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**Citation:** M. Lattanzi and G. Montani. "A separable solution for the oscillatory structure of plasma in accretion disks." EPL (Europhysics Letters) 89.3 (2010): 39001.

**As Published:** <http://dx.doi.org/10.1088/0741-3335/51/12/124007>

**Publisher:** Institute of Physics

**Persistent URL:** <http://hdl.handle.net/1721.1/52508>

**Version:** Author's final manuscript: final author's manuscript post peer review, without publisher's formatting or copy editing

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# High energy plasmas, general relativity and collective modes in the surroundings of black holes

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**Abstract.** Theoretical finding of composite disk structures around compact objects (e.g. black holes) and recent experimental observations indicate that highly coherent and dynamically important magnetic field configurations exist in the core of these plasma structures. Coherent configurations involving closed magnetic surfaces provide a means to resolve the “accretion paradox” for a magnetized disk while the formation of jets emitted in the close vicinity of the compact object is related to these configurations. The absence of vigorous accretion activity in the presence of black holes in old galaxies can be attributed to the loss of the surrounding coherent magnetic configurations during their history. The relevant dynamics include, axisymmetric (ballooning) modes as well as tri-dimensional spirals which can be excited from disks with a “seed” magnetic field, under the effects of differential rotation and of the vertical plasma pressure gradient. The properties of these spirals are strongly dependent on their vertical structure. Axisymmetric modes can produce vertical flows of thermal energy and particles in opposing directions that can be connected to the winds emanating from disks in Active Galactic Nuclei (AGN’s). A similarity with the effects of temperature gradient driven modes in magnetically confined laboratory plasmas is pointed out. Spiral modes that are oscillatory in time and in the radial direction can produce transport of angular momentum toward the outer region of the disk structure, a necessary process for the occurrence of accretion. The excitation of radially localized density spirals co-rotating with the plasma, at a distance related to the Schwartzchild radius  $R_s = 2GM_*/c^2$ , is proposed as the explanation for High Frequency Quasi Periodic Oscillations (HFQPOs) of non-thermal X-ray emission from compact objects.

## 1. Introduction

The plasma structures that are associated with compact astrophysical objects can have properties that are definitely different from those of the fluid-dynamic disks which are well described in the existing literature [1]. In the following sections we shall outline the theoretical issues that have to be dealt with in identifying these properties. A special attention is devoted to the spectrum of modes that can be excited in the most elementary disk structure, to the processes that can be associated with these modes, and to the relevance of these processes (e.g. particle flows, density enhancements and dips, accretion) to current experimental observations.

## 2. The Stripped Plasma Disk

The axisymmetric “stripped” plasma disk is one in which there is no accretion process present, the only flow velocity is the rotation velocity  $V_\phi = \Omega(R)R$ , a nearly constant magnetic field  $\mathbf{B} = B_z \mathbf{e}_z$  is present and the relevant two-fluid description is characterized

by the electric fields  $\mathbf{E}_p = -\nabla\Phi_p = -(\nabla p_e)/(en)$  and  $\mathbf{E}_\Omega = -\nabla\Phi_\Omega = -(\nabla_\phi B_z/c)\mathbf{e}_R$ . Clearly, gravity acts on the ions directly and the electrons are coupled to them by the electric field  $\mathbf{E}_p$ .

In particular  $0 \approx \partial p_i / \partial z + enE_z - z\Omega_k^2 n m_i$  and  $\Omega(R) \approx \Omega_k(R) \equiv (GM_*/R^3)^{1/2}$ , where  $\Omega_k$  is the Keplerian frequency  $M_i$  is the ion mass,  $M_*$  the mass of the central object (e.g. black hole), and we use the coordinates  $R, \phi, z$ . Here  $p = p_e + p_i \equiv 2Tn$  and  $n = \rho/m_i$ . The parameter  $\eta_T$  representing the relative gradient of the plasma temperature to that of the density  $\eta_T = (d \ln T / dz) / (d \ln n / dz)$  is of special interest [2] as the differential rotation [3] and the vertical gradient of the plasma pressure  $dp/dz$  play an important role in the nature of the modes that are found. Clearly, polytropic profiles  $p \propto \rho^{5/3}$ , corresponds to  $\eta_T = 2/3$ . We consider the disk to be symmetric around  $z=0$  and the density profile to be represented by  $n \approx n_0(1 - z^2/H_0^2)$  where  $R = R_0$  is a reference distance from the axis of symmetry and  $H_0(R_0) \ll R_0$  (thin disk approximation). The solution of the vertical equilibrium equation is  $p(R_0, z) \approx p_0(R_0) \exp\left[-(m_i \Omega_k^2 / 4) \int_0^z dz^2 / T(z^2)\right]$  and  $H_0^2 = 4T(z=0) / (m_i \Omega_k^2)$ . The stripped disk model is a simplified initial plasma structure from which significant plasma modes and relevant collective processes can be demonstrated to emerge. Finally, we note that there is a large class of black holes that are “inactive” and for which the presence of vigorous accretion processes can be excluded.

