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Validation of a Novel Catheter Guiding Method for the Ablative Therapy of Ventricular Tachycardia in a Phantom Model

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Abstract—Accurate guidance of an ablation catheter is critical in the RF ablation (RFA) of ventricular tachycardia (VT). With current technologies, it is challenging to rapidly and accurately localize the site of origin of an arrhythmia, often restricting treatment to patients with hemodynamically stable arrhythmias. We investigated the effectiveness of a new guidance method, the inverse solution guidance algorithm (ISGA), which is based on a single-equivalent dipole representation of cardiac electrical activity and is suitable for patients with hemodynamically unstable VT. Imaging was performed in homogeneous and inhomogeneous saline-filled torso phantoms in which a catheter tip was guided toward a stationary electrical dipole source over distances of more than 5 cm. Using ISGA, the moving catheter tip was guided to within 0.61 ± 0.43 and 0.55 ± 0.39 mm of the stationary source in the homogeneous and inhomogeneous phantoms, respectively. This accuracy was achieved with less than ten movements of the catheter. These results suggest that ISGA has potential to provide accurate and efficient guidance for RFA procedures in the patient population with hemodynamically unstable arrhythmias.

Index Terms—Catheter ablation, equivalent moving dipole, ventricular tachycardia (VT).

I. INTRODUCTION

More than 200 000 deaths a year in the United States may be attributable to ventricular tachycardia (VT) postmyocardial infarction [1]. RF ablation (RFA) of the site of origin of the arrhythmia may permanently abolish the VT, and is therefore currently the optimal treatment [2], [3]. We have previously described a method to guide the electrophysiologist to the optimal site for ablation, the *inverse solution guidance algorithm* (ISGA). This algorithm utilizes a single-equivalent moving dipole (SEMD) model of electrical activity to localize the exit site of the reentrant circuit [4]–[7]. To accurately guide an ablation catheter to the exit site, ISGA then estimates the location of the ablation catheter tip by calculating the location and orientation of a current dipole generated between two electrodes at its tip [5]. *Real space* contains the real catheter tip and real arrhythmogenic dipole; *image*

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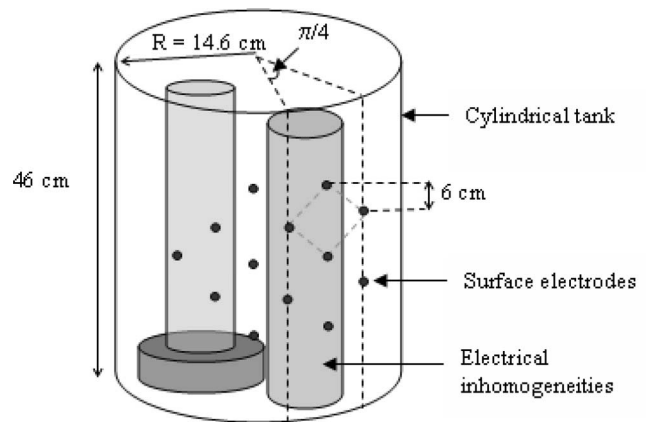


Fig. 1. Experimental setup showing the phantom model, the electrode layout, and an example configuration of the three objects used to create an electrically inhomogeneous phantom (these were of dimensions $r = 7.5$ cm, $h = 37$ cm; $r = 9$ cm, $h = 28$ cm; and $r = 8.3$ cm, $h = 8$ cm, and were arranged in a different configuration for each trial).

space contains the dipole solutions for the electrical activity at these two locations estimated using ISGA. By manipulating the catheter so as to bring the dipole positions together in image space, the ablation catheter tip may be guided to the exit site of the reentrant circuit for the accurate delivery of ablative energy. The localized electrical activity at the exit site of the reentrant circuit shall be referred to as the *arrhythmogenic dipole*. The dipole solutions for electrical activity at the catheter tip and exit site shall be referred to as the *catheter dipole image* and *arrhythmogenic dipole image*. Since a computationally simple infinite, homogeneous model is used to estimate the potentials on the torso surface due to a dipole source in real time, the arrhythmogenic dipole image is displaced from the *true* position of the arrhythmogenic dipole [8]. Previous studies have indicated the potential of ISGA to colocalize two electrical sources, and thus, to be used as a guidance method [4], [5], [7], [9]. However, experimental measurements with inhomogeneous phantom models have not been conducted.

