MAGNETOHYDRODYNAMIC ANALYSIS OF THE STABILITY OF THE PLASMAPAUSE

by

ADOLFO FIGUEROA VIÑAS

B.S. University of Puerto Rico (1970)

M.S. University of Puerto Rico (1973)

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Signature of Author ______________________________
Department of Earth and Planetary Sciences

Certified by ______________________________
Theodore Madden
Thesis Supervisor

Accepted by ______________________________

To My Wife, Maria Elena

It is not knowledge, but the act of learning

not possession, but the act of getting there

which grants the greatest enjoyment

Carl Friedrich Gauss

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ABSTRACT

We have presented a unified MHD treatment of the Kelvin-Helmholtz and the interchange (or "ballooning") instabilities in what we called the shear flow-ballooning instability and we have applied these results to the stability analysis of the plasmapause. This approach is analogous to the investigation of the stability of hydrodynamic flows and for this reason we have adopted the hydrodynamic terminology, but adding the term magnetic to its vocabulary. In the hydrodynamic problem the gravitational effects exert stabilizing influences on the flows. Its influence is represented by the quantity called the Brunt-Väisälä frequency (Ω_{BV}). In the hydromagnetic context, the "magnetic buoyancy" due to curvature of the field lines behaves very much like the gravitational buoyancy and it is the principal term in the expression for the magnetic Brunt-Väisälä frequency in the magnetospheric environment. It is also found that the stability criterion depend on the ratio of the gravitational, thermal, rotational and magnetic effects to the shear flow influence and this condition is called the magnetic Richardson number ($Ri = \Omega^2_{BV}(r)/(\partial v_o/\partial r)^2$). The unstable waves in the hydromagnetic case are found to be drift waves. These waves are very much analogous to gravity waves, its hydrodynamic counterpart, and similarly the unstable modes are trapped in the shear zone.

This treatment was applied, in particular, to the analysis of the stability of the plasmapause to investigate the erosion process that occurs during periods of enhanced magnetic activity. The injection of hot ring current particles across the plasmapause is a destabilizing mechanism for which a convective instability could be established. However, using real density, pressure and magnetic field data we find that it is unlikely for this region to become convectively unstable. Estimates of the magnetic Richardson number show that shear instability can arise for velocity gradients which are confined to a zone of 10 to 40 km thick. However, at present there is no data to estimate the velocity (or electric field) gradient

across the plasmapause. We can, however, predict the wave parameters associated with the drift waves that would be excited if there were an instability. These estimates give wave periods in the range of 1 to 4 minutes with wavelengths of 65 to 365 km. This mechanism may explain the plasma erosion across the plasmapause but two important problems, which deserve more experimental and theoretical investigation, emerge from the study of this instability. A more complete analysis requires the explanation of the penetration of the electric field and the steepness of its gradient across the plasmapause.

Finally, we should mention that this treatment of the shear flow-ballooning instability could be applied to other shear flow boundaries in the space environment, as for example, the magnetopause and the shear flow boundaries in the Jovian magnetosphere.

Thesis Supervisor: Prof. Theodore R. Madden
Professor of Geophysics

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CHAPTER I

INTRODUCTION TO MAGNETOHYDRODYNAMIC PLASMA INSTABILITIES

I.1 Introduction

The objective of this work is to investigate the stability of the plasmapause. The stability analysis of this region, which includes the study of the distribution and the dynamics of the thermal plasma, are of fundamental importance for understanding many magnetospheric processes, e.g. large scale plasma motions. The currently accepted definition of the plasmapause is the region between the high density thermal plasma of the inner Earth's magnetosphere and the low density hot plasma of the outer magnetosphere, e.g. see Figure 1.1. The region enclosed by the plasmapause is called the plasmasphere and it contains the cold plasma (e.g. particles with energies of 0.1 to 1 ev) in corotation with the Earth (Carpenter, 1963, 1966; Angerami et al., 1966; Carpenter et al., 1973; Mozer, 1973 and Vickers, 1976). It is generally accepted that the origin of the plasma inside the plasmasphere is mainly due to the outflow of ionospheric particles. Park (1970) estimated the outflow rate of ionospheric particles into the plasmasphere as 3×10^8 particles/cm^2-sec at an altitude of 1000 km during the daytime when the ionosphere is heated by the Sun. During the night, an inward flow of particles from the plasmasphere into the ionosphere at a rate of 1.5×10^8 particles/cm^2-sec has also been observed. The difference between this outward and inward flow of particles implies a net excess of plasma particles in the plasmasphere. It has also been observed that the position of the plasmapause region is influenced by the level of magnetic activity (e.g. Chappell, et al., 1970a,b). The plasmapause shows a general decrease in radius (i.e. an inward motion) with increasing magnetic activity, e.g. see Figure 1.2. Thus, the bulk of the thermal plasma in the plasmasphere which is constantly changing in size and shape with magnetic activity present important clues to large scale magnetospheric dynamics. This is a clear indication that the solar wind interaction with the geomagnetic field plays an important role in the behavior of the plasmapause (e.g. Nishida, 1966; Brice, 1967; Chappell et al., 1972 and Frank, 1971). This motion of the plasmapause also suggests the unstable nature of the system.

Before the discovery of the plasmapause (initially called the "knee") by Carpenter (1963) it was already known that magnetospheric electric field distributions play an important role in the behavior of the plasma in the magnetosphere (Axford and Hines, 1961; Dungey, 1961). Axford and Hines (1961) proposed a model based on the effect of an induced electric field on the magnetospheric thermal plasma. In such a model, due to the viscous interaction of the solar wind at the magnetospheric boundary, i.e. the magnetopause, and the geomagnetic field; an electric field is induced across the magnetosphere. The induced electric field is directed from dawn to dusk in the midnight equatorial plane causing the sunward convection of the thermal plasma in the interior of the magnetosphere. The electric field induced by the solar wind gives the low energy particles a drift velocity independent of their sign. However, the effect of this induced electric field on the high energy particles in the magnetosphere is different. The convective velocity $\underline{V}$ produced by the induced electric field is given by

$$\underline{V} = \frac{\underline{E} \times \underline{B}}{B^2} \tag{1.1}$$

where $\underline{E}$ is the induced electric field and $\underline{B}$ is the geomagnetic field. Axford and Hines argue that this convective flow is modified by the rotation of the Earth since the neutral atmosphere, the ionosphere and the plasmasphere corotate with it (see Figure 1.3). The magnitude of the convection electric field influences the precise position and shape of the plasmasphere (see Figure 1.4), and a change in this field is reflected in the dynamics of the plasmapause. Therefore, continual observations of the plasmasphere-plasmapause morphology give an excellent indirect measurement of variations which take place in the convection field and also in the magnetosphere.

Figures 1.3 and 1.4 show a plot of different local time sectors of the plasmapause as a function of the radial distance from the Earth. The solid line in Figure 1.3 and the outer edge of the shaded area in Figure 1.4 shows the average plasmapause position. Figure 1.4 also shows the equatorial sections of the equipotential lines which corresponds to McIlwain's (1972, 1974) electric field distribution inside and outside the plasmasphere. Outside the plasmasphere the convection pattern leads to a loss of thermal plasma at the magnetopause . Inside the plasmasphere corotation effects are most important. The local time sector 15:00 to 22:00 corresponds to the bulge region of the plasmasphere. This bulge is due to the difference between the convective electric field and the corotation electric field. Plasma elements which lie inside the solid line in Figure 1.3, will rotate with an angular velocity smaller than the Earth's angular velocity. This is due to the penetration of the convective electric field deep inside the plasmasphere, therefore reducing the corotation electric field. Nishida, et al. (1972) has suggested that this extension of the convective electric field inside the plasmasphere is probably related to the unsteady character of the convection.

In spite of the influence of these electric fields; the low energy ambient plasma population of the plasmasphere is not transported to outer regions where the plasma loss is possible, by virtue of the dominance of the corotational motion over external electric fields. Thus, the position of the plasmapause, namely the sharp density gradient of the thermal plasma is determined by the balance between the corotational and the convective electric fields.

An interesting aspect implied by Axford and Hines' model of the convective and corotational electric fields is that the flow velocity $\underline{V}$ given in equation (1.1) changes rapidly with position in space across the plasmapause. Following this model we infer an apparent discontinuity in the streaming velocity between the inner plasma, i.e. inside the plasmasphere, and the outer plasma. We deduce also from their model that the "boundary" between these two regions, which is the plasmapause, may be susceptible to a form of magnetohydrodynamic instability such as the shear-flow or Kelvin-Helmholtz instability. This is an important geophysical phenomenon which is crucial in the investigation of the plasmaspheric and magnetospheric dynamics. Some authors, e.g. Southwood, 1968 and Hasegawa, 1972;

considered this instability to be responsible for the excitation of low frequency micropulsations such as in the Pc 2 to Pc 5 frequency range (i.e. from 5 to 600 seconds). However most of the micropulsations have very long wavelengths and are mostly related to compression of the whole magnetosphere by the solar wind.

The Kelvin-Helmholtz is a well-known phenomenon in fluid mechanics that occurs when two fluids are in contact, streaming relative to each other. This instability is the mechanism by which small perturbations of the boundary between fluids having tangential motion with respect to each other can be made to grow exponentially with time. The source of this instability clearly lies in the kinetic energy of the relative motion of the fluids. In other words, the energy of the relative streaming is converted to wave motion of the perturbation, and eventually turbulence and mixing results. Therefore, in order to understand the general topology and dynamics of the plasmasphere we must incorporate the effects of the rapid variation on the flow velocity (or the convective electric field) in the stability analysis of the plasmapause.

A different kind of motion of the thermal plasma in the magnetosphere was proposed by Gold (1959). Gold studied the conditions determining the dynamical behavior of the plasma by means of the ideal magnetohydrodynamic approximation which considers the plasma as a perfectly conducting fluid. Under this representation the magnetic field and the plasma fluid are "frozen" together through the motion of the plasma. Gold suggested the existence of a class of motions that will not change or overcome any magnetic force. He proposed the adiabatic interchange motion of magnetic flux tubes containing the plasma, as a consequence of the insulating layer represented by the atmosphere which decouples the magnetospheric field lines from the Earth. Such motion would be unstable if the interchange of magnetic flux tubes causes a decrease in the total energy of the system. In particular, Gold pointed out that magnetic tubes of force convect in the magnetosphere in a manner dependent on the energy density of the plasma contained by the tubes. If the energy density of the plasma diminishes at a particular rate (e.g. slower than the adiabatic gradient) with increasing radial distance, the system will be stable against convection or interchange. If the energy density gradient of the plasma in the tubes is increased (e.g. faster than the adiabatic gradient), the tube will symmetrically convect upwards until the energy density gradient required for stability is reached (see Figure 1.5). The instability criterion derived by Gold for the thermal plasma confined in a dipole magnetic field is given by

$$- \frac{r}{P_o} \frac{\partial P_o}{\partial r} > 4\gamma \qquad (1.2)$$

where γ is the adiabatic constant of expansion (i.e. $\gamma = 5/3$), P_o is the thermal energy density and r is the radial distance. This expression shows that if the thermal energy density of the magnetic tubes is steeper than $r^{-4\gamma}$, i.e. the adiabatic gradient, the tubes will be unstable and the interchange occurs spontaneously.

An important observation of this type of motion is that it gives a mechanism by which erosion of plasma particles across the magnetic field lines can be achieved. That is, this type of motion could allow the transport of plasma from the plasmasphere into the magnetosphere. It has been observed by Smith,et.al.(1973) that during magnetically active times the hot particles of the toroidal ring current that circulates around the outer regions of the plasmapause can penetrate inside the plasmasphere increasing the thermal energy density of this system (see Figures 1.6 to 1.7). This process could lead to the onset of the convective or interchange instability described by Gold. Cladis (1968) suggested that this instability may be responsible for the P_i 2 micropulsation which corresponds to periods of about 40 to 150 seconds. It has also been suggested, Hasegawa (1971, 1972), that this instability could give rise to drift waves associated with spatial non-uniformity inherent in a plasma. In order to complete the general picture of the dynamics of the plasmapause, the effect of the injection of hot particles of the ring current must be considered.

We have mentioned two different approaches that have been used independently to study the motions or instabilities of the plasmapause. One approach has been to study the dynamic or shear instability as a Kelvin-Helmholtz instability problem (Fejer, 1964; Southwood, 1968; Laster, 1970). These results are disappointing as they always lead to instability at short wavelengths. The other approach has been to consider the interchange or convective instability problem (Gold, 1959; Sonnerup and Laird, 1963; Richmond, 1973; Lemaire, 1974). This approach produces a much more meaningful picture of the actual behavior of the plasmapause, but is incomplete as the shear flow is ignored.

It appears that the approaches followed by many other investigators do not account for the interaction between the instabilities. These approaches have resulted in the neglect of the Kelvin-Helmholtz instability on the convective instability and vice versa. On the contrary, there are extensive indications that both phenomena play an important role in the stability of a system (Pierce, 1967; Claerbout, 1967; Madden _et al._, 1968, Bretherton, 1966; and Booker et al., 1967).

An analogous problem is found in the atmosphere (Bretherton, 1966, 1969, Booker et al., 1967). Ample evidence of stable vertical wind shears point out the inadequacy of the Kelvin-Helmholtz analysis. For vertical wind shears such as we find in the jet streams in the atmosphere, the atmospheric buoyancy, i.e., interchange or convective, stability plays an important role in countering the Kelvin-Helmholtz instability. When gravitational terms are included, a stable regime exists for conditions specified by a factor known as the Richardson number R_i. This number depends on the ratio of buoyancy-free oscillation frequency, i.e., Brunt-Väisälä frequency Ω_{BV}, and the vertical wind shear as follows:

$$R_i = \frac{\Omega^2_{BV}(z)}{\left(\frac{\partial V_{0x}}{\partial z}\right)^2} \quad , \quad \Omega^2_{BV}(z) = \frac{g}{\rho_0}\left(\frac{1}{C_s^2}\frac{\partial P_0}{\partial z} - \frac{\partial \rho_0}{\partial z}\right) \tag{1.3}$$

The stable regime is found for values of R_i greater than 0.25. As long as the maximum wind shear is less than twice the Brunt-Väisälä frequency, the wind profile is stable. In the troposphere the Brunt-Väisälä period is typically about ten minutes. Therefore, wind shears less than 20 m sec^{-1} km^{-1} are stable. This is a typical maximum wind shear for the tropospheric jet stream which may indicate that the instability plays a role in shaping the jet stream profile (Madden and Claerbout, 1968). By similarity with the atmospheric problem we notice that the magnetic field line interchange or magnetic buoyancy is something very much analogous to buoyancy effects. Thus, there must exist a magnetohydrodynamic shear flow instability analysis analogous to the hydrodynamic case, with a magnetic Richardson number which measures the relative importance of the interchange stability to the shear plasma flow instability. Rudraiah, et al., (1972a,b,c; 1976; 1977) studied the stability of a perfectly conducting fluid in the presence of a magnetic field and a shear flow using an approach parallel to the atmospheric case. In their analysis they found an analogous Richardson criterion which includes the effect of the magnetic field. However, their definition of the Richardson number is not quite conventional because this characteristic number has been always defined to be dependent on the properties of the medium and on the flow velocity, but not on the wave properties, as for example, the wavevector. The Richardson number is a quantity that indicates whether turbulent motions will persist or decay in an inhomogeneous fluid. This number provides a measure of the stabilizing or destabilizing influence of gravitational, thermal, rotational or magnetic forces in comparison with the destabilizing effects of the shear flow. According to the analysis of Rudraiah et al.

(1972b), in the case of a uniform magnetic field transverse to the flow velocity, a decrease of the wavelength along the magnetic field will have stabilizing influences and will increase their Richardson number. They also include the effects of rotation, however a series of approximations made in their investigation lead their analysis to show stabilizing effects in the rotational forces. On the contrary, when the rotational velocity is increased the system should become more convectively unstable due to the sign reversal in the effective gravity and also thus in the Brunt-Väisälä frequency. Another interesting observation of Rudraiah's result is that the most unstable mode corresponds to the case where the wavevector was along the flow, i.e. the wavevector along the magnetic field was zero. However, when this condition is satisfied the effect of the magnetic field in the Richardson number vanishes in their model and the problem reduces exactly to the hydrodynamic case. This is due to the fact that in their analysis, they consider only a uniform magnetic field and therefore the effects of gradient or curvature of the magnetic field were absent. In the Earth's situation we shall find that both the gradient and curvature effects of the magnetic field play an important role since they dominate the gravitational terms. This accounts for the use of the term "magnetic buoyancy" to describe the convective motion.

In this work we propose to carry out a magnetohydrodynamic analysis of both the Kelvin-Helmholtz and the interchange instabilities combined in what we shall call the shear flow-magnetic field line interchange instability. The analysis of this instability will be applied to the plasmapause region. We will extend the problem, in comparison with previous authors (e.g. Rudraiah, et al., 1972a,b,c; 1976, 1977 and Acheson, 1972a,b, 1973, 1978) in order to consider the effects of a full non-uniform medium by allowing gradients in density, pressure, flow velocity and magnetic field. Also the gravitational and rotational forces will be considered. In relation to the plasmapause region, the presence of a finite conductive medium, i.e., the ionosphere, at the feet of the magnetic tubes, further complicates the stability problem (Dungey, 1968; Lemaire, 1975) since the interchange motion of plasma tubes are only possible when the integrated Pedersen conductivity is significantly reduced below its commonly assumed infinite value (i.e. "line-tying" effect). This implies that this kind of motion is more susceptible to occur in the nightside of the plasmasphere. We propose to include in a qualitative way the effect of this finite conductive medium on the instabilities.

In the next section we shall describe briefly the principal characteristic physical properties of the plasma-pause. Then, in latter sections we shall review some of the most important papers related to the Kelvin-Helmholtz and the interchange plasma instabilities. We shall briefly point out the principal results relevant to our stability analysis. Finally, an outline of the presentation of this thesis will be presented.

I.2 Plasmapause Environment

As an introduction to our stability problem we shall describe briefly our theoretical understanding of the plasmasphere and its physical parameters inferred from ground-based and satellite measurements.

An important feature of the plasmasphere is its density structure. From whistler studies we have indications of the sharp decrease in density of plasma particles at distances of 4 to 6 Earth radii (Carpenter, 1966; Angerami and Carpenter, 1966). This boundary of decreasing density was called first the plasma "knee" and later the plasmapause. Carpenter inferred densities inside the plasmapause to be about 100 particles per cubic centimeter and outside the boundary about 1 to 5 particles per cubic centimeter (all these values in the equatorial plane). He also showed that the plasmapause is a permanent feature of the magnetosphere, indicating that the region must be stable.

Mayr and Voland (1968) estimate that the temperature of the thermal plasma at a height of 1,000 km (the top of the ionosphere) is about 3,000°K and just outside the plasmapause about 20,000°K. Thus, the thermal energy of the particles is about 0.5 to 2 electron-volts. If we assume that the plasma can be approximated as an ideal gas at a mean temperature value of about 10,000°K, then the sound velocity inside the plasmapause will be about 30 km sec^{-1}. It is also interesting to estimate the Alfven speed inside the plasmapause. If we take the equatorial value of the magnetic field at the Earth's surface as 0.32 Gauss and we extrapolate this value along the dipole field lines to an equatorial distance of about 4 Earth radii, the Alfven speed is computed to be in the range of 200 to 400 km sec^{-1}. But this technique does not take into account the compression of the Earth's magnetic field by the solar wind and more reasonable values for the Alfven velocity lie in the range of 500 to 1,000 km sec^{-1} (Dungey, 1968).

Another parameter of interest is the distance over which the particle density decrease occurs, i.e., the plasmapause thickness. Angerami and Carpenter (1966) roughly estimated that the change in plasma density occurred over a distance of less than 0.15 Earth radii (i.e. about 900 km). However, from the satellite data measured by OGO-5 and presented in Fig. 1.2 taken from Chappell, et al., (1970a,b) we can observe that the plasmapause thickness is of the order of 2 Earth radii during quiet times (i.e. $k_p \lesssim 1$). Also notice that when the magnetic activity increases (i.e. $k_p \gtrsim 1$) the thickness of the plasmapause decreases. Laster (1970) and Ong et al.,

(1972) suggested that the thickness parameter becomes an impotant factor in any stability analysis. Roth (1976) made a theoretical analysis of the thickness of the plasmapause. He computed the minimum possible plasmapause thickness to be of the order of 2.1 km (i.e. about five times the cold ion Larmor radius). In this analysis he shows that for a thin region like this one, the cold plasma is electrostatically unstable and the instability will lead to a broadening of the plasmapause.

I.3 Previous Investigations of the Kelvin-Helmholtz Instability

In the last section we presented two characteristic properties of the plasmapause region, i.e., a sharp change in density and relative streaming velocity of the tenuous plasma with respect to the denser plasma. Due to the electromagnetic forces caused by the motions of the plasma particles, the ions and electrons moving relative to each other will have motions that are not independent of each other. Thus, collective effects dominate the behavior of the plasma. Therefore, as we shall show later, the low-energy plasma particles can be treated as a continuous conductive fluid. By similar reasoning it is reasonable to expect a magnetohydrodynamic behavior of the plasmapause.

The Kelvin-Helmholtz instability has been known for about a century in non-conductive fluids. However the study of this instability in conductive fluids is more recent. One of the early papers related to this instability was presented by T. Northrop (1956). He considered a model consisting of a transverse magnetic field, i.e., relative to the plasma streaming, immersed in a vacuum on one side of the boundary and a plasma with no magnetic field at the other side. Northrop shows that for short enough wavelengths parallel to the plasma streaming, the disturbance at the boundary was always unstable. Unlike the case of ordinary fluids, in his model there was no effect such as surface tension which was known to stabilize the disturbance.

In recent years there have been many studies of models quite different from Northrop's model. Some of them have comparable results. The models have differed from Northrop's model by considering magnetic fields of general orientations on both sides of the boundary, by assuming compressible plasmas, and by assuming anisotropic plasmas at both sides of the boundary. Interest in the magnetopause has led to the investigation of the Kelvin-Helmholtz instability. We shall review some of these studies briefly in the present section. Subsequently, we shall compare some of the relevant results with our investigation.

An introductory review of the Kelvin-Helmholtz instability is given by Chandrasekhar (1961). He covers both the hydrodynamic stability of non-conductive, i.e. ordinary fluids and the hydromagnetic stability of conductive fluids. Chandrasekhar shows that in the hydromagnetic case of an incompressible fluid, a magnetic field transverse to the streaming flow does not stabilize the Kelvin-Helmholtz instability. This instability will show up for short enough wavelengths parallel to the streaming flow as in Northrop's case. He also shows that the magnetic field along the streaming flow will stabilize for all wavelengths if and only if the flow velocity does not exceed the root mean squared Alfven speed along the stream. Thus, he suggested that there will be a critical plasma flow velocity for which the system will be unstable.

Sen (1964, 1965) introduced the effects of compressibility. He found that a slight amount of compressibility destabilizes an otherwise marginally stable perturbation. He considers arbitrary orientation in the magnetic field at both sides of the boundary, he also assumes densities, sound speeds and magnetic fields identical in magnitude and direction in both plasmas. Sen then studied the effect of supersonic flow speeds. In fact, in that part of his paper, with the magnetic field exactly parallel to the flow, he found the interface to be stable to all wavelengths for all ratios of the flow speed to the Alfven speed. However, he also found that modes propagating at a small angle to the flow are still stable, as long as the flow velocity was less than twice the Alfven velocity. In the case in which the flow is not parallel to the magnetic field, Sen found the interface to be always unstable.

Fejer (1964) had studied independently the same model as Sen, solving numerically the dispersion relation for an arbitrary amount of compressibility. He obtained some results that differ from Sen's. Both authors consider the case of identical densities, sound speeds and magnetic field strengths on either side of the interface. They agreed on the fact that compressibility decreases the stabilizing effect of the parallel magnetic field. Fejer found that when both the magnetic field and the plasma flow velocity are in the same direction modes propagating along the flow will have some stabilizing effects if the Alfven velocity is greater than the flow velocity. His condition for stability in a slightly or highly compressible case is given by

$$C_A > \frac{V_o}{2} \left(1 + \frac{V_o^2}{16C_s^2} \right) \qquad (1.4)$$

where C_A, V_o and C_s are the Alfven, plasma flow and sound velocities, respectively. To the best of my knowledge, the difference between Fejer's and Sen's stability criterion is still apparently unresolved. A point favoring Fejer's result is that his numerical calculation shows this stability condition going over smoothly with increasing sound speed into the stability condition known to hold for incompressible plasmas.

Lerche (1966) discussed some of the results obtained by Sen and Fejer and pointed out that they should not be taken too seriously if the wavelengths in consideration were less than or equal to the ion Larmor radius. He argues that when this occurs the magnetohydrodynamic theory fails and the collisionless Vlasov equations, together with Maxwell's equations should be used to describe the plasma. In his analysis, Lerche found similar relationships to Fejer's condition described above. There was no complete agreement between Lerche and Sen as to the interpretation of the results.

Talwar (1965) was apparently the first to formulate the Kelvin-Helmholtz instability problem assuming an anisotropic plasma and using the Chew-Goldberger-Low theory, i.e., CGL theory. This theory is a modification of the magnetohydrodynamic theory. The fundamental difference is based on the assumption of anisotropic pressures (Chew, Goldberger and Low, 1956; Krall and Trivelpiece, 1973). Talwar showed that when the parallel pressure was small enough compared with the perpendicular pressure, i.e. $2P_{\parallel} < P_{\perp}$, stability was possible. Like Fejer (1964), Talwar found a range of relative streaming speeds for which an unstable mode could always be found. The range of streaming velocities is given by

$$V_o^2 < \frac{C_{s_\perp}^4 (C_A^2 + C_{s_\perp}^2 - 4C_{s_\parallel}^2)}{12C_{s_\parallel}^2 (C_A^2 + 2C_{s_\perp}^2)} \tag{1.5}$$

where V_o, C_A, $C_{S_\perp}$ and $C_{S_\parallel}$ are the plasma flow, Alfven, perpendicular sound velocities and parallel sound velocity, respectively. Fundamentally, the only new results introduced by the CGL theory were the possibility of two new instabilities named the "fire-hose" and "mirror" instabilities; but these are of no interest for our problem.

Rao, Kalra and Talwar (1968) considered the case where the magnetic field and the streaming velocity are perpendicular to each other, and showed that this system is unstable. Talwar and Kalra (1967) also introduced the effect of the Hall conductivities. They found them to be a destabilizing factor for the Kelvin-Helmholtz instability.

One of the more complete treatments of the Kelvin-Helmholtz instability is that of Southwood (1968). He allows the magnetic field on either side of the boundary to have arbitrary orientation with respect to the one another and with respect to the streaming velocity. He found critical conditions in the plasma streaming velocity for stability that are comparable to Fejer's conditions.