### 3. Axisymmetric Modes

We adopt the ideal MHD approximation and note that axisymmetric normal modes [4,5] are represented in general as  $\hat{v}_\phi = \tilde{v}_\phi(R, z) \exp(-i\omega t)$  where  $\hat{v}_\phi$  is the perturbed toroidal velocity. The modes of interest are localized vertically in the sense that  $\tilde{v}_\phi(R, (z/H_0) \rightarrow \infty) \rightarrow 0$ . Given the nature of the operator that characterizes the relevant linearized equations, modes that are evanescent in the vertical direction are oscillatory in the radial direction. Therefore,  $\hat{v}_\phi = \tilde{v}_\phi(z) \exp[-ik_R(R - R_0) + \gamma_0 t]$  as in this case  $\text{Re } \omega = 0$  and  $k_R \approx k_0$  where  $k_0 \equiv \Omega_D / v_A$ ,  $\Omega_D^2 = -d\Omega^2(R)/dR$  and  $v_A^2 = B_z^2 / (4\pi n_0 m_i)$ . These modes do not have a characteristic width of localization in the radial direction over scale distances that are smaller than the macroscopic scale distances of the disk. The relevant growth rates and the vertical profiles depend in a significant way [2,6] on the temperature profile parameter  $\eta_T$ . Therefore disks surrounded by a hotter corona for which  $\eta_T \leq 0$  will sustain different modes than those for which the temperature profile is peaked on the equatorial plane and  $\eta_T > 2/3$ . The  $\eta_T - 2/3$  factor combined with  $\Omega_D$  drives a new kind of modes [2] that is “ballooning” in the vertical direction. The lowest relevant eigenfunction is  $\tilde{\xi}_z(z) = \tilde{\xi}_z^0 \exp(-z^2/2\Delta_z^2)$ , where  $\tilde{\xi}_z$  is the amplitude of the vertical plasma displacement,  $\Delta_z^2 \sim H_0/k_0$  and the corresponding growth rate is given by

$$\gamma_0^2 \simeq \frac{6}{35} \frac{v_A}{H_0} \Omega_D \left( \eta_T - \frac{2}{3} \right), \quad (3.1)$$

for  $\Omega_D^2 > k_R^2 v_A^2$ , that corresponds to limited bending of the magnetic field lines. We label this as the “Thermo-Rotational-Mode” [2]. We observe that the higher eigenfunctions remain unstable even when  $\eta_T < 2/3$  and the stabilizing factor  $2/3 - \eta_T$  becomes progressively less important as the order of the eigenfunction increases. In particular, for the next (odd) eigenfunction

$$\gamma_0^2 \simeq \frac{6}{7} \frac{v_A}{H_0} \left\{ \frac{3}{5} \Omega_D \left( \eta_T - \frac{2}{3} \right) + \frac{v_A}{H_0} \right\}, \quad (3.2)$$

and the following (even) eigenfunction remains unstable for  $\eta_T < 2/3 - 7/(2H_0 k_0)$ .

These modes are nearly “isobaric” in that  $\hat{n}/n \simeq -\hat{T}/T$  where  $\hat{n}$  is the density perturbation and  $\hat{T}$  the temperature perturbation (i.e.  $|\hat{p}| \ll |\hat{n}|$ ,  $\hat{p}$  being the pressure perturbation). Therefore, the onset of modes of this kind is expected to have a strong effect on quantities such as the mean free path  $\lambda_{mfp} \propto T^2/n$ , the collision frequency  $\nu \propto n/T^{3/2}$  and the critical runaway field  $E_R^c \propto n/T$  and could trigger the transition to regimes where the non-thermal features can be exhibited by the affected plasma. We note that the relevant density perturbations are related to  $\hat{\xi}_z$  by

$$\frac{\hat{n}}{n} \simeq -\frac{z}{H_0^2} \frac{6}{5} \left( \eta_T - \frac{2}{3} \right) \hat{\xi}_z. \quad (3.3)$$

When  $\eta_T < 2/3$  and for  $k_R^2 v_A^2 > \Omega_D^2$ , that is when the restoring effect of magnetic field line bending prevails, a class of oscillatory modes can be found [6] that are localized vertically and are weakly damped. Consequently these modes may be excited as a result of non-linear coupling with modes that are unstable, having  $\Omega_D^2 > k_R^2 v_A^2$  and the same values for  $\eta_T$ .

