor to a radius that does not exceed two-tenths of the radius of the outer conductor, then in the plasma region the E-field varies approximately linearly with the radius, and

$$\alpha \approx \frac{\int_0^R r\, dr}{\int_0^R r^3\, dr} = 2R^2$$

$$R_1 = \frac{1}{2}\left(\frac{r_b}{R}\right)^2 \left(\frac{\omega}{\nu_m}\right)^2 \qquad (11)$$

It is then only necessary to make $\omega/\nu_m$ of the order of $(R/r_b)$ for the resonance to be easily measurable.

A more stringent condition is the necessity of an exact alignment of the E- and

B-fields. If there is a small angle $\theta$ between them, a "true" cyclotron resonance effect resulting from the component of E at right angles to B may mask the resonance described above. The change in the Q value will then be

$$\Delta\left(\frac{1}{Q}\right) \approx \frac{\omega_p^2}{\omega^2}\left[\frac{\nu_m/\omega}{1+(\nu_m/\omega)^2}\,\frac{\int_p E^2\,dv}{\int_c E^2\,dv} + \theta^2\,\frac{\nu_m/\omega}{(\nu_m/\omega)^2+[(\omega-\omega_b)/\omega]^2}\,\frac{\int_p E^2\,dv}{\int_c E^2\,dv}\right.$$

$$\left. + \left(\frac{r_b}{2}\right)^2 \frac{\nu_m/\omega}{(\nu_m/\omega)^2+[(\omega-\omega_b)/\omega]^2}\,\frac{\int_p [(dE)/(dv)]^2\,dv}{\int_c E^2\,dv}\right] \tag{12}$$

And we must have

$$\theta < \frac{1}{\sqrt{2}}\left(\frac{r_b}{R}\right) \tag{13}$$

for the temperature measurement to be accurate.

A More Rigorous Analysis

The simple analysis given above is in terms of the motion of an average electron and is restricted to constant collision frequencies. A rigorously correct description is obtained from Boltzmann's equation

$$\frac{\partial f}{\partial t} + v \cdot \nabla f + \frac{e}{m}(E + v \times B) \cdot \nabla_v f = \frac{\partial f}{\partial f_{coll}} \tag{14}$$

Expanding the distribution function f and the fields in the standard manner (1)

$$f = f_o + f_1, \quad E = E_o + E_1, \quad B = B_o + B_1 \tag{15}$$

we obtain, following Rosenbluth (2),

$$f_1 = -\frac{e}{m}\exp(-\nu_m t)\int_0^t \exp(\nu_m t)\,(E_1 + v \times B_1) \cdot \nabla_v f_o\,dt \tag{16}$$

where the perturbed distribution function $f_1$ is caused by the fields $E_1$ and $B_1$ and where the integral follows the particle orbit in the six-dimensional phase space. In our case we have

$$B_1 \approx 0 \quad E_1 = E[r(t)]\,e^{j\omega t} \approx \left\{E(r_o) + \frac{dE}{dr_o}\,r_b\cos(\omega_b t)\right\} e^{j\omega t}$$

Equation 16 is easily integrated:

$$f_1 = -\frac{e}{m}\left\{\frac{\vec{E}(r_o)}{v_m + j\omega} + \frac{r_b}{2}\frac{d\vec{E}}{dr_o}\left[\frac{\exp(j\omega_b t)}{\nu_m + j(\omega+\omega_b)} + \frac{\exp(-j\omega_b t)}{\nu_m + j(\omega-\omega_b)}\right]\right\} \cdot \nabla_v f_o \tag{17}$$

The microwave current $J_1$ is obtained from

$$J_1 = -\frac{4\pi}{3} e \int_0^\infty f_1 v^3 \, dv \tag{18}$$

To integrate Eq. 18 the unperturbed distribution function $f_o$ must, in general, be known. If $\nu_m$ is independent of v, Eq. 18 reduces to Eq. 4.

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References

1. W. P. Allis, Handbuch der Physik XXI, p. 404.
2. M. Rosenbluth, II$^{nd}$ Magnetohydrodynamic Symposium, Stanford, 1957.