The main difference in relation to Fejer's paper is that Southwood assumed more realistic values for the densities, magnetic fields and sound speeds in application to the magnetospheric boundary. The most important result appears to be that at low and middle latitudes, the first unstable modes propagate across the Earth's field, along the streaming plasma with a very low phase velocity and wave fronts closely aligned to the meridian planes.

Gerwin (1968) reviewed some basic properties of the Kelvin-Helmholtz instability and some of the papers dealing with this phenomenon. He suggested that the origin of the discrepancy between Sen's and Fejer's papers may be that either (or both) of these studies fail to check that the modes whose stability is being considered actually remain bounded at long distances from the interface.

All the above studies have considered the transition zone between the two plasma fluids to be of zero thickness. It is more realistic to assume that the streaming velocity changes continuously across the finite thickness transition zone. Such behavior is referred to by fluid dynamicists as "shear flow" or "plane Poiseuille flow". Laster (1970) studied the effect of a continuously varying plasma stream velocity in a finite thickness transition zone for the Kelvin-Helmholtz instability problem employing the CGL theory. He considers a compressible plasma with a varying stream velocity perpendicular to a constant magnetic field and a variable plasma density. Laster found no stabilizing effect of the finite-thickness transition zone for the Kelvin-Helmholtz instability. Ong and Roderick (1972) had also studied the effect of a continuously varying plasma flow velocity across a finite thickness transition zone employing the magnetohydro-dynamic theory. They considered a magnetic field perpendicular to the streaming flow and variable across the transition layer. They also included the effect of a very small constant magnetic field along the flow. They concluded that in the limit where the small constant magnetic field along the plasma flow vanishes the growth rate of the instability is decreased for all wavelengths. This result is in contradiction with Laster's results. We have proved that Ong and Roderick's paper has a series of mistakes which may have led to this discrepancy. Thus, we tend to favor Laster's results.

Dobrowolny (1972) studied the Kelvin-Helmholtz instability by using the two MHD equations. He considered a high-β collisionless plasma, where β is the ratio of the plasma particle kinetic pressure to the magnetic pressure. Dobrowolny's model assumes a constant magnetic field along the plasma streaming, with a velocity flow and a plasma density spatially varying in the direction perpendicular to the field. He found that for waves characterized by parallel wavelengths much larger than the perpendicular ones, *i.e.*, parallel or perpendicular refers to the direction relative to the magnetic field, the Alfven wave and the slow magneto-acoustic wave branches of the dispersion relation can be separated. He also found that the velocity shear can cause instability at both branches. An important result presented in this paper is that the density gradients exert stabilizing effects and these effects become more important upon increasing β , *i.e.* $\beta >> 1$.

One of the most complete treatments of the Kelvin-Helmholtz instability of anisotropic plasmas in a magnetic field is presented by Duhau and Gratton (1975). In this paper they study this instability by the anisotropic CGL and MHD theory. They summarize most of the conclusions found by previous authors and emphasize the important role of anisotropy in the stability conditions.

Nagano (1978) studied the effect of the finite ion Larmor radius (FLR) on the Kelvin-Helmholtz instability problem for incompressible and compressible plasmas. He concluded that when a wave vector is parallel to a magnetic field, the effect of FLR tends to stabilize perturbations with shorter wavelengths. However, this stabilization will depend on the configuration of the plasma flow velocity and the magnetic field.

All the previous work reviewed in this section show fundamental characteristics of the Kelvin-Helmholtz instability under different conditions. The principal property given by these investigations is that the most unstable Kelvin-Helmholtz mode is the one with the shortest wavelength propagating along the flow velocity. In the next section we shall review some of the principal papers concerning the interchange or convective instability. Furthermore, at the end of that section we shall discuss the need to incorporate both the Kelvin-Helmholtz and interchange phenomena in a unified treatment for the stability analysis of the plasmapause.

I.4 Previous Investigations of the Interchange Instability

As we mentioned at the beginning of this chapter, the interchange instability of the Earth's magnetosphere was first proposed by Gold (1959). He pointed out two important characteristics of this type of motion. First, Gold proposes that adiabatic interchange motion of magnetic flux tubes containing plasma will occur, as a consequence of the insulating atmosphere which decouples the magnetic field lines from the Earth without overcoming any magnetic force. Secondly, he established that such motions would be unstable if the interchange of magnetic flux tubes causes a decrease in the total energy of the system. In other words, he considers the interchange of two adjacent magnetic flux tubes filled with plasma, extending from one end or "foot" at the bottom of the ionosphere to its conjugate point. This interchange involves deformation or rearrangement of the plasma in each tube, therefore suggesting that such motions may be driven by variations in the thermal energy of the plasma. If after a slight perturbation of the plasma tube its internal energy density is higher than its surroundings, then the plasma tube will continue moving and therefore an unstable situation has been created. Although this motion leaves the magnetic field unchanged, it is still controlled by this field.

A later study of this instability by Sonnerup and Laird (1963), by means of energy considerations, found that these interchanges are very important at low latitudes. They analyzed the interchange motions in two limits: where the gravitational energy is much smaller or much larger in absolute value than the internal energy density of the plasma. Smaller gravitational energy applies to the outer tubes of force for which the energy content is dominated by the internal energy of the plasma. Under this limit, Sonnerup and Laird found that spontaneous interchanges occur only if the energy content of the magnetic tubes decreases sufficiently rapid in the radial direction. In the limit of larger gravitational energy which applies to the inner tubes of force, the plasma is considered as a cold gas. They found that under this limit, spontaneous adiabatic interchanges may occur if the temperatures in the upper ionosphere increases sufficiently rapidly with increasing latitude.

The interchange instability is a form of Rayleigh-Taylor instability, however it does not require the presence of a real gravitational field. The Rayleigh-Taylor instability occurs when a dense incompressible plasma is supported against gravity by a magnetic field immersed in a less dense plasma (Rosenblith and Longmire, 1957; Coppi, 1964, 1976, 1977; Schmidt, 1966; Krall et al., 1973; Mikhailovskii, 1974). When perturbations of the interface between these plasmas are present, the denser plasma will interchange positions with the lighter one, thus creating an unstable situation. Therefore, the Rayleigh-Taylor instability is driven by an inverted density gradient.

However, in the interchange instability the driving mechanism is the buoyant force due to the gradients in the thermal energy and in the magnetic field. This buoyant force is created by an effective gravity due to curvature of the magnetic field lines. Suppose we move adiabatically a parcel of compressible plasma from one level in the plasma fluid to a higher level. The parcel of plasma should expand or compress in order to adjust its total pressure, i.e. thermal plus magnetic, equal to that of the surrounding plasma fluid. If after the expansion or compression the elevated plasma has a larger thermal energy density that the surrounding, the resulting magnetic buoyancy will continue to drive up the parcel and therefore will drive an instability. This mechanism that drives the interchange motion is called "magnetic buoyancy" (Parker, 1955; Bateman, 1976, 1978; Roberts and Stewartson, 1977; Acheson, 1978; Acheson and Gibbons, 1978; Moffatt, 1978). However, in some of the recent investigations concerning the equilibrium of a plasma in thermonuclear devices this phenomenon has been denominated as "ballooning" instability (Coppi, 1976, 1977). In some of the former studies previously mentioned, the authors have found a criterion for "magnetic buoyancy" instability for a magnetic field without distortion, given by:

$$L_{Bo}^{-1} > L_{\rho o}^{-1} \tag{1.6}$$

where L_{B_o} and L_{ρ_o} are the scale-heights for the magnetic field and density defined respectively by

$$L_{Bo}^{-1} = -\frac{1}{B_o}\frac{\partial B_o}{\partial r} \quad , \quad L_{\rho_o}^{-1} = -\frac{1}{\rho_o}\frac{\partial \rho_o}{\partial r} \tag{1.7}$$

Expression (1.6) merely shows that the medium is prone to instability when the magnetic field strength decreases sufficiently with height in comparison with the density variation. This condition takes into account neither the distortion nor the curvature of the field lines. In addition, it assumes that the thermal effects are very small. However, in the Earth's situation this last assumption can not be established since its thermal effects are very important.

Greene and Johnson (1968) studied the interchange instability with the ideal magnetohydrodynamic theory. They used the energy principle to analyze the instability, and showed that the energy sources that drive the instability are associated with the change in thermal and magnetic energies. They also suggested that the magnetic configuration plays an important role in the stability analysis.

Richmond (1973) examined the importance of the interchange instability at the plasmapause assuming an electrostatic approximation. His analysis was based on the evolution of the distribution function and the drift motions of charged particles on the Earth's magnetic field. Richmond shows that the energy density of the thermal plasma plays a significant role in driving small-scale convective motions and that in particular drive an instability at the plasmapause which will tend to limit the steepness of the density gradient there. He also computed a maximum growth rate for the night time interchange instability on the order of 1.9×10^3 sec^{-1} and a convective velocity of a magnetic flux tube at 4 Earth radii (i.e. 4Re) on the order of 0.7 Re/hr. Richmond presented instability curves (i.e. plots of the growth rate versus the longitudinal wavelength) for three different plasmapause models. In his first model he assumes an isothermal plasmapause with no rotation belt plasma but including convective shear. This first model shows that for very short longitudinal wavelengths the plasmapause is unstable. This result is in agreement with the theoretical requirements for the Kelvin-Helmholtz instability. His second model considers an isothermal plasmapause including radiation belt plasma but no convective shear and in his last model he only assumes the presence of a strong outward temperature gradient. These two final models show a slight reduction in the growth rate as the wavelength decreases from its maximum growth rate. Richmond discussed three possibilities that may explain how a steep density gradient is maintained. First, he considers that due to an enhanced ionospheric conductivity there will be a reduction in the growth rate of the instability. Secondly, some kind of diffusive destructive turbulence mechanism due to non-linearities is sufficient to limit the widening rate. And third, rapid radial convergence of a large scale flow due to the presence of an Alfven layer at the plasmapause may also reduce the growth rate.

Hudson and Kennel (1975) studied the interchange instability in a partly ionized collisional plasma. They found that the Alfven mode is strongly stabilized for finite wavelengths along the magnetic field. They also showed, like other authors, that when gravity and density gradients are anti-parallel to each other, the interchange mode is destabilized. This conclusion agrees with the Rayleigh-Taylor instability mentioned previously.

Lemaire (1974, 1975a,b) has suggested that for moderately active magnetic conditions, the plasmasphere of the Earth is peeled off near the midnight sector at a distance of about 4.5 Earth radii by plasma element interchange motion driven by centrifugal or inertial forces. He proposes that field aligned hydrostatic distributions of the plasma are convectively unstable for all magnetic flux tubes beyond a critical distance referred to as the "Roche-Limit". He estimated this critical distance of 5.78 Earth radii, however his definition of the "Roche-Limit" is not correct. His definition differs from the conventional description of the Roche Limit in astrophysics. According to Lemaire, these magnetic flux tubes interchange motions are only possible when the integrated Pedersen conductivity is significantly reduced below its commonly assumed infinite value which is in agreement with Dungey (1968), Richmond (1973) and other authors.

Although all the previously mentioned papers for the Kelvin-Helmholtz and interchange instabilities may describe in their own way some of the behavior of the plasmapause boundary, their analysis has been incomplete since the combined effect of both phenomena has been ignored. To the best of my knowledge, there has been no complete treatment of these two instabilities in a self-consistent way in application to the plasmapause boundary. Furthermore, a unified analysis of these two instabilities using the magnetohydrodynamic normal mode theory has not been presented in the literature. The only analogous situation found in the scientific literature but in application to arbitrary conducting fluids, is the work by Rudraiah, et al. (1972a,b,c; 1976, 1977). In principle, Rudraiah, et al. presents an extension of the work done by Booker and Bretherton (1967) in hydrodynamics but applied to conducting fluids immersed in a magnetic field. They considered perturbations from equilibrium of a rotating, perfectly conducting, slightly compressible (i.e. Boussinesq approximation), inviscid, adiabatic fluid moving with a mean horizontal velocity $V_{ox}(z)$ in the presence of a transverse uniform magnetic field. They concluded that the flow and the waves excited in it will be stable if the fluid satisfy the Richardson criterion given by

$$R_i = \frac{\Omega_{BV}^2(z) + K_y^2 C_A^2 + \Omega_R^2}{(\frac{\partial V_{ox}}{\partial z})^2} > 0.25 \tag{1.8}$$

where Ω_{BV} is the Brunt-Väisälä frequency, C_A is the Alfven velocity, K_y is the wavevector along the magnetic field and Ω_R is the rotational angular velocity. As we previously mentioned, their definition of the Richardson number is not quite conventional due to the presence of wave properties. Nevertheless, a very important aspect of their analysis shows that the most unstable mode corresponds to short wavelengths along the flow velocity, i.e. $K_x = \infty$. Notice that when this condition is satisfied the effects of the magnetic field in the Richardson number vanishes. This is due to the lack of curvature in their magnetic field model and we can expect different results for a dipole field.

In spite of the great efforts made by previously mentioned authors in explaining the stability of the plasmapause in terms of the interchange or Kelvin-Helmholtz instabilities, their treatment of the global stability of this system has been incomplete. This is due to the fact that both phenomena has been studied in isolation and their combined effect has been neglected. To achieve this unified treatment including some of the most important effects previously mentioned shall be one of the principal aspects of this investigation. The physical explanation of the dynamics of the plasmapause on the basis of this unified treatment represent the main purpose of this thesis.

I.5 Outline of the Thesis

In the next chapter we present the magnetohydrodynamical equations from which the stability problem will be discussed. A mathematical formalism on the basis of the normal mode analysis will be applied to the MHD equations in order to describe the disturbance of the plasmapause. These analyses lead to the derivation of the MHD - wave equations which describe the system. Those theoretical results which are original contributions to the stability problem are emphasized.

In Chapter III, the plasmapause is modeled as a continuous formation zone. Afterwards, we transform the wave equation in such a way that its solution could be expressed analytically. Assuming physically reasonable boundary condition together with the analytic wave solutions, a dispersion relation is derived. From the numerical solution of the dispersion relation we could evaluate the range of unstable eigenmodes of the system. Finally a discussion of the stability of the plasmapause on the basis of the real available data is presented. Important conclusions in relation to the stability of the region will be drawn from this final analysis.

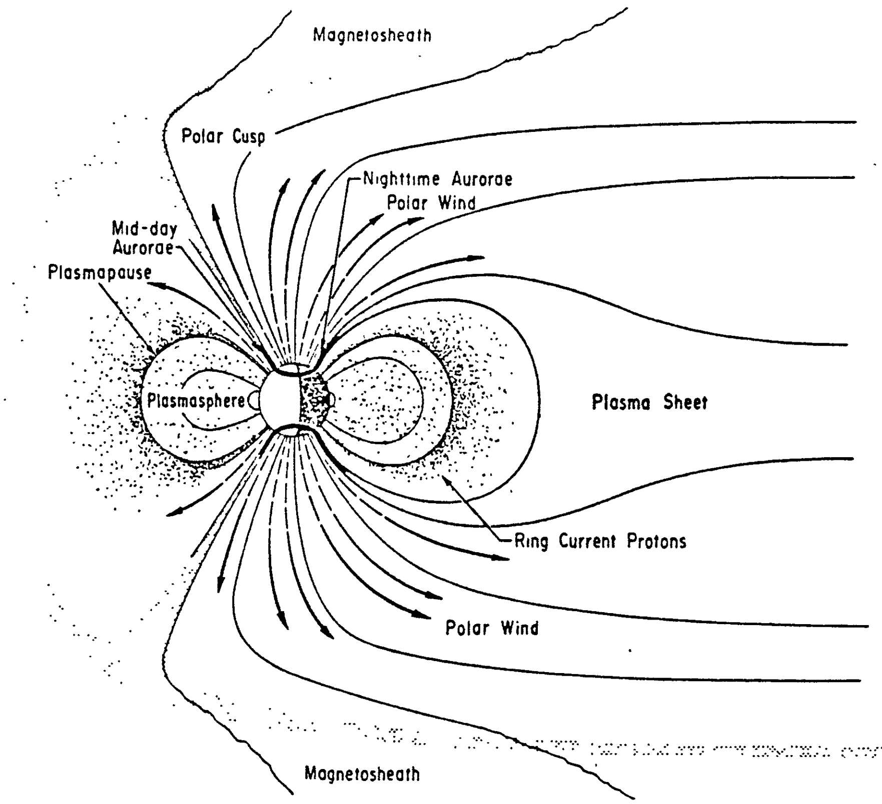

FIGURE 1.1-MODEL OF THE EARTH'S MAGNETOSPHERE IN THE MERIDIAN PLANE

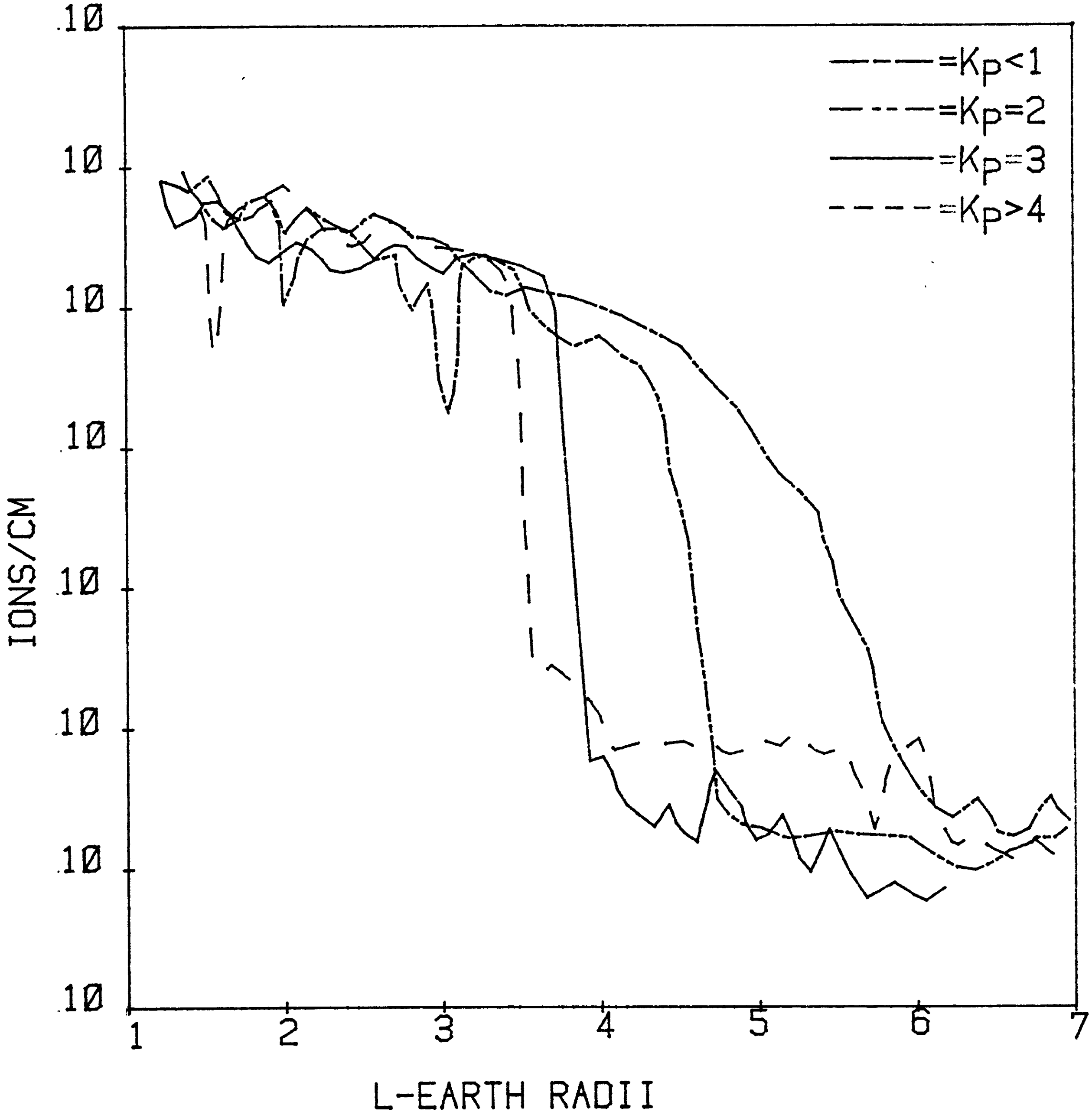

FIGURE 1.2-EXPERIMENTAL OGO-5 RESULTS SHOWING THE PARTICLE DENSITY PROFILE OF THE PLASMAPAUSE AND ITS LOCATION FOR DIFFERENT LEVELS OF MAGNETIC ACTIVITY (CHAPPELL'S et. al. 1970).

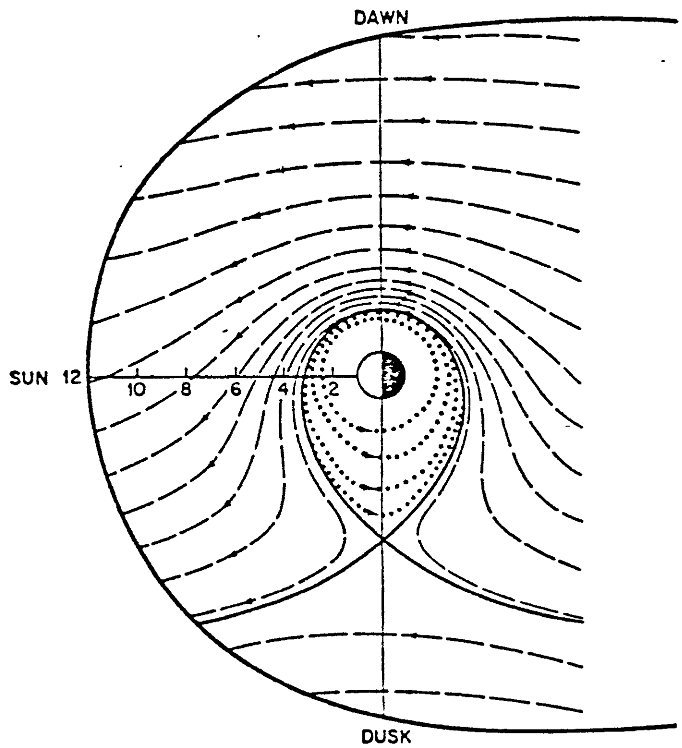

FIGURE 1.3-PLASMA DRIFT MOTIONS OR EQUIPOTENTIALS VIEWED IN THE EQUATORIAL PLANE (KAVANAGH et. al. 1968).

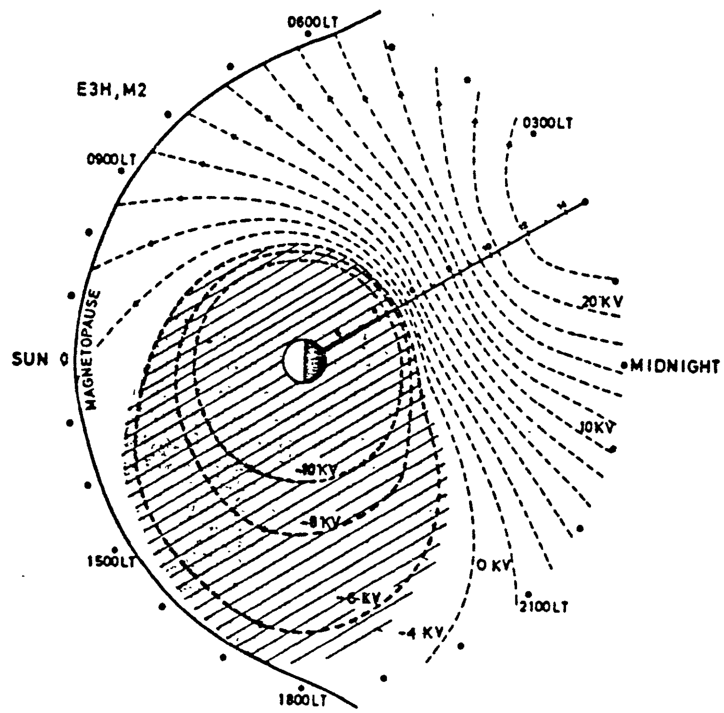

FIGURE 1.4-MAGNETOSPHERIC ELECTRIC FIELD OF McILWAIN (1974). THE DASHED CURVES REPRESENT THE EQUATORIAL SECTIONS OF EQUIPOTENTIAL SURFACES.

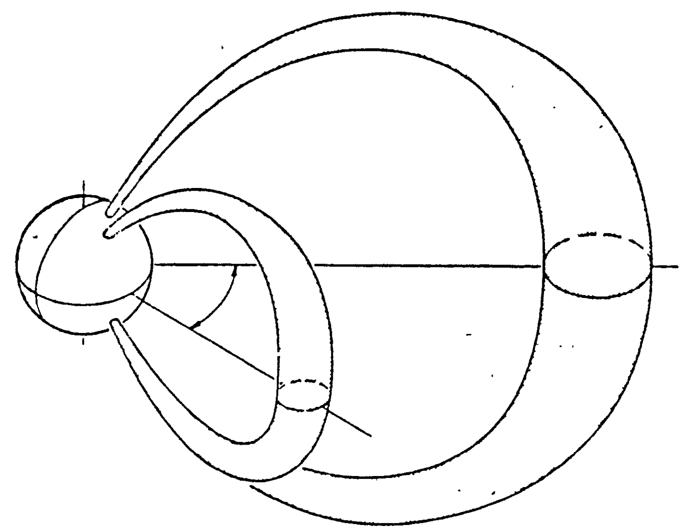

FIGURE 1.5-SCHEMATIC VARIATION OF A MAGNETIC FLUX TUBE
(INTERCHANGE OF MAGNETIC TUBES).

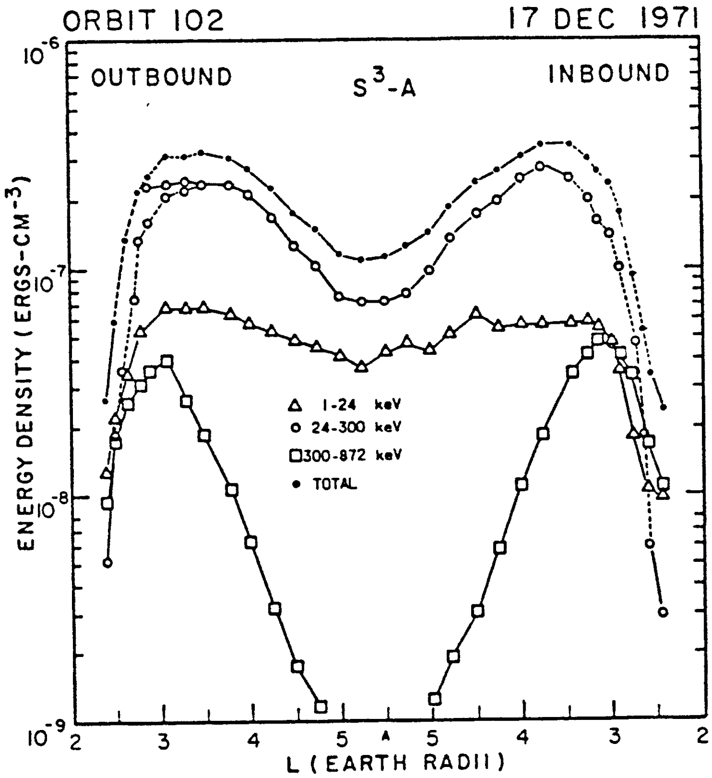

FIGURE 1.6-PROTON ENERGY DENSITY RADIAL PROFILE DURING A MAGNETIC STORM (SMITH, et. al. 1973).