#### 4. Vertical Particle Transport (e.g. Wind) and Thermal Energy Convection

The considered modes can produce particle density transport [2], in the vertical direction, that is of contrary sign to that of the temperature transport and modify the density and temperature profiles in such a way as to lead  $\eta_T$  toward  $2/3$ , corresponding to a polytropic. Thus, if  $\eta_T > 2/3$  a particle inflow toward the equatorial plane is induced. When  $\eta_T < 2/3$ , including the case where  $\eta_T = 0$  or where the surface of the disk can be hotter than the interior, the particle transport is away from the equatorial plane. These arguments are based on the quasilinear analysis that gives the vertical particle flux produced by unstable modes as

$$\Gamma_{pz} = \langle \langle \hat{n} \hat{v}_z \rangle \rangle = -\frac{4}{5} \gamma_0 \langle \langle \left| \hat{\xi}_z \right|^2 \rangle \rangle \times \left[ \frac{\partial}{\partial z} n - \frac{3}{2} \frac{n}{T} \frac{\partial}{\partial z} T \right], \quad (4.1)$$

where  $\langle\langle \rangle\rangle$  indicates an average over a radial distance  $\Delta R$  such that  $1/k_R < \Delta R < R_0$ . The corresponding temperature flux is  $\langle\langle \hat{T}\hat{v}_z \rangle\rangle \simeq -\langle\langle \hat{n}\hat{v}_z \rangle\rangle T/n$ .

The outflows produced by these modes when  $\eta_T < 2/3$  can be considered as candidates to explain the origin of the particle fluxes (winds) that have been observed to emanate from disk structures such as those at the core of AGN's [7].

The transport process described by Eq. (4.1) is similar to that proposed for the theoretical explanation of the experimentally observed particle inflow in magnetically confined toroidal plasmas that is associated [8] with the outflow of electron thermal energy and is related to the ratio of the gradients of the radial electron temperature and the particle density. To illustrate this case we refer, for simplicity, to a cylindrical or plane plasma confinement configuration where the magnetic field is in the  $z$  direction. Electrostatic modes can be excited with longitudinal phase velocities  $|\omega/k_{\parallel}| < v_{the}$ ,  $v_{the}$  being the electron thermal velocity. Considering the limit where the electron longitudinal thermal conductivity is relatively large but finite it is possible to show that the radial transport velocity for the electrons is equal to and of opposite sign of that of the electron thermal energy. In particular, when  $\eta_e \equiv (d \ln T_e / dr) / (d \ln n / dr) > 2/3$ , that is a condition most commonly verified in well confined plasma experiments, the thermal energy flux is outward and the particle flux is inward. We note that the experimental evidence of the particle inflow process in well confined plasmas mentioned above has expanded over the years and by now it is well documented.

### 5. 3-D Spirals

Spirals modes whose properties depend strongly on their structure in the vertical direction are found [6] in addition to the axisymmetric modes discussed in Section 3. These tridimensional modes are localized radially over scale distances  $\Delta_R$  such that  $1/k_0 < \Delta_R < R_0$ , have low toroidal modes numbers  $m_{\phi}$ , are tightly wound, are trailing spirals [9] and co-rotate with the plasma at  $R = R_0$ . The relevant analytical representation is

$$\hat{\xi}_z \simeq \tilde{\xi}_z(z) \exp \left\{ -\left( z^2 / 2\Delta_z^2 \right) - \left( R - R_0 \right)^2 / \Delta_R^2 \right\} \exp \left\{ -im_0(\Omega_0 t - \phi) + ik_R(R - R_0) + \gamma_0 t \right\} \quad (5.1)$$