In this study, we first performed the investigation of ISGA's accuracy and efficiency as a guidance algorithm in an inhomogeneous phantom. Three measures were used to evaluate its success. First, the accuracy with which the moving catheter tip could be superposed with the stationary dipole in both homogeneous and inhomogeneous phantom models was calculated. Second, a quantitative comparison was made of the distances and directions moved in real and image spaces. Third, the magnitude of the displacement between a dipole's position in real and image spaces was calculated.

II. METHODS

A. Phantom Model and Signal Measurement

The phantom model consisted of an open cylindrical tank of radius 14.6 cm, filled with standard saline solution to a height of 46 cm to approximate the dimensions of an average male torso (see Fig. 1). Sixty Ag–AgCl electrodes were arrayed over the vertical surface of the phantom in a rhombic lattice configuration. One electrode at the base of the phantom was chosen as the reference electrode. The phantom and the proximally unshielded electrode leads were enclosed within a Faraday cage to minimize 60 Hz noise. A rigid catheter with two platinum electrodes 3 mm apart at its tip was vertically mounted on a 3-D positioning system and placed inside the phantom. The positioning system allowed the catheter tip location to be measured with an accuracy of

0.2 mm in real space, and moved within a 20-cm sided “heart volume” approximately centered within the phantom. A sinusoidal signal of frequency 100 Hz and amplitude 10 V was generated between the catheter tip electrodes by an electrically isolated BK Precision model 4011A function generator. This created potentials ranging from 1 to 10 mV at the surface electrodes, approximating the amplitude of a surface ECG. A sinusoidal signal was chosen instead of a VT waveform for computational simplicity. It should be noted that only *instantaneous* signal amplitudes are used by the ISGA, as previously described [4]–[6]. Therefore, it was not necessary to replicate the frequency components of an ECG signal.

The differential signals between the 59 measurement electrodes and one reference electrode were amplified using WPI ISO-DAM8 isolated bioamplifiers. Electrical isolation of the amplifiers ensured that current leakage through the surface electrodes was insignificant compared to the amplitude of the current between the catheter tip electrodes. The data in the 59 channels were sampled at 1 kHz and filtered with a fifth-order Butterworth bandpass filter to retrieve the sinusoidal signal amplitudes.

A graphical user interface enabled real-time assessment of signal quality from each channel. It also displayed the estimated locations and orientations of the stationary target dipole and moving catheter tip dipole. A dipole’s location and orientation were represented using a ball and arrow, respectively. The distance and vector between the two dipole images (the *interimage vector*) were displayed numerically.

B. Inverse Solution Guidance Algorithm

The ISGA has been described previously [4]–[6]. In the application of the ISGA, voltages created by a dipole within the phantom are measured at the multiple electrodes on the phantom surface. A χ^2 measure of the *goodness-of-fit* between measured and modeled surface voltages is then used to calculate the dipole position and orientation; a *brute-force* search of the χ^2 objective function space (which avoids local minima) is then conducted within the volume conductor, and the dipole whose parameters minimize the objective function is the solution to the inverse problem. Due to the proximity of the reference electrode to the measurement electrodes in the phantom model, the accuracy of the SEMD solution was improved by taking into account the significant 100-Hz signal at the reference electrode. Therefore, the forward model describing the estimated potential ϕ^i in the i th channel due to a single dipole in an infinite, homogeneous volume conductor (1) was adapted

$$\phi^i = \frac{\mathbf{p}(\mathbf{r} - \mathbf{r}'_i)}{4\pi g |\mathbf{r} - \mathbf{r}'_i|^3} - \frac{\mathbf{p}(\mathbf{r} - \mathbf{r}'_{\text{ref}})}{4\pi g |\mathbf{r} - \mathbf{r}'_{\text{ref}}|^3} \quad (1)$$

where \mathbf{r}_{ref} represents the reference electrode location.

