

XII. PHYSICAL ACOUSTICS

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A. NONLINEAR DISTORTION OF SOUND WAVES IN LIQUID HELIUM*

Some initial results in this investigation were presented in Quarterly Progress Report No. 54, pages 152-154. The photographs (Fig. XIII-2, p. 153) indicated a roughly linear spatial growth in the second harmonic of an initially monochromatic finite wave, followed by a steep exponential decay. It is possible that some of this nonlinear distortion occurs at the crystal surface as the wave bounces back and forth between

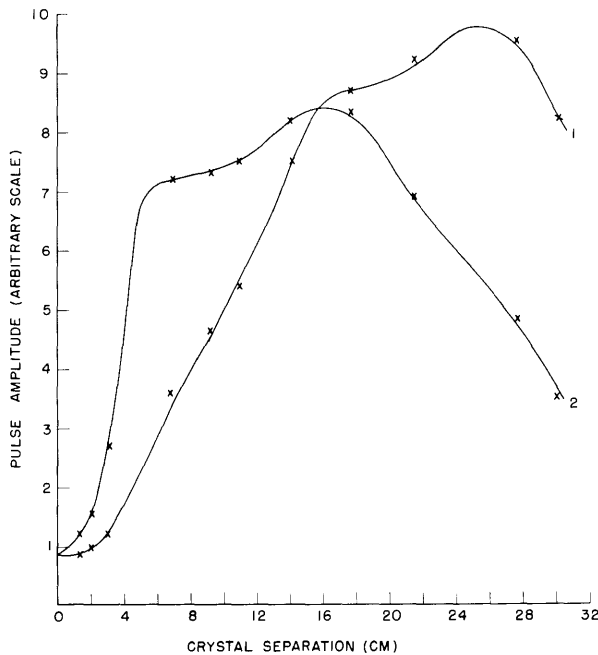


Fig. XII-1. Wave pulse amplitude of second harmonic as a function of separation between source and pickup transducers. $T = 2.94 \pm .03^\circ\text{K}$; the driving voltage was 11.4 volts rms.

the crystals. In order to avoid this possible mixing at the crystal surfaces, the multiple-reflection method of measurement was avoided, and the spatial distortion of the wave was studied as a function of travel distance by variation of the location of the pickup crystal.

Such spatial measurements were taken over a range of source strengths and helium temperatures. The poor reproducibility of these measurements led to partial redesign of the electronic apparatus. The first set of data taken with the redesigned equipment is plotted in Fig. XII-1.

Curve 1 in Fig. XII-1 shows the amplitude of the pulse at the second harmonic as it strikes the pickup crystal. Curve 2 in Fig. XII-1 shows the amplitude of the pulse at its second reflection from the

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pickup crystal. If the horizontal scale is changed by a factor of 3 for curve 2, and if the nonlinear distortion at the crystal surfaces is considered negligible, then because the acoustic path is three times longer for the second reflection than for the first, we would expect curve 2 to be an attenuated version of curve 1. In fact, the first curve does begin to flatten at a point near 16 cm, while the second curve flattens near 4-6 cm, which is consistent with these expectations. The anomalous second peak, on the other hand, departs significantly from the 3/1 adjustment, since the peaks occur at 25 cm and 16 cm. If forthcoming measurements bear out the present data, it will appear that this second peak, whose existence is unexplained thus far, is dependent on factors other than the distance of acoustic travel. Conditions at the reflecting surfaces will be investigated further.

H. L. Willke, G. W. Myers, U. Ingard

B. SCATTERING OF SOUND BY SOUND*

Work is progressing on the scattering of two sound pulses by each other. The pulses are generated in water and collide at right angles. In the absence of hard objects in the interaction region no scattered wave has been observed. However, when hard objects are placed in the interaction region, strong signals are observed.

Measurements were taken with a glass rod placed in the interaction region perpendicular to the plane of interaction. They are being analyzed to determine whether or not they represent nonlinear scattering.

