XIII. NOISE IN ELECTRON DEVICES*

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RESEARCH OBJECTIVES

A detailed study of the noise performance of linear amplifying systems having multiple input and output signal bands will be carried out. The object is to find the limits on the noise performance of such devices when the excitations in the signal bands are partly or fully correlated.

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A. WAVE REPRESENTATION OF AMPLIFIER NOISE

A wave representation of noise in a linear two-port, which has been given by Bauer and Rothe,[1] apparently has not been described in English. The representation has the advantages that both equivalent noise generators are at the input to the amplifier (or linear two-port), and that they are uncorrelated.

1. Previous Representations

Several schemes have been used to represent noise at a given frequency f in a linear two-port.[2] By Thévenin's theorem, the noise generated internally by the two-port can be represented by two noise voltage sources in series with the input and the output. These two generators are, in general, correlated, and four real numbers are necessary to specify the noise properties of the two-port; these may be the mean-square values of the two voltages and their complex correlation coefficient.

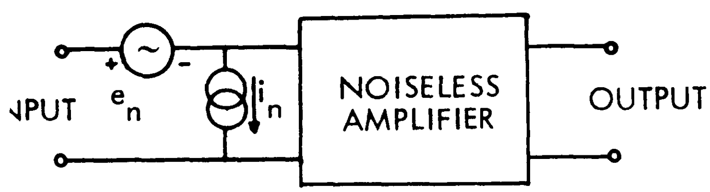

Fig. XIII-1. The Rothe-Dahlke noise model for a noisy two-port has two correlated generators at the input. The advantage is that the noiseless part can be disregarded in calculating the noise figure of the amplifier.

Unfortunately, in this representation there is one equivalent generator at each port.

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For many purposes (particularly calculating noise figures of amplifiers) it is advantageous to have both generators at one port, the input. Rothe and Dahlke[3] have shown how to do this, with the equivalent circuit of Fig. XIII-1. The two generators are correlated; one can define the four real numbers associated with this representation as the mean-square values of e_n and i_n, and their complex correlation coefficient ρ or, alternatively, the quantities

$$R_n = \frac{\overline{|e_n|^2}}{4kT_0\Delta f} \tag{1}$$

$$G_n = \frac{\overline{|i_n|^2}}{4kT_0\Delta f} \tag{2}$$

and

$$\rho = \frac{\overline{i_n e_n^*}}{\sqrt{\overline{|i_n|^2}\;\overline{|e_n|^2}}}. \tag{3}$$

The correlation coefficient ρ is less than or equal to 1 in magnitude.

The advantage of this representation is the ease of calculating the amplifier noise figure. The noiseless amplifier shown in Fig. XIII-1 does not affect the noise figure because it treats all noises and signals identically. The noise figure expression is, therefore, independent of the properties of the noiseless amplifier except insofar as they help to determine the quantities R_n, G_n, and ρ. If the source impedance is $Z_s = R_s + jX_s$, the excess noise figure[2, 3] is

$$F - 1 = \frac{\overline{|e_n+Z_s i_n|^2}}{4kT_0\Delta f R_s}$$

$$= \frac{R_n + |Z_s|^2 G_n + 2\sqrt{R_n G_n}\,\mathrm{Re}(\rho Z_s)}{R_s} \tag{4}$$

As Z_s is varied over all values in such a way that R_s is positive, the noise figure goes through the minimum value

$$(F-1)_{min} = 2\sqrt{R_n G_n}\left(\sqrt{1-(\mathrm{Im}\,\rho)^2} + \mathrm{Re}\,\rho\right). \tag{5}$$

This minimum occurs for the particular value of source impedance $Z_{s,opt} = R_{s,opt} + jX_{s,opt}$

$$|Z_{s,\,opt}|^2 = \frac{R_n}{G_n} \tag{6}$$

$$X_{s,\,opt} = \sqrt{\frac{R_n}{G_n}}\, \mathrm{Im}\, \rho. \tag{7}$$

It has been suggested[2] that the noise characteristics of a linear two-port, such as an amplifier, be specified by the four numbers $(F-1)_{min}$, R_n, and $Z_{s,\,opt}$ ($Z_{s,\,opt}$ is a complex number). Probably such a specification is more convenient than simply R_n, G_n, and ρ, since the quantities of engineering interest are $(F-1)_{min}$ and the optimum source impedance.

2. Wave Representation

An alternate representation, suggested by Bauer and Rothe,[1] uses wave, or scattering[4] variables, and retains the advantages of the Rothe-Dahlke representation. It has the further advantage that the two noise sources are uncorrelated, so that the expression for noise figure is particularly simple.

