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Electromagnetic Detection of a Perfect Invisibility Cloak

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A perfect invisibility cloak is commonly believed to be undetectable from electromagnetic (EM) detection because it is equivalent to a curved but empty EM space created from coordinate transformation. Based on the intrinsic asymmetry of coordinate transformation applied to motions of photons and charges, we propose a method to detect this curved EM space by shooting a fast-moving charged particle through it. A broadband radiation generated in this process makes a cloak visible. Our method is the only known EM mechanism so far to detect an ideal perfect cloak (curved EM space) within its working band.

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Physical laws that describe nature govern the behavior of an object in space and time. For example, two sets of fundamental physical laws, Newton's laws of motion and Maxwell's equations, are both functions of space and time, but apply to different physical objects—one commonly applies to concrete objects with inertia, while the other applies to electromagnetic (EM) waves, or photons with zero stationary mass. In general, the space where these physical laws apply is flat if gravity is ignored. However, analogous to general relativity where space and time are curved, the new blooming field of transformation optics shows that the space for light can also be bent in an almost arbitrary way [1,2] by transforming an original EM space [3,4] in the absence of gravity.

The most attractive prediction of transformation optics might be the possibility of perfect invisibility cloaking [3–12], where the cloak mimics the coordinate transformation, squeezing the originally flat EM space and guiding light smoothly along curved trajectories around a hidden object. Perfect cloaking is believed to be undetectable at its working frequency because a perfect cloak is equivalent to a curved but empty EM space. Although the first cloak models can achieve invisibility only in a very narrow frequency band [3,13], efforts to extend the bandwidth are currently proceeding [9–12,14]. While it is well known that nothing can hide from a pulse detection covering the whole frequency spectrum [15], the possibility of realizing invisibility cloaking within a nonzero bandwidth has great significance on both theoretical and practical levels and has attracted a lot of attention from scientific community. However, the other side of this issue—how to detect a truly ideal perfect invisibility cloak, or a curved EM space, within its working band—has not been solved in previous studies.

It is the purpose of this Letter to develop a method to detect a perfect invisibility cloak electromagnetically, no matter what frequency band it works in. Our results demonstrate the asymmetry of coordinate transformation: Though this transformation of EM space can completely control the motion of photons, it is not directly applicable

to the motion of charges. The consequence is that the curved EM space and the flat mechanical space overlap at the cloak's location. Thus a fast-moving charged particle moving in both spaces can perceive the bending of EM space because of the reference function of the flat mechanical space. A broadband radiation will be generated in this process, making the cloak visible. The whole radiation process is explained by comparing the motion of the charged particle in both EM and mechanical spaces. Detailed calculation is based on an extension of Frank and Tamm's theory on Čerenkov radiation, together with dyadic Green functions.

To explain our detection mechanism, we need to first recall the theoretical foundation behind transformation optics, which is the symmetry [16] of Maxwell's equations: the coordinate transformation does not change the form of Maxwell's equations but only changes the constitutive parameters and field values. It can be argued that a photon, which perceives only the EM space, does not know that the EM space is curved since there is no other reference for it to make judgement. However, one should notice that when the space for Maxwell's equations (EM space) is squeezed, the space for Newton's laws of motion (mechanical space) is still flat. Before the coordinate transformation, they share the same physical space, but after transformation, they are separated. In other words, at the same physical location, the curved EM space and the flat mechanical space overlap. If one chooses the flat mechanical space as the reference, the bending of EM space is perceivable.

The only bridge from the flat mechanical space to the curved EM space is the charge attached to a particle. A charged particle possesses both mechanical and EM properties, enabling it to perceive both spaces simultaneously. Take a spherical cloak as an example, as shown in Fig. 1(a), whose constitutive parameters inside the cloak layer satisfy the relations proposed in Ref. [3]. The spherical cloak is created by squeezing the virtual EM space such that a “hole” of $r < R_1$ in the physical space is generated. In the physical space, light rays are bent around the concealed region $r < R_1$ to make the cloak “invisible.” The charge,