L. W. Dean III

C. GENERATION OF HYPERSONIC WAVES*

Two years ago, generation of sound waves of frequencies of the order of 1000 mc was reported by Baranskii, at the University of Moscow (1). About a year ago, we set up an experiment designed to reproduce Baranskii's results, and to go to higher frequencies.

Our technique was similar to Baranskii's – driving a quartz crystal bar by the electric field in a resonant cavity. The presence of sound in the quartz crystal is detected by diffraction of light. Several cavities have been built with resonant frequencies between 2000 mc and 500 mc. Our initial work was performed at frequencies near 1000 mc, but no positive results were obtained. Meanwhile, Bömmel (2) succeeded

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in generating 1000 mc/sec sound in quartz by using the same method as Baranskii. The purpose of this note is to report that we have now been able to generate 570 mc/sec sound in quartz. Details of this work are reported in the author's thesis (3).

M. B. Moffett

References

1. K. N. Baranskii, Doklady Akad. Nauk S.S.S.R. 114, 517 (1957); English translation, Soviet Physics Doklady 2, 237 (1957).
2. H. E. Bömmel and K. Dvansfeld, Phys. Rev. Letters 1, 234 (1958).
3. M. B. Moffett, Optical detection of elastic waves in quartz, S.B. and S.M. Thesis, Department of Physics, M.I.T., 1959.

D. THE INFLUENCE OF ATMOSPHERIC TURBULENCE ON THE SOUND FIELD OVER A PLANE BOUNDARY*

1. Introduction

The influence of atmospheric turbulence on sound propagation in free space, with no consideration of reflecting boundaries, has been discussed in numerous papers. The addition of boundaries, of course, does not introduce anything basically new; the problem is roughly equivalent to that of radiation from an extended sound source in a turbulent, infinite medium. However, in the analysis of the sound field from a source over ground, it has been customary to consider only the effects of the (reflecting) ground alone, with no regard to the fluctuations of the atmosphere. The sound fields obtained on this basis, in both homogeneous and inhomogeneous atmospheres, have been found to be in good agreement with the results of experiments in the laboratory, where the medium above the reflecting boundary can be kept quiescent. But similar experiments performed outdoors give results that generally deviate considerably from the field obtained from the customary analysis. For example, the measured sound pressure in shadow zones created by temperature or wind is considerably greater than that predicted from diffraction theory. Similarly, in the absence of shadow zones, the sound pressure squared is predicted to decrease as $1/x^4$ with the horizontal distance x from a point source (12 db per doubling of distance) when x is sufficiently large. Field measurements rarely confirm this result. It is the purpose of this note to investigate this problem and to determine to what extent atmospheric turbulence modifies the conventional field distribution. It will be shown that even modest fluctuations of the atmosphere produce marked changes of the field distribution.

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2. The Field from a Point Source

We shall consider the field from a point source located a distance h above a plane boundary, as shown in Fig. XII-2. The boundary is specified acoustically by a normal admittance, η , which is assumed to be independent of the angle of incidence. The sound field above the boundary can be expressed as the sum of the direct and reflected fields, $p = p_d + p_r$. If the source height is larger than a half-wavelength, this field has an interference pattern with maxima and minima that extend a certain distance from the source.

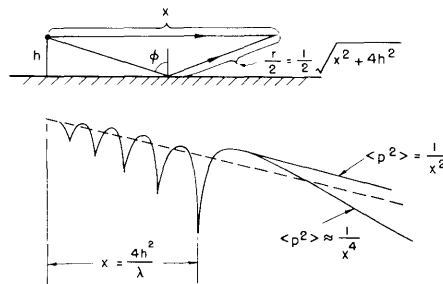


Fig. XII-2. Sound source over a plane boundary. If the atmosphere is homogeneous and quiescent, the sound field (if $h > \lambda/4$) consists of an interference zone (approximately defined as $x < 4h^2/\lambda$) with maxima and minima. For large values of x , $\langle p^2 \rangle \approx 1/x^2$ when the boundary is totally reflecting, and $\langle p^2 \rangle \approx 1/x^4$ when the boundary is partially reflecting.