We define the noise wave generators[5] a_n and b_n at the input of the amplifier, in terms of the parameters of the Rothe-Dahlke model,

$$a_n = -\frac{e_n + Z_\nu i_n}{2\sqrt{\mathrm{Re}\, Z_\nu}} \tag{8}$$

$$b_n = -\frac{e_n - Z_\nu^* i_n}{2\sqrt{\mathrm{Re}\, Z_\nu}} \tag{9}$$

where Z_ν is a normalization impedance. Thus, if the scattering matrix of the two-port is S, we obtain

$$\begin{bmatrix} b_1 - b_n \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \times \begin{bmatrix} a_1 + a_n \\ a_2 \end{bmatrix}. \tag{10}$$

An equivalent circuit of this representation is shown in Fig. XIII-2. The wave generators are represented by directional couplers and ordinary sources; this schematic is accurate in the limit of small coupling to the line and large strengths of the sources. Note that this "transmission line" has a complex "characteristic impedance," Z_ν.

The normalization impedance Z_ν used in Eqs. 8 and 9 is chosen so as to make

a_n and b_n uncorrelated,

$$\overline{a_n b_n^*} = 0. \tag{11}$$

It happens that this impedance is precisely $Z_{s,\,opt}$ given in Eqs. 6 and 7. This point is of importance: If the optimum source impedance is used as the normalization impedance in defining noise wave generators from the Rothe-Dahlke noise generators, the

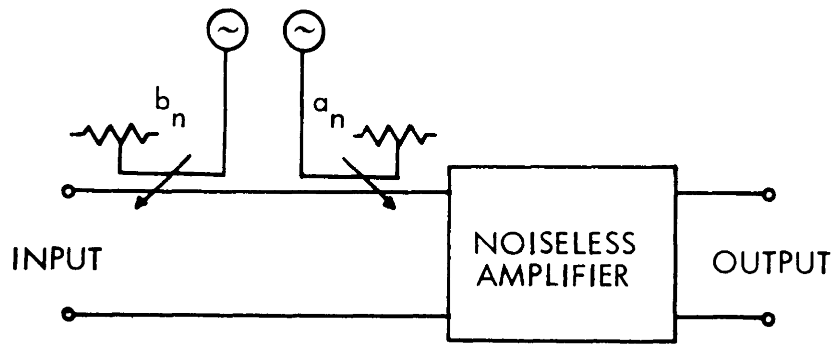

Fig. XIII-2. Proposed wave model of a noisy two-port consists of a noiseless two-port with uncorrelated wave generators at the input. The noiseless two-port can be disregarded in calculating the noise figure of the amplifier.

resulting wave generators are uncorrelated. The simplicity of the formulas given below is entirely the result of this fact. The wave representation here fails when $R_{s,\,opt} = 0$; that is, only when $R_n = 0$, $G_n = 0$, or $\rho = \pm j$. In each of these cases, $(F-1)_{min}$ equals zero and is achieved with a reactive source. Only one equivalent noise source is necessary if it is properly placed.

The strength of a_n defines a temperature T_a.

$$T_a = \frac{\overline{|a_n|^2}}{k\Delta f}, \tag{12}$$

and similarly, the strength of b_n defines a temperature T_b.

$$T_b = \frac{\overline{|b_n|^2}}{k\Delta f}. \tag{13}$$

In terms of the Rothe-Dahlke model, these are

$$T_a = 2T_0\sqrt{R_nG_n}\left(\sqrt{1-(\mathrm{Im}\,\rho)^2} + \mathrm{Re}\,\rho\right) \tag{14}$$

$$T_b = 2T_0\sqrt{R_nG_n}\left(\sqrt{1 - (\mathrm{Im}\,\rho)^2} - \mathrm{Re}\,\rho\right) \tag{15}$$

so that T_a is related to the minimum noise figure of Eq. 5:

$$T_a = T_0(F-1)_{min}. \tag{16}$$

The expression for the noise figure F − 1 is very simple when written in terms of these wave variables. Consider the amplifier run from a source with impedance $Z_s = R_s + jX_s$. The reflection coefficient of the source, Γ_s, is

$$\Gamma_s = \frac{Z_s - Z_\nu}{Z_s + Z_\nu^*} \tag{17}$$

when the wave variables are defined with Z_ν as the normalization impedance. Then the noise figure is simply

$$F - 1 = \frac{T_a + T_b|\Gamma_s|^2}{T_0(1-|\Gamma_s|^2)}$$

$$= \frac{T_a}{T_0} + \frac{T_a + T_b}{T_0}\,\frac{|\Gamma_s|^2}{1 - |\Gamma_s|^2}$$

$$= \frac{T_a}{T_0} + \frac{T_a + T_b}{T_0}\,\frac{|Z_s - Z_\nu|^2}{4R_s\,\mathrm{Re}(Z_\nu)}. \tag{18}$$

The choice of source impedance to minimize the noise figure, from Eq. 18, is obviously the value that sets Γ_s to zero, or simply Z_ν. The minimum excess noise figure so achieved is T_a/T_0. The temperatures T_a and T_b can be easily measured, at least in principle, even at microwave frequencies.

It has been suggested[2] that T_a [or $(F-1)_{min}$], Z_ν, and one other parameter (say, R_n) be used to characterize noisy two-ports, since these are the parameters of most engineering interest. Perhaps a more meaningful fourth parameter to specify would be T_b, or a quantity related to it, rather than R_n.

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