C. Experimental Protocol

The catheter was placed at a random position within the phantom, and its 3-D coordinates in real space (the *stationary dipole* location) and image space (the *stationary dipole image* location) were noted. The catheter was then moved in all three dimensions by a minimum of 5 cm to a second random position. The catheter’s new 3-D coordinates in real space (the initial *moving dipole* location) and image space (the initial *moving dipole image* location) were recorded. The interface simultaneously displayed the stationary and moving dipole images. A second operator then attempted to guide the catheter tip to the location of the stationary dipole using only the information displayed on the user interface and moving the catheter tip each time in real space by between one-half and two-thirds of the remaining distance between the

images. Superposition of the dipole images was indicated by a distance between them of less than 0.5 mm on the graphical interface. Since the same catheter was used to generate both the stationary and moving dipoles, there was no collision feedback to the operator if the moving dipole was brought to the location of the stationary dipole inside the phantom.

Fifteen trials were conducted in a homogeneous phantom, each time using a randomly chosen stationary dipole location and randomly chosen initial moving dipole location. A further 15 trials were conducted in an inhomogeneous phantom. Torso inhomogeneities were simulated using three cylindrical plastic objects (of dimensions $r = 7.5$ cm, $h = 37$ cm; $r = 9$ cm, $h = 28$ cm; and $r = 8.3$ cm, $h = 8$ cm) placed in different configurations within the phantom (see Fig. 1). The objects were arranged at the start of each trial in a random configuration that would not obstruct the movement of the catheter tip toward the stationary dipole.

III. RESULTS

A. Accuracy of Dipole Superposition

The catheter tip was guided to within 0.61 ± 0.43 and 0.55 ± 0.39 mm of the stationary dipole in the homogeneous and inhomogeneous phantom studies, respectively. This difference in guidance accuracy is *not* significantly different at the 0.05 level, indicating that guidance accuracy does not change in the presence of electrical inhomogeneities. A single outlier with an end-point accuracy of 28.2 mm is excluded from our stated end-point accuracy for the inhomogeneous phantom. For this outlier, unlike the other data points, it was observed that when the dipole images were superposed, their orientations were *not* aligned. Furthermore, when the catheter tip was moved to the real location of the stationary dipole, the dipole image locations and orientations were *both* superposed. It was ascertained that the outlier result was not, in fact, a result of a nonlinear parameter-fitting error, in which the algorithm had become caught in a local minimum of the χ^2 goodness-of-fit objective function. In fact, our previously described *brute-force* search method correctly found the absolute χ^2 minimum within the volume conductor, as it is designed to do [10]. Instead, because of severe distortions in image space caused by the electrical inhomogeneities, two dipoles at different locations in real space mapped to dipoles at the same location but different orientations in image space.

B. Comparison of Real and Image Spaces

The distances of the catheter tip from the arrhythmogenic dipole in image space (i_d) and real space (r_d) were compared over multiple trials for m positions of the catheter tip in the homogeneous ($m = 95$) and inhomogeneous ($m = 119$) phantom models. The results are shown in Fig. 2(a) and (b). The linear relationship of i_d to r_d shown in the figures indicates that distances in real and images space are almost equal even in the presence of significant nonidealities (the scattering of data points about the best-fit polynomial is more limited in the homogeneous case). The number of movements of the catheter required to superpose the images of the moving and stationary dipoles was also small— 7.33 ± 2.09 and 7.81 ± 2.14 for the homogeneous and inhomogeneous trials, respectively (the difference is not statistically significant). The difference between the angles made in real space versus image space was $5.38 \pm 1.94^\circ$ and $4.65 \pm 1.76^\circ$ for the homogenous and inhomogeneous phantoms, respectively.