Beyond this distance, the field decreases monotonically with distance. Since the field depends on the phase difference between the direct and the reflected waves, it is clear that fluctuations of this phase difference can cause changes of the rms value of the sound pressure field which should be particularly significant in the minima of the interference pattern and in the $1/x^4$ (12 db per doubling of distance) region.

In a homogeneous, quiescent atmosphere the sound pressure in the direct wave at a horizontal distance x from the source can be expressed as $p_d = \exp[i(kx - \omega t)]/x$. Then, for the purpose of this report, it is sufficient to use as an approximation for the reflected wave the value $p_r = R \exp[i(kr - \phi)]/r$, where $R = (\cos \phi - \eta)/(\cos \phi + \eta)$ is the plane-wave reflection coefficient, and x , r , and ϕ are the distances and angle indicated in Fig. XII-2. Then, if we restrict ourselves to a real value of η (and R), the variance of the total sound pressure becomes

$$\langle p^2 \rangle = \left(\frac{1}{x}\right)^2 \left[1 + R^2 \left(\frac{x}{r}\right)^2 + 2R \frac{x}{r} \langle \cos \beta \rangle \right] \quad \text{for } R \text{ real} \quad (1)$$

where β is the phase difference between the direct and the reflected waves, and angular

brackets represent time average.

If the index of refraction is denoted $(1+\mu)^{-1}$, the speed of sound is $c = c_0(1+\mu)$, where c_0 is the speed of sound in the quiescent atmosphere. If the temperature fluctuations in the atmosphere are negligible, the index of refraction is simply the component of the (vector) Mach number \underline{m} in the direction of sound propagation; that is, $\mu = m_s$. Then, if we use ray or geometric approximation, the phase difference between the direct and the reflected rays is

$$\beta = k_0(r-x) + k_0 x \int (\mu_1 - \mu_2 / \sin \phi) dx = \beta_0 + \delta \quad (2)$$

in which μ_1 and μ_2 are the values of the index of refraction along the paths of the direct and the reflected waves, respectively, and $\sin \phi = x/(x^2+4h^2)^{1/2}$. We have denoted the steady part of the phase difference β_0 , and the fluctuating part δ . Then, if we assume equal probability of positive and negative values of δ , the variance of the pressure can be rewritten as

$$\langle p^2 \rangle = \left(\frac{1}{x}\right)^2 \left[1 + R^2 \left(\frac{x}{r}\right)^2 + 2R \frac{x}{r} \langle \cos \delta \rangle \cos \beta_0 \right] \quad (3)$$

It has been assumed in taking the time average that the fluctuations of δ have a much lower frequency than the acoustic frequency. If the amplitude of the δ fluctuations is assumed to increase with distance, it is clear that at a certain distance x_1 the amplitude of δ reaches a value approximately equal to 2π . Beyond this distance, the average of $\cos \delta$ will approach zero, the direct and reflected waves will be uncorrelated, and the rms value of the sound pressure will decrease with distance as in free field, that is, with 6 db per doubling of distance.

In order to determine the effect of the fluctuations on the sound pressure field in more detail, we must express $\langle \cos \delta \rangle$ in terms of the statistical properties of the index of refraction. We have already indicated that when the amplitude of the fluctuation of δ is larger than 2π we have $\langle \cos \delta \rangle \approx 0$. Therefore, we are interested mainly in the region in which δ is small. When δ is sufficiently small, we can approximate $\langle \cos \delta \rangle$ by $\cos(\langle \delta^2 \rangle)^{1/2}$. The variance $\langle \delta^2 \rangle$ can be expressed in terms of the index of refraction as

$$\langle \delta^2 \rangle = k^2 \iint \langle (\mu_1 - \mu_2 / \sin \phi)' (\mu_1 - \mu_2 / \sin \phi)'' \rangle dx' dx'' \quad (4)$$