where  $k_R \simeq k_0$ ,  $\Delta_z^2 \sim c_s v_A / \Omega_k^2$ ,  $c_s^2 \sim T/m_i$ ,  $\Delta_R^2 \simeq (\Omega_k \gamma_0) / (m_{\phi} k_R d\Omega/dR) \sim (\gamma_0 v_A R_0)^{1/2} / \Omega$ ,  $m_{\phi} k_R d\Omega/dR > 0$  (trailing spiral) and  $\gamma_0$  is the same growth rate as the corresponding axisymmetric mode with same vertical profile  $\hat{\xi}_z(z)$ . Thus if  $\tilde{\xi}_z = \tilde{\xi}_z^0 = \text{const}$ ,  $\gamma_0$  is given by Eq. (3.1), while if  $\tilde{\xi}_z = \tilde{\xi}_z^0 z / \Delta_z$ ,  $\gamma_0$  is given by Eq. (3.2). Given that these modes are “contained” radially within a distance of the order of  $\Delta_R$  they may be expected to reach larger amplitudes than the corresponding axisymmetric modes.

Another class of spiral modes is found that is related to the (time) oscillatory axisymmetric modes discussed at the end of Section 3. These spiral modes are not spatially localized but oscillatory in the radial direction and are represented by

$$\hat{\xi}_z = \hat{\xi}_z^0 \exp(-z^2/2\Delta_z^2) \exp\left\{i\left[m_\phi(\phi - \Omega_0 t) + ik_R(R - R_0)\right] + i\left[\sigma_R(R - R_0)^2/2 - \delta\omega_0 t\right]\right\} \quad (5.2)$$

where  $\sigma_R = \frac{7}{3} m_\phi k_R \frac{d\Omega}{dR} \frac{1}{\delta\omega_0} \sim m_\phi \frac{k_0}{R} \frac{\Omega}{\delta\omega_0}$ , and  $\delta\omega_0$  is the frequency shift obtained from the dispersion relation of oscillatory axisymmetric mode following the same kind of derivation as that for the growth rate  $\gamma_0$  of unstable modes [6].

Modes packets (convective modes) can be constructed out of elementary oscillatory modes and a diffusion coefficient can be derived that has the following expression

$$D_{eff} = \frac{\partial}{\partial \sigma_R} \delta\omega_0 \approx -\frac{3}{7} \frac{(\delta\omega_0)^2}{m_\phi k_R (d\Omega/dR)} > 0, \quad (5.3)$$

for  $m_\phi k_R (d\Omega/dR) > 0$ , corresponding to trailing spirals. These convective modes can provide an effective means to transport energy and angular momentum (an adequate process of this kind has been long sought in view of its necessity [1] for the occurrence of accretion) as  $D_{eff} \sim (v_A R_0 / m_\phi)(v_A / c_s)$ . The detailed theory of these modes is given in Ref. [6].

## 6. Crystal Magnetic Configuration

A basic problem is that of identifying the possible equilibrium configurations of axisymmetric disk structures where self gravity and accretion can be neglected. The relevant analysis [10] has led to find a class of structures in which the currents carried by the plasma are significant and the vertical Lorentz force (“Lorentz compression”) can prevail over the vertical component of the gravitational force. The resulting “crystal magnetic configuration” consist of a periodic sequence of “field reverse configurations” produced by pairs of counter-streaming currents around the axis of symmetry of the disk structure as illustrated in Fig.1.

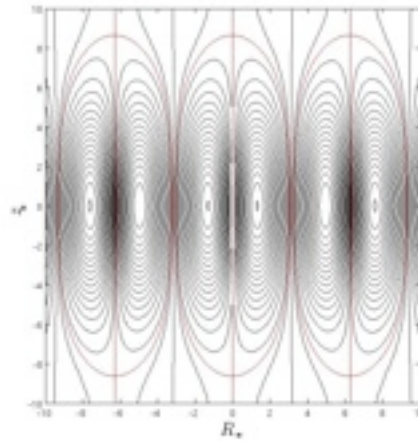


Fig.1

Moreover, when the magnetic field pressure and the thermal plasma pressure are of comparable magnitude [11], the resulting disk structure is made of a periodic series of plasma rings (Fig.2) that is reminiscent of the rings observed in the (Chandra) X-ray image of the Crab Pulsar [12].

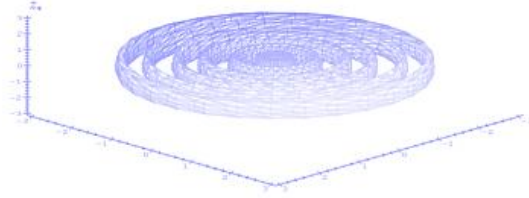


Fig.2

Recently, the non-linear theory [10,11] of crystal configurations has been extended to describe a surrounding hot region where the vertical force due to the gravity is important.