## E. OPTICAL DIAGNOSTICS

An optical method has been devised for measuring the relative change of electron energy or temperature of a changing plasma. The "effective" temperature, T, is so defined that the tail of the actual electron distribution in the neighborhood of the excitation energies matches the tail of a theoretical Maxwellian distribution of temperature T.

In a low-pressure hydrogen discharge, the intensity of a line in the atomic spectra is given by

$$I_\ell \propto n_e n_1 X_\ell(T)$$

where $n_e$ and $n_1$ are the electron and atomic specie densities, and $X_\ell(T)$ is the unit excitation frequency of the principal quantum level of the atom which leads to the monochromatic radiation of the line in question. For electron energies less than 5 volts, the logarithm of the ratio of two line intensities, such as the $\alpha$ and $\beta$ lines of the Balmer series, can be approximated by

$$\ln\frac{I_\alpha}{I_\beta} = (V_\beta - V_\alpha)\frac{1}{T} + C$$

where $V_\beta$ and $V_\alpha$ are the threshold excitation energies of the respective atomic levels, and C is determined by instrumental and physical constants. Thus the ratio of two line intensities is independent of electron and atomic hydrogen densities.

In practice, the constant C is difficult to evaluate, and only relative changes in temperature can be measured as the line intensities vary. Thus

$$\frac{1}{T} - \frac{1}{T_o} = \frac{1}{(V_\beta - V_a)} \left[ \ln\left(\frac{I_a}{I_\beta}\right) - \ln\left(\frac{I_a}{I_\beta}\right)_o \right]$$

This technique has been used to measure temperature changes in a pulsed microwave discharge superposed on a steady-state microwave discharge. The electron density was measured with microwaves, and the ratio of density in the pulse to the density in the steady state was set at 10. The time variation of the β and γ line intensities were photographed and analyzed. The results of the calculated change in 1/T are plotted in Fig. II-6.

The plot of 1/T clearly shows that the electron energy decreases for high electron densities, and increases for low electron densities. The decrease in energy can easily account for the extinction of the plasma at high densities which is an observed fact.

The total change in reciprocal temperature is 0.2. Although the absolute temperature has not been measured, it is clear from the expression

$$\frac{1}{T} - \frac{1}{T_o} = 0.2$$

that T cannot be greater than 1/0.2 = 5 volts, regardless of the value of $T_o$. T is more likely to be in the neighborhood of 2 or 3 volts. This evaluation, therefore, puts an upper limit on the value of T. Figure II-6 also indicates that the temperature T varies over a period of many milliseconds even though the electron density changes

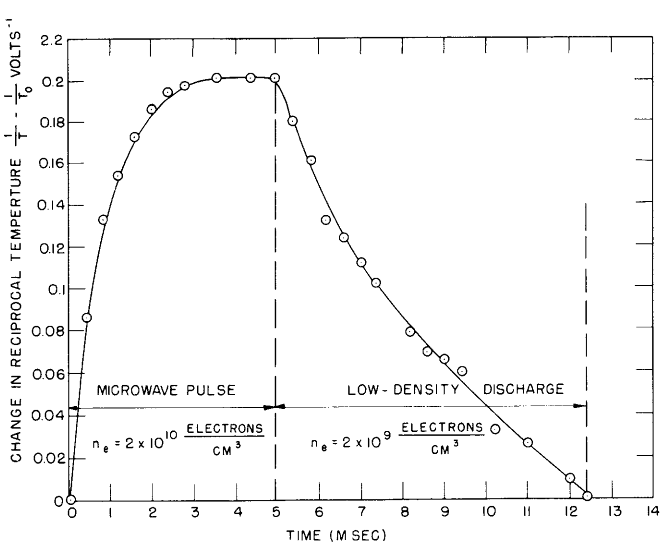


Fig. II-6. Pulsed microwave discharge (hydrogen gas, $p_o = 0.93\mu$).

almost instantaneously. This fact is the result of the build-up of atomic hydrogen density, and possibly water vapor, which provide a sink of energy for the electrons and decrease their energy.

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