For the purpose of estimating the effect of the fluctuations of the index of refraction, we shall neglect $\langle \mu_1 \mu_2 \rangle$ in comparison with $\langle \mu_1^2 \rangle$ and $\langle \mu_2^2 \rangle$, an approximation that is justified if the eddy size is sufficiently small in comparison with the height of the source over ground (μ_1 refers to the position at the height $y = h$, whereas μ_2 refers to the position $y = h - x \cot \phi$ along the path of the reflected ray). Then, using an autocorrelation function of the form $\langle \mu^2 \rangle = \mu_0^2 \exp(-\xi^2/L^2)$, where $\xi = |x'' - x'|$, we obtain the approximate

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expression for $\langle \mu^2 \rangle$:

$$\langle \delta^2 \rangle \approx 2\sqrt{\pi} kx kL \mu_0^2 \quad (5)$$

where L is a measure of the eddy size and correlation length. Using expression 5 for the calculation of the rms value of the phase difference, we estimate that the direct and the reflected waves will be incoherent at a distance X that is given approximately by $2\sqrt{\pi} kX kL \mu_0^2 \approx \pi$, and hence

$$X = \frac{1}{8\sqrt{\pi}} \frac{1}{\mu_0} \frac{\lambda^2}{L} \quad (h \ll X) \quad (6)$$

For example, with $L = 5$ ft, $\mu_0 = 0.01$, and $\lambda = 1$ ft (1000 cps), the critical distance will be approximately $X \approx 150$ ft.

In Figs. XII-3 and XII-4 are shown the results of some numerical calculations of the sound field, as given by Eq. 3, in which we have used the approximate value of $(\langle \delta^2 \rangle)^{1/2}$ given in Eq. 5.

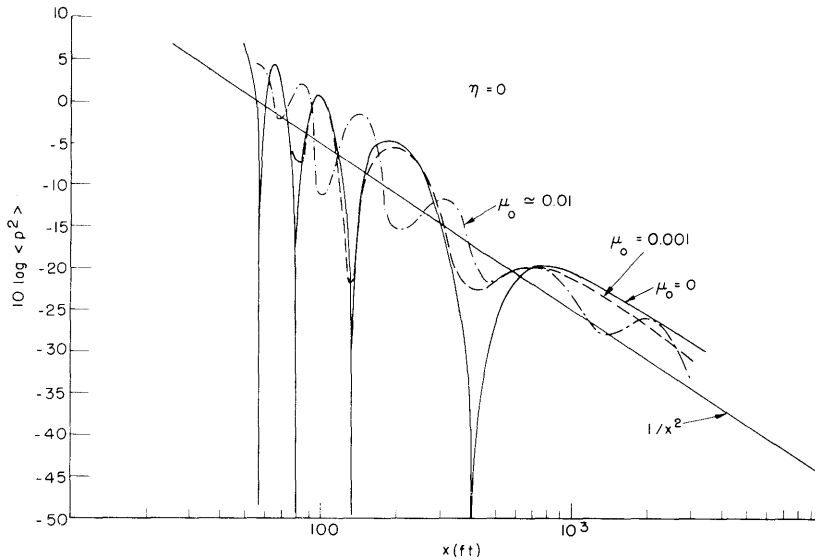


Fig. XII-3. Effect of turbulent fluctuations on the sound field from a point source over a plane, totally reflecting ground surface.

For values of $(\langle \delta^2 \rangle)^{1/2}$ larger than π , we have assumed incoherence between the direct and reflected waves. Three different values of μ_0 : 0, 0.001, and 0.01, and two different boundary admittances, $\eta = 0$ and $\eta = 0.2$, have been used, corresponding to the specific acoustic impedances $\zeta = \infty$ and $\zeta = 5$ (in units of the characteristic impedance