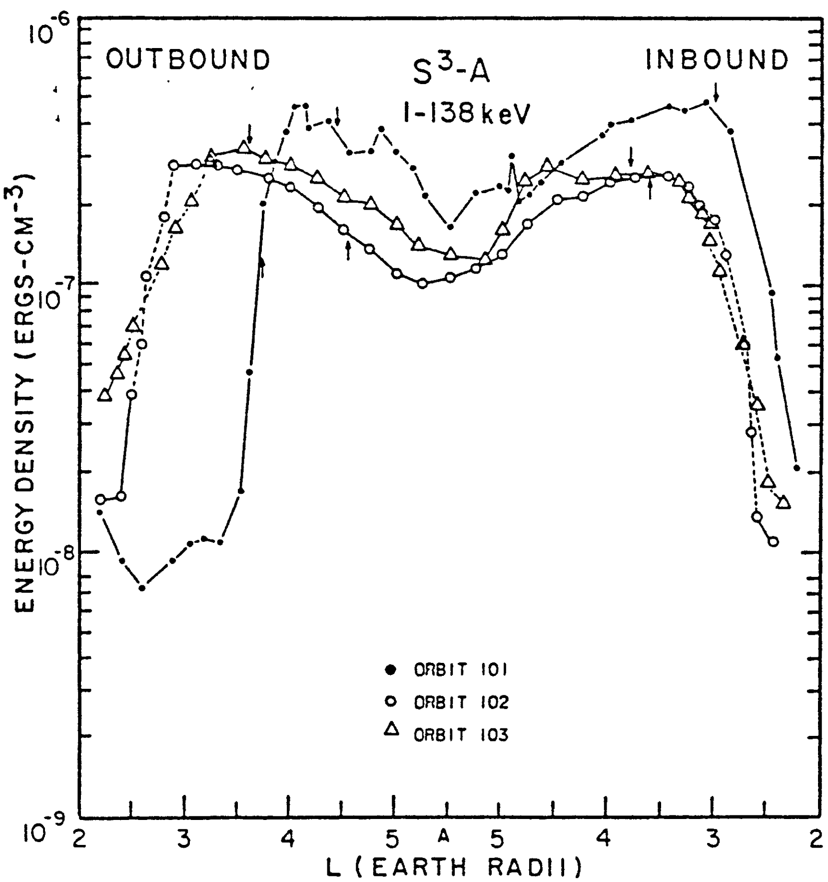

FIGURE 1.7-RADIAL PROFILE FOR PROTONS IN THE ENERGY RANGE OF 1 TO 138 kev FOR THREE ORBITS. THE ARROWS INDICATE THE LOCATION OF THE PLASMAPAUSE (SMITH, et. al. 1973).

CHAPTER II

MAGNETOHYDRODYNAMIC WAVE EQUATION AND DISPERSION ANALYSIS

II.1 Introductory Remarks

In this chapter we shall derive the MHD wave equation from which we shall describe the plasmapause behavior. The derivation of the wave equation will be based on the ideal MHD equations. Derivation of the MHD equations has been presented in many other texts, e.g., Shkarofsky, et al., (1966); Rossi and Olbert, (1970), thus, we shall not pursue a discussion of their origin in this thesis. An intensive local analysis of the derived wave equation, shall be presented. From the wave equation, a dispersion relation for a quasi-uniform medium will be obtained. Analyzing the dispersion relation, the different wave modes and their properties will be investigated. A study of the propagation properties of these modes and their instability conditions in relation to the plasmapause region is considered. Finally, a comparison and discussion of our theoretical results with other investigations shall be presented.

II.2 Mathematical Development of the Magnetohydrodynamic Wave Equation

In this section we shall present a mathematical analysis of the MHD equations with which the stability of the plasmapause region is investigated. The general wave equation which describes the system will be derived from this mathematical procedure.

We shall consider the plasmapause-plasmasphere regions in a spherical coordinate system represented by (r, θ, ϕ) centered at the Earth. In this coordinate system r is the radial distance and (θ, ϕ) are the colatitude and longitudinal components, respectively. In order to consider the effect of rotational forces, we shall assume that the Earth is rotating around the z-axis, in the positive ϕ-direction with a constant angular velocity Ω_E. We also assume that the plasmapause is corrotating with the Earth. Therefore, our analysis will be given in a rotating frame of reference. We will approximate the plasmapause in low and medium latitudes regions by a zone of transition (i.e. a spherical shell zone) between two moving, perfectly conducting fluids of different densities, temperatures, velocities and magnetic fields. A series of simplifying assumptions are needed in order to make the problem tractable. We shall assume that the wavelengths of the disturbance in consideration are much larger than the ion Larmor radius (which is about 400 meters at a distance of 4 Earth radii assuming a cold plasma), so that the use of the MHD equations is justified (Shkarofsky *et al.*, 1966; Rossi *et al.*, 1970). However, we will assume that the wavelengths are short compared to the curvature of the plasmapause at its particular location. In other words, the plasmapause can be represented as a spherical shell (at least for low and medium latitudes) enclosing the Earth. The Earth's magnetic field shall be assumed in the (θ,ϕ) plane and we will consider that it has a "weak" curvature. By "weak" curvature we mean that the ccenter of the spherical shell is far enough from its surface (*i.e.* the plasmapause layer). However, this assumption will break down for the real dipole field at very high latitudes. We shall also consider that the Earth's magnetic field has a gradient in the radial direction. The magnetic field curvature and gradient are important since they are probably the main cause of the ring current system which penetrates inside the plasmasphere, causing an increase in the thermal energy of the region. It is assumed that the convected plasma from the outer magnetosphere curve around the Earth so that the direction of the streaming velocity will be taken in the (θ,ϕ) plane. This curving action of the streaming velocity introduces additional centrifugal forces on the plasma motion. In addition it is assumed that the streaming velocity has a gradient in the radial direction. This model for the streaming velocity is consistent with Axford and Hine's (1961) and McIlwain's (1972, 1974) models for the magnetospheric plasma flow on the basis of the electric field distribution. We shall also assume a sharp radial density gradient across the plasmapause consistently with Carpenter's (1966) and Chappell's *et al.*, (1970a,b). It will be assumed that a radial pressure gradient exists across the transition layer, *i.e.*, the plasmapause. This pressure gradient effects are very important if we want to understand the behavior of the plasmapause to the injection of the hot particles of the ring current. This is consistent with Smith's *et al.* (1973) data which shows the distribution of thermal energy of the ring current during geomagnetic storms. Finally, gravitational and centrifugal forces due to the rotation of the Earth will also be included.

We shall now proceed to solve the ideal MHD equations in the previously mentioned coordinate system and with the previous considerations. The ideal MHD equations form a closed system for the variables: density ρ, pressure P, magnetic field $\underline{B}$ and velocity $\underline{V}$. These quantities are related by conservation equations of mass, momentum and energy. In a rotating coordinate system, the ideal MHD equations are given by:

$$\rho\left[\frac{D\underline{V}}{Dt} + 2\underline{\Omega}_E \times \underline{V} + \underline{\Omega}_E \times (\underline{\Omega}_E \times \underline{r})\right] = -\underline{\nabla}P + \underline{J} \times \underline{B} + \rho\underline{g} \quad (2.1)$$

$$\frac{\partial\rho}{\partial t} + \underline{\nabla} \cdot (\rho\underline{V}) = 0 \quad (2.2)$$

$$\underline{\nabla} \times \underline{E} = -\frac{\partial\underline{B}}{\partial t} \quad , \underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad , \underline{\nabla} \cdot \underline{B} = 0 \quad (2.3)$$

$$\frac{D}{Dt}\left[P/\rho^{\gamma}\right] = 0 \quad (2.4)$$

$$\underline{E} = -\underline{V} \times \underline{B} \quad (2.5)$$

where the operators D/Dt is the convective derivative defined by:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \underline{V} \cdot \underline{\nabla} \quad (2.6)$$

Equations (2.1) to (2.5) represent the momentum and mass conservation equations, the Maxwell's equations for the electromagnetic field, the adiabatic equation of state and the "Frozen-In" Law (or constitutive relation) respectively.

The system of equations (2.1) to (2.5) is non-linear and therefore their analytic solution is extremely difficult. In order to solve these equations we shall use the so-called "normal mode" analysis. This method assumes that the non-linear equations can be linearized around some known equilibrium state. In other words, it assumes that we have a system whose forces are on equilibrium and that this system is subjected to a perturbation that alters the forces acting on it. If these modified forces act to increase the initial perturbation, then the system is unstable. For example, in the case of the plasmapause, we suppose to know all the variables, e.g., density, pressure, etc., in the unperturbed or equilibrium state. Also, it supposes that we know the equilibrium forces acting on the plasmapause. Now, if we introduce a small perturbation at this layer, the equilibrium will be modified and if the resulting forces act to change the initial unperturbed state, then the system is unstable. The study of plasma instabilities with this method is usually based on perturbation theory or Fourier-Laplace integral transformations. This approach makes sense only if there is an initial steady state plasma equilibrium about which it is possible to consider small departures. Inevitably, non-linear effects cause that growing perturbations alters the properties of the plasma, e.g. density, pressure, etc. However, these modifications may lead to a new equilibrium state that is stable to the mode that destroyed the previous state. Nevertheless, the linear normal mode method allows us to compute the initial growth rate of the instability. The normal mode analysis assumes that the linearized plasma equations for the time development of the perturbation can be solved, subject to the appropriate boundary conditions, assuming a time dependence $\exp(-i\omega t)$. This procedure gives an equation for ω, i.e. the wave frequency, in terms of the equilibrium parameters and the wavevector $\underline{k}$. The ω's from the equation may be real, imaginary or complex. If all the ω's are real, then all perturbed variables oscillate harmonically, and the plasma is stable. If any or all of the ω's have some positive imaginary part, then the system is unstable since the normal mode will grow in time. Normal mode analysis provides complete information about instabilities associated with a particular plasma equilibrium. The development of any initial perturbation can be followed up to the limits imposed by the linearization of the ideal MHD equations. Unfortunately, normal mode method can be applied only to linear systems of equations and to those cases where the plasma equilibrium is simple enough to allow solutions of the plasma equations. There is another method widely used in the study of plasma instabilities which is called the "energy principle". This method is very useful in determining general stability criterion, however the initial growth rate of a particular instability cannot be determined. In this investigation we shall only consider the "normal mode" method for the analysis of the instabilities, therefore we shall not pursue the discussion of the "energy principle" method.

Let us linearize the system of equations (2.1) to (2.6) by the use of perturbation method. The perturbed quantities are given by

$$\begin{vmatrix} \rho(r,\theta,\phi,t) \\ P(r,\theta,\phi,t) \\ \underline{V}(r,\theta,\phi,t) \\ \underline{B}(r,\theta,\phi,t) \end{vmatrix} = \begin{vmatrix} \rho_o(r) \\ P_o(r) \\ \underline{V}_o(r) \\ \underline{B}_o(r) \end{vmatrix} + \begin{vmatrix} \delta\rho(r,\theta,\phi,t) \\ \delta P(r,\theta,\phi,t) \\ \delta\underline{V}(r,\theta,\phi,t) \\ \delta\underline{B}(r,\theta,\phi,t) \end{vmatrix} \qquad (2.7)$$

where variables with subscript "o" are the known equilibrium values and those preceded by "δ" are the perturbations. We shall assume in the linearization that perturbed quantities are very small in comparison to the equilibrium values, so that second order terms in the perturbation can be neglected. This approximation is valid as long as the growth rate of the disturbance is small. Otherwise the non-linear terms become important and these second order terms cannot be neglected.

Using the Maxwell's equations in (2.3) together with the constitutive relation (2.5) we can rearrange the momentum conservation equation (2.1). Similarly, we can also find a more simple expression for the magnetic field which we shall call the "magnetic equation". The modified momentum conservation equation is

$$\rho\left[\frac{D\underline{V}}{Dt} + 2\underline{\Omega}_E \times \underline{V} + \underline{\Omega}_E \times (\underline{\Omega}_E \times \underline{r})\right] = -\underline{\nabla}\left[P + \frac{B^2}{2\mu_o}\right] + \frac{1}{\mu_o}(\underline{B}\cdot\underline{\nabla})\underline{B} + \rho\underline{g} \qquad (2.8)$$

and the "magnetic equation is:

$$\frac{\partial \underline{B}}{\partial t} = \underline{\nabla} \times (\underline{V} \times \underline{B}) = (\underline{B}\cdot\underline{\nabla})\underline{V} - \underline{B}(\underline{\nabla}\cdot\underline{V}) - (\underline{V}\cdot\underline{\nabla})\underline{B} \qquad (2.9)$$

An important assumption in these equations is that the magnetic field is not a "curl-free" field at the point where the instability is analyzed, i.e. the plasmapause. Therefore, the effects of currents are produced due to the inhomogeneities in the magnetic field and the plasma properties, e.g. Chandrasekhar, (1962); Alfven et al., (1962); Schmidt, (1966) and Boyd, et al. (1969).

As we previously mentioned, we shall consider radial gradients in all the equilibrium variables. Therefore, in component form, the unperturbed quantities are:

$$\rho_o = \rho_o(r) \quad , \; P_o = P_o(r)$$

$$\underline{V}_o(r) = V_{o\theta}\,\hat{\boldsymbol{e}}_\theta + V_{o\phi}\,\hat{\boldsymbol{e}}_\phi$$

$$\underline{B}_o(r) = -\,B_{o\theta}\,\hat{\boldsymbol{e}}_\theta + B_{o\phi}\,\hat{\boldsymbol{e}}_\phi \tag{2.10}$$

$$\underline{\Omega}_E = \Omega_E \cos\theta\,\hat{\boldsymbol{e}}_r - \Omega_E \sin\theta\,\hat{\boldsymbol{e}}_\theta$$

where $(\hat{\boldsymbol{e}}_r, \hat{\boldsymbol{e}}_\theta, \hat{\boldsymbol{e}}_\phi)$ are unit vectors along the (r,θ,ϕ) directions in a sperical coordinate system. Substituting the equilibrium variables (2.10) and the perturbed variables (2.7) into equations (2.2), (2.4), (2.8), and (2.9) and keeping up to first order terms we can find the equilibrium state of the system and the conservation equations for the perturbed quantities. The equilibrium state of the system is obtained from the radial component of the zero oder perturbation equation. After some algebraic manipulations using some vector identities the general equilibrium equation of the system is given by

$$\frac{\partial}{\partial r}\left[P + \frac{B_o^2}{2\mu_o}\right] = -\rho_o\left[\bar{g} - \frac{1}{\rho_o\mu_o}\,[(\underline{B}_o \cdot \underline{\nabla})\underline{B}_o]_r\right] \tag{2.11}$$

where $\bar{g}(r)$ is an effective gravity given by

$$\bar{g}(r) = g - \Omega_E^2\, r \sin^2\theta - \frac{V_o^2}{r} - 2\Omega_E V_{o\phi} \sin\theta \tag{2.12}$$

$V_0(r)$ is the total flow velocity, $g(r)$ is the gravitational acceleration, $B_0(r)$ is the total magnetic field and is the longitudinal flow velocity. Note from equation (2.11) that the magnetic field seems to modify the distribution of thermal energy density. If we consider the case of the real Earth's magnetic dipole field and assuming the ideal gas law for the plasma, i.e., $P_0(r) = \frac{K_B}{m} T_0(r) \; \rho_0(r)$, we can determine the distribution of thermal energy density as a function of radial distance as

$$P_0(r) = \bar{P}(r=R_E) \; \exp\left\{ - \int_{R_E}^{r} \frac{dr'}{H(r')} \right\} \tag{2.13}$$

where $r = R_E$ is the position of the Earth's surface and $H(r')$ is the scale height given by:

$$H(r) = \frac{K_B T_0(r)}{mg} \tag{2.14}$$

Assuming typical values for the plasmasphere medium we could estimate the scale height at the equatorial plane (i.e. $\theta \approx 90°$). Let us consider that the plasmapause is about 4 Earth radii in corrotation with the planet, with a typical particle density of 3×10^3 particles/cm^3 and with a temperature of 2×10^5 °Kelvin. For these typical values the scale height is compared to be of the order of 3.46×10^6 km, which is roughly in agreement with the observed density fall-off at the plasmasphere (see Chappell, et al. 1970a,b).

From the first order terms we can find the linearized equations for the perturbed quantities. These equations are:

$$\rho_o\left[\frac{D\delta\underline{V}}{Dt} + (\delta\underline{V}\cdot\underline{\nabla})\underline{V}_o + 2\underline{\Omega}_E x\delta\underline{V}\right] = -\underline{\nabla}\left[\delta P + \frac{\underline{B}_o\cdot\delta\underline{B}}{\mu_o}\right] + \frac{1}{\mu_o}(\underline{B}_o\cdot\underline{\nabla})\delta\underline{B} \quad (2.15)$$

$$+\frac{1}{\mu_o}(\delta\underline{B}\cdot\underline{\nabla})\underline{B}_o + \delta\rho\left[\underline{g} - 2\underline{\Omega}_E x\underline{V}_o - \underline{\Omega}_E x(\underline{\Omega}_E x\underline{r})\right]$$

$$\frac{D\delta\underline{B}}{Dt} = (\underline{B}_o\cdot\underline{\nabla})\delta\underline{V} + (\delta\underline{B}\cdot\underline{\nabla})\underline{V}_o - (\underline{\nabla}\cdot\delta\underline{V})\underline{B}_o - \delta\underline{B}(\underline{\nabla}\cdot\underline{V}_o) - (\delta\underline{V}\cdot\underline{\nabla})\underline{B}_o \quad (2.16)$$

$$\frac{D\delta\rho}{Dt} = -\rho_o\underline{\nabla}\cdot\delta\underline{V} - \delta\underline{V}\cdot\underline{\nabla}\rho_o - \delta\rho\ \underline{\nabla}\cdot\underline{V}_o \quad (2.17)$$

$$\frac{D\delta P}{Dt} + \delta\underline{V}\cdot\underline{\nabla}P_o = C_s^2\left[\frac{D\delta\rho}{Dt} + \delta\underline{V}\cdot\underline{\nabla}\rho_o\right] \quad (2.18)$$

where C_S is the sound velocity defined by

$$C_s = \left[\frac{\gamma P_o(r)}{\rho_o(r)}\right]^{1/2} \tag{2.19}$$

and γ is the adiabatic constant, i.e. $\gamma = 5/3$.

The system of equations (2.15) to (2.18) is now linear in the perturbations and we can solve it by the use of "normal mode" technique or Fourier-Laplace integral transform. By expanding the perturbed variables in normal modes given by

$$\begin{vmatrix} \delta\rho(r,\theta,\phi,t) \\ \delta P(r,\theta,\phi,t) \\ \delta \underline{V}(r,\theta,\phi,t) \\ \delta \underline{B}(r,\theta,\phi,t) \end{vmatrix} = \begin{vmatrix} \tilde{\rho}(r) \\ \tilde{P}(r) \\ \tilde{\underline{V}}(r) \\ \tilde{\underline{B}}(r) \end{vmatrix} \cdot \exp(-i\omega t+im\theta+in\phi) \tag{2.20}$$

we can obtain the equations for the disturbance. Note that equations (2.15) and (2.16) are vector equations which represent a system of six equations for both the components of the velocity and the magnetic field perturbations. By substitution of equation (2.20) into the linearized equations (2.15) to (2.18) and using a series of vector identities, the transformed equations in component form are given by:

$$-i\Omega\rho_o\tilde{V}_r-\frac{2\rho_o}{r}(\underline{V}_o\cdot\tilde{\underline{V}})-2\rho_o\Omega_E\tilde{V}_\phi=-\frac{\partial\tilde{P}_T}{\partial r}+\frac{i(\underline{K}_t\cdot\underline{B}_o)}{\mu_o}\tilde{B}_r-\frac{2(\underline{B}_o\cdot\tilde{\underline{B}})}{r\mu_o}-\tilde{\rho}\bar{g} \qquad (2.21)$$

$$-i\Omega\rho_o\tilde{V}_\theta+\frac{\rho_o\tilde{V}_r}{r}\frac{\partial}{\partial r}(rV_{o\theta})=-iK_\theta\tilde{P}_T+\frac{i(\underline{K}_t\cdot\underline{B}_o)}{\mu_o}\tilde{B}_\theta+\frac{\tilde{B}_r}{r\mu_o}\frac{\partial}{\partial r}(rB_{o\theta}) \qquad (2.22)$$

$$-i\Omega\rho_o\tilde{V}_\phi+\frac{\rho_o\tilde{V}_r}{r}\frac{\partial}{\partial r}(rV_{o\phi})+2\rho_o\Omega_E\tilde{V}_r=iK_\phi\tilde{P}_t+\frac{i(\underline{K}_t\cdot\underline{B}_o)}{\mu_o}\tilde{B}_\phi+\frac{\tilde{B}_r}{r\mu_o}\frac{\partial}{\partial r}(rB_{o\phi}) \qquad (2.23)$$

$$-i\Omega\tilde{\rho}+\tilde{V}_r\frac{\partial\rho_o}{\partial r}=-\rho_o\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tilde{V}_R)+i(\underline{K}_t\cdot\tilde{\underline{V}})\right] \qquad (2.24)$$

$$-i\Omega\tilde{B}_r=i(\underline{K}_t\cdot\underline{B}_o)\tilde{V}_r \qquad (2.25)$$

$$-i\Omega\tilde{B}_\theta=i(\underline{K}_t\cdot\underline{B}_o)\tilde{V}_\theta+\tilde{B}_r\left[\frac{\partial V_{o\theta}}{\partial r}-\frac{V_{o\theta}}{r}\right]-\tilde{V}_r\left[\frac{\partial B_{o\theta}}{\partial r}-\frac{B_{o\theta}}{r}\right]-B_{o\theta}(\underline{\nabla}\cdot\tilde{\underline{V}}) \qquad (2.26)$$

$$-i\Omega\tilde{B}_\phi=i(\underline{K}_t\cdot\underline{B}_o)\tilde{V}_\phi+\tilde{B}_r\left[\frac{\partial V_{o\phi}}{\partial r}-\frac{V_{o\phi}}{r}\right]-\tilde{V}_r\left[\frac{\partial B_{o\phi}}{\partial r}-\frac{B_{o\phi}}{r}\right]-B_{o\phi}(\underline{\nabla}\cdot\tilde{\underline{V}}) \qquad (2.27)$$

$$-i\Omega\tilde{P}_T+\tilde{V}_r\frac{\partial P_o}{\partial r}=C_s^2\left[-i\Omega\tilde{\rho}+\tilde{V}_r\frac{\partial\rho_o}{\partial r}\right]-\frac{i\Omega}{\mu_o}(\underline{B}_o\cdot\tilde{\underline{B}}) \qquad (2.28)$$

where Ω is the Doppler shifted frequency, $\underset{\sim}{K}_t$ is the tangential wavevector and $\tilde{P}_T$ is the total pressure perturbation defined respectively by

$$\Omega(r) = \omega - (\underset{\sim}{K}_t \cdot \underset{\sim}{V}_o) \tag{2.29}$$

$$\underset{\sim}{K}_t = K_\theta \hat{\boldsymbol{e}}_\theta + K_\phi \hat{\boldsymbol{e}}_\phi \quad , \quad K_\theta = m/r \quad , \quad K_\phi = n/r \tag{2.30}$$

$$\tilde{P}_T = \tilde{P} + \frac{(\underset{\sim}{B}_o \cdot \tilde{\underset{\sim}{B}})}{\mu_o} \tag{2.31}$$