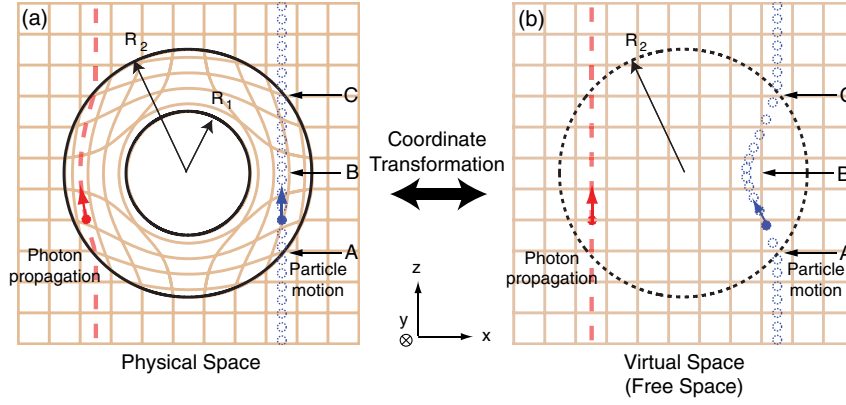


FIG. 1 (color online). Trajectory of a fast-moving charged particle inside the cloak compared with the trajectory of a photon in (a) the physical space and (b) the virtual space. B is the midpoint of the trajectory. Background is the mesh of EM space. Mechanical space is not shown.

which goes together with the mass, moves uniformly along a straight line in the physical space (mechanical space) due to its inertia. However, if we treat the same problem in the virtual EM space as shown in Fig. 1(b), the particle's trajectory is bent, meaning it moves nonuniformly in the transformed EM space. We can also see that the velocity of the particle is larger close to the outer boundary and smaller in the middle. This motion generates radiation.

Historically, the radiation generated by a fast-moving charged particle such as Čerenkov radiation [17] was mostly studied for simple geometries such as plates or layers, and for simple anisotropy such as homogenous anisotropy. Few studies considered both anisotropy and inhomogeneity. Here, we extend Frank and Tamm's theory [17] to an invisibility cloak that is both inhomogeneous and anisotropic with a fast-moving charged particle going through it uniformly. Detailed calculation is provided in [18], and its summary is given below.

Suppose the charge of the particle is q and the velocity is v in the \hat{z} direction with $x = x_0$ and $y = y_0$. We can write down its current density as follows

$$\vec{J}(\vec{r}, t) = \hat{z} q v \delta(x - x_0) \delta(y - y_0) \frac{1}{\sigma \sqrt{2\pi}} e^{-(z-vt)^2/(2\sigma^2)} \quad (1)$$

where in \hat{z} direction the profile is a Gaussian function. It is similar to the case in which many electrons are being shot along the same line and are moving together. In practice, this kind of electron bunching has been used to study Čerenkov radiation [19]. When $\sigma = 0$, the Gaussian function returns to a delta function.

We first Fourier transform the time-varying current into frequency domain, as follows

$$\begin{aligned} \vec{J}(\vec{r}) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \vec{J}(\vec{r}, t) e^{i\omega t} dt \\ &= \hat{z} \frac{q}{2\pi} \delta(x - x_0) \delta(y - y_0) e^{-\sigma^2 \omega^2 / 2v^2} e^{i\omega z/v} \end{aligned} \quad (2)$$

where we decompose the original time-varying current into many time-harmonic current line sources, each having its own frequency. For each time-harmonic current line

source, the field distribution can be solved by virtue of the dyadic Green function.

From the transformation point of view, the current and charge densities need to be rescaled in the transformation [2]. Therefore, the dyadic Green functions take different forms accordingly. Here, we derive the dyadic Green functions in the physical space directly, whose solution can be extended to consider both loss [6] and dispersion [13].