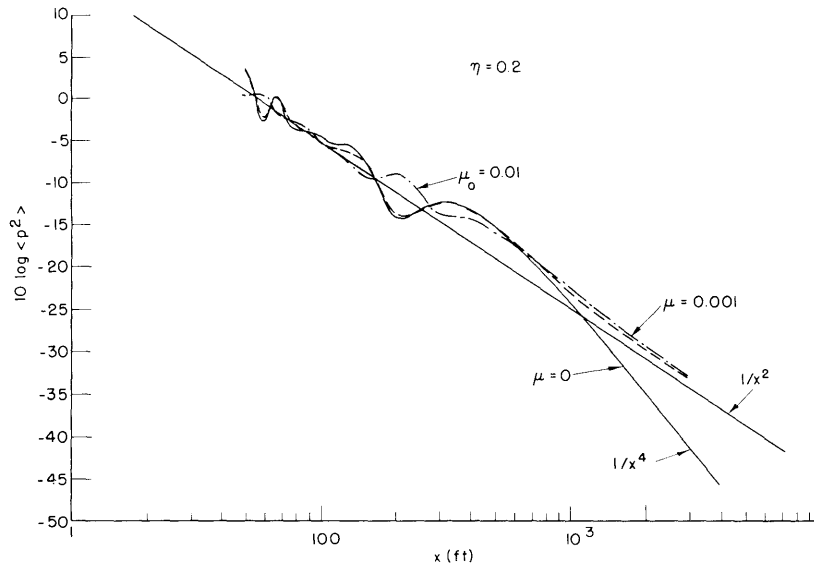


Fig. XII-4. Effect of turbulent fluctuations on the sound field from a point source over a plane, partially reflecting ground surface.

of the medium). With a totally reflecting boundary ($\eta=0$), we see that the deep pressure minima in the interference zone are blurred out by the fluctuations, and with a partially reflecting boundary ($\eta=0.2$), the decrease of the sound pressure level with distance for large values of x is reduced by the turbulent fluctuations from 12 db to 6 db per doubling of distance.

3. Fluctuations in the Sound Field

The fluctuations in the sound pressure field in a plane wave, or about a point source in an unbounded turbulent region, can be shown to increase monotonically with the distance from the source. In the presence of a reflecting ground surface, on the other hand, when the sound field exhibits an interference pattern with maxima and minima, the measured fluctuations are found to be much larger than those obtained in free field and do not always increase with distance. Often we find that the fluctuations reach a maximum at a relatively short distance from the source. We can readily demonstrate this effect by measuring the fluctuations of a pure-tone signal from a reasonably directive source (an ordinary loud-speaker) when the signal is transmitted, first, parallel with the ground, and then straight up (in which case reflections from the ground are negligible). With a signal frequency of approximately 2000 cps (or higher) and a source and receiver height of approximately 5 ft, the fluctuations at a distance of 50 ft can be as great as 15 to 20 db, whereas at the same distance above the source and with the signal directed upward, the fluctuations are found to be approximately 2-4 db.

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These experimental results can be understood qualitatively by noticing that at a minimum in the interference pattern, where the sound pressure is practically zero, even a slight contribution of scattered energy will produce a considerable fluctuation in the sound pressure level. Consequently, large fluctuations of the sound pressure level are expected in the interference region, which for a perfectly reflecting plane is defined by $x < 4h^2/\lambda$ (if we assume the receiver height to be the same as the source height, h).

To compute the corresponding peak-to-peak fluctuations in the sound pressure level over ground, we need to determine the ratio between the maximum and minimum values of $\langle p^2 \rangle$ in Eq. 1, because $\cos(\beta_0 + \delta)$ varies (slowly) with time on account of the fluctuations of the phase difference δ . The fluctuation amplitude obtained on this basis will depend on the distance from the source. We shall not carry through a detailed numerical analysis now but limit ourselves to a study of the fluctuation amplitude only at the locations of destructive interference, where $\cos \beta_0 = -1$. At these points the minimum and maximum levels are obtained at the instants when δ passes through zero and a maximum value, respectively.