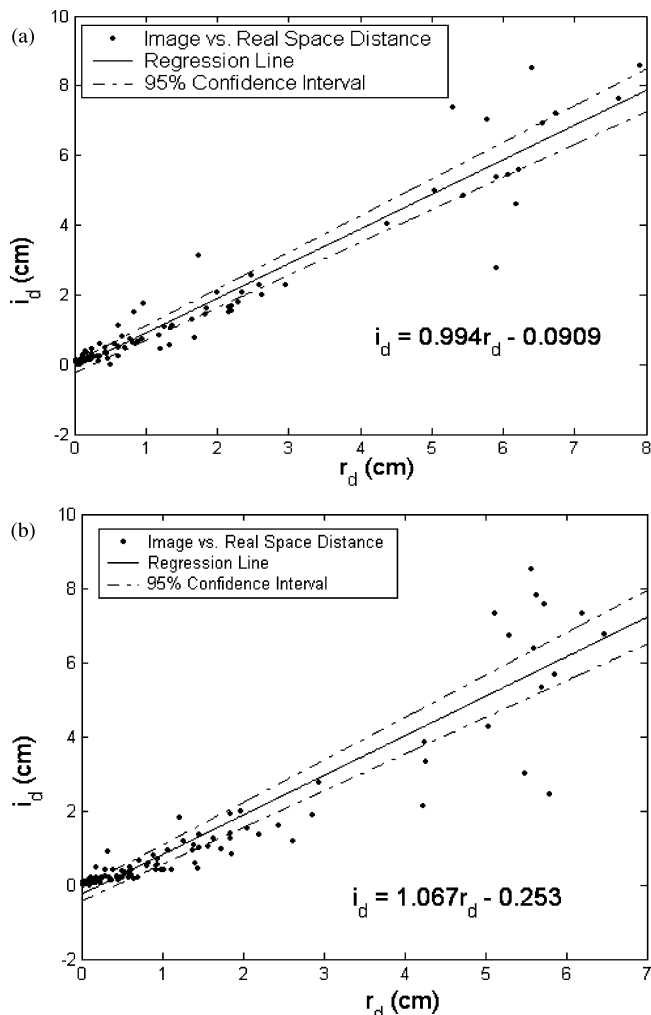


Fig. 2. Comparison of the distance of the catheter tip from the arrhythmogenic dipole in image space (i_d) and real space (r_d) for m positions of the catheter tip as it moves toward the arrhythmogenic dipole. (a) Homogeneous ($m = 95$) phantom model. (b) Inhomogeneous ($m = 119$) phantom model. The least squares first-order polynomial fit of i_d versus r_d (solid line), whose function is shown with each figure, indicates that the ratio of i_d to r_d is approximately 1 in both cases, while the 95% confidence intervals (dashed-dotted line) indicate a good quality of fit.

IV. DISCUSSION

In this study, the accuracy with which our ISGA could be used to guide a catheter tip to the site of a cooriented stationary dipole was investigated in homogeneous and inhomogeneous phantom models. An accuracy of significantly less than 1 mm was attained in all 15 experiments in the homogeneous phantom and in 14 out of 15 experiments in the inhomogeneous phantom. Furthermore, our results indicated that movements of the catheter tip dipole in real and image spaces correspond both in distance and direction even when significant inhomogeneities and boundary effects are present in the volume conductor.

The single outlier in the inhomogeneous phantom trials indicates that the ISGA may falsely indicate the spatial superposition of two dipoles. However, for this outlier, while the dipoles were *spatially* superposed in image space, their orientations were *different*. Therefore, in the presence of significant nonidealities and sources of systematic error, *dipoles at multiple locations in real space may map to dipoles with*

the same location in image space but different orientations. Therefore, a method to compensate for the effect of dipole orientation in the presence of sources of systematic error is necessary. An algorithm utilizing a catheter design with multiple tip electrodes, arranged such that three independent dipoles may be generated between them, has been developed. By linearly superposing the three independent dipoles, a catheter tip dipole may be created whose orientation matches that of the target at each step. This algorithm has been tested in simulations, and has been found to improve convergence in the presence of sources of systematic error [10]. Future work will involve development of such a catheter and testing of the method in a phantom torso.

There are several limitations to this study. First, the human torso contains electrical anisotropies that have not been simulated in this study. In addition, it is likely that in an animal or a human, exact electrode positions would not be known. However, these are both simply additional forms of systematic error, and it is therefore expected that the proposed algorithm would work nevertheless. While the phantom model used in this study does not have a realistic torso shape, we would expect our results to *improve* if a more realistic geometry were used; surface electrodes on the front of the chest are closer to the heart than those in our phantom model, and the convex surface of a real torso would improve the algorithm's accuracy along the torso axis. However, it should be noted that in a real system, sources of systematic error will not be constant. Breathing and cardiac movement cause variations in electrode positions and the distribution of torso inhomogeneities. To overcome this, the use of controlled breath-holding and image gating by surface ECG signals would need to be explored in a clinical setting.