We now proceed to obtain the general wave equation for the perturbed amplitudes which will be the governing equation for the model. We shall manipulate the system of equations (2.21) to (2.28) to obtain a set of two first order differential equations coupled by a matrix system (see Pierce, 1967a,b; Claerbout, 1968; Laster, 1970). For dependent variables we shall use those quantities which must be conserved at a boundary in the plasma fluid. The quantities to be used are the radial component of the perturbed displacement $\tilde{\xi}_r$ defined in terms of the radial velocity as follows:

$$\tilde{\xi}_r = \frac{i\tilde{V}_r}{\Omega} \tag{2.32}$$

and the total stress $\tilde{P}_T$ defined in equation (2.31) (see Laster, 1970; Gossard, et al., 1975). For the purpose of obtaining the couple set of differential equations, we shall first eliminate the density perturbation $\tilde{\rho}$ and the components of the magnetic field fluctuations $\tilde{B}_r$, $\tilde{B}_\theta$, and $\tilde{B}_\phi$ by using equations (2.25) to (2.27) and (2.28) in the remaining equations. Then solving for $\tilde{V}_\theta$ and $\tilde{V}_\phi$ from equations (2.22) and (2.23) and substituting all together into equations (2.21) and (2.24) we finally get the coupled matrix system of first order differential equations of the form:

$$\frac{d\chi(r)}{dr} = \begin{vmatrix} a_{11}(r) & a_{12}(r) \\ a_{21}(r) & a_{22}(r) \end{vmatrix} \chi(r) \tag{2.33}$$

where X(r) is a column vector of the dependent variables given by

$$\chi(r) = \begin{vmatrix} \tilde{P}_T \\ r^2 \tilde{\xi}_r \end{vmatrix} \tag{2.34}$$

and A(r) is a two-by-two coupling matrix whose elements are defined by:

$$a_{11}(r) = -\Big[\frac{\Omega^2(\bar{g}+\frac{2C_A^2}{r})-2(\underset{\sim}{K}_t\cdot \underset{\sim}{C}_A)^2 C_s^2/r}{(1+M^2)C_s^2\ \Omega_M^2} - \frac{2\Omega(\underset{\sim}{K}_t\cdot\underset{\sim}{V}_o)}{r\Omega_A^2}$$

$$\frac{2\Omega^3(\underset{\sim}{K}_t\cdot\underset{\sim}{C}_A)(\underset{\sim}{V}_o\cdot\underset{\sim}{C}_A)}{r(1+M^2)C_s^2\Omega_M^2\Omega_A^2} + \frac{2\Omega\Omega_*}{(1+M^2)\Omega_M^2}\left\{ \frac{\Omega^2 C_{A\theta}(\underset{\sim}{K}_t\times\underset{\sim}{C}_A)_r}{\Omega_A^2 C_s^{\ 2}} - K_\phi\right\}\Big]$$

$$a_{12}(r) = \frac{\rho_o}{r^2}\Big[\Omega_A^2-\Omega_o^2-\Omega_R^2-\frac{2V_o^{\ 2}}{r^2}\left\{\frac{\Omega^2+(\underset{\sim}{K}_t\cdot\underset{\sim}{C}_A)^2}{\Omega_A^2}\right\} - \frac{4(\underset{\sim}{K}_t\cdot\underset{\sim}{C}_A)(\underset{\sim}{V}_o\cdot\underset{\sim}{C}_A)}{r^2\Omega}$$

$$+\frac{2\Omega(\underset{\sim}{K}_t\cdot\underset{\sim}{C}_A)(\underset{\sim}{V}_o\cdot\underset{\sim}{C}_A)}{r\ C_s^2\ \Omega_A^2}\left\{\bar{g}-\frac{2C_s^2}{r}\right\} + \Big\{\frac{2(\underset{\sim}{K}_t\cdot\underset{\sim}{C}_A)(\underset{\sim}{V}_o\cdot\underset{\sim}{C}_A)}{r\Omega} \tag{2.35}$$

$$-M^2(\bar{g}-2C_s^2/r)\Big\}\cdot\Big\{-\frac{\Omega^2}{(1+M^2)C_s^2\Omega_M^2\rho_o}\frac{\partial}{\partial r}\left(P_o+\frac{B_o^2}{2\mu_o}\right) + \frac{2M^2(\Omega^2-(\underset{\sim}{K}_t\cdot\underset{\sim}{C}_A)^2/M^2)}{r(1+M^2)\Omega_M^2}$$

$$+\frac{2\Omega^3(\underset{\sim}{K}_t\cdot\underset{\sim}{C}_A)(\underset{\sim}{V}_o\cdot\underset{\sim}{C}_A)}{r(1+M^2)C_s^2\Omega_M^2\Omega_A^2}\Big\}\Big]$$

$$a_{21}(r) = -\frac{r^2}{\rho_o}\left[\frac{\Omega^4}{(1+M^2)C_s^2\Omega_M^2\Omega_A^2} - \frac{K_t^{\ 2}}{\Omega_A^2}\right]$$

$$a_{22}(r) = -\,a_{11}(r)$$

In the previous equations the parameters Ω_A^2, Ω_M^2, Ω_o^2 and Ω_R^2 which represent certain natural frequencies defined by

$$\Omega_A^2 = \Omega^2 - (\underline{K}_t \cdot \underline{C}_A)^2 \tag{2.36}$$

$$\Omega_M^2 = \Omega^2 - \frac{(\underline{K}_t \cdot \underline{C}_A)^2}{1+M^2} \tag{2.37}$$

$$\Omega_o^2(r) = \frac{\bar{g}}{\rho_o}\left\{\frac{1}{C_s^2}\frac{\partial}{\partial r}\left[P_o + \frac{B_o^2}{2\mu_o}\right] - \frac{\partial \bar{\rho}_o}{\partial r} - \frac{\rho_o M^2}{r}\right\} \tag{2.38}$$

$$+ \frac{2C_A^2}{r^2} - \frac{2C_A^2}{rB_o}\frac{\partial B_o}{\partial r} + \frac{2V_o}{r}\frac{\partial V_o}{\partial r}$$

$$\Omega_R^2 = 4\Omega_E^2\left[1 + \frac{K_\theta C_{A\theta}(\underline{K}_t \cdot \underline{C}_A)}{\Omega_A^2} + \frac{K_\phi C_{A\phi}(\underline{K}_t \cdot \underline{C}_A)}{(1+M^2)\Omega_M^2}\right. \tag{2.39}$$

$$\left. - \frac{C_{A\theta}C_{A\phi}(\underline{K}_t \cdot \underline{C}_A)(\underline{K}_t \times \underline{C}_A)_r \Omega^2}{(1+M^2)C_s^2\Omega_M^2\Omega_A^2}\right] + 4\Omega\Omega_\varepsilon\left[\frac{\Omega V_{o\phi}}{r\Omega_M^2}\right.$$

$$+ \frac{C_{A\phi}(\underline{K}_t \cdot \underline{C}_A)}{r\Omega_M^2} - \frac{C_{A\phi}(\underline{K}_t \cdot \underline{C}_A)}{(1+M^2)C_s^2\Omega_M^2}\left\{\bar{g} + \frac{C_A^2 - C_s^2}{r}\right\}$$

$$\left. - \frac{\Omega(\underline{K}_t \cdot \underline{C}_A)^2\,(C_{A\theta}(\underline{V}_o \times \underline{C}_A)_r + C_{A\phi}(\underline{V}_o \cdot \underline{C}_A))}{r(1+M^2)C_s^2\Omega_M^2\Omega_A^2}\right]$$

In addition we also defined the quantities M and C_A as the magnetic Mach number and the Alfven velocity respectively together with the parameters $C_{A\theta}$ and $C_{A\phi}$ by

$$M = \frac{C_A(r)}{C_s(r)} \tag{2.40}$$

$$C_A(r) = \frac{B_o(r)}{\sqrt{\rho_o(r)\mu_o}} \tag{2.41}$$

$$C_A\ (r) = \frac{B_{o\theta}(r)}{\sqrt{\rho_o(r)\mu_o}} \quad , \quad C_A\ (r) = \frac{B_{o\phi}(r)}{\sqrt{\rho_o(r)\mu_o}} \tag{2.42}$$

The wave equation (2.33) has some interesting properties. First, the trace of the matrix A(r) is zero which implies that the eigenvalues have the same magnitude but with opposite signs. The eigenvalues of the matrix A are associated with the propagation properties of the wave. Another feature of this matrix system is that for certain values of r, some of its elements may become singular. These singularities are given by

$$\begin{aligned} \Omega(r) &= 0 \\ \Omega(r) &= \pm \frac{(\underline{K}_t \cdot \underline{C}_A)}{\sqrt{1+M^2}} \\ \Omega(r) &= \pm (\underline{K}_t \cdot \underline{C}_A) \end{aligned} \tag{2.43}$$

and are associated with the existence of drift waves, slow magnetoacoustic waves and guided (or shear) Alfven waves in the plasma, respectively.

A local analysis of the wave equation in relation to the plasmapause region will be discussed in the next section.

II.3 Local Analysis and Dispersion Relation of the Wave Equation

In this section we shall present an extensive local analysis of our theoretical results, i.e. the general wave equation (2.33), in relation to characteristic properties of the plasmapause region. In order to simplify the investigation of the wave equation it is convenient to establish a series of approximations in accordance with their validity at the plasmapause. First, let us define the parameter α, as the angle between the plasma flow velocity and the magnetic field; and the parameters χ_b and χ_v as the angles between the wavevector $\underline{K}_t$ and the total magnetic field and flow velocity, respectively. We shall assume for simplicity that our theoretical results are localized at the equatorial plane. Also, we will consider the magnetic field $\underline{B}_o(r)$ as a dipole field along the meridian plane, so that $B_{o\theta}(r) >> B_{o\phi}(r)$. This approximation is very consistent with magnetic field data at low and medium latitudes. Another approximation will be to consider the plasma flow velocity $\underline{V}_o(r)$ basically along the positive ϕ-direction at the equatorial plane, so that $V_{o\phi}(r) >> V_{o\theta}(r)$. This is consistent with Axford et al. and McIlwain's models previously mentioned. Consequently, quantities which involve the dot product $\underline{V}_o \cdot \underline{C}_A = V_o C_a \cos\alpha$ can be neglected. This implies that the angle α is near 90°, therefore the magnetic field and the flow velocity vectors are basically perpendicular to each other. Using these approximations in the wave equation (2.33) we find the simplified wave equation

$$\frac{d}{dr}\chi(r) = A(r)\chi(r) \tag{2.44}$$

where the elemtns of the matrix A(r) are given by

$$a_{11}(r) = -\left[\frac{\Omega^2\left\{\bar{g}+\frac{2C_A{}^2}{r}\right\} - 2(\underline{K}_t\cdot\underline{C}_A)^2 C_s^2/r}{(1+M^2)C_s^2\Omega_M^2} - \frac{2\Omega(\underline{K}_t\cdot\underline{V}_o)}{r\Omega_A^2} \right.$$

$$\left. + \frac{2\Omega\Omega_\varepsilon}{(1+M^2)\Omega_M^2}\left\{\frac{\Omega^2 C_{A\theta}(\underline{K}_t\times\underline{C}_A)_r}{\Omega_A^2 C_s^2} - K_\phi\right\}\right]$$

$$a_{12}(r) = \frac{\rho_o}{r^2}\left[\Omega_A^2 - \Omega_o^2 - \Omega_R^2 - \frac{2V_o{}^2}{r^2}\left\{\frac{\Omega^2+(\underline{K}_t\cdot\underline{C}_A)^2}{\Omega_A^2}\right\}\right. \tag{2.45}$$

$$\left. + M^2\left\{\bar{g}-\frac{2C_s{}^2}{r}\right\}\left\{\frac{\Omega^2}{(1+M^2)C_s^2\Omega_M^2\rho_o}\frac{\partial}{\partial r}(P_o+\frac{B_o{}^2}{2\mu_o}) - \frac{2M^2(\Omega^2-(\underline{K}_t\cdot\underline{C}_A)^2/M^2)}{r(1+M^2)\Omega_M^2}\right\}\right]$$

$$a_{21}(r) = -\frac{r^2}{\rho_o}\left[\frac{\Omega^4}{(1+M^2)C_s^2\Omega_M^2\Omega_A^2} - \frac{K_t^2}{\Omega_A^2}\right]$$

$$a_{22}(r) = -a_{11}(r)$$

The simplified wave equation has some interesting properties. Note that for certain values of r some of the elements of the reduced matrix A(r) in equation (2.45) can become singular. These singularities are given by

$$\omega = K_t V_o(r)\ \cos\chi_v$$

$$\omega = K_t V_o(r)\ \cos\chi_v \pm \frac{K_t C_A(r) C_s(r)\ \cos\chi_b}{\sqrt{C_A^2 + C_s^2}} \tag{2.46}$$

$$\omega = K_t V_o(r)\ \cos\chi_v \pm K_t C_A(r)\ \cos\chi_b$$

and they correspond to the propagation of drift waves, slow magnetoacoustic waves and guided Alfven waves, respectively. The singularity associated with the guided Alfven waves occurs due to the presence of a magnetic field in the plasma. However, the singularity associated with the slow magnetoacoustic waves requires both, the presence of a magnetic field and of finite thermal effects in the plasma. Another feature that we shall discuss in a later section is the association of these waves to the Kelvin-Helmholtz instability problem. We shall demonstrate that the presence of the magnetic field perpendicular to the flow velocity has some stabilizing effects. From now on we shall refer to the singularities associated with the guided Alfven waves and the slow magnetoacoustic waves as hydromagnetic singularities and the one associated with drift waves as the hydrodynamic singularity. If we localize the hydromagnetic singularities in the ω-axis we find that depending on whether the sound velocity is smaller or larger than the Alfven velocity, i.e. low or high β-plasma respectively, the position of this singularity changes. Also, note that each of the hydromagnetic singularities bifurcates into two components that corresponds to propagation parallel or anti-parallel to the magnetic field. However, these bifurcations do not occur at the same spatial location and are not symmetrical relative to the hydrodynamic singularity since the plasma is inhomogeneous. Note, from equation (2.46) that the hydromagnetic singularities converge to the hydrodynamic singularity when the angle χ_b approaches 90°, i.e. when the wave propagation is perpendicular to the magnetic field. In addition note that the component of the phase velocity along the magnetic field is much greater than the perpendicular one, since the Alfven velocity is much greater than the plasma flow velocity.

As we previously mentioned in the first chapter, the Kelvin-Helmholtz instability is more unstable for short wavelengths modes propagating along the flow velocity. We have also mentioned that for the interchange instability the most unstable modes are those that do not change or deform the magnetic field lines. This suggests that the wavelength along the magnetic field is much greater than the wavelength perpendicular to it. It also implies that the shear flow-magnetic field line interchange problem is mostly associated with the hydrodynamic singularity which is related to drift waves, since a finite component along the magnetic field is stabilizing. Therefore, our problem reduces to the study of modes such that satisfy the condition $\chi_b \simeq 90°$. Nevertheless, for reasons of completeness, a study of the solution of the wave equation near the hydromagnetic singularities shall be presented in Appendix A. From the study of the hydromagnetic singularities we find that in a medium whose properties change with position, the radial wavelength decreases to zero as a wave approaches any of these critical levels. From the analysis of the hydromagnetic singularities we showed that for a purely convective system the most unstable mode is perpendicular to the magnetic field. We have also shown that these hydromagnetic modes can be propagating or evanescent, depending upon the magnitude of the frequency $|\omega_A^2|$ with respect to the magnetic Brunt-Väisälä frequency $|\Omega_{BV}^2|$.

Let us now confine our attention to those modes which propagate perpendicular to the magnetic field, i.e. $\chi_b = 90°$. In this situation both hydromagnetic singularities converge to the hydrodynamic case. After rearranging the wave equation (2.44) in order to see the effect of the hydrodynamic singularity, the simplified equation becomes

$$\frac{d}{dr}\tilde{\chi}(r) = A(r)\tilde{X}(r) \tag{2.47}$$

where $\tilde{\chi}(r)$ is now defined in terms of the radial velocity fluctuation $\tilde{V}_r$ and the pressure fluctuation as follows:

$$\chi(r) = \begin{vmatrix} \tilde{P}_T \\ r^2\tilde{V}_r \end{vmatrix} \tag{2.48}$$

and where A(r) is a two-by-two matrix whose elements are given by:

$$a_{11}(r) = -\left[\frac{(\bar{g}+2C_A^{\,2}/r)}{C_A^{\,2}+C_s^{\,2}} - \frac{2K_t}{r\Omega}(V_o+\Omega_\varepsilon r)\right]$$

$$a_{12}(r) = \frac{i\rho_o}{r^2\Omega}\left[\Omega^2-\Omega_{BV}^2 - \frac{4}{r^2}(V_o+\Omega_\varepsilon r)^2 - \frac{2}{r}(V_o+\Omega_\varepsilon r)\left\{\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right\}\right] \tag{2.49}$$

$$a_{21}(r) = \frac{ir^2\Omega}{\rho_o}\left[\frac{1}{C_A^{\,2}+C_s^2} - \frac{K_t^{\,2}}{\Omega^2}\right]$$

$$a_{22}(r) = -\left[a_{11}(r) + \frac{K_t}{\Omega}\left\{\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right\}\right]$$

and where $\Omega_{BV}^2(r)$ is defined as the magnetic Brunt-Väisälä frequency given by

$$\Omega_{BV}^2(r) = -\bar{g}\,\frac{\partial \ell n \rho_o}{\partial r} + \frac{(\bar{g}+2C_A^2/r)}{\gamma(1+M^2)}\frac{\partial \ell n P_o}{\partial r} + \frac{M^2(\bar{g}-2C_s^2/r)}{(1+M^2)}\left\{\frac{\partial \ell n B_o}{\partial r} - \frac{1}{r}\right\} - \frac{2\Omega_E}{r}\frac{\partial}{\partial r}(rV_o) \qquad (2.50)$$

Note that in this matrizant wave equation the only singularity is given by $\Omega(r) = 0$, i.e. $\omega = K_t V_o(r)$. As we have previously mentioned, this singularity is associated with the propagation of drift waves. If a plasma medium has gradients of density, temperature, magnetic field, etc. these inhomogeneities give rise to particle drifts (or currents). Due to these particle drifts, plasma oscillations which are called drift waves, may be excited and move across the magnetic field with a phase velocity on the order of the diamagnetic drift velocity of the plasma particles. These plasma oscillations are modes that depend on the inhomogeneities of the plasma medium. Drift waves are supported by these gradients and the kinetic energy of these drifts can be transferred to the plasma oscillations, thus creating an instability (Mikhailovskii, 1967; Krall, 1968; Chen, 1974; Haseqawa, 1975; Cap, 1978 and Schmidt, 1979).

Let us now derive the dispersion relation for a quasiuniform medium. As in hydrodynamics, in MHD theory it is difficult, in a non-uniform medium to specify which part of an oscillatory motion corresponds to an upward travelling wave and which to a downward travelling wave since there is a continual interchange between the two. In the uniform medium, on the other hand, precise and physically important identifications can be made. In this subsequent analysis we shall consider a quasi-uniform medium, i.e. a medium whose properties do not vary very much over a radial wavelength. This condition on the medium is equivalent to a WKB approximation on the wave equation (2.47). In addition we shall also consider that the Doppler shifted frequency $\Omega(r)$ varies very slowly as a function of the radial coordinate. To simplify the analysis, let us modify the wave equation (2.47) by transforming it into the so-called "transmission-line" form (Madden, 1972). These forms rearrange the matrix in such a way that the main diagonal elements are zero (see Appendix A). After some algebraic manipulations the rearranged matrizant wave equation becomes:

$$\frac{d}{dr}\begin{vmatrix} e^{-\int a_{11}(r')dr'}\,\tilde{P}_T \\ e^{-\int a_{22}(r')dr'}\,r^2\tilde{V}_r \end{vmatrix} = \begin{vmatrix} 0 & e^{-\int (a_{11}-a_{22})dr'}\,a_{12}(r) \\ e^{\int (a_{11}-a_{22})dr'}\,a_{21}(r) & 0 \end{vmatrix} \begin{vmatrix} e^{-\int a_{11}(r')dr'}\,\tilde{P}_T \\ e^{-\int a_{22}(r')dr'}\,r^2\tilde{V}_r \end{vmatrix} \tag{2.51}$$

Therefore, from the eigenvalues of the transformed matrizant wave equation we find the dispersion relation

$$K_r^{\,2}(r) = K_t^{\,2}\left[\frac{\Omega_{BV}^2}{\Omega^2} - 1\right] + \frac{1}{C_A^2+C_s^2}\left[\Omega^2 - \Omega_{BV}^2\right] \qquad (2.52)$$

Therefore, for a quasiuniform medium the general solution of the transformed wave equation (2.51) is given by

$$\chi(r) = C_1 V_+ e^{ik_r r} + C_2 V_- e^{-ik_r r} \qquad (2.53)$$

where C_1 and C_2 are arbitrary constants of integration, V_+ and V_- are the column eigenvectors of the transformed matrix A(r) in equation (2.51) and k_r is the radial wavevector defined in equation (2.52) which basically corresponds to the eigenvalues of the matrix A(r).

We shall assume that $C_A^2 + C_s^2$ is large. To study the nature of k_r on the basis of quasi-uniformity, two situations shall be considered separately. In the first case the Doppler shifted frequency Ω is much greater than the magnetic Brunt-Väisälä frequency Ω_{BV}, i.e. high frequency eigenmodes. Thus, the dispersion relation (2.52) becomes

$$\omega = K_t V_o(r) \pm \left[\left\{K_r^2 + K_t^2\right\}\cdot\left\{C_A^2 + C_s^2\right\}\right]^{1/2} \tag{2.54}$$

The second situation occurs when the Doppler shifted frequency is much smaller than the magnetic Brunt-Väisälä frequency, i.e. low frequence eigenmodes. In this case the dispersion relation (2.52) gives

$$\omega = K_t V_o(r) \pm \frac{K_t \Omega_{BV}(r)}{\sqrt{K_t^2+K_r^2}} \tag{2.55}$$

The dispersion relationship in equation (2.54) for high frequency eigenmodes gives the propagation properties of fast magnetoacoustic waves. These waves are non-dispersive and they propagate perpendicular to the magnetic field.
These waves will propagate only if

$$\left| \frac{\Omega}{K_t} \right| > (C_A^2 + C_s^2)^{1/2} \tag{2.56}$$

In the case of low frequency eigenmodes, the dispersion relation (2.55) gives the propagation properties of drift waves. These waves are dispersive since, as we shall discuss in a later section, their phase and group velocities are different. These waves will propogate only if the condition

$$\left| \frac{\Omega}{K_t} \right| < (C_A^2 + C_s^2)^{1/2} \tag{2.57}$$

is satisfied.

By dispersion we mean that the phase velocity is dependent on the wavelength causing both the group and phase velocities to be different. Dispersion can arise from two distinct causes. It can be structural in origin, i.e. it will depend on the properties of the medium, or it can be geometric in origin and arise from interference effects due to reflections of the boundary of the plasma medium.

The dispersion relation (2.52) for both low and high frequency waves can be represented in a different way. Assuming that the medium properties and the Doppler shifted frequency vary very slowly as a function of the radial distance and considering rotational effects are very small, we can rearrange the dispersion relation (2.52) in the form

$$\frac{\hat{K}_r^2}{a^2} + \frac{\hat{K}_t^2}{b^2} = 1 \qquad (2.58)$$

which represents the equation for a general conical surface and where the coefficients are defined by

$$\hat{K}_r = \frac{K_r \, (C_A^2 + C_s^2)^{1/2}}{\Omega_{BV}} \quad , \quad \hat{K}_t = \frac{K_t (C_A^2 + C_s^2)^{1/2}}{\Omega_{BV}}$$

$$a^2 = \left[\frac{\Omega^2}{\Omega_{BV}^2} - 1 \right] \quad , \quad b^2 = \hat{\Omega}^2 = \frac{\Omega^2}{\Omega_{BV}^2} \qquad (2.59)$$

Observe that $\hat{K}_r$ and $\hat{K}_t$ represents the normalized radial and horizontal wavevectors components or the "magnetic index of refraction" in analogy with electromagnetic waves in a transparent medium. The surface generated by equation (2.58) describe the propagation properties of the eigenmodes of the system. The propagation surfaces are plotted in Figure 2.1. For fast magnetoacoustic or unguided Alfven modes $a^2 > o$ and $b^2 > o$ and the contours are ellipses of constant normalized Doppler shifted frequency in the range from 1.2 to 1.8. This normalized frequency $\hat{\Omega}$ is the ratio between the Doppler shifted and the magnetic Brunt-Väisälä frequencies. These ellipses represent the high frequency eigenmodes, i.e. $|\Omega| > |\Omega_{BV}|$ with major semi-axis along $\hat{K}_t$. For drift waves, $a^2 < o$ and $b^2 > o$ and the contours are hyperbolae of constant normalized Doppler shifted frequencies in the range from 0.2 to 0.8. These hyperbolae represent the low frequency eigenmodes, i.e. $|\Omega| < |\Omega_{BV}|$ and they are concave toward the positive and negative K_t -axis. Notice the analogy of these curves in Figure 2.1 to the one described by Hines (1960) and Claerbout (1967) for acoustic-gravity waves.

In Figure 2.2 we show the $\hat{\Omega}$ - $\hat{K}_r$ dispersion plot for constant $\hat{K}_t$ contours and in Figure 2.3 we give the $\hat{\Omega}$ - $\hat{K}_t$ diagram for constant $\hat{K}_r$ contours. These curves can be used to analyze qualitatively the radial and horizontal phase and group velocities for unguided (fast magnetoacoustic) Alfven waves and drift waves. The most significant observation to be made comes from Figure 2.2 where it may be seen that for drift waves, the radial phase and group velocities are oppositely directed. Thus, if the energy that sustains the drift waves has propagated upwards, then observations of the waves themselves would show a downward movement.

Equations (2.54) and (2.55) can also be used to study the components of the phase and group velocities for fast magnetoacoustic waves and for drift waves. The group velocity gives the direction of energy flow (except in a highly dispersive medium) whereas the phase velocity is the observed movement of the peaks and troughs of the wave. In a highly dispersive medium, the concept of group velocity becomes meaningless because there are not enough waves of nearly equal phase velocities to allow a wave packet to form. Therefore each component will travel individually with its own phase velocity.

Assuming a slowly varying medium and using equation 2.54 we can get the components of the phase velocity for unguided Alfven waves as

$$\frac{\omega}{K_t} = V_o \pm \left| \frac{(C_A^2 + C_s^2)^{1/2}}{\cos\theta_K} \right|$$

$$\frac{\omega}{K_r} = V_o \cot \theta_K \pm \left| \frac{(C_A^2 + C_s^2)^{1/2}}{\sin\theta_K} \right| \tag{2.60}$$

where θ_K is the angle of propagation defined by

$$\theta_K = \tan^{-1}\left[\frac{K_r}{K_t}\right] \tag{2.61}$$

The group velocity components for these waves are given by:

$$\frac{\partial\omega}{\partial K_t} = V_o \pm \left| (C_A^2 + C_s^2)^{1/2} \cos \theta_K \right|$$

$$\frac{\partial\omega}{\partial K_r} = \pm \left| (C_A^2 + C_s^2)^{1/2} \sin \theta_K \right| \tag{2.62}$$

In the case of drift waves, the phase velocity components are obtained from equation (2.54) as:

$$\frac{\omega}{K_t} = V_o \pm \left| \frac{\Omega_{BV}}{K_t} \cos \theta_K \right|$$

$$\frac{\omega}{K_r} = V_o \cot \theta_K \pm \left| \frac{\Omega_{BV} \cos^2 \theta_K}{K_t \sin \theta_K} \right| \tag{2.63}$$

and the group velocity components are:

$$\frac{\partial\omega}{\partial K_t} = V_o \pm \left| \frac{\Omega_{BV} \sin^2 \theta_K \cos \theta_K}{K_t} \right|$$

$$\frac{\partial\omega}{\partial K_r} = \mp \frac{\Omega_{BV} \cos^2\theta_K \sin \theta_K}{K_t} \tag{2.64}$$

Note that in the case of unguided Alfven waves, when the angle of propagation θ_K approaches 90°, both the radial component of the phase and group velocities approaches the same limit, i.e. $\pm (C_A^2+C_s^2)^{1/2}$. However, the horizontal phase and group velocities approaches $\pm \infty$ and the flow velocity V_o, respectively. These should be expected since along the radial direction, due to compression and rarefactions of both the plasma and the magnetic field, the fluctuations will propagate with an effective velocity given by the root mean squared of the sound velocity plus the Alfven velocity. In the horizontal direction the behavior is quite different since the only motion consists of the plasma flow velocity. Similarly, when the angle of propagation θ_K approaches 0°, both the horizontal component of the phase and group velocity approaches the same limit, i.e. $V_o \pm (C_A^2 + C_s^2)^{1/2}$, In this case the radial components of the phase velocity approaches infinity while the group velocity approaches zero.

For the case of drift waves, when the angle of propagation θ_K approaches 90°, both the horizontal phase and group velocities approach the plasma flow velocity V_o. In addition, both the radial phase and group velocity approach zero. However as the angle of propagation θ_K approaches 0° the horizontal phase velocity approaches $V_o \pm |\Omega_{BV}/K_t|$ while the corresponding group velocity tends to V_o. Similarly, the radial phase velocity component approaches infinity while the corresponding group velocity approaches zero. Observe that in this case an effective velocity Ω_{BV}/K_t due to "buoyant" effects modify the plasma flow velocity.