The minima in the interference pattern are located at

$$x_n = \frac{2n-1}{4} \left[\left(\frac{4}{2n-1} \frac{h}{\lambda} \right)^2 - 1 \right] \lambda \quad n = 1, 2, \dots, N \quad (7)$$

Equation 7 applies only when $\eta = 0$ and the source height is equal to the receiver height. The minimum farthest from the source corresponds to $n = 1$, and is located at $x_1 = 4h^2/\lambda - \lambda/4$. (Clearly, in order to have a minimum point at all, the height of the sound source above the ground must be larger than a quarter-wavelength when the boundary is totally reflecting.) The (time-dependent) ratio between the maximum and the minimum values of the variance of the sound pressure at such a point becomes

$$\gamma_n = \left(\frac{p_{\max}^2}{p_{\min}^2} \right)_n = 1 + \frac{A_n^2}{2} (1 - \langle \cos \delta \rangle) \quad (8)$$

$$A_n = \left(\frac{4}{2n-1} \cdot \frac{h}{\lambda} \right)^2$$

For large values of A_n and small values of δ^2 , Eq. 8 can be written

$$\gamma_n = 1 + \frac{x_n^2}{h^2} \langle \delta^2 \rangle \quad (A \gg 1) \text{ and } (\delta^2 \ll 1) \quad (9)$$

Since, according to Eq. 5, $\langle \delta^2 \rangle$ is proportional to $kx \cdot kL$, it follows that the $(\gamma_n - 1)$ increases as the third power of the distance x_n and as the second power of frequency.

In the case of a totally reflecting boundary, the interference zone reaches out to a

distance of $x = 4h^2/\lambda - \lambda/4$, which at a frequency of 1000 cps and a source height of 10 ft is approximately 150 ft. After this last minimum is reached the sound pressure field decreases monotonically with x in the same way as it does from a point source in free space. The fluctuations in the sound amplitude are then expected to be approximately the same as for a plane wave in a turbulent medium, a result which is well known.

If the sound pressure amplitude in a plane wave is denoted by $A = A_0 + a$, where A_0 is the unperturbed amplitude, and a is the perturbation caused by turbulence, the variance of the perturbation is $\langle a^2 \rangle / A_0^2 \approx 0.5\sqrt{\pi} kx kL \mu_0^2$ for sufficiently large values of x , $x > 4\sqrt{L}\lambda$. If we add a factor 2 to account for the reflecting plane, and rewrite this expression to conform with Eq. 9, we have, for $x > (4L^2/\lambda - \lambda/4)$ and $x > \sqrt{L}\lambda$,

$$\frac{A_0^2 + \langle a^2 \rangle}{A_0^2} \approx 1 + \sqrt{\pi} kx kL \langle \mu^2 \rangle \quad (10)$$

However, when the boundary is not perfectly reflecting, we shall not approach free-field conditions even for large values of x . The variance of the unperturbed sound pressure field will decrease as $1/x^4$, rather than $1/x^2$, as in free field. Consequently, the ratio between the maximum and the minimum values of the fluctuating variance will be proportional to x^2 , and the fluctuations will be considerably larger than those predicted for the free-field condition in Eq. 10. In other words, the amplitude of fluctuation of the variance of the sound pressure over a boundary should increase with distance as x^3 in the interference region and approach x^2 beyond this region for large values of x .

4. Experimental Results

The experimental results presented here were obtained in the summer of 1953 at Ipswich Beach, Ipswich, Massachusetts, in connection with studies of sound propagation in the atmosphere. The object of these particular measurements was the study of sound propagation into shadow zones created by temperature and wind; the results obtained on fluctuations were incidental and incomplete. Nevertheless, it is interesting to see how the results compare with the analysis which is presented here.

In Fig. XII-5 the peak-to-peak fluctuations of pure tones transmitted over ground (wet sand with sparse grass) from a sound source located 6 ft above the ground are shown. These fluctuations, measured 6 ft above ground, and read directly on the record of a logarithmic sound-pressure-level recorder, represent the peak-to-peak variations of $10 \log \langle p^2 \rangle$. The average wind velocity measured at 6 ft above ground was 9 ft per second, and the direction was the same as the direction of the course of the sound propagation over which the measurements were taken. Similar results, obtained when the direction of the sound propagation was opposite to the wind direction, are shown in Fig. XII-6.