We have tested the ISGA in a phantom model, and shown that it provides an accurate and efficient method for the guidance of a catheter tip to the location of a stationary dipole. Since this method is able to image the site of origin of the arrhythmia from a single beat of the tachycardia, ISGA is suitable for use even in patients with hemodynamically unstable VT. Therefore, ISGA may allow RFA to be performed not only more accurately, but also in a much wider segment of the population affected by VT than is currently possible.

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Determining the Visual Angle of Objects in the Visual Field: An Extended Application of Eye Trackers

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Abstract—Many eye-tracker systems display the point of central gaze fixation on video images of the viewed environment. We describe here a method for determining the visual angles of objects located in the periphery. Such data are needed to study the potential contributions of peripheral vision during cognitive and motor tasks.

Index Terms—Central vision, eye movements, gaze behavior, peripheral vision, saccades, visual angles.

I. INTRODUCTION

Eye trackers (ETs) are commonly used to study gaze behavior in a wide range of research fields (e.g., visual-vestibular systems, visual attention, motor control) and activities (e.g., locomotion, sports, driving, reading) [1]. While some ET systems simply record eye movements, others display the point of gaze fixation on video images of the environment, as recorded by a head-mounted or remote "scene" camera [1]. Such systems allow the object that was fixated to be identified, but provide no quantitative information about the locations of objects in peripheral regions of the visual field.

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The human eye focuses incoming light rays most accurately on the fovea, the retinal area of the greatest visual acuity (visual angles $<2.5^\circ$). The surrounding macula (visual angles $<9^\circ$) also provides high acuity [2]. Although acuity decreases as the visual angle of the object moves farther into the periphery [2], studies have demonstrated that peripheral regions of the visual field do provide information that is important in many cognitive and motor tasks (e.g., spatial location [3]–[5] or detecting the movement of objects [6]). Typically, such findings have been derived from studies in which portions of the visual field are occluded (using special goggles, contact lenses, or computer displays [5], [6]) or subjects are instructed to maintain fixation on a specified point [3], [4]. To study potential contributions of the central and peripheral visual fields under more natural task conditions, it is necessary to be able to determine the visual angles of various objects of interest over the duration of the cognitive or motor task being studied.

This communication presents a method for augmenting the point-of-gaze data provided from an ET by plotting associated visual angle information on the scene-camera image, thus allowing the visual angles of objects of interest in the subject's field of view to be determined. The method presented herein is valid for an ET system for which the spatial relationship between the eye and the ET remains constant. This could include a head-mounted ET on a mobile subject or an environment-mounted ET when the subject's head is immobilized. Example data are presented, and the assumptions and sources of error are discussed.

II. METHODS

The visual field of the eye may be thought of as a cone, of visual angle θ , centered on the line of gaze [Fig. 1(a)]. As a person views the surroundings, anything that falls within that cone will be visible within the visual angle θ . When using an ET, the 3-D scene viewed by the subject is recorded as a 2-D video image in the "image plane" of the ET scene camera. The ET calculates the intersection of the line of gaze with the image plane, and plots this "point of gaze" P on the recorded scene. To determine, with respect to P , the visual angles of objects in the scene, one must find the coordinates of the "gaze ellipse" created by the intersection of the visual-angle cone and the image plane.

A. Derivation of Basic Method

The fundamental objective is to convert distances recorded in the scene-camera image plane to visual angles θ given the point of gaze P that is provided by the ET software [Fig. 1(b)]. We define the point at which a perpendicular line of gaze would intersect the scene-camera image plane as D . Thus, the distance from any point of gaze to the perpendicular gaze point is PD . If the focal length of the scene-camera lens is OD and gaze is inclined at angle α , then

$$\alpha = \tan^{-1} \left(\frac{PD}{OD} \right). \quad (1)$$

If points A and C define the extents of the gaze ellipse, then

$$AD = OD \tan(\alpha + \theta) \quad (2)$$

$$CD = OD \tan(\alpha - \theta) \quad (3)$$

and the length of the major axis of the ellipse AC can be calculated as

$$AC = AD - CD. \quad (4)$$

This information then allows the location of the center of the ellipse B to be found using

$$BD = CD + \frac{AC}{2}. \quad (5)$$