We note that the crystal configuration can evolve out of the “stripped disk” configuration as a result of the excitation of axisymmetric ballooning modes of the type described in Section 3 at first and then of a magnetic reconnection process like that described in the formation of FRCs in the laboratory. By this process a new family of closed magnetic surfaces is produced. A simple analytical representation [11] of a crystal magnetic configuration is the magnetic surface function  $\bar{\psi} = \varepsilon_B R_* + (\sin R_* + \sin 2 R_*/8) F_0(z_*^2)$  over the interval  $-\pi < R_* < \pi$  where  $R_* = (R - R_0)/\delta_R$  and  $F_0(z_*^2) \approx \exp(-z_*^2/2)$ ,  $z \equiv z/\Delta_z$ ,  $\Delta_z^2 = \delta_R H_0$  and  $\delta_R$  is the radial periodicity distance that depends on  $k_0$  and the magnetic field amplitude.

## 7. Resolution of the Accretion Paradox

We refer to a “stripped disk” where, under strictly stationary conditions ( $\partial \mathbf{A}/\partial t = 0$ ) an accretion velocity  $V_R$  is present and  $E_\phi = 0$ . Considering the electron momentum conservation equation,  $\mathbf{E} + \mathbf{V} \times \mathbf{B}/c = \eta \mathbf{J}$  where  $\eta$  is the electrical resistivity and  $\mathbf{J} = (c/4\pi) \nabla \times \mathbf{B}$ ,

$$V_R \approx D_m \frac{1}{B_z} \left( \frac{\partial}{\partial R} B_z - \frac{\partial}{\partial z} B_R \right), \quad (7.1)$$

where  $D_m = \eta c^2/(4\pi)$  is the magnetic diffusion coefficient. Therefore, given the large scale distances involved, the values of  $V_R$  that are compatible with a classical resistivity  $\eta$  are not significant  $\left[ D_m^{cl} \approx 1.3 \times 10^{-3} (10 \text{ keV}/T_e)^{3/2} (\ln \Lambda/15) \text{ cm}^2/\text{sec} \right]$ . Then, the presence of a large anomalous resistivity over large scales distances would need to be postulated (see, for instance Ref. [13]) but a microscopic process capable of producing it, such as a current density driven instability, is most difficult to envision. Moreover, if that were the case all modes that rely on

the hyperconductivity condition,  $\mathbf{E} + \mathbf{V} \times \mathbf{B}/c = 0$ , such as those described earlier, would have to be excluded.

The resolution that is proposed for the accretion paradox is based on considering a disk structure characterized by a “crystal” magnetic field configuration (Fig. 1) and assuming that the hyperconductivity condition, implying that

$$\mathbf{V} = \alpha_v \mathbf{B} + \Omega(\psi) R \mathbf{e}_\phi, \quad (7.2)$$

constrains the plasma flow velocity  $\mathbf{V}$  over nearly all of the disk structure while a process for the corresponding outward flux of angular momentum [1, 14] is present. Clearly,  $\mathbf{B}_p \cdot \nabla \psi = 0$  and  $\alpha_v = \chi(\psi)/\rho$  as  $\nabla \cdot (\rho \mathbf{v}) = 0$ . In particular, the plasma is considered to flow along channels following the separatrices as indicated in Fig. 3

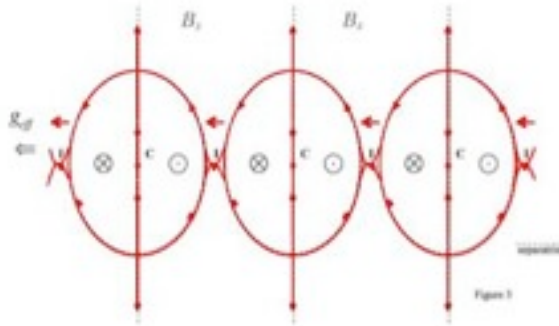


Fig.3

and we propose that:

i. tridimensional plasma modes involving small scale distances are excited near the equatorial plane in an intermediate region (I) around successive separatrices. ii. these ballooning modes [15] are associated with the effects of finite electrical resistivity, letting the plasma slip through the magnetic fields lines. iii. a modulated (in time) accretion process can be produced as a result of the periodic onset of these modes. The relevant instability [16] is driven by the combined effect of the density gradient resulting from the plasma accumulation and the difference between the radial components of the gravity and of the centrifugal force. We may argue that accretion is likely to occur in a magnetized plasma only if a coherent structure like that described in Figs. 1 and 3 can be maintained. In fact, this circumstance may justify the observation that inactive black holes are more frequent in older galaxies that should have suffered collisions and not have the conditions necessary to sustain coherent structures.