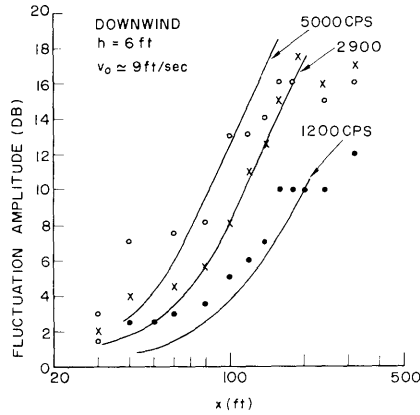


Fig. XII-5. Measured and calculated fluctuation amplitudes of the sound pressure level in a turbulent atmosphere over a plane boundary.

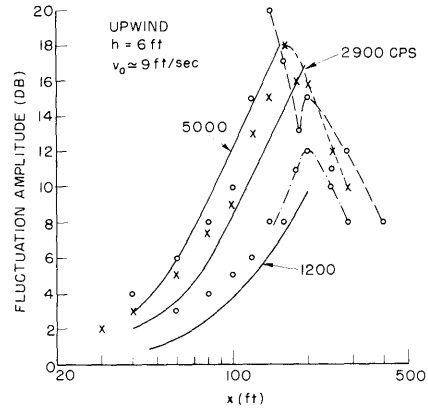


Fig. XII-6. Measured and calculated fluctuation amplitudes of the sound pressure level in a turbulent atmosphere over a plane boundary.

To discuss these results, we note that at the lowest frequency shown, 1200 cps, the interference zone reaches out to approximately $x = 160$ ft. Here, we expect to find the beginning of a transition in the fluctuation curve to an x^2 -like dependence for large values of x . The experimental curve shows a leveling off in this region, and there is an indication of a later increase. However, there are not enough experimental points on the curve for drawing any conclusions regarding the continued slope of the curve. A similar behavior is shown at 2900 cps. At this frequency the distance to the last minimum in the interference pattern is expected to be approximately 380 ft. We find an indication of a transition at a position close to 200 ft, that is, closer to the source than expected. At 5000 cps the scatter of the points is considerable, and there is an indication of a transition at 200 ft. With fluctuations as great as 20 db it is conceivable that background noise might have limited any continued increase of the curve at this point.

The results obtained when the sound propagates into the wind indicate a marked decrease in the fluctuation beyond a certain distance. This behavior presumably is a result of the acoustic shadow formed in the upwind direction. In this case the shadow boundary is located approximately 100 ft from the source. Inside the boundary there is no interference between a direct and a reflected wave, and the fluctuation is therefore expected to be somewhat smaller in the shadow than in the interference region.

In order to compare the experimental results in the interference region with Eq. 3, we select a value for the unknown product of the scale L and the variance of the index of refraction $\langle \mu^2 \rangle$ so that Eq. 3 will be in agreement with the experimental values at one particular distance and one particular frequency. We choose for reference a distance of 100 ft and a frequency of 2900 cps, which correspond to a measured value of the 8-db

fluctuation in the downwind direction (see Fig. XII-2). The corresponding value of $L \langle \mu^2 \rangle$ then is $0.2 \cdot 10^{-6}$. If we use this value in Eq. 3 for the calculation of the fluctuation at other locations and other frequencies, we obtain the curves shown in Fig. XII-4. Although considerable deviation occurs at certain points, our analysis of the fluctuations at the minimum points in the interference pattern gives an over-all x-dependence of the fluctuation amplitude that is roughly in agreement with measurements. At frequencies much lower than 1000 cps, the length of the interference region becomes very short and the number of minima too small to make a comparison between predicted and measured x-dependence meaningful. At sufficiently low frequencies, such as $\lambda > 4h$, there will be no interference region at all. In this case the fluctuation amplitude of the variance of the sound pressure should increase with distance approximately as x^2 after a comparatively short distance that is determined by the impedance of the ground.

U. Ingard