The expression for the scalar potentials inside a linearly transformed spherical cloak has been derived in our previous publication [6], and is $\Psi = k_t(r - R_1)j_n(k_t(r - R_1))P_n^m(\cos\theta)e^{im\phi}$, where $k_t = k_0 R_2/(R_2 - R_1)$ and j_n is the spherical Bessel function of the n th order. By introducing an auxiliary tensor $\vec{T} = \hat{r}\hat{r}(r - R_1)^2/r^2 + \hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi}$, we define the vector eigenwaves inside the cloak,

$$\vec{M}_{n,m} = \vec{\nabla} \times \hat{r}\Psi \quad (3)$$

$$\vec{N}_{n,m} = \frac{1}{k_t} \vec{\nabla} \times (\vec{T} \cdot \vec{M}_{n,m}). \quad (4)$$

The electric dyadic Green function $\vec{\bar{G}}_E^{11}$ can be obtained as follows,

$$\begin{aligned} \vec{\bar{G}}_E^{11} &= -\delta(\vec{r} - \vec{r}') \vec{T}^{\prime-1} + \frac{i\pi}{2k_t} \vec{T}^{-1} \sum_{n,m} \frac{(2n+1)(-1)^m}{2\pi^2 n(n+1)} \\ &\times \begin{cases} (\vec{M}_{n,m}^{(1)} \vec{M}_{n,-m}' + \vec{N}_{n,m}^{(1)} \vec{N}_{n,-m}') \cdot \vec{T}^{\prime-1}, & r > r' \\ (\vec{M}_{n,m} \vec{M}_{n,-m}'^{(1)} + \vec{N}_{n,m} \vec{N}_{n,-m}'^{(1)}) \cdot \vec{T}^{\prime-1}, & r < r' \end{cases} \end{aligned} \quad (5)$$

where the superscript $(\cdot)'$ means replacing (r, θ, ϕ) by (r', θ', ϕ') (the source's location) and $(\cdot)^{(1)}$ means replacing the spherical Bessel function j_n by the spherical Hankel function of the first kind $h_n^{(1)}$. This derivation utilized the Ohm-Rayleigh method [20].

The transmitted field in the region of $r > R_2$ can be written by the dyadic Green function $\vec{\bar{G}}_E^{12}$, which is

$$\begin{aligned} \vec{\bar{G}}_E^{12} &= \frac{i\pi}{2k_t} \sum_{n,m} \frac{(2n+1)(-1)^m}{2\pi^2 n(n+1)} \cdot (\vec{M}_{n,m}^{\text{out}} \vec{M}_{n,-m}' \\ &+ \vec{N}_{n,m}^{\text{out}} \vec{N}_{n,-m}') \cdot \vec{T}^{\prime-1} \end{aligned} \quad (6)$$

where \vec{M}^{out} and \vec{N}^{out} are directly obtained from \vec{M} and \vec{N} by letting $R_1 = 0$.

The electric field generated by the current source inside the cloak ($R_1 < r' < R_2$) can thus be written as

$$\vec{E}(\vec{r}) = \begin{cases} i\omega\mu_t \iiint \vec{G}_E^{12} \cdot \vec{J}(r') dV', & r > R_2 \\ i\omega\mu_t \iiint \vec{G}_E^{11} \cdot \vec{J}(r') dV', & R_1 < r < R_2 \\ 0, & r < R_1 \end{cases} \quad (7)$$

Regarding the current source in free space outside the cloak ($r' > R_2$), we treat it as the difference of the total current source $\vec{J}(\vec{r})$ and an imaginary part located inside the cloak. The electric field from the former [total current source $\vec{J}(\vec{r})$] in absence of the cloak can be calculated analytically by shifting the z axis to the line at $x = x_0$ and $y = y_0$ and establishing a new local cylindrical coordinate system (ρ_p, ϕ_p, z) . The electric field from the latter (the imaginary current source in free space but located inside the cloak, $R_1 < r' < R_2$) can be obtained by using the well-known dyadic Green function in free space [17].

Then the total field $\vec{E}_{\text{tot}}(\vec{r})$ generated by the complete time-harmonic line source is obtained by combining all the source terms' contributions. After inverse Fourier transformation, we can obtain the field in time domain $\vec{E}_{\text{tot}}(\vec{r}, t)$.