Relations (2.60) to (2.62) for fast magnetoacoustic waves and equations (2.63) to (2.64) for drift waves show how the energy of these waves is distributed as the wave propagates through the medium and the angle of propagation changes.

Note from the dispersion relation (2.55) that in the case of drift waves, the angle of propagation can also be expressed in terms of the Brunt-Väisälä frequency as

$$\tan \theta_K = \frac{K_r}{K_t} = \left[\frac{\Omega_{BV}^2}{\Omega^2} - 1 \right]^{1/2} \tag{2.65}$$

which implies that

$$\Omega = \pm \; |\Omega_{BV} \cos\theta_K| \tag{2.66}$$

Expression (2.66) is a surprisingly simple and useful relation between the wave frequency, the propagation angle and their dependence on the fundamental hydromagnetic Brunt-Väisälä frequency. Observe that radial variations in the model parameters will vary the value of k_r in equation (2.55) and (2.65). The most important variations are those imposed by gradients in the medium properties. If $k_r{}^2$ becomes negative and remains negative for at least half a wavelength, then the radially propagating waves will not be transmitted or propagated through the region. This means that the waves are evanescent or trapped and they can only propagate horizontally in that region. Therefore there will be a point, called the Alfven cut-off at which $k_r^2 = 0$, thus giving rise to reflections and separating both the evanescent region from the propagating region. This point in the frequency domain is given when

$$\Omega(r) = \Omega_{BV}(r) \tag{2.67}$$

and it represents the reflection condition for both high and low frequency eigenmodes. This condition also implies that for a given maximum flow velocity (max V_o) there will be a minimum wavevector K_t for which the waves will remain trapped and no propagation outside of the shear flow zone will occur. This wavevector is given by

$$K_t \geq \min (K_t) = \frac{\omega - \Omega_{BV}}{\max(V_0)} \qquad (2.68)$$

where Ω_{BV} is the local hydromagnetic Brunt-Väisälä frequency at the point where the flow velocity is maximum.

The two modes, fast magnetoacoustic and drift waves are analogous to the sound wave and the gravity wave respectively of the hydrodynamic problem in the Earth's atmosphere. It is possible to show that our problem goes over smoothly to the atmosphere situation as the magnetic field goes to zero. For this reason we have used the terminology of the atmospheric acoustic-gravity wave problem by merely adding the magnetic field effects and therefore introducing in anology the concepts of magnetic Brunt-Väisälä frequency and the magnetic Richardson number.

II.4 Wave Impedance and Boundary Conditions

In this section we shall discuss the electromagnetic and the mechanical boundary conditions that propagating waves must satisfy at the boundary between two medium.

When a propagating wave in a plasma medium impinges on the boundary of a contiguous second medium, a reflected wave is generated in the first medium and a transmitted wave in the second medium. The ratios of the respective intensities and pressure amplitudes of the reflected and transmitted waves to those of the incident wave depend on the characteristic impedances of the two media and on the angle of incidence of the incident wave.

The electromagnetic boundary conditions are the continuity of the radial component of the magnetic field and continuity of the tangential electric fields. It is found that the normal component of the magnetic field is indeed continuous, and in fact vanishes. This is not a surprising result since in the ambient state, there was no normal component of the magnetic field. Furthermore, the field lines are struck to the infinitely conductive plasma, and since no plasma crossed the perturbed boundary, neither did the magnetic field.

There are two mechanical boundary conditions that must be satisfied at all times and at all points on the plane surface separating the two media. These are: i) the dynamic condition which states that the total pressures, i.e., thermal plus magnetic pressure, on the two sides of the boundary are equal; and ii) the kinematic condition which states that the plasma parcel velocities normal to the interface are equal. The first condition of continuity of pressure results from the fundamental law that the pressure in a fluid plasma is a continuous, single valued function. The second condition is equivalent to the requirement that the two media remain in constant contact at the boundary.

Some subtleties arise upon the application of the kinematic condition. We know that linear wave theory is only strictly applicable to waves of infinitesimal amplitude. If the amplitude is sufficiently small compared with the wavelength, the normal to the interface tends to be radial and the continuity of radial velocities becomes

$$(\delta V_r)_1 = (\delta V_r)_2 \tag{2.69}$$

This is the form in which the kinematic condition is often applied in wave theory, but it has some limitations. Rigorously, the kinematic condition should have been stated as

$$(\delta \xi_r)_1 = (\delta \xi_r)_2 \tag{2.70}$$

where $\delta\xi_r = \frac{i\delta V_r}{\Omega}$ is the radial displacement of the plasma fluid on each side of the boundary. When the unperturbed plasma flow velocity in the two sides is not equal, i.e., $(\underline{V}_o)_1 \neq (\underline{V}_o)_2$ then condition (2.69) can only be satisfied if because

$$\delta V_r = \frac{D\delta\xi_r}{Dt} = \frac{\partial\delta\xi_r}{\partial t} + \underline{V}_o \cdot \underline{\nabla}\delta\xi_r \qquad (2.71)$$

Another way of representing the continuity of pressure and velocity is by means of the wave impedance. The ratio of the total pressure in a medium to the associated plasma velocity (or displacement) is defined as the characteristic impedance Z of the medium for the particular type of wave motion present. In general the specific impedance is complex and its real part is called the specific resistance and the imaginary part, the specific reactance. This is because the total pressure is not always in phase with the fluid velocity at the boundary. The characteristic impedance of a medium for magnetohydrodynamic wave is analogous to the index of refraction of a transparent medium for light waves.

The characteristic wave impedance Z can be computed using the wave equation (2.51) assuming a quasi-uniform medium. After some algebraic manipulation of the wave equation, the wave impedance is given by:

$$Z^2 + C = 0 \qquad (2.72)$$

where Z is defined by

$$Z = \frac{\tilde{P}_T}{r^2\tilde{V}_r} \qquad (2.73)$$

and the coefficient C is given by

$$C = \frac{\rho_o{}^2}{r^4\Omega^2} \frac{(\Omega^2 - \Omega_{BV}^2)}{\left[\frac{1}{C_A^2+C_s^2} - \frac{K_t^2}{\Omega^2}\right]} \qquad (2.74)$$

If we assumed, as we previously mentioned, that the rotational effects are very small in comparison to the gradient effects, then a simple expression for the characteristic wave impedance for fast magnetoacoustic waves and for drift waves could be obtained. Considering the case of fast magnetoacoustic waves where the condition $|\Omega|>>|\Omega_{BV}|$ is satisfied, the wave impedance is given by

$$r^2Z = \pm \rho_o(C_A^2 + C_s^2)^{1/2} \qquad (2.75)$$

and for drift waves, where the condition $|\Omega| << |\Omega_{BV}|$ is assumed the wave impedance becomes

$$r^2 Z = \pm \rho_o \Omega_{BV}/K_t \tag{2.76}$$

By the use of the impedance matching boundary condition we could find an approximate dispersion relation in the regime where we would expect the Kelvin-Helmholtz instability to occur. Similarly with previous calculations, we can transform the wave equation (2.44) into the so-called "transmission line" or canonical form, in order to simplify the dispersion analysis (Madden, 1972; Claerbout, et al., 1968). Therefore the transmission line from of equation (2.44) becomes

$$\frac{d}{dr}\left|\begin{array}{c} e^{-\int a_{11}(r')dr'}\,\tilde{P}_T \\ \\ e^{-\int a_{22}(r')dr'}\,r^2\tilde{\xi}_r \end{array}\right| = \left|\begin{array}{cc} 0 & e^{-\int (a_{11}-a_{22})dr'}\,a(r) \\ \\ e^{\int (a_{11}-a_{22})dr'}\,a(r) & 0 \end{array}\right| \left|\begin{array}{c} e^{-\int a_{11}(r')dr'}\,\tilde{P}_T \\ \\ e^{-\int a_{22}(r')dr'}\,r^2\tilde{\xi}_r \end{array}\right| \tag{2.77}$$

where the matrix elements $a_{ij}(r)$ are given in equation (2.45). The simplicity of this form allows us to study the qualitative behavior of guided Alfven waves or drift waves and has the advantage of allowing continuity in the dependent variables even if singularities in the matrix elements exist.

For simplicity we shall assume that the medium properties are slowly varying functions of r. Therefore curvature and rotational effects can be neglected in comparison with the gradient terms. From the analogy to transmission lines we have for the radial propagation

$$K_r^2 = K_t^2 \left[\frac{\Omega_{BV}^2}{\Omega_A^{\ 2}} - 1 \right] + \frac{\Omega^4}{(C_A^2 + C_s^2)\,\Omega_M^2} \left[1 - \frac{\Omega_{BV}^2}{\Omega 2} \right] \tag{2.78}$$

and the characteristic impedance

$$\frac{\tilde{P}_T}{\tilde{\xi}_r} = \pm\, \rho_o \frac{\left\{ \Omega_A^2 - \Omega_{BV}^2 \right\}}{iK_r} \tag{2.79}$$

We shall now assume that the plasmapause could be represented as a discontinuity between two different constant media. Therefore the correct boundary condition at the discontinuity $r = r_p$ is given by the matching of the wave impedance at this point. Since we assume that the medium has constant properties, then Ω_{BV}^2 must vanish. Also we could neglect the second term at the right hand side of the dispersion relation (2.78) since the Alfven velocity is very large for the plasmapause region. In this case the dispersion relation (2.78) simple reduces to

$$K_r = \pm\, iK_t \tag{2.80}$$

which is the same value obtained by Chandrasekhar (1961) and by Laster (1970). Since the value of K_t is common for both media at the plasmapause and assuming that the wave solution must decay away from the discontinuity, then by matching the impedances of the two media we finally get

$$\frac{\rho_1 \, \Omega_{A1}^2}{K_t} = -\frac{\rho_2 \, \Omega_{A2}^2}{K_t} \quad \text{at} \quad r = r_P \tag{2.81}$$

where the subscripts 1 and 2 refer to the properties above and below the plasmapause interface. Recallling that we have taken a coordinate system fixed in the center of the Earth and corrotating with the denser plasmasphere, we have $\Omega = \omega - (K_t \cdot C_A)_1^2$ and $\Omega = \Omega - (K_t \cdot C_A)_2^2$. The expression given in (2.81) is the dispersion relation for the two half-space plasmapause problem. It provides a relation between ω and K_t which must be satisfied in order for a solution to exist for the problem as posed. We shall now study the dispersion relation by solving this equation for the eigenfrequency $\omega(K_t)$. Solving equation (2.81) for we simply get

$$\omega = \frac{\rho_2}{\rho_1+\rho_2} K_t V_{o2} \cos\chi_{v_2} \pm \frac{K_t\sqrt{\rho_1\rho_2}}{\rho_1+\rho_2} \left[\left(\frac{\rho_1+\rho_2}{\rho_2}\right) C_{A1}^2 \cos^2\chi_{b1} + \left(\frac{\rho_1+\rho_2}{\rho_1}\right) C_{A2}^2 \cos^2\chi_{b2} - V_{o2} \cos^2\chi_{v_2} \right]^{1/2}. \tag{2.82}$$

These modes can become unstable if the quantity under the radical is negative. This occurs if

$$|V_{o2}\cos\chi_V| > \left[\left(\frac{\rho_1+\rho_2}{\rho_2}\right) C_{A_1}^2\cos^2\chi_{b_1} + \left(\frac{\rho_1+\rho_2}{\rho_1}\right) C_{A_2}^2\cos^2\chi_{b_2}\right]^{1/2} \tag{2.83}$$

and in this case the growth rate for the instability is given by

$$\mathrm{Im}\,\omega=\omega_i=\pm\left|\frac{K_t\sqrt{\rho_1\rho_2}}{\rho_1+\rho_2}\left[\left(\frac{\rho_1+\rho_2}{\rho_2}\right) C_{A_1}^2\cos^2\chi_{b_1} + \left(\frac{\rho_1+\rho_2}{\rho_1}\right) C_{A_2}^2\cos^2\chi_{b_2} - V_{o_2}^2\cos^2\chi_{V_2}\right]^{1/2}\right| \tag{2.84}$$

Observe that the condition for instability given in equation (2.83) is mostly satisfied when the propagation vector is closely perpendicular to the magnetic field. Note that when the streaming vanishes, these solutions are guided Alfven waves with speeds determined by the average density of the form

$$\omega=\pm\frac{K_t}{\sqrt{\rho_1+\rho_2}}\left[\rho_1 C_{A1}^2\cos^2\chi_{b1} + \rho_2 C_{A2}^2\cos^2\chi_{b2}\right]^{1/2} \tag{2.85}$$

II.5 Dispersion Analysis in the Vicinity of the Hydrodynamic Critical Level

In this section we shall present an anlysis of the hydrodynamic singularity $\Omega(r_c) = 0$ in the wave equation (2.47). We will discuss the behavior of the solution of the wave equation and its dispersion relation in the vicinity of the singularity $r = r_c$. This singularity has come to be known as critical level since as the mathematical singularity suggests, the wave behavior in that vicinity can be rather dramatic (Booker et al., 1967; Claerbout, 1967; Acheson, 1972; Rudraiah et al., 1972a,b,c, 1976, 1977). This critical level can be defined as a site where strong coupling between a drift wave and the background plasma flow occurs. The singularity $\Omega(r_c) = 0$ represents a critical level r_c at which the horizontal phase velocity ω/K_t matches exactly the plasma flow velocity $V_o(r_c)$.

Notice that there will be a rapid variation of the Doppler shifted frequency across the medium since the flow velocity changes as a function of the radial distance. Let us initially confine our attention to the solution of the wave equation (2.47) in the neighborhood of a rapidly varying medium. For this let us expand $\Omega(r)$ in a Taylor series around some point $\bar{r}$ in the varying medium as follows:

$$\frac{1}{\Omega(r)} \approx \frac{\left[-K_t\left\{\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right\}_{r=\bar{r}}\right]^{-1}}{r - \bar{r} - \Delta(\bar{r})} \tag{2.86}$$

where $\Delta(r)$ is defined by

$$\Delta(r) = \frac{\omega_r + i\omega_i - K_t V_o(\bar{r})}{K_t\left\{\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right\}_{r=\bar{r}}} \tag{2.87}$$

and where ω_r and ω_i are the real and imaginary part of the wave frequency ω. Then substituting the expansion (2.86) into the wave equation (2.47) we get the modified wave equation valid around the rapidly varying medium as

$$\frac{d}{dr}\tilde{X}(r) = \frac{B(\bar{r})}{r - r - \Delta(r)}\tilde{X}(r) \tag{2.88}$$

where $\tilde{X}(r)$ is the vector amplitude defined in equation (2.48) and $B(\bar{r})$ is a two by two non-singular matrix evaluated at the point $\bar{r}$ and whose elements are given by

$$b_{11}(r) = \frac{\left[\frac{\Omega(\bar{g}+2C_A^2/r)}{C_A^2+C_s^2} - \frac{2K_t}{r}(V_o+\Omega_E r)\right]_{r=\bar{r}}}{K_t\left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right]_{r=\bar{r}}}$$

$$b_{12}(r) = -\frac{i\rho_o}{r^2}\frac{\left[\Omega^2-\Omega_{BV}^2\right]_{r=\bar{r}}}{K_t\left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right]_{r=\bar{r}}}$$

$$b_{21}(r) = -\frac{ir^2}{\rho_o}\frac{\left\{\frac{\Omega^2}{C_A^2+C_s^2} - K_t^2\right\}_{r=\bar{r}}}{K_t\left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right]_{r=\bar{r}}} \tag{2.89}$$

$$b_{22}(r) = -\frac{\left\{\frac{\Omega(\bar{g}+2C_A^2/r)}{C_A^2+C_s^2} - \frac{2K_t}{r}(V_o+\Omega_E r) - K_t\left(\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right)\right\}_{r=\bar{r}}}{K_t\left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right]_{r=\bar{r}}}$$

Therefore, the solution of the wave equation in the varying medium can be easily found by integrating equation (2.88) to give

$$\tilde{\chi}(r) = \left| e^{B(\bar{r})\,\ln\left(\frac{r-\bar{r}-\Delta(\bar{r})}{r_o-\bar{r}-\Delta(\bar{r})}\right)} \right| \tilde{\chi}(r_o) \tag{2.90}$$

where $\tilde{X}(r_o)$ is an initial boundary condition at some initial point r_o. Observe that the quantity in bracket is a function of a matrix which can be easily determined by Sylvester's theorem which gaves

$$e^{B\ell n(\frac{\Delta r}{\Delta r_o})} = \sum_{i=1}^{n} e^{\lambda_i \ell n(\frac{\Delta r}{\Delta r_o})} \frac{\prod_{s=i}^{n}(\lambda_s I - B)}{\prod_{s=i}^{n}(\lambda_s - \lambda_i)} \quad (s \neq i) \tag{2.91}$$

where the λ_i's are the eigenvalues of the matrix $B(r)$ and I is the identity matrix.

Observe that during the integration of equation (2.88) we cross the point where the singularity exists, i.e. $\Omega(r)=K_t V_o(r_c)$. To integrate around the singularity we must use contour integration techniques, however it is extremely important to know whether the path of integration goes above or below the singularity. This problem was resolved by Booker and Bretherton (1966) and the correct solution is to select the path of integration below the singularity. The choosing of the path of integration is related to whether or not the causality condition is satisfied (see Briggs, 1961; Krall, et al., 1972). During the integration we found that in the limit of the vanishing of the imaginary part of ω, as we integrate from $r<r_c$ to $r<r_c$ the phase of the argument of the logarithm changes from $-\pi$ to 0. Thus, in the case of a small but finite imaginary part and

in the limit of $r \to r_c$ we find that $\Omega(r) \to K_t V_o(r)$ and also that

$$\lim_{r \to r_c} \Delta(r) \to \frac{i\omega_i}{K_t \left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r} \right]_{r=r_c}} \tag{2.92}$$

We also find, neglecting the imaginary part, that the elements of the matrix B reduce to

$$b_{11}(r_c) = -\frac{2}{r_c} \frac{\left[V_o + \Omega_E r\right]_{r=r_c}}{\left\{ \frac{\partial V_o}{\partial r} - \frac{V_o}{r} \right\}_{r=r_c}}$$

$$b_{12}(r_c) = \frac{i\rho_o}{r_c^2} \frac{\Omega_{BV}^2(r_c)}{K_t \left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r} \right]_{r=r_c}} \tag{2.93}$$

$$b_{21}(r_c) = \frac{i r_c^2}{\rho_o} \frac{K_t}{\left(\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right)_{r=r_c}}$$

$$b_{22}(r_c) = 1 + \frac{2(V_o + \Omega_E r)_{r=r_c}}{r_c \left(\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right)_{r=r_c}}$$

Thus, the solution at the critical level will be given by equation (2.90) expanded by Sylvester's theorem, together with equations (2.92) and (2.93). The eigenvalues of the matrix B at the critical level are determined from the maxtri elements in equation (2.93) as

$$\lambda_{\pm} = 1/2 \pm \mu \quad , \qquad \mu = \sqrt{1/4 - R_i} \tag{2.94}$$

where R_i is defined as the magnetic Richardson number evaluated at the critical point r_c and given by

$$R_i = \frac{\Omega_{BV}^2(r)}{\left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r}\right]^2} \tag{2.95}$$

and where Ω_{BV} has been previously defined in equation (2.50) as the magnetic Brunt-Väisälä frequency.

An important result related to the stability of the system, i.e. the plasmapause, can be obtained from the analysis of the eigenvalues in equation (2.94) and the complex argument of the logarithm in equation (2.90). Note that if $R_i < 0.25$ and the imaginary part of the frequency ω_i is positive, the equation (2.90) will show unstable solutions. Thus, the magnetic Richardson number will indicate whether turbulent motions will persist or decay in the inhomogeneous plasma. Note that, as it was previously mentioned, this number provides a measure of the stabilizing influence of the magnetic buoyancy modified by pressure and density gradients in comparison with the destabilizing effects of the velocity gradients. Therefore, the instability criterion for drift waves propagating in an inhomogeneous plasma with a velocity shear becomes

$$R_i < 0.25 \tag{2.96}$$

Another important result that can be obtained from the analysis of the wave equation around the critical level is the behavior of the dispersion relation in this vicinity. From equations (2.52) and (2.86) we find that as r approaches the critical value r_c, then $\Omega(r_c) \to 0$ and the dispersion relation in the vicinity of the critical level becomes:

$$K_r^2(r) \approx \frac{R_i}{(r-r_c{}^2)} \tag{2.97}$$

which implies that as $r \to r_c$, $K_r(r) \to \infty$ *i.e.* the radial wavelength tends to zero as the wave approaches the critical level. By examining the behavior of the components of the group velocity in the vicinity of the critical level we find how the energy is distributed. Therefore, inserting equation (2.86) into the components of the group velocity in equation (2.64) and rewriting these equations we get:

$$\frac{\partial\omega}{\partial K_t} = V_o \pm \left| \left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r} \right]_{r=r_c} (r-r_c) \right|$$

$$\frac{\partial\omega}{\partial K_r} = \mp \left| \frac{K_t}{R_i^{1/2}} \left[\frac{\partial V_o}{\partial r} - \frac{V_o}{r} \right]_{r=r_c} (r-r_c)^2 \right| \tag{2.98}$$

Note from equations (2.98) that as the wave approaches the critical level r_c it is absorbed by the plasma flow, i.e. it cannot be reflected or transmitted. Another way to examine the absorption phenomena is by considering the time required for a wave energy to propagate from one level r_1 to another level r_2 across the critical level. This time is simply calculated as

$$\Delta t = \int_{r_1}^{r_2} \left[\frac{\partial\omega}{\partial K_r} \right]^{-1} dr = \frac{R_i^{1/2}}{K_t \left\{ \frac{\partial V_o}{\partial r} - \frac{V_o}{r} \right\}_{r=r_c}} \left[\frac{1}{r_1 - r_c} - \frac{1}{r_2 - r_c} \right] \tag{2.99}$$

which implies that as $r \to r_c$, $\Delta t \to \infty$ approaches infinity; therefore the wave energy approaches but never reaches the critical level since it would take an infinite time to arrive.

An important result emerges from the analysis of the instability criterion for drift waves. Observe that the criterion requires the Richardson number to be small (less than 0.25) in order for the system to become unstable. This condition can be achieved by two means: first by an increase in the velocity gradient, or second, by a reduction in the hydromagnetic Brunt-Väisälä frequency. It can be shown that in a situation where the hydromagnetic Brunt-Väisälä becomes small so that the condition

$$\Omega_{BV}^2(r) < 0 \tag{2.100}$$

is satisfied, a purely convective instability is established. In other words, when condition (2.100) is present, the interchange or "ballooning" instability will result. We can also show by examining this condition, that Gold criterion for instability in equation (1.2) is satisfied. Assuming that the effects due to gravitation g are very small in comparison to the effective gravity resulting from curvature effects of a dipole magnetic field and using equations (2.50) and (2.100) we find the instability criterion

$$-\frac{r}{P_o}\frac{\partial P_o}{\partial r} > 4\gamma - \frac{\gamma(1+M)^2}{C_A^2}\ \Omega_E r \frac{\partial}{\partial r}(rV_o) \tag{2.101}$$

which reduces to Gold's result when the rotation of the planet is neglected. This relation implies that if the energy density decreases much faster than a certain rate, the system will become convectively unstable. Observe that this instability condition (2.101) is more susceptible to occur for a fast rotating body than for a non-rotating one. This obviously shows that the centrifugal force has a destabilizing effect in the system. We shall find that for the plasmapause problem, the effect of the rotation of the Earth is extremely small even for situations where this layer is far away from the planet.

Another very important result that we obtained from the instability criterion (2.96) is that before the plasmapause can become convectively unstable (i.e. $\Omega^2_{BV}(r) < 0$), the region will be unstable due to the decrease of the Richardson number (i.e. $R_i < 0.25$). This is an extremely important result since we shall find that the terms which include the thermal energy density gradient in equation (2.50) for the hydromagnetic Brunt-Väisälä frequency will have a dominant effect. Therefore, we shall find that during the injection of the ring current particles, a negative pressure gradient across the plasmapause is established which accounts for the reduction in the Brunt-Väisälä frequency, thus leading to a decrease in the hydromagnetic Richardson number.

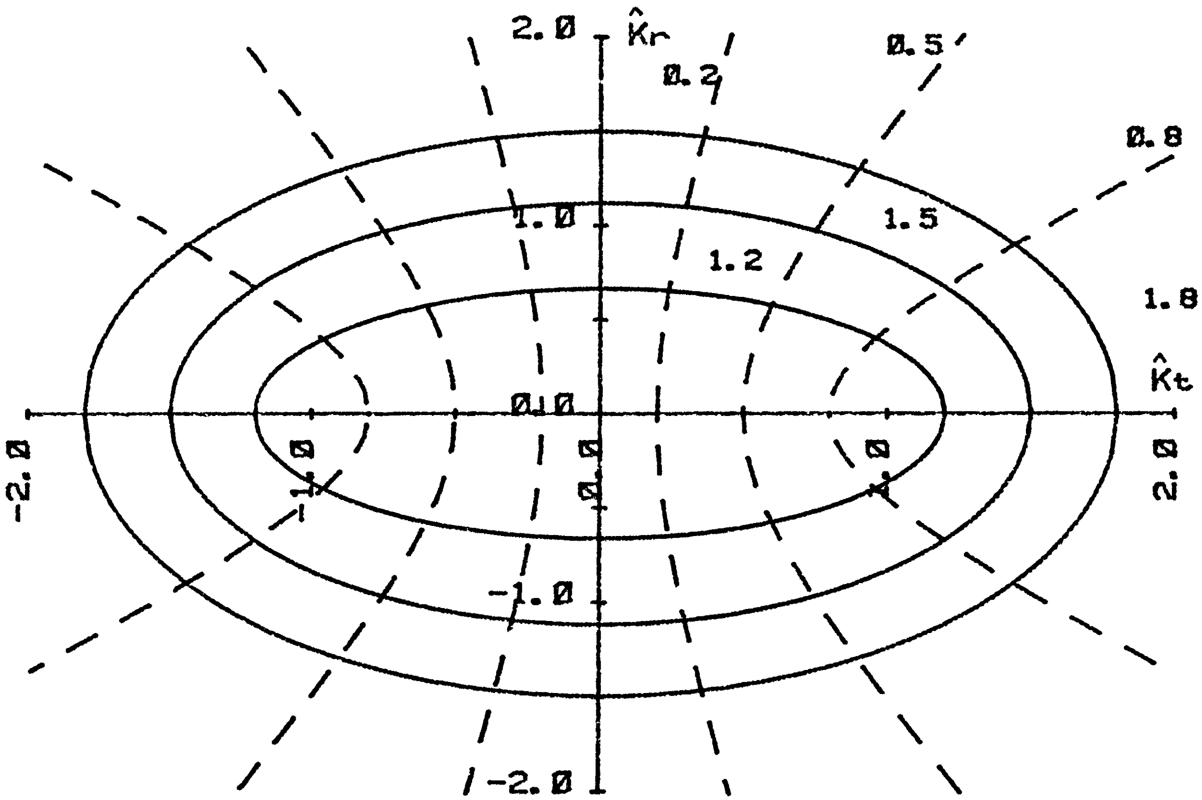

FIGURE 2.1-PROPAGATION SURFACES ($\hat{K}_r$ Vs. $\hat{K}_t$) FOR FAST MAGNETOACOUSTIC WAVES (ELLIPSES) AND FOR DRIFT WAVES (HYPERBOLAE) FOR CONSTANT NORMALIZED DOPPLER SHIFTED FREQUENCY ($\Omega / \Omega_{BV} = \hat{\Omega}$).