Moreover, we suggest that the accreting plasma structure surrounding a compact object should be characterized by two components, the inner component being the Lorentz compressed structure with the magnetic crystal configuration and the outer (enveloping) component being a hotter low density disk that is confined vertically by the gravitational force. Recent experimental observations appear to confirm this kind of theoretical conclusion.

Then we may argue that the vertical expulsion of matter that is observed as jets emitted in the vicinity of a black hole is modulated syntonically with the accretion process. Therefore, we may



envision that jets are formed by the ejection of the innermost rings [17] in the vicinity of a given black hole and consist of a vertical sequence of stable “smoke rings”.

We consider the plasma regions surrounding a black hole to consist of a Relaxation Region whose inner edge is at the radius of the relevant marginally stable orbit where the plasma density vanishes. This region is assumed to be strongly turbulent and surrounded by a weakly turbulent region whose inner edge is close to that of the radius of the orbital frequencies corresponding to those of the High Frequency Quasiperiodic Oscillations (HFQPOs) discussed in Section 8. These oscillations are assumed to be excited when the rings are dissipated before reaching the outer edge of the Relaxation Region and a “conventional” disk structure is formed. Accordingly, the experimental observations show that emission of jets and HFQPOs do not co-exist for a given object [18].

### 8. Plasma QPOs Connected to 3D Collective Modes

The excitation of 3D radially localized spirals of the type described earlier has been proposed [19] to explain the observation of High Frequency Quasi Periodic Oscillations (HFQPOs) of non-thermal X-ray emission from compact galactic objects. These modes co-rotate with the plasma in the vicinity of a black hole at the edge of the Relaxation Region where their growth rate is maximum, compatibly with the necessary conditions for their excitation. The lowest (toroidal) harmonics  $m_\phi = 2, 3$  are considered to prevail. The periodic radiation emission of QPOs is clearly identified as the result of the plasma density and temperature variations due to these modes whose excitation factors  $d\Omega/dR$  and  $\partial p/\partial z$  are well understood theoretically. The radiation coming from the spiral source has been computed [20] by an appropriate three-dimensional ray tracing code described in Ref. [21]. The spiral theory developed in the Newtonian limit in Ref. [6] has been extended [20] to include relevant general relativistic corrections by adopting appropriate effective gravitational potentials, and in particular the Paczynsky-Wiita potential for a Schwarzschild black hole, that are adequate for the applications of the theory.

An important issue, in this context, is that of envisioning plausible parameters for the characteristic plasma structures that can exist in the vicinity of black holes, typically at distances of the order of  $10 R_s$  where  $R_s$  is the Schwarzschild radius [ $R_s = GM_*/(2c^2)$ ,  $M_*$  = black hole mass]. Thus a 2-component plasma model is adopted for the emitting region consisting of a “hot”, relatively low density component and “low” temperature high density component. The energy transferred from the “hot” component to the “cold” component compensates for the energy loss by bremsstrahlung from the latter component. The height of the disk at the radius where the plasma modes are localized is evaluated from the temperature of the hot component and is related to the radius of the marginally stable orbit for a given black hole. From this an expression for the QPOs frequencies in terms of basic physical quantities can be obtained [20].

That is  $\Omega_0 \approx [(2m_\phi c)/(\alpha_* R_s)] [m_e/(2m_i)]^{1/2}$  where  $m_e$  is the electron mass,  $\alpha_* = \alpha_{MS}/(\bar{T}_h)^{1/2}$ ,  $\alpha_{MS}$  is finite coefficient that depends on the lowest value of the radius  $R_{MS}$  of the marginally stable orbit and  $\bar{T}_h = T_i/(m_e c^2)$  is an uncertainty parameter for the hot ion temperature. The scaling of the frequencies in terms of  $M_*$ , their numerical values and the characteristic 3:2 ratio are consistent with the relevant experimental observations. A considerably colder plasma, with higher

densities than that of the “cold” component, is envisioned to surround the region from which the QPO radiation emission occurs.

### **Acknowledgments**

It is a pleasure to thank E. Gallo for her comments and P. Rebusco for sharing her knowledge of QPOs. This work was sponsored in part by the U.S. Department of Energy.

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