Now we are able to discuss the physical process. The spherical cloak we choose has inner radius $R_1 = 1 \mu\text{m}$ and outer radius $R_2 = 2 \mu\text{m}$. The particle's velocity is fixed at $v = 0.9c$. The parameter σ in the Gaussian function of the current description is chosen as $\sigma = 0.05 \mu\text{m}$. The total charge q is set to be equivalent to 1000 electrons.

We first plot the radiation process in the physical space as shown in Fig. 2. We can divide the radiation process into two stages, one for $t < 0$ and the other for $t > 0$, if we specify that when $t = 0$, the particle is located in the middle of its trajectory, i.e., at $((R_1 + R_2)/2, 0, 0)$ (point B in Fig. 1). We can see that in the beginning, at $t = -7$ fs, the charged particle is approaching the cloak but is still outside. There is no radiation generated. As the particle enters into the cloak, some radiation is generated. At $t = 0$ fs, when the charged particle is located halfway through the cloak, the first wave front has already passed the particle and is propagating away, meaning that the first stage of radiation generation has finished. At $t = 3$ fs, the radiation generated in the first stage has completely separated from the charged particle. However, the charged particle starts to generate more radiation, meaning the second stage of radiation generation has started. At $t = 7$ fs, the charged particle has come out from the cloak, so the second stage of radiation generation has finished. It is seen that at $t = 7$ fs, the radiation from the second stage is composed of two particular shapes. One has the rough shape of a cone while the other has the rough shape of a sphere. At $t = 11$ fs, all radiation propagates away from the cloak and is separated from the charged particle.

We can explain the above process by comparing the motion in the physical space and in the virtual space. When the particle enters the cloak, its velocity in the virtual

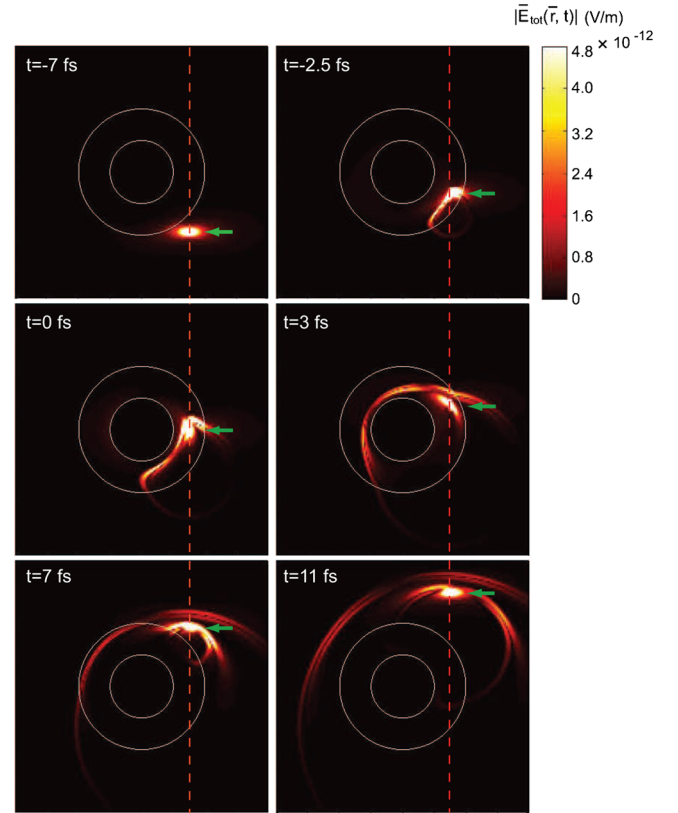


FIG. 2 (color online). $|E_{\text{tot}}(\vec{r}, t)|$ during the radiation from a charged particle going through a spherical invisibility cloak. The dotted line represents the trajectory of the particle. The small arrow indicates the exact position of the particle's center along its trajectory. The inner radius and outer radius of the cloak are 1 and 2 μm , respectively. The net charge corresponds to 1000 electrons. $v = 0.9c$.