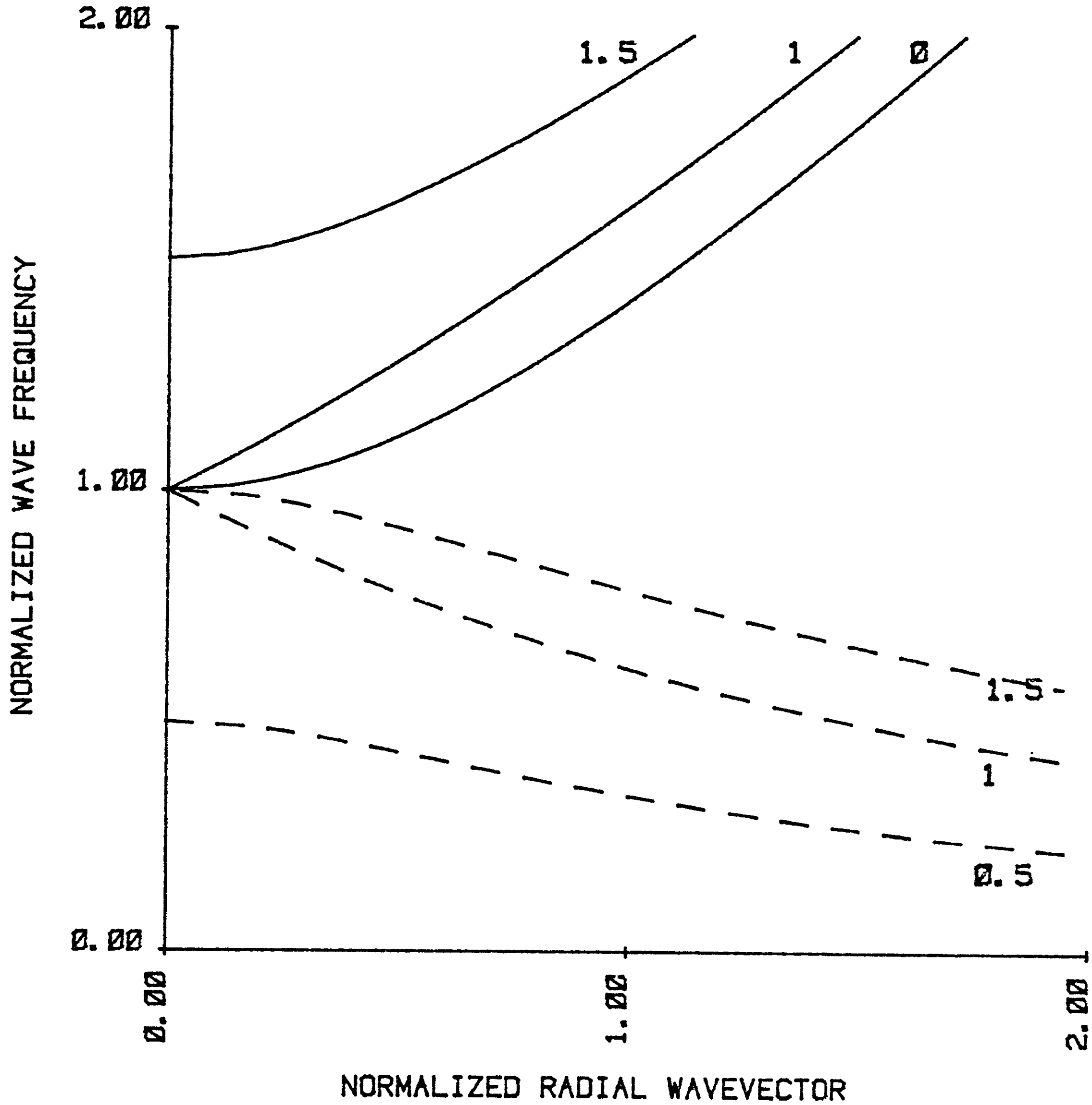

FIGURE 2.2-DISPERSION PLOT ($\hat{\Omega}$ Vs. $\hat{K}_r$) AT CONSTANT $\hat{K}_t$ VALUES, FOR FAST MAGNETOACOUSTIC ($\hat{\Omega}>1$) AND FOR DRIFT ($\hat{\Omega}<1$) WAVES.

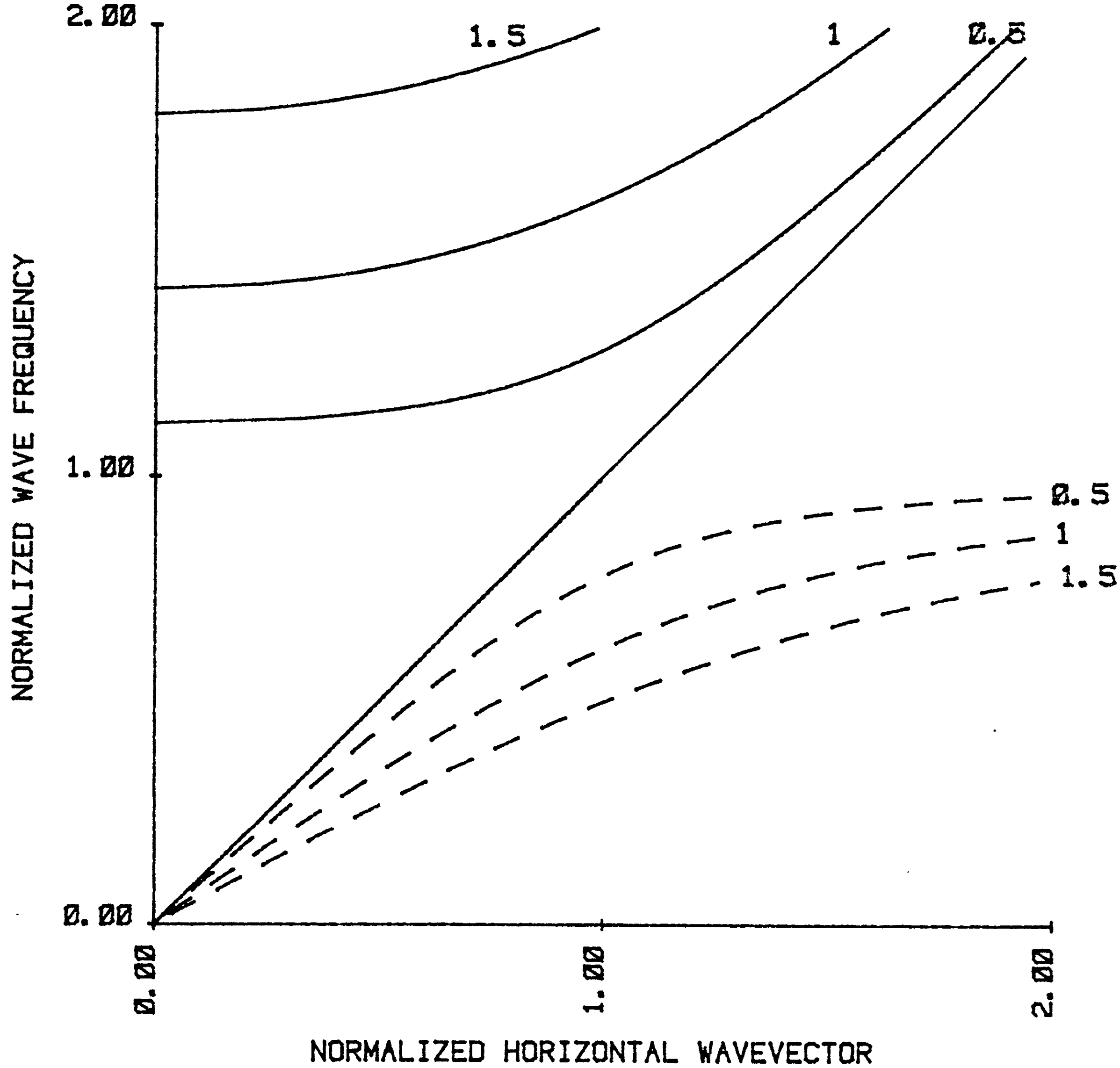

FIGURE 2.3-DISPERSION PLOT ($\hat{\Omega}$ Vs. $\hat{K}_t$) FOR CONSTANT $\hat{K}_r$ VALUES, FOR FAST MAGNETOACOUSTIC ($\Omega>1$) AND FOR DRIFT ($\hat{\Omega}<1$) WAVES.

CHAPTER III

STABILITY ANALYSIS OF CONTINUOUS MODELS AND NUMERICAL RESULTS

III.1 Introductory Remarks

In this chapter we shall modify the matrix wave equation in such a way that its solution can be expressed in terms of simple analytic functions. We shall present the analytic solution of this modified wave equation for a constant medium and in a varying medium, particularly in the vicinity of the critical level. Using these solutions and some physically admissible boundary conditions, a dispersion relation will be derived. This will be our starting point for the numerical procedure. Afterwards, using a simplified model to describe the plasma environment of the plasmapause, a solution for the unstable eigenmodes, their growth rate and their physical significance will be presented. Finally, we incorporate the effects of the finite conductivity of the ionosphere. A qualitative analysis of the ionospheric drag on the magnetic flux tubes and the stability of the plasmapause is investigated. The presence of the hot plasma injection into the plasma pause and its consequence in relation to the ionospheric drag will also be discussed.

III.2 Wave Equation and Dispersion Relation For a Continuous Linear Model

In this section we shall modify the matrix wave equation (2.45) by expressing it into a simple second order differential equation. Then using a simple linear model for the plasmapause environment, together with physically reasonable boundary conditions we shall find the dispersion relation.

The plasmapause will be modeled as a thin transition region of thickness d with linearly varying physical properties, i.e. constant gradients in density, pressure, etc. The transition region will be bounded above by an infinitely thick layer of hot plasma and below by the plasmasphere. The plasmasphere will terminate at the Earth's surface and will be assumed to be thick enough (e.g. about 4 Earth radii) so that the plasmapause region is far from the Earth. We will also consider that the plasma properties of the plasmasphere are constants. To reduce the matrix wave equation (2.45) into a second order differential equation we shall express the total pressure P_T in terms of the radial displacement $\tilde{\xi}_r$ and its derivative. Since we are interested in studying the most unstable modes which corresponds to propagation perpendicular to the magnetic field we consider that $(\underline{k}_t \cdot \underline{B}_o) = 0$.

Let us also assume that rotational effects are very small in comparison with gradient terms. This assumption seems to be in contradiction with previous results (see Lemaire, 1974, 1975) since the rotational effects appears only on the expression for the effective gravity $\overline{g}$ (see equation 2.12) which multiplies the gradients in the Brunt-Väisälä frequency equation (2.50). Therefore significant changes in these gradients may be produced if the centrifugal term overcomes the gravitational term. However, if we compare this effective gravity $\overline{g}$ with the effective gravity terms produced by the curvature of the magnetic field lines and given by $2C_A^2/r$ and $2C_s^2/r$ we find that these latter terms have a much stronger influence on the gradients. In other words the terms and are typically much greater than $\overline{g}$. To simplify this computation we have also neglected the term $(C_A^2 + C_s^2)^{-1}$ since $\Omega^2/k_t^2 << (C_A^2 + C_s^2)$ in the matrix wave equation. Then, after some algebraic manipulations we get

$$\frac{d^2W}{dr^2} + p(r)\frac{dW}{dr} + q(r)\,W = 0 \tag{3.1}$$

where we define

$$\widetilde{W} = r^2 \, \widetilde{\xi}_r \qquad (3.2)$$

$$p(r) = \left(-L_{\rho_o}^{-1} + 2\,\frac{d\ln\,\Omega}{dr}\right) \qquad (3.3)$$

$$q(r) = \frac{k_t^2 \Omega_{BV}^2}{\Omega^2} - k_t^2 + \frac{2k_t}{r\Omega}\left(\frac{\partial V_o}{\partial r} + \Omega_E - \frac{2}{r}(V_o + \Omega_E r)\right) \qquad (3.4)$$

and L_{ρ_o} was previously defined as the density scale height (see equation (1.7)).

Let us consider now the solution of the wave equation (3.1) above and below the plasmapause. Since the plasma properties for our model are constant above and below the plasmapause, the gradient term vanishes. Rotational and curvature terms can also be neglected since their contributions are very small. Under these considerations, the general solution of the wave equation (3.1) above and below the plasmapause region becomes

$$\widetilde{W}(r) = C_1 \exp(k_t r) + C_2 \exp(-k_t r) \qquad (3.5)$$

where C_1 and C_2 are constant of integration to be determined from the boundary conditions.

We shall now calculate the solution of the wave equation (3.1) in the transition region, i.e. the plasmapause. We rearrange equation (3.1) by expanding $\Omega(r)$

in Taylor series around some particular point r' inside the plasmapause, as follows

$$\frac{1}{\Omega(r)} \simeq \frac{(-k_t\{\partial V_o/\partial r - V_o/r\}_{r=r'})^{-1}}{r-r_o} \tag{3.6}$$

where r_o is defined by

$$r_o = r' + \frac{\Omega(r') + i\,\omega_i}{k_t(\partial V_o/\partial r - V_o/r)_{r=r'}} \tag{3.7}$$

Finally, making the substitutions (Chandrasekhar, 1961; Jones, 1968)

$$\tilde{W}=(r-r_o)^{-1}\,\tilde{\psi}\,\exp(r/2L_\rho)\ ,\ \hat{k}=k_t d\ ,\ \hat{z} = \frac{4\hat{k}^2+ d^2/L^2_{\rho_o}}{d}\,(r-r_o)$$

$$R_i= \frac{\Omega^2_{BV}}{(\frac{\partial V_o}{\partial r} - \frac{V_o}{r})^2}\ ,\quad \mu= \frac{1}{4} - R_i\ ,\quad \alpha= \frac{d/L_\rho}{4\hat{k}^2+ d^2/L^2_{\rho_o}} \tag{3.8}$$

the wave equation (3.1) is transformed to

$$\frac{d^2\tilde{\psi}}{d\hat{z}^2} + \left(- \frac{1}{4} + \frac{\alpha}{\hat{z}} + \frac{1/4 - \mu^2}{\hat{z}^2} \right) \tilde{\psi} = 0 \tag{3.9}$$

Note that equation (3.9) is the well-known Whittaker's differential equation. Its general solution may be written in terms of the confluent hypergeometric function or Kummer's solution as follows

$$\tilde{\psi}(\hat{z})= A\, M_{\alpha,\mu}(\hat{z}) + B\, M_{\alpha,-\mu}(\hat{z}) \tag{3.10}$$

where A and B are integration constants. The function $M_{\alpha,\pm\mu}(\hat{z})$ is defined

$$M_{\alpha,\pm\mu}(\hat{z}) = \hat{z}^{\frac{1}{2}\pm\mu}\exp(-\hat{z}/2)\ {}_1F_1\left(\frac{1}{2}-\alpha\pm\mu,\ 1\pm 2\mu,\ \hat{z}\right) \tag{3.11}$$

where the function ${}_1F_1(a,b,\hat{z})$ $\left(a=\frac{1}{2}-\alpha\pm\mu,\ b=1\pm 2\mu\right)$ is the confluent hypergeometric function given by

$${}_1F_1(a,b,z) = 1 + \frac{a}{b}\frac{\hat{z}}{1!} + \frac{a(a+1)}{b(b+1)}\frac{\hat{z}^2}{2!} + \ldots \tag{3.12}$$

Note that the solution of the wave equation (3.10) has a branch point at $\hat{z} = o$ (i.e. $r=r_o$). Since the real part of the Doppler shifted frequency $\Omega(r)$ can vanish inside the transition layer, then $\hat{z} = o$ corresponds to a critical level. However, since we are looking for unstable modes such that $\omega_i>0$ then we require the eigenfunction $\tilde{\Psi}$ to be continuous across the branch point $\hat{z} = o$. Therefore, if there exist any unstable solution in the upper half ω-plane we shall take the path of integration below the branch point as we cross it, thus restricting the argument of $\hat{z}$ according to (see Miles, et al., 1964)

$$-\pi < \arg\hat{z} < 0\ ,\quad \omega_i > 0 \tag{3.13}$$

With this condition, the analytic continuation of the solution (3.10) around the branch point $\hat{z} = o$ can be determined according to

$$M_{\alpha,\pm\mu}(\hat{z}) = M_{-\alpha,\pm\mu}(-\hat{z})\ \exp\left(-i\pi\left(\frac{1}{2}\pm\mu\right)\right) \tag{3.14}$$

Once the solutions inside and outside the plasmapause region are known, the dispersion relationship can be determined using physically reasonable boundary conditions. The boundary conditions that need to be met at the upper and lower boundary of the transition layer is the continuity of the wave impedance. This condition is equivalent to the continuity of the logarithmic derivative of the function $\tilde{W} = r^2 \tilde{\xi}_r(r)$. Additional boundary conditions are applied to the general solution (3.5) in the regions above and below the plasmapause. We use outgoing solutions which are evanescant in these regions. The physical implication of this condition is that the wave is totally reflected in the evanescant region. This is justified by the fact that the Doppler shifted frequency is much greater than the Brunt-Väisälä frequency in these regions.

We have presented the solutions and the boundary conditions to be met at the interfaces of the plasmapause and at far distances. Now we can calculate the dispersion relation or the eigenvalue equation for the plasmapause region. Using the solutions given in equation (3.15) and the general solution in the transition region given by equations (3.10) and (3.8) we match the logarithmic derivatives of $\tilde{W}(r)$ at the upper (r_+) and lower (r_-)

boundaries of the plasmapause. After some extensive algebraic manipulations and using the analytic continuation equation (3.14) of the solution inside the transition region we get the dispersion relation

$$D(\hat{c}_r,\hat{c}_i,\hat{k},R_i)= \exp(i\pi\mu)\left(\frac{\hat{z}_+}{\hat{z}_-}\right)^{\mu} f(\hat{z}_+,\hat{p}_+,\alpha,\mu)\; f(\hat{z}_-,\hat{p}_-,-\alpha,-\mu) - \exp(-i\pi\mu)\left(\frac{\hat{z}_+}{\hat{z}_-}\right)^{-\mu} f(\hat{z}_+,\hat{p}_+,\alpha,-\mu)\; f(\hat{z}_-,\hat{p}_-,-\alpha,\mu)= 0. \tag{3.15}$$

In the dispersion relation (3.15) we have defined the following parameters:

$$\hat{z}_+= \frac{d}{\alpha L_{\rho_o}}(1-\hat{C}) \quad , \quad \hat{z}_-= \frac{d}{\alpha L_{\rho_o}}\hat{C} \tag{3.16}$$

$$\hat{p}_\pm = \frac{\hat{k} \pm d/2L_{\rho_o}}{4\hat{k}^2+ d^2/L_{\rho_o}^2} \tag{3.17}$$

$$f(\hat{z}_\pm,\hat{p}_\pm, \pm\alpha, \pm\mu)= \left(\hat{p}_\pm \hat{z}_\pm - \frac{1}{2} \pm \mu\right) \Phi_{\pm\alpha,\pm\mu}(\hat{z}_\pm) + \hat{z}_\pm \frac{d\Phi_{\pm\alpha,\pm\mu}}{d\hat{z}}(\hat{z}) \tag{3.18}$$

$$\Phi_{\pm\alpha,\pm\mu}(\hat{z}_\pm)= \exp(-\hat{z}_\pm/2)\; {}_1F_1\left(\frac{1}{2}\mp\alpha\pm\mu,\ 1\pm 2\mu\ ,\ \hat{z}_\pm\right) \tag{3.19}$$

where $\hat{c}$ is the normalized complex phase velocity given by

$$\hat{c} = \hat{c}_r + i\,\hat{c}_i = \hat{\omega}/\hat{k}\ ,\ \hat{\omega} = \omega d/U_o \tag{3.20}$$

and U_o is the maximum flow velocity.

Equation (3.15) will be the starting point for the numerical procedure. This equation, together with the condition

$$-\pi\mu < \arg\left(\frac{\hat{z}_+}{\hat{z}_-}\right)^{\mu} < 0 \tag{3.21}$$

shall be used in order to determine the eigenmodes of the system.

In the next section we present the numerical procedure followed in order to determine the unstable eigenmodes of the plasmapause system. We also show the numerical results obtained from the dispersion relation (3.15).

III.3 Numerical Procedure and Results For a Linear Model

In this section we shall describe the procedure followed in order to determine the eigensolutions from the dispersion relation (3.15). These results were obtained for a simple reasonable linear model of the plasmapause region.

Note that the dispersion relation (3.15) is written in terms of the dimensionless parameters

$\hat{k}$ and R_i which allow us to study a great variety of physical models of the plasmapause environment. However, since we know from previous analysis that the necessary conditions for instability requires $\hat{c}_i = \text{Im}\,\hat{c}$ to be positive and the magnetic Richardson number Ri to be less than 0.25 we could then specify values in these regions. Afterward, we carried out a search for values of $\hat{k}$ and $\hat{C}r$ until the dispersion relation $D(\hat{c}_r, \hat{c}_i, \hat{k}, R_i)$ vanishes.

Since we know the analytic form of the dispersion relation in terms of simple analytic functions, we prepared a computer program to evaluate the dispersion relation (3.15). The numerical procedure followed was more or less a predictor-corrector iterative scheme. First, particular values of $\hat{c}_i$ and R_i were chosen. Then we evaluated the dispersion relation for an initial guess on the parameters $(\hat{k}\,,\,\hat{c}_r)$. Since the evaluation of the dispersion relation requires the computation of the confluent hypergeometric function, a subroutine which determines these functions carrying the expansion up to terms less than 10^{-10} was prepared. Afterwards we computed numerically the derivative of the dispersion relation with respect to each parameter $\hat{k}$ and $\hat{c}_i$. Using the Newton-Raphson's iterative procedure, corrections to these parameters were computed until the vanishing of the dispersion relation $D(\hat{c}_r, \hat{c}_i, \hat{k}, R_i)$ was obtained. This procedure was again repeated for new values of $\hat{c}_i$ and R_i.

In this procedure we have assumed the Boussinesq approximation by setting (i.e. $d/L_{\rho_0}=0$) equal to zero. The Boussinesq approximation is a convenient framework in which to develop concepts which depend essentially on the buoyancy forces and their interplay with the shear. These concepts may probably be extended into a wider context, but for the present the approximation is adopted without comment. We have also assumed that the plasma flow velocity, density, pressure and the magnetic field vary in a linear fashion.

The results of the search for unstable modes in a parametric form are presented in Figures (3.1) and (3.2). Figure (3.1) shows contour-plots of the magnetic Richardson number at the center of the plasmapause layer versus the normalized horizontal wavevector k for constant values of the normalized imaginary part of the phase velocity (c_i). These eigensolutions have a marginal stable boundary consisting of singular neutral modes. Singular neutral modes are eigensolutions for which c_i vanishes. This boundary represents solutions which are marginally stable. It divides the region of stability given by $c_i=\text{Im}c < 0$ from the instability region given by $c_i=\text{Im}c > 0$. The range of normalized wavevectors represents all the wavelengths which are marginally unstable if the Richardson number lies in the interval $R_i < 0.25$. The normalized wavevectors lies in the interval $0 < \hat{k} < 1.278$ as shown by the enclosing boundary (i.e. the singular neutral modes boundary). The first initial marginal unstable normalized wavevector corresponds to the maximum peak at which the magnetic Richardson number is 0.25. This normalized wavevector is shown in Figure (3.1) to be at $\hat{k} = 0.877$. We also show in Figure (3.1) instability curves for which the normalized imaginary phase velocity $\hat{c}_i$ is greater than zero ($\hat{c}_i > 0$). This corresponds to the contour-curves given by $\hat{c}_i = 0.1$ and $\hat{c}_i = 0.2$. The surface $\hat{c}_i = 0.1$ has its maximum peak for $\hat{k} = 0.689$ and $R_i = 0.18$. Its boundary encloses the unstable normalized wavevectors in the range $0 < \hat{k} < 1.187$. The instability surface $\hat{c}_i = 0.2$ has its maximum peak at $\hat{k} = 0.525$ and $R_i = 0.11$ and it encloses the range of unstable wavevectors given by $0 < \hat{k} < 0.877$. A very interesting feature found during the evaluation of these eigenmodes is that all the eigensolutions have the same normalized real phase velocity, that is $\hat{c}_r = \mathrm{Re}\ \hat{c} = 0.5$. This corresponds to the case where the critical level is at the center of the velocity profile.

Figure (3.2) shows contour-plots of the normalized growth rates $\hat{\omega}_i = \text{Im}\ \hat{\omega} = \hat{k}\hat{c}_i$ versus the normalized horizontal wavevectors $\hat{k}$ for constant values of the magnetic Richardson number. These curves represent the rate at which the instability grows for a particular wavelength and Richardson number. We show two contour plots for constant magnetic Richardson number. The contour plot at which the Richardson number is 0.0025 shows a maximum growth rate of 0.2 at the normalized wavevector of 0.8. Similarly the contour plot at which the Richardson number is 0.1 shows a maximum growth rate of 0.13 at the normalized wavevector of 0.77. We have also calculated the contour plot at which the Richardson number is 0.2 but this is not shown in Figure (3.2). In this situation the maximum growth rate is 5.7×10^{-2} at a normalized wavevector of 0.81.

In the next section we shall apply these results to the real plasmapause environment in order to describe the physical processes which may lead to the onset of the instability.

III.4 Numerical Results of the Stability of the Plasmapause

In this section we present a discussion of the MHD instability on the basis of real data of the plasmapause environment. For those unstable situations we shall find the regime of eigenmodes using the eigensolutions determined in the last section. A discussion of the physical mechanism which gave rise to the instability will be presented.

We have seen that the necessary condition for the plasmapause to become unstable requires for the magnetic Richardson number to be less than 0.25 (Miles, 1960, 1963; Howard, *et al.*, 1962; Howard, 1963, 1964). We have also mentioned that this number provides a measure of the stabilizing or destabilizing influence of the pressure, density and magnetic field gradients in comparison with the destabilizing effect of the shear flow. The influence of the pressure, density and magnetic field gradients, together with rotational and gravitational effects is contained in the expression for the magnetic Brunt-Väisälä frequency given in equation (2.50). Therefore we shall use the available data of the plasmapause environment in order to evaluate and to study those conditions which may lead to small values of the magnetic Richardson number.

From the definition of the magnetic Richardson number and the Brunt-Väisälä frequency we can infer two possible conditions for which the instability criterion can be satisfied. One is due to the effects of large velocity shears across the plasmapause. This condition requires large electric field gradient across the transition layer. The second condition is due to small values of the magnetic Brunt-Väisälä frequency. This can be obtained if some kind of cancelation between the different gradient terms occurs. Our investigation will provide us of an insight for which of these two (or both) conditions is relevant to the instability of the plasmapause.

Before we discuss the problem of stability or instability of the system, let us review the available data for which the plasmapause models will be obtained. The essential set of data parameters for which the MHD-stability of the plasmapause will be investigated are the density, pressure, flow velocity and magnetic field gradients. Typical particle density models can be obtained from Chappell's et al. (1970a, b; 1972) and Harris' et al. (1970) data taken aboard the OGO-5 satellite. Some of these density profiles, representative of different magnetic conditions, are shown in Figure 1.2. These plots show characteristic particle densities profile from 1000 particles/cm^3 to 1 particle/cm^3 across the plasmapause in the midnight sector (i.e. from 24:00 to $\pm$3 hr L.T.). Similar profiles for the evening sector (i.e. from 18:00 $\pm$3 hr L.T.) ranging from 200 particles/cm^3 to 1 particle/cm^3 across the plasmapause has been observed (Chappell's *et al.* 1970a,b; 1972). We can also infer from the density data as shown in Figure 1.2 the distance by which the density changes as well as the plasmapause position for different magnetic conditions. Chappell's *et al.* (1970a,b; 1972) results also shows the variation in position of the plasmapause as a function of the local time. Typical plasmapause positions, for the midnight sector, lie in the range of 6.5 to 3.5 Earth radii, with increasing magnetic activity. In the evening sector, the average position lies in the range of 4 to 9 Earth radii depending upon the magnetic conditions. Estimates of the distance over which the particle density changes show values in the range of 0.1 to 2 Earth radii (*i.e.* from 600 km to 13,000 km) depending upon the level of magnetic activity.

Calculations for the thermal energy density gradient across the plasmapause were made on the basis of Smith's *et al.* (1973) data taken aboard the S^3 satellite. Typical plots of these data are shown in Figure 1.6 and 1.7. From these plots we find that during periods of geomagnetic storms (*i.e.* very active conditions) the hot particles of the plasmasheet (*i.e.* the ring current particles) are well inside the plasmasphere. Protons in the energy range of 1 to 138 kev become the dominant energy contributors (*e.g.* about 90%) to the storm. However, during quiet times, this energy regime contributes only by 20% or less to the total energy density. An important feature that we can infer from these data is the presence of a decreasing energy density profile across the plasmaspause. This suggests the possibility for a convectively unstable situation since the pressure gradient is negative and it could give rise to an imaginary Brunt-Väisälä frequency (Gold 1959; Sonnerup *et al.*, 1963; Richmond, 1973; Lemaire, 1974, 1975, 1976). Typical values of the energy density from 4.5×10^{-7} erg/cm^3 to 6.5×10^{-8} erg/cm^3 across the plasmapause are estimated. Assuming a perfect gas law, together with the previous estimates of energy and particle densities we calculated the sound velocities to be in the range of 200 km/sec to 2,500 km/sec inside and outside the plasmasphere respectively. Similarly as for the particle density profile, we estimated the distance over which the pressure changes to be in the range of 0.7 to 1.65 Earth radii (*i.e.* from 4,000 km to 11,000 km).

Note, from both the particle and energy density data, that during the progress of a geomagnetic storm (*i.e.* during periods of increased magnetic activity) an inward movement of the plasmapause toward lower L-shells is observed. In addition we notice a steepening of the particle density gradient and a local enhancement of the thermal energy at the plasmapause. This inward motion has been associated with the plasma erosion process that occurs across the field lines during periods of enhanced activity at the plasmapause (see Chappell's *et al.* 1970b, 1972). This implies a possible convective activity in the outer edges of the plasmasphere during geomagnetic storms (Carpenter, 1970; Chappell's *et al.* 1970b, 1972). As we previously mentioned (see Chapter I), two different mechanisms have been suggested in order to explain this convective activity or erosion process at the plasmapause. The first mechanism is explained on the basis of an interchange or "ballooning" instability in which convection of plasma tubes occurs spontaneously due to an enhancement on the local thermal energy density (Gold, 1959; Sonnerup, *et al.* 1963; Richmond, 1973 and Lemaire, 1974, 1975, 1976). This mechanism may correlate very well with the energy density data previously shown (Schmidt's *et al.*, 1973) in Figures 1.6 and 1.7 and thus it also relates to the possibility of an imaginary Brunt-Väisälä frequency (or small Richardson number). The second suggested mechanism, by which erosion activity occurs is due to the penetration of the convective electric field across the plasmapause. This penetration allows for the peeling off of the plasma in the outer edges of the plasmasphere (Carpenter, 1970; Chappell's *et al.* 1970b, 1972). Once the plasma has been torn due to the electric field it convects into the outer magnetosphere where it is lost at the magnetopause. In other words, since inside the plasmasphere the corrotating electric field dominates, a penetration of the convective electric field across the plasmapause produces a shear flow velocity. If the shear flow velocity is large enough to reduce the magnetic Richardson number to small values, then an instability could develop and plasma is removed at the outer edges of the plasmasphere. All these processes seem to imply a mechanism by which plasma is supplied to the outer magnetosphere.

An interesting problem which arises from the investigation of the last mechanism is the efficiency with which electric fields of magnetosphere origin penetrates the plasmapause. The fundamental idea behind this problem is that the hot particles at the plasmasheet (i.e. the ring current particles) extend earthward during enhanced convective periods creating a layer which shields the inner magnetosphere from the convective electric field. This layer is known as an Alfven layer and it represents a region of charge separation which arises because of the different drift paths of ions and electrons moving in a combined electric field and a non-uniform magnetic field (Alfven, 1963; Schield et al. 1969; Kavanagh, et al. 1968; Wolf, 1975). The electric field produced by this charge separation reduces the convective electric field shielding the inner magnetosphere.

Considerable experimental evidence exists to show that electric field of magnetospheric origin penetrate the plasmapause (Carpenter, 1972; Vasiliunas, 1972; Mozer, 1973). It is found that the shielding time constant of the ring current particles is greater than six hours and therefore electric fields of magnetospheric origin with time scale of about one hour should penetrate the plasmapause. This is because the inertia of the ring current causes it to respond to changes of the magnetospheric electric field slowly (Mozer, 1973). Therefore, it appears that there is no inconsistency between the plasmapause shielding and the penetration of the convective electric field.

An important parameter which seems to have been neglected by other investigators in relation to the magnetospheric and corrotational electric fields is the steepness of the shear flow velocity or electric field gradient across the plasmapause. This parameter is very significant since, as we shall see later, the magnetic Richardson number is very sensitive to its magnitude allowing a possible unstable condition. To the best of our knowledge, measurements of this parameter have not been made. Therefore we could only predict those levels of the shear flow velocity which are necessary in order to obtain small Richardson numbers. Although we have no information of the shear flow or electric field gradient, we can find data which supplies information of the magnitude and direction of the electric field and plasma flow velocity. The plasma flow velocity is approximately based on McIlwain's (1972, 1974) magnetospheric potential distribution taken aboard the ATS-5 satellite and Carpenter's (1970) whistler data. Figure (1.4) shows a sample of a magnetospheric electric field model of McIlwain (1974). From the equipotential lines we can infer electric field in the range of 0.4 to 0.9 mV/m. Carpenter's observations shows electric fields in the range of 0.5 to 2 mV/m for the midnight sector and 1 to 4 mV/m for the evening sector during geomagnetic storms. These electric fields produce average convective velocities in the order of 1.5 to 10 km/sec depending upon the radial equatorial distance at which the plasmapause is located.

There has been some theoretical calculations in relation to the minimum thickness that the plasmapause can support (Sestero, 1964; 1966; Roth, 1976). Roth has calculated that the minimum thickness that the plasmapause could support is of the order of five times the cold Larmar radius (about 2.1 km). Their calculations also suggest that a hydrodynamic beam instability of electrostatic origin can develop if the relative velocity of the ions and electrons exceeds the thermal ion speed in this transition zone (see also Papadopoulos, 1973). These results suggest a lower bound limit for the shear flow velocity because the electrostatic instability will destroy the steady state transition zone. Roth analysis seems to be valid only for magnetically quiet conditions since his model does not show the thermal energy enhancement due to hot particle injection at the plasmapause, during active periods. On this basis, it appears that his calculations can be significantly modified. We should also point out that Roth's results can not be taken too seriously since no discussion on the electric field penetration or the Alfven layer shielding is presented.

The Earth's magnetic field will be considered as a dipole with a surface value of 0.32 gauss. We computed the Alfven velocity, using the density data to be about 240 km/sec and 3,200 km/sec inside and outside the plasmasphere respectively.

On the basis of the previously mentioned data parameters we show in Table 3.1 the plasma environment for six different models of the plasmapause. Models 1 to 3 correspond to typical conditions in the midnight sector whereas models 4 to 6 are representative situations in the evening sector. These models represent typical conditions at the particular local time of the plasmapause during magnetically active periods (i.e. k_p 2). We have combined the data parameters according to the plasmapause position since the different data values do not correspond to simultaneous measurements of the medium. Therefore, with this limitation on the data, we can only rely on typical average conditions. Since no measurements of the shear flow velocity has been made we shall assume that its length of variation is equal to the plasmapause thickness (i.e. the particle density scale length). The electric field value will be taken to be about 1 mV/m since this is a typical average value during periods of enhanced activity. The remaining set of parameters will be fixed from the measurements of the plasma environment. Computations of the magnetic Richardson number and the Brunt-Väisälä period for the models shown in Table 3.1 yield stable conditions. Estimates of the Richardson number gives values greater than 0.25 for identical velocity and density scale lengths. On this basis we could conclude that the models in Table 3.1 are very stable. It is then apparent from these results and the data parameters that the convective or interchange instability given by condition (2.99) is improbable. However, we cannot argue that the Richardson number could be small since we have no measurements of the shear flow velocity. With this limitation we can only predict what levels of velocity or electric field gradient are required to obtain Richardson numbers less than 0.25. In addition we can also calculate the range of unstable wave parameters corresponding to those unstable conditions. These results are shown in Table 3.2. The wave parameters are the range of unstable wavelengths and frequencies (or periods). Since linear theory can only predict the initial growth rate of the disturbance we cannot infer how the system will behave after the instability has grown. Therefore any discussion on the growth rate is meaningless. However, we can infer those data parameters for which the instability will be sensitive. It is apparent that the steeper the velocity gradient, the faster the instability growth. It also appears, from Table 3.2 that the range of unstable wavelengths always lies between the shear flow thickness and the density scale lengths. Typical unstable wavelengths for the midnight sector are in the range of 65 to 370 kilometers whereas for the evening sector are in the range of 70 to 270 kilometers. Similarly, typical wave periods for the midnight sector of the order of 1 to 4 minutes whereas for the evening sector we have periods ranging from half a minute up to 2 minutes. Hasegawa (1971) studied the drift wave instability for low energy particles at the plasmapause. His drift wave analysis is basically studied as a convective instability since the effects of a velocity shear are neglected and only the effects of the density gradient are included. He found two conditions required for instability. The first condition states that those unstable wavelengths perpendicular to the magnetic field must be smaller than the density scale length (i.e. $(\frac{1}{\rho_o}\frac{\partial \rho_o}{\partial r})^{-1} > \lambda_\perp$). The second condition requires for the parallel phase velocity to be between the ion and electron thermal velocities (i.e. $V_{thp} < \omega/K_{11} < V_{the}$). Hasegawa estimated unstable perpendicular wavelengths of the order of 350 meters. He also shows that during active conditions an increase in the electron temperature will enhance the unstable growth rate. However our results confirm that the contribution of both the pressure and magnetic field gradient terms together with the effective gravity due to the curvature of the magnetic field are more significant to the plasma-pause instability problem than the density gradient. Although the Richardson number is sensitive to these gradients through the magnetic Brunt-Väisälä frequency, it appears that the shear flow velocity will be the main mechanism by which the instability will develop. Nevertheless, it is very interesting to calculate the pressure variation thickness which could give rise to small Richardson numbers and therefore to an instability. If we select, for example, models 1 and 4 which are typical conditions for the midnight and evening sectors respectively, we find that pressure variation thickness of the order of 0.512 and 0.645 Earth radii can give rise to Richardson number of about 0.24. For this calculations we assumed that the velocity variation distance is equal to the particle density scale lengths. Calculations of the magnetic Brunt-Väisälä period for models 1 and 4 yields values of 2 and 4 minutes respectively. We have also calculated the wave parameters for these models. For model 1 we find that wavelengths in the range of 814 to 1,140 kilometers are unstable. Estimates of the wave periods yield values of 13 to 20 minutes. Similar results were obtained for model 4 in Table 3.1. For this model we find unstable wavelengths in the range of 2,545 to 3,560 kilometers. Evaluations of the wave periods gives values in the region of 18 to 27 minutes. In comparison with Gold's instability criterion given in equation (1.2), we also estimated the rate at which the pressure profile decreases. From these results we find that the pressure decreases as $r^{-6.64}$ across the plasmapause. This value is still smaller than the adiabatic gradient (i.e. $r^{-20/3}$) discussed by Gold. This immediately implies that it is very unlikely for the Brunt-Väisälä frequency to become imaginary since the pressure profile does not change fast enough. Thus, on the basis of these results and the available data we can infer that the interchange or "ballooning" instability does not correspond to the main mechanism from which we could explain the erosion process at the plasmapause.

In summary, we have presented a mechanism that may account for the plasma erosion process and the excitation of drift waves at the outer edges of the plasmasphere.

Measurements indicate that the interchange or "ballooning" instability is not likely to occur, since we have only found large values of the magnetic Brunt-Väisälä frequency. Therefore it seems that the only mechanism for which convection of plasma may be accounted for is due to the large electric field (or velocity) gradients. However, we can only predict those velocity gradients required for the instability, since apparently measurements of this parameter has not been made in the past. Nevertheless, we submit in this investigation a range of velocity gradients that could give rise to the instability. We also predict the range wave periods and wavelengths that may be excited during the unstable process. An important factor which has not been mentioned and which plays an important role in the convective motions is the effect of the finite conductivity of the ionosphere. In the next section we shall qualitatively discuss this effect and hopefully a more complete picture of the overall instability may be established.

TABLE 3.1

Plasma Parameters/ Model No.	Plasmapause Position L_{pp} [Earth Radii]	Electric Field E_o [Milivolt/ meter]	Flow Velocity U_o [km/sec]	Particle Density n_1 Inside Plasmasphere [mrt./cm^3]	Particle Density n_2 Outside Plasmasphere [part./cm^3]	Density Variation Distance [Earth Radii] ΔL_n	Energy Density P_1 Inside Plasmasphere [erg/cm^3]	Energy Density P_2 Outside Plasmasphere [erg/cm^3]	Pressure Variation Distance [Earth Radii] ΔL_p	Density Scale Lengths [km]	Plasma Parameter $\beta(L_{pp})$
1	3.78	1	2	1675	.2	0.18	3.49×10^{-7}	1.42×10^{-7}	1.27	127	.16
2	3.9	1	2.2	1850	.15	0.46	3.49×10^{-7}	1.42×10^{-7}	1.27	311	.19
3	4.63	1	3.4	1000	.1	0.27	1.74×10^{-7}	7.28×10^{-8}	1.64	249	.27
4	4.52	1	3.3	100	.1	0.43	3×10^{-7}	1.16×10^{-7}	1.2	397	.39
5	4.42	1	3.1	150	.1	0.4	4.54×10^{-7}	1.6×10^{-7}	0.89	349	.50
6	4.89	1	4.1	215	.1	0.33	1.29×10^{-7}	6.7×10^{-8}	0.745	274	.31

TABLE 3.2

Model No.	Velocity Variation Distance [km]	Magnetic Richardson Number Ri	Magnetic Brunt-Väisälä Period T_{BV} [seconds]	Range of Unstable Wavelengths λ [km]	Range of Unstable Periods T [seconds]
1	10.52	0.24	10.66	67.5-94.4	67.5-94.4
2	11.48	0.24	10.74	73.6-103.0	66.9-93.6
3	40.82	0.24	24.5	262.0-366.4	154.12-215.5
4	11.48	0.24	7.12	73.6-103.0	44.6-62.5
5	14.35	0.24	14.35	92.0-129.0	59.35-83.23
6	29.34	0.24	14.65	188.1-263.4	92-128.5

III.5 The Effect of the Ionospheric Conductivity on the Instability

The coupling of the inner magnetosphere to the ionosphere may have very important effects on the convective motion at the plasmapause. The effect of the ionospheric conductivity on the plasmapause represent another mechanism by which the steep density gradient can be sustained in spite of the destructive tendency of the instability. The ionospheric conductivity produces a dragging effect at the feet of the magnetic field lines which slows down the instability growth rate. This process has been referred to in the literature as a "line-tying" or a "foot-dragging" effect (Gold, 1959; Dungey, 1968; Richmond, 1973; Lemaire, 1975).

In order to understand how this mechanism acts to slow down any unstable convective motion we shall consider two purely hypothetical physical situations. The results obtained from these situations will be applied to understanding a more realistic case. The purpose of this approach is not to develop a quantitative theory but merely to establish qualitatively the physical explanation of the influence of the conductivity and its effects on the unstable plasma motions. For the sake of simplicity we shall consider a steady-state situation. As a consequence, the electric field $\underline{E}$ may be considered as a curl-free field and can therefore be expressed in terms of a gradient of a potential ϕ . In our first hypothetical case we consider the Earth as a "perfect" conductor and we neglect the existance of the atmosphere and the ionosphere. In consequence the plasma is in direct contact with the Earth. Therefore, since the conductivity of the plasma is infinitely large, the frozen-in law given by $\underline{E} + \underline{V} \times \underline{B} = 0$ can be applied. This condition implies that all points of a given field line are at the same potential. However, since the magnetic field lines are in direct contact with the Earth's surface, which has been presumed a good conductor, it follows that in the frame of the Earth there cannot be any potential difference between different field lines since they are "shorted out". Therefore, the electric field is everywhere zero and from the "frozen-in" law, the component of the plasma perpendicular to the field lines is also zero. Then, the plasma is permanently attached to the field lines of the Earth and is obliged to rotate with it. Another physical reason for this to occur is that any attempt to move the plasma across the field lines tends to produce an "infinite" current which will oppose this motion. Therefore the "infinitely" large conductivity of the Earth, in a sense acts as an infinitely viscous medium which causes a drag in any motion of a flux tube and its plasma content.

We now take into consideration the existance of the atmosphere but still neglect the ionospheric layer. This corresponds to our second hypothetical case. Let us now examine how the previous results are modified. For simplicity we shall assume the atmosphere as a perfect insulator, separating the "perfectly" conductive Earth from the plasma. As before, in the plasma medium above the atmosphere, all points at a given field line are at the same potential since the conductivity along the field lines is infinitely large. However, different magnetic field lines may now have different potentials. This is because the field lines are now separated from the Earth and from themselves by the atmosphere. In whatever frame of reference we place ourselves we may find an electric field perpendicular to the field lines. In the Earth's situation for example, this may be the corrotating electric field. Therefore the plasma may move now perpendicular to the field lines with a velocity $\underline{V}(=(\underline{E} \times \underline{B})/B^2)$ due to this electric field. Notice also that since the conductivity along the magnetic field lines is still very large. Any perpendicular electric field will map exactly along the field lines. Thus, any motion of a plasma at one point of a field line uniquely determines the motion at all other points on the same line. It should be noted now that the plasma no longer needs to remain confined within a magnetic tube, but may slowly drift with it. In other words, a particular flux tube projected down through the atmosphere cannot be identified with any tube projecting through the surface of the Earth and therefore a slippage between the two systems is permissible.

Finally let me discuss the more realistic case which allows for the presence of a finite conducting region, the ionosphere, between the atmosphere and the perfectly conductive plasma. This case represents an intermediate situation between the two previously discussed cases. We can now physically interpolate how the motion of any flux tube will be modified by the presence of the ionosphere. Since the ionosphere has a conductivity which is smaller than the plasmaspheric medium but higher than the atmosphere its effect will be to slightly slow down any motion of a flux tube as in the first hypothetical case. These effects are dependent upon the conductivity level of the ionosphere. We could then infer that any unstable motion which tends to move any flux tube across the field lines will experience the dragging effect of the ionosphere. We should point out that since the night time conductivity is in general smaller than the day time conductivity, any unstable motion is most probable to occur at the midnight sector. However the night time conductivity tends to increase during magnetic disturbances up to levels as high as 30 mho (see Schuman _et al._, 1980). This process, then, could be relevant to the fact that the plasmapause steepens during magnetically active times.

Since the ionosphere tends to slow down any flux tube motion, it then acts as a dissipative medium. Therefore, we can roughly estimate the power dissipated at the feet of a magnetic flux tube. The power dissipated at the feet of the ionisphere is given by

$$P_D = \int_V \underset{\sim}{\tilde{J}} \cdot \underset{\sim}{\tilde{E}} \quad dV \tag{3.22}$$

where the integral is taken over a volume V of a flux tube at the ionosphere. Since $\underset{\sim}{\tilde{J}}$ and $\underset{\sim}{\tilde{E}}$ are both complex vector functions of time and position, we could average the losses over time. To evaluate the power dissipated from equation (3.22) we shall assume that the conductivity along the field lines is infinitely large and the ionosphere is then enough so that the cross-sectional area of the tube remains constant. To estimate the fluctuating quantities $\underset{\sim}{\tilde{J}}$ and $\underset{\sim}{\tilde{E}}$ we made use of the polarization relations given in Appendix B. We choose a cross-sectional tube area equal to 1 km^2 and we assume that a constant magnetic field fluctuation $\tilde{B}_\theta$ of one gamma (i.e. $\tilde{B}_\theta$ = 1 gamma) is used as a normalization constant for the perturbations. Thus, assuming that the perturbations do not vary very much with radial distance, equation (3.22) yields

$$P_D = \frac{1}{2} \frac{\sum p}{\sin I} A_I B_{o\theta}^2 \operatorname{Re} |\tilde{V}_r|^2 \tag{3.23}$$

where Σ_p is the height-integrated Pedersen conductivity, I is the dip angle of the magnetic field at the ionosphere, $\tilde{V}_r$ is the average radial velocity fluctuation, A_I is the magnetic tube cross-sectional area at the ionosphere and $B_{o\theta}$ is the ionospheric magnetic field. Estimates of the power dissipated at the ionosphere can be calculated using typical values for the plasma tube environment. In particular, let us consider that the ionosphere is about 100 Km ($L_I = 1.016$) above the Earth's surface and it has an integrated Pedersen conductivity ($\Sigma_p/\sin I$) of about 0.3 mho to represent quiet magnetic conditions in the evening to late midnight sector. Estimates of the fluctuating velocity for a one gamma perturbed magnetic field at the ionosphere using the polarization relations in Appendix B yield a value of about 0.22 m/sec using a typical growth rate of about 10^{-3} rad/sec. The power dissipated is estimated to be about 3×10^{-5} watts over an area of 1 km^2 at the ionosphere.

Similarly we could determine the power input by the disturbance on a flux tube in a localized region around the equatorial plane. Using the expression for the total energy of the disturbance, the power is given by

$$P = 2\omega_i \int_V \left(\frac{1}{2}\rho_o \tilde{V}^2 + \frac{\tilde{B}^2}{2\mu_o}\right) dV \qquad (3.24)$$

where ω_i is the growth rate of the perturbation and the terms in the integrand represent the total kinetic and magnetic energies of the disturbance. We have neglected the gravitational and the thermal energy fluctuations since their contribution is very small. The integral in equation (3.24) is taken over a localized volume of a flux tube in the neighborhood of the equatorial plane so that the length element $d\ell$ can be expressed in terms of the latitudual angle λ as $rd\lambda$ (i.e. $d\ell \approx rd\lambda$). With this condition the expression (3.24) for the power reduces to

$$P \approx 2\omega_i \, A_T \left(\frac{1}{2}\rho_o \, \mathrm{Re}\, |\tilde{V}|^2 + \frac{\mathrm{Re}|\tilde{B}|^2}{2\mu_o}\right) \int_0^{\Delta\lambda} r_p \, d\lambda \qquad (3.25)$$

where A_T is the tube cross-sectional area and r_p is the plasmapause position. Estimating the velocity fluctuation from Appendix B for a one gamma magnetic field perturbation at 4 Earch radii yield a velocity of about 53 m/sec. Numerical estimates of the total input power assuming a localized region of about 20^o gives typical values of about 0.01 watts over a tube cross-sectional area of $1K_m^2$. A comparison of the total power dissipated with the total input power clearly shows that the ionosphere will not reduce the instability. Thus, the instability may proceed with a negligible small dragging effect at the feet of the magnetic tubes.