space in Fig. 1(b) has an abrupt change at point A, which generates some radiation in virtual space. This radiation corresponds to the transition radiation that occurs at the incident point at the outer boundary of the cloak in physical space [point A in Fig. 1(a)]. Similarly, when the particle comes out at the other side of the cloak, another abrupt velocity change occurs in the virtual space [point C in Fig. 1(b)], which corresponds to the transition radiation emitted at the outer boundary of the cloak in physical space [point C in Fig. 1(a)]. Throughout its motion inside the cloak in the physical space [segment AC in Fig. 1(a)], the particle generates transition radiation due to the inhomogeneity and anisotropy of the cloak. This particle's motion in the virtual space [curved segment ABC in Fig. 1(b)] can be divided into two parts—motion along AB and motion along BC, corresponding to the two radiation stages. While moving, the particle's velocity keeps changing direction as well as magnitude, which gives rise to Bremsstrahlung and synchrotron radiation. Another kind of radiation that is generated in the regions close to the outer boundary of the cloak [close to A or C in both Figs. 1(a) and 1(b)], is Čerenkov radiation when the velocity of the particle is larger than the speed of light.

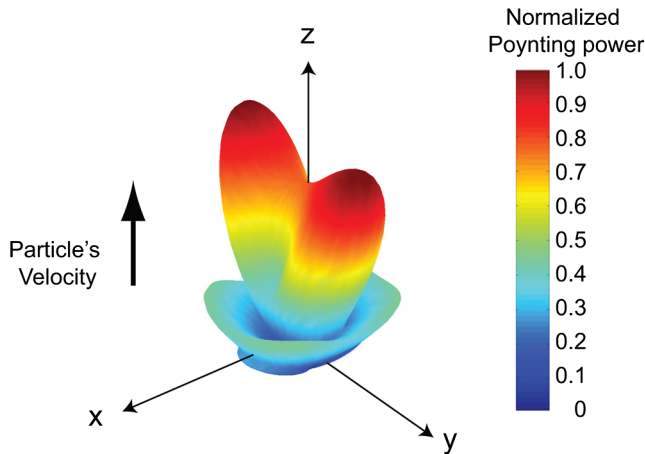


FIG. 3 (color online). Far field spatial radiation pattern at 500 THz from the fast-moving charged particle through the invisibility cloak. The inner radius and outer radius of the cloak are 1 and 2 μm , respectively.

The shape of the radiation generated can be explained also. When the particle enters the cloak in the virtual space [point A in Fig. 1(b)], the radiation is due to the abrupt velocity change at point A and the particle's motion within the cloak near point A. The corresponding radiation in the physical space is composed of transition radiation from the particle impinging on the outer boundary and transition radiation from the particle's motion inside the inhomogeneous medium near the outer boundary. When the particle moves close to point B in the virtual space [Fig. 1(b)], its velocity in the virtual space slows down. Thus, the light from the first radiation stage has a chance to catch up and overtake the particle. After the particle leaves point B, it is accelerating in the virtual space. When its velocity gets higher, it is able to generate obvious radiation, corresponding to the radiation in the second stage. The radiation generated along the segment BC in virtual space is similar to that calculated in previous studies on accelerating charges [21]. It is interesting to see that when the particle comes out, the radiation from the second stage exhibits two different shapes. The rough shape of a cone is the transmission of radiation generated along the segment BC in the virtual space before hitting point C. The spherelike shape is the radiation due to the abrupt velocity change at point C.

So far, we have shown the total field at different times. We can also plot the radiation pattern at a particular frequency. For example, at 500 THz or a wavelength of 600 nm, the far field radiation is shown in Fig. 3. It can be seen that the radiation is mainly in the forward direction. The two lobes are due to the bending in the virtual space on the xz plane containing the trajectory of the particle. The total radiated energy for the example is 2.89×10^{-14} J. Because of the broadband property of the radiation, a perfect cloak working within any frequency band can always be detected by shooting fast-moving charged particles through the cloak.

In conclusion, we have demonstrated the broadband radiation process of a fast-moving charged particle going through a perfect invisibility cloak that is equivalent to a curved EM space. Dyadic Green function is derived and used in the calculation. The radiation is explained by comparing the motion of the charge in both physical and virtual spaces. We believe this is the only known mechanism thus far to detect a perfect invisibility cloak or a curved EM space within its working band electromagnetically.

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