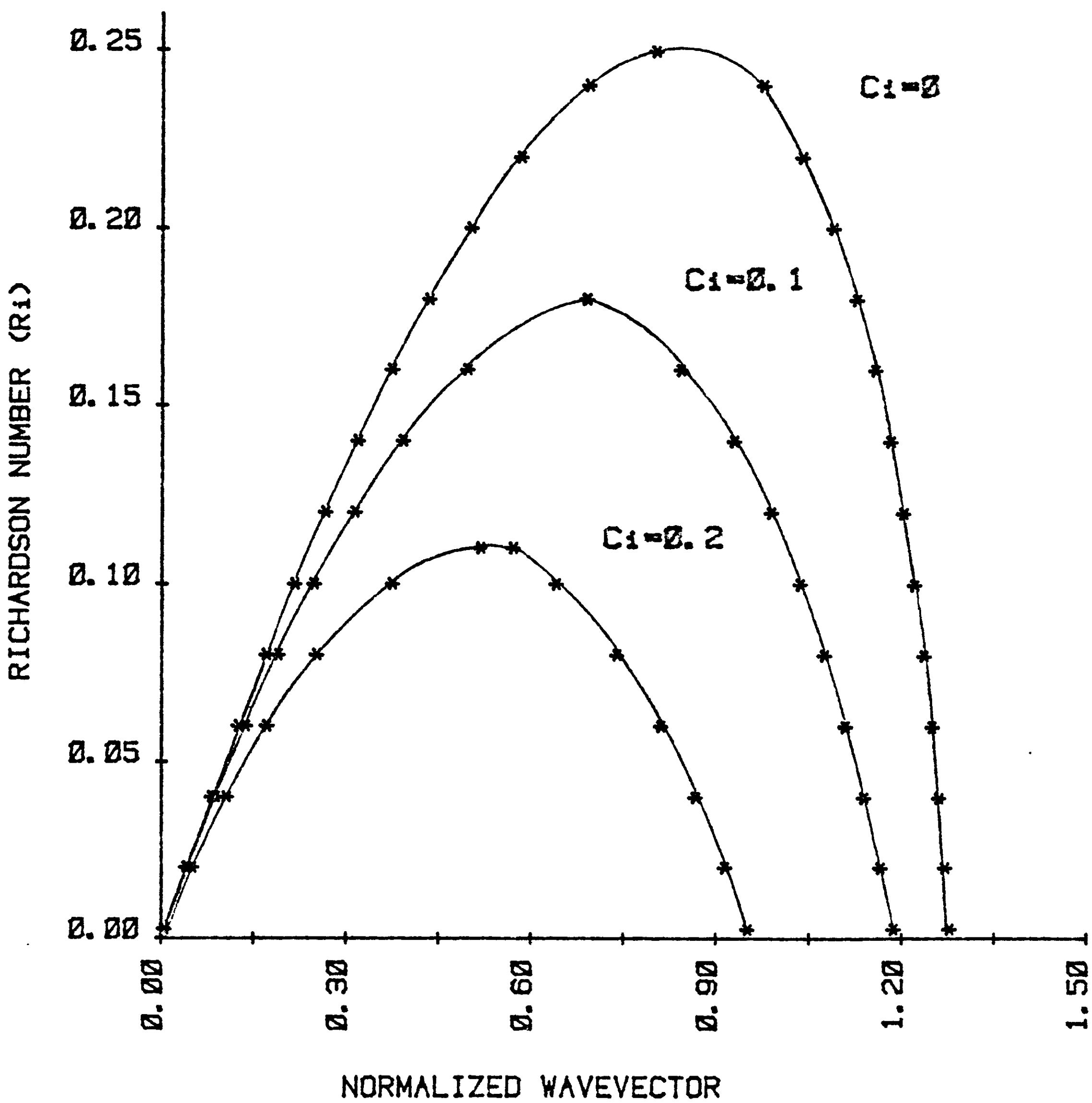

FIGURE 3.1-INSTABILITY SURFACES (Ri Vs. Kt) FOR CONSTANT NORMALIZED IMAGINARY PHASE VELOCITY Ci.

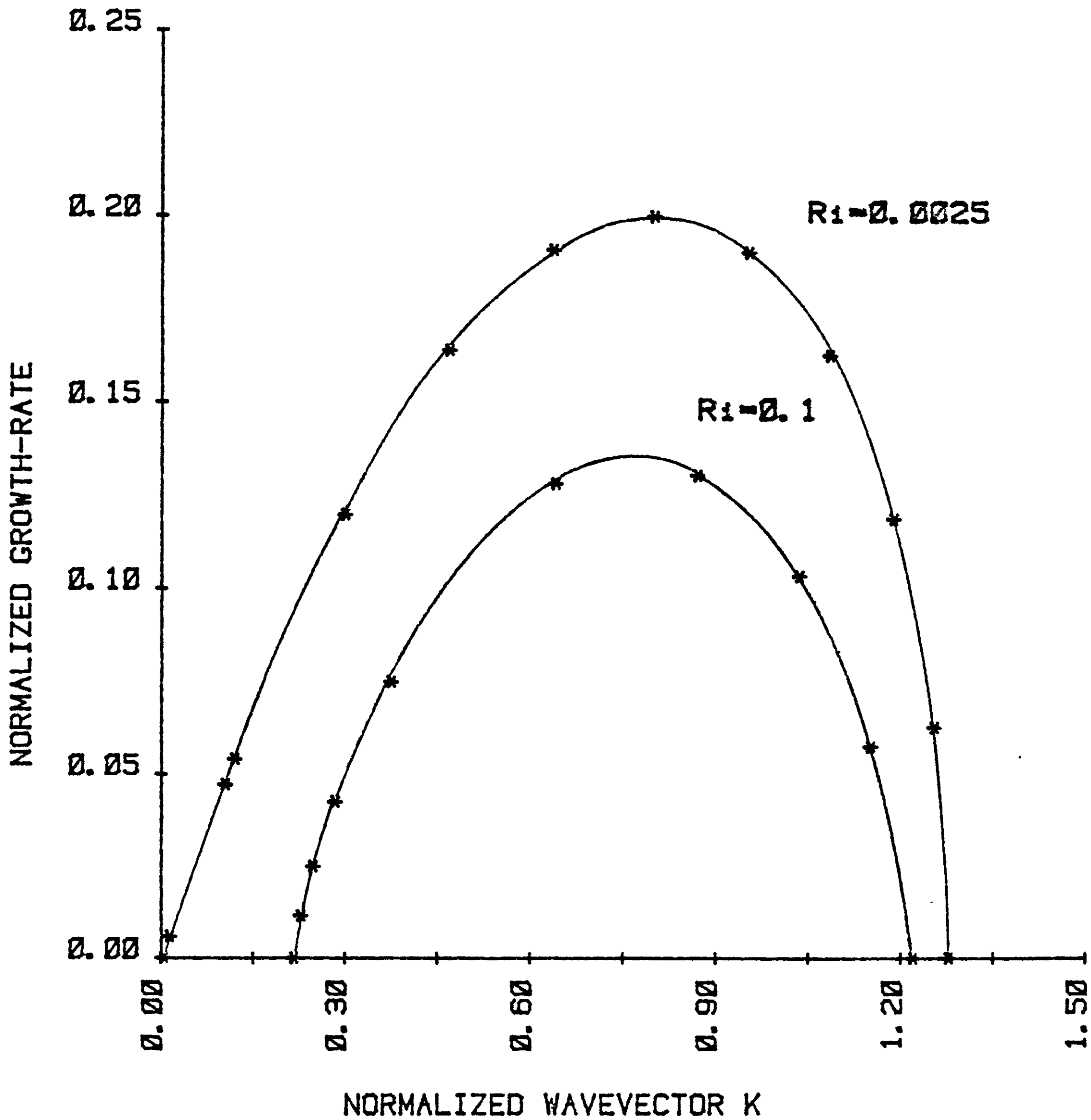

FIGURE 3.2-GROWTH RATE SURFACES (Vs. Kt) FOR CONSTANT RICHARDSON NUMBER Ri.

CHAPTER IV

CONCLUSIONS

IV.1 Summary

In this dissertation we have presented a magnetohydrodynamical stability analysis for the Earth's plasmapause environment. The plasmapause was assumed as a continuous transition zone where the streaming velocities, particle densities, energy densities and magnetic field vary in a linear profile across it.

We have been able to present a unified treatment of both the interchange or "ballooning" instability and the shear flow or Kelvin-Helmholtz instability. With this approach we showed that previous treatment of the Kelvin-Helmholtz and the interchange instabilities were inadequate since their interplay role has significant effects in the description of the dynamics of the system. To the best of our knowledge, this unified treatment has not been presented in relation to the physics of space plasmas.

We were able to find complete data from which the possibility for convective or interchange instability could be tested through the magnetic Brunt-Väisälä frequency. From the available data of the plasmapause environment during geomagnetic storms we show that it seems improbable to have a purely convective instability since calculations always show large positive values of the magnetic Brunt-Väisälä frequency. Therefore it appears that the interchange or "ballooning" instability is not the appropriate mechanism by which the erosion processes at the plasmapause occurred.

In respect to the shear instability, represented by the magnetic Richardson instability criterion, we do not have any conclusive evidence which may confirm or deny that this mechanism could describe the erosion process at the plasmapause. The reason for this is because there are still two important problems for which no definite resolution and measurements have been made. These problems are related first to the question of how deep the electric field could penetrate the plasmapause and secondly to how steep the electric field could be across it. Until these two problems have been resolved we won't be able to definitely confirm or deny the shear mechanism as the process for which erosion takes place. We can predict from our theory which velocity shears are required in order for the onset of the instability to occur. We found that thickness on the order of 10 to 40 kilometers are required. We have also predicted the wave periods and wavelengths of the drift wave excited by the instability. Our estimates show that wave periods in the range of 1 to 4 minutes for the midnight sector and half a minute up to 2.5 minutes for the evening sector are excited. Similarly we found longitudinal wavelengths in the range of 65 to 365 kilometers for the midnight sector and from 70 up to 260 kilometers for the evening sector.

IV.2 Suggestions for Further Investigations

On the basis of our results, it seems to be appropriate to suggest to future researchers to investigate more thoroughly other available electric field data in order to clarify the questions about the electric field penetration and the steepening of this field across the plasmapause. In addition, theoretical investigations pertaining to those two problems and their relationship to the ring current particle injection and the Alfven layer should be motivated.

From the theoretical point of view it is obvious that the signficance of the non-linear terms for the stability should be investigated. However the analytical difficulties are tremendous. Some information can be obtained using purely numerical techniques such as finite difference. However, the models in this case would have to be very simple and a highly turbulent state could require very long computation time. Another approach to the non-linear problem could be to consider the non-steady state equilibrium for the unperturbed variables. We have evidence that this approach could give very important results even for large Richardson numbers in relation to the finite amplitude wave interaction with the velocity shear and its time evolution (Paul, 1977).

Finally, some of the results of this thesis could be tested experimentally. We have predicted disturbances with periods of a few minutes and phase velocities on the order of one to two kilometers per second. It could be interesting to review the available ground recorded and satellite data to see if such disturbances are actually observed.

APPENDIX A

ANALYSIS AROUND THE HYDROMAGNETIC CRITICAL LEVELS

In this appendix we shall present a brief discussion of the hydromagnetic singularities of the matrix wave equation (2.33). These singularities which correspond to the vanishing of Ω_A^2 and Ω_M^2 are defined by

$$\Omega = \pm(\underline{K}_t \cdot \underline{C}_A) \quad , \quad \Omega = \pm\frac{(\underline{K}_t \cdot \underline{C}_A)}{\sqrt{1+M^2}} \tag{A.1}$$

where Ω is the Doppler shifted frequency $(\Omega = \omega - \underline{K}_t \cdot \underline{V}_o)$, $\underline{K}_t$ is the horizontal wavevector and M is the magnetic Mach number defined in terms of the ratio of the Alfven velocity (C_A) and the sound velocity (C_S). As we mentioned in chapter two, these singularities are associated with the propagation of guided Alfven waves and slow magnetoacoustic waves (i.e. or sound waves) respectively. The plus and minus sign in expressions (A.1) corresponds to propagation parallel and anti-parallel to the magnetic field respectively.

Since these singularities are fundamentally related to wave modes along the field lines, we shall simplify our investigation by only considering these particular eigenmodes. Thus the horizontal wavevector K_t becomes K_{11} to represent parallel to the magnetic field lines. With this simplification the Doppler shifted frequency Ω

reduces to ω, since the plasma flow velocity is prependicular to the magnetic field lines. Similarly Ω_A^2 and Ω_M^2 reduces to ω_A^2 and ω_M^2 respectively. Furthermore, we shall ignore the centrifugal terms due to the Earth's rotation and the curling action of the plasma flow, since they represent very small effects in comparison with the density, pressure, and magnetic field gradients terms. This approximation does not altered the physics of this analysis since the plasma flow velocity effects vanishes for field aligned modes. With these conditions, let us modify the matrix wave equation (2.33) by transforming it into the so-called "transmission-line" form which corresponds to the canonical form of a matrix differential system. This transformation simplifies the algebraic procedure without any modification on the important physical parameters (Madden, 1972). As a result and after some algebraic manipulations, the matrizant wave equation (2.33) becomes

$$\frac{d}{dr}\begin{vmatrix} \tilde{\mathcal{V}} \\ \tilde{\mathcal{G}} \end{vmatrix} = \begin{vmatrix} 0 & M_{12}(r) \\ M_{21}(r) & 0 \end{vmatrix} \begin{vmatrix} \tilde{\mathcal{V}} \\ \tilde{\mathcal{G}} \end{vmatrix} \qquad (A.2)$$

where $\tilde{\nu}$ and $\tilde{\vartheta}$ are defined by

$$\tilde{\nu} = \tilde{P}_T \exp\left\{-\int a_{11}(r)\,dr\right\} \quad , \quad \tilde{\vartheta} = r^2\tilde{\xi}_r \exp\left\{-\int a_{22}dr\right\} \qquad (A.3)$$

and the matrix elements are given by

$$M_{12}(r) = a_{12}(r) \exp\left\{-2\int a_{11}(r)\,dr\right\} , \; M_{21}(r) = a_{21}(r) \exp\left\{2\int a_{11}(r)\,dr\right\} \qquad (A.4)$$

The matrix elements a_{ij} (i,j=1,2) were defined in equation (2.35) but with the previous simplifications they reduced to

$$a_{11}(r) = -\left[\frac{\omega^2(g+2C_A^2/r) - 2K_{11}^2 C_A^2 C_s^2/r}{(1+M^2)C_s^2\Omega_M^2}\right]$$

$$a_{12}(r) = \frac{\rho_o}{r^2}\left[\omega_A^2 + \frac{2C_A^2}{rB_o}\frac{\partial B_o}{\partial r} - \frac{2C_A^2}{r^2} - \frac{\bar{g}}{\rho_o}\left\{\frac{1}{C_s^2}\frac{\partial}{\partial r}\left(P_o + \frac{B_o^2}{2\mu_o}\right) - \frac{\partial\rho_o}{\partial r} - \frac{\rho_o M^2}{r}\right\}\right.$$

$$\left. + M^2\left(g - \frac{2C_s^2}{r}\right)\left\{\frac{\omega^2}{\rho_o(1+M^2)C_s^2\omega_M^2}\frac{\partial}{\partial r}\left(P_o + \frac{B_o^2}{2\mu_o}\right) - \frac{2M^2(\omega^2 - K_{11}^2 C_s^2)}{r(1+M^2)\,\omega_M^2}\right\}\right] \qquad (A.5)$$

$$a_{21}(r) = -\frac{r^2}{\rho_o}\left[\frac{\omega^4}{(1+M^2)C_s^2\omega_M^2\omega_A^2} - \frac{K_{11}^2}{\omega_A^2}\right]$$

$$a_{22}(r) = -a_{11}(r)$$

The "transmission-line" form of the wave equation (A.2) is very useful in order to determine the dispersion relation for field aligned modes. Since the parallel propagation vector K_{11} is smaller than the perpendicular one (K_ϕ, K_r) , then field aligned eigenmodes are very fast. Therefore it is convenient to investigate those modes which lie in the range

$$C_s < \omega/K_{11} < C_A \tag{A.6}$$

since for the plasmapause region the Alfven velocity is greater than the sound velocity. With this condition and assuming that the medium properties vary very slowly in comparison with the radial wavelength, we can apply the WKB approximation to the matrix system (A.2) to derive the dispersion relation given by

$$K_r^2 = K_{11}^2 \left(\frac{\Omega_{BV}^2}{|\omega^2 - K_{11}^2 C_A^2|} - 1\right) \tag{A.7}$$

where the symbol "| |" implies the absolute value and Ω_{BV} represents the magnetic Brunt-Väisälä frequency defined by

$$\Omega_{BV}^2(r) = -\bar{g}\frac{\partial \ell n\rho_o}{\partial r} + \frac{(\bar{g}+2C_A^2/r)}{\gamma(1+M^2)}\frac{\partial \ell nP_o}{\partial r} + \frac{M^2(\bar{g}-2C_s^2/r)}{(1+M^2)}\left[\frac{\partial \ell nB_o}{\partial r} - \frac{1}{r}\right] \tag{A.8}$$

Note the similarity between the dispersion relation (A.7) for field aligned guided Alfven modes to the dispersion relation (2.54) for low frequency drift modes.

As in the case of drift waves, guided Alfven waves become evanescent if the magnetic-Brunt-Väisälä frequency $\Omega^2_{BV}(r)$ is smaller than the Doppler shifted Alfven frequency ω^2_A given by $|\omega^2 - K^2_{11} C^2_A|$. This implies that there will be a radial level r_A from which guided Alfven waves will be reflected. In other words, waves that can propagate at one side of r_A will become evanescent upon crossing this level. Thus the point $r = r_A$ represents a cut-off condition since the radial phase velocity becomes infinitely large.

An important feature in relation to the stability or instability of the system can be determined by rearranging the dispersion relation (A.7) in the form

$$\omega^2 = K^2_{11} C^2_A + \Omega^2_{BV}(r) \qquad \text{(A.9)}$$

where we have assumed that $K_r >> K_{11}$. Since the magnetic Brunt-Väisälä frequency can become negative due to the gradients, the possibility for complex wave frequency arises. This is related to the stability or instability of the system. If the magnetic Brunt-Väisälä frequency satisfies the condition

$$\Omega^2_{BV}(r) < -K^2_{11} C^2_A \qquad \text{(A.10)}$$

then the system will become convectively unstable spontaneously. This result represents the criterion of instability for "ballooning-modes" as described by Coppi

(1978)[1] and Hasegawa (1980)[2]. "Ballooning" instability is fundamentally an interchange mode which is driven by the presence of a pressure gradient in an unfavorable magnetic field curvature. Another important observation that can be obtained from the instability criterion (A.10) or the dispersion relation (A.9) is that the presence of a finite wavelength along the magnetic field produces stabilizing effects to this instability. Therefore we can infer that the most unstable modes are those for which the magnetic field remains unchanged and no finite parallel wavelength stabilization is present.

Another important characteristic of the general dispersion relation (A.7) for field aligned modes is that, as in the case of drift waves, there will be a critical level at which the radial wave propagation vector will go to infinity. This corresponds to a resonance condition since the radial phase velocity becomes zero. This condition is satisfied whenever $\omega/K_{11} = \pm C_A(r_c)$, i.e. this critical level is a point at which a field aligned mode matches the local Alfven velocity. Examining the behavior of the dispersion relation (A.7) near the critical level r_c we find

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[1] Coppi, B. - Class Notes of a Plasma Physics Course offered at MIT (8.641-8.642).

[2] Hasegawa, A. - Progress Report in "Ballooning Modes in the Jovian Magnetosphere", Bell. Lab. (unpublished) (1980).

$$K_r^2 \approx \frac{\Omega_{BV}^2(r_c)}{\left[\frac{dC_A}{dr}\Big|_{r_c}\right]^2 (r-r_c)^2} \qquad (A.11)$$

where we have made use of the Taylor series to expand the Alfven velocity around the critical point. Similar results were obtained in the case of drift waves propagating perpendicular to the magnetic field.

APPENDIX B

POLARIZATION RELATIONS

In this appendix we shall present the so called "polarization relations". These equations give the relations and the phase difference between the pressure fluctuations, density variations, the components of the velocity variations and the components of the magnetic field fluctuations. To find these relations, it is necessary to use and combine the system of equations (2.21) to (2.28) together with the matrix differential equation (2.33). For simplicity, let us assume that the flow velocity $\underline{V}_o$ is perpendicular to the magnetic field $\underline{B}_o$, thus terms containing ($\underline{V}_o \cdot \underline{B}_o$) will vanish (i.e. $\underline{V}_o(r) = V_{o\phi}(r)\ \hat{e}_\phi$ and $\underline{B}_o(r)=B_{o\theta}(r)\ \hat{e}_\theta$). Therefore, after some algebraic manipulations the resulting polarization relations becomes:

$$\tilde{V}_\theta = \frac{\Omega \tilde{P}_T}{\rho_o \Omega^2}\left[K_\theta - \frac{B_{o\theta}(\underline{K}_t\cdot \underline{B}_o)\,\Omega^2}{\rho_o\mu_o(1+M^2)C_S^2\,\Omega_M^2}\right] + \frac{\tilde{V}_r}{i\Omega}\left[\frac{2B_{o\theta}(\underline{K}_t\cdot\underline{B}_o)}{\rho_o\mu_o r\Omega} - \frac{B_{o\theta}(\underline{K}_t\cdot\underline{B}_o)}{\Omega\rho_o\mu_o}\left(\frac{\Omega^2(\bar{g}+2C_A^2/r)-2(\underline{K}_t\cdot\underline{C}_A)^2C_S^2/r}{(1+M^2)\ C_S^2\ \Omega_M^2}\right) - \frac{2\Omega_E B_{o\theta}(\underline{K}_t\cdot\underline{B}_o)}{(1+M^2)\Omega_M^2\ \rho_o\mu_o}\left(\frac{C_{A\theta}(\underline{K}_t\cdot\underline{C}_A)^2(\underline{K}_t\times\underline{C}_A)_r}{\Omega_A^2\ C_S^2}+\frac{K_\phi M^2(\underline{K}_t\cdot\underline{C}_A)^2}{\Omega_A^2}\right)\right] \tag{B.1}$$

$$\tilde{V}_\phi = \frac{K_\phi \Omega \tilde{P}_T}{\rho_o \Omega_A^2} + \frac{\tilde{V}_r}{i\Omega} \left[\frac{\partial V_{o\phi}}{\partial r} + \frac{V_{o\phi}}{r} \left(\frac{\Omega^2 + (\underline{K}_t \cdot \underline{C}_A)^2}{\Omega_A^2} \right) + 2\Omega_E \right.$$

$$\left. + \frac{2\Omega_E K_\theta B_{o\theta} (\underline{K}_t \cdot \underline{B}_o)}{\rho_o \mu_o} \right] \tag{B.2}$$

$$\tilde{B}_r = - \frac{(\underline{K}_t \cdot \underline{B}_o)}{\Omega} \tilde{V}_r \tag{B.3}$$

$$\tilde{B}_\theta = \frac{\tilde{P}_T}{\rho_o \Omega_A^2} \left[\frac{B_{o\theta} \Omega^4}{(1+M^2) C_s^2 \Omega_M^2} - K_\theta (\underline{K}_t \cdot \underline{B}_o) \right] + \frac{\tilde{V}_r}{i\Omega} \left[\frac{\partial B_{o\theta}}{\partial r} - \frac{B_{o\theta}}{r} \right.$$

$$+ B_{o\theta} \left(\frac{\Omega^2 (\bar{g} + 2C_A^2/r) - 2(\underline{K}_t \cdot \underline{C}_A)^2 C_s^2/r}{(1+M^2) C_s^2 \Omega_M^2} + \frac{2\Omega\Omega_E}{(1+M^2)\Omega_M^2} \left\{ \frac{C_{A\theta} (\underline{K}_t \cdot \underline{C}_A)^2 (\underline{K}_t \times \underline{C}_A)_r}{\Omega_A^2 C_s^2} \right. \right.$$

$$\left. \left. \left. + \frac{K_\phi M^2 (\underline{K}_t \cdot \underline{C}_A)^2}{\Omega_A^2} \right\} \right) \right] \tag{B.4}$$

$$\tilde{B}_\phi = - \frac{K_\phi \tilde{P}_T (\underline{K}_t \cdot \underline{B}_o)}{\rho_o \Omega_A^2} - \frac{\tilde{V}_r}{i\Omega} \left[\frac{2\Omega V_{o\phi} (\underline{K}_t \cdot \underline{B}_o)}{r \Omega_A^2} \right.$$

$$\left. + \frac{2\Omega \, \Omega_E K_\theta B_{o\theta}}{\Omega_A^2} \right] \tag{B.5}$$

$$\tilde{\rho} = \frac{\Omega^2 \tilde{P}_T}{(1+M^2)\, C_s^2 \Omega_M^2} - \frac{\rho_o \tilde{V}_r}{i\Omega} \Bigg[\frac{1}{\rho_o C_s^2} \frac{\partial (P_o + \frac{B_o^2}{2\mu_o})}{\partial r} - \frac{1}{\rho_o} \frac{\partial \rho_o}{\partial r} - \frac{M^2}{r}$$

$$+ \frac{2K_\phi M^2 \Omega_E}{(1+M^2) C_s^2 \Omega_M^2} + \frac{2K_\phi M^2 \Omega_E K_\theta B_{o\theta} (\underline{K}_t \cdot \underline{B}_o)}{\Omega \Omega_A^2 \rho_o \mu_o} + M^2 \Bigg(\frac{\Omega^2 (\bar{g} + 2C_A^2/r) - 2(\underline{K}_t \cdot \underline{C}_A)^2 C_s^2/r}{(1+M^2)\; C_s^2\; \Omega_M^2}$$

$$+ \frac{2\Omega\Omega_E}{(1+M^2)\Omega_M^2} \left\{ \frac{C_{A\theta} (\underline{K}_t \cdot \underline{C}_A)^2 (\underline{K}_t \times \underline{C}_A)_r}{\Omega_A^2\; C_s^2} - K_\phi (1+M^2) \right\} \Bigg) \Bigg] \qquad \text{(B.6)}$$

In addition we can determine the components of the electric field fluctuations $\tilde{E}$ by the use of the "frozen-in" law. These electric field fluctuations are given by

$$\tilde{E}_r = - \frac{\tilde{P}_T}{\rho_o \Omega_A^2} \Bigg[\Omega (\underline{K}_t \times \underline{B}_o)_r + \frac{\Omega^4 (\underline{V}_o \times \underline{B}_o)_r}{(1+M^2) C_s^2 \Omega_M^2} + (\underline{K}_t \cdot \underline{B}_o)(\underline{K}_t \times \underline{V}_o)_r \Bigg]$$

$$+ \frac{\tilde{V}_r}{i\Omega} \Bigg[- \frac{\partial (\underline{V}_o \times \underline{B}_o)_r}{\partial r} - \frac{2(\underline{K}_t \cdot \underline{C}_A)^2 (\underline{V}_o \times \underline{B}_o)_r}{r\, \Omega_A^2} + \frac{2\Omega_E\, \Omega^2 B_{o\theta}}{\Omega_A^2}$$

$$- (\underline{V}_o \times \underline{B}_o)_r \left(\frac{\Omega^2 (\bar{g} + 2C_A^2/r) - 2(\underline{K}_t \cdot \underline{C}_A)^2 C_s^2/r}{(1+M^2)\; C_s^2\; \Omega_M^2} \right) \qquad \text{(B.7)}$$

$$- \frac{2\Omega\Omega_E (\underline{V}_o \times \underline{B}_o)_r}{(1+M^2)\Omega_M^2} \left(\frac{C_{A\theta} (\underline{K}_t \cdot \underline{C}_A)^2 (\underline{K}_t \times \underline{C}_A)_r}{\Omega_A^2\; C_s^2} + \frac{K_\phi \dot{M}^2 (\underline{K}_t \cdot \underline{C}_A)^2}{\Omega_A^2} \right) \Bigg]$$

$$\tilde{E}_\theta = \left(\frac{V_{o\phi}(\underline{K}_t \cdot \underline{B}_o)}{\Omega}\right) \tilde{V}_r \tag{B.8}$$

$$\tilde{E}_\phi = - B_{o\theta} \tilde{V}_r \tag{B.9}$$

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BIOGRAPHICAL NOTE

The author was born on 11 November 1949 in Arecibo, Puerto Rico and attended the Dra. Maria Cadilla de Martinez High School. He received his Bachelor's and Master's Degrees in Physics from the University of Puerto Rico in 1970 and 1972 respectively. In 1972 he joined the University of Puerto Rico as an instructor. Since 1974 he has attended M.I.T. where he became interested in the field of magnetospheric plasma physics.