

XXIX. SENSORY AIDS RESEARCH*

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RESEARCH OBJECTIVES

The long-range objective of this group is to develop useful artificial sensory mechanisms for people afflicted with sensory deprivation – in particular, vision or vision-plus-audition deprivation.

From our present viewpoint, one important long-range technical problem is to develop "sensor" systems that extract small amounts of information from, for example, an optical image, and "display" systems that feed the processed information into the skin or the ear. Among the sensor-extractor systems that are being considered are: (a) edge tracers for the extraction of a "cartoon" from an image; (b) various two-dimensional autocorrelation schemes for extracting position-independent and size-independent qualities of an image; and (c) color sensors for the exploitation of color information in an image. A multiplicity of different "filters" that simultaneously extract different crude aspects of an image may turn out to be a useful sensor-system for a mobility aid and perhaps for a reading machine.

S. J. Mason

A. BOUNDS ON INFORMATION TRANSFER FOR A PARTICULAR CLASS OF CHANNELS

In this report we discuss the derivation of bounds on information transfer for a model channel that is useful for characterizing many sensory-aid systems. Experimental measurements have substantiated the validity of the model channel and the derived bounds.

This model channel is proposed in an attempt to make a first-order approximation to an actual sensory-aid channel. Upper and lower bounds have been derived in terms of two variables defined from the channel matrix. An experimentally measured channel will be compared with the model channel, and the calculated information transfer compared with the derived upper and lower bounds.

1. Description of the Class of Channels

Most sensory aids can be characterized by information channels whose inputs are stimuli and whose outputs are the responses. The majority of the channels associated with useful sensory aids will have terms that are nearly unity on the major diagonal,

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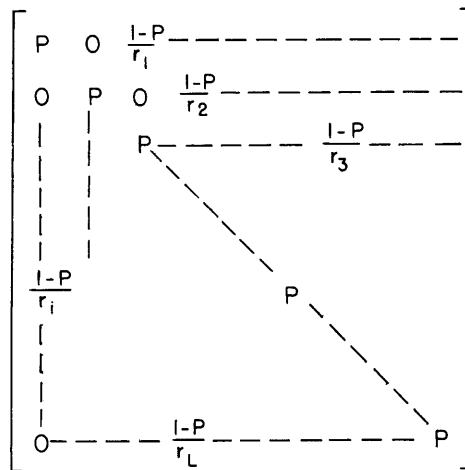


Fig. XXIX-1. Model channel.

since the rather limited memory span of the human receiver precludes the use of complex codes. Furthermore, if the channel is large (32 stimuli, or larger), the dominant features will be an abundance of zeros for the off-diagonal terms and a smattering of rather small error terms.

A first-order approximation to such a channel is given in Fig. XXIX-1. Each of the major diagonal terms of this conditional probability matrix is taken to be equal to p . The off-diagonal terms in each row may be either zero or a constant. If r_i is defined as the number of nonzero off-diagonal terms in the i^{th} row, then this constant is $(1-p)/r_i$. The order of the matrix is L , so that we have $1 \leq r_i \leq L - 1$. No assumptions have been made as to the probability that any off-diagonal term will be zero. The zeros inserted in Fig. XXIX-1 are there only to serve as an example. Although a more general class of channels could be defined by allowing each major diagonal term to be different, it is of interest to investigate this simpler channel, first.

2. Derivation of the Lower Bound

Fano¹ has derived a lower bound on the equivocation, which is

$$H(X|Y) \leq H(e) + P(e) \log (L-1) \quad (1)$$

where

$$H(e) = -P(e) \log P(e) - (1-P(e)) \log (1-P(e)),$$

$P(e)$ is the probability of error, and L is the order of the channel matrix. We shall assume that the input probability distribution $p(x)$ is uniform for the derivation of both bounds so that

$$H(X) = \log L. \quad (2)$$

The information transfer can be expressed as

$$I(X;Y) = H(X) - H(X|Y). \quad (3)$$

Substituting (1) and (2) in (3), we obtain a lower bound of

$$I(X;Y) \geq \log L - P(e) \log (L-1) - H(e). \quad (4)$$

If L is large and $P(e)$ reasonably small, the lower bound given in (4) is not decreased significantly if we apply the inequality $\log L > \log (L-1)$ to (4) to obtain

$$I(X;Y) \geq (1-P(e)) \log L - H(e). \quad (5)$$

The lower bound (5) is subject to the rather interesting interpretation that the transmitted information is at least as great as the percentage of the offered information that is identified correctly, minus the information required to specify when an error was made.

Note that the bounds (4) and (5) hold for all of the channels when the input probability distribution is uniform; that is, they do not require the rather special properties of the proposed model channel. However, if the model channel is considered for the special case $r_i = L - 1$, then the equality sign holds for the bound given by (4).

3. Derivation of the Upper Bound

Since the input $p(x)$ is uniform, Eq. 2 still holds, and we always have

$$H(Y) \leq \log L. \quad (6)$$

$H(XY)$ will now be evaluated as the negative sum over the whole matrix of $\frac{P_{ij}}{L} \log \frac{P_{ij}}{L}$. Consider, first, the sum on row i .

$$\begin{aligned} S_i &= \sum_j \frac{P_{ij}}{L} \log \frac{P_{ij}}{L} \\ &= \frac{p}{L} \log \frac{p}{L} + r_i \frac{1-p}{Lr_i} \log \frac{1-p}{Lr_i} \\ &= \frac{p}{L} \log \frac{p}{L} + \frac{1-p}{L} \log \frac{1-p}{L} - \frac{1-p}{L} \log r_i \end{aligned}$$

Then,

$$-H(XY) = \sum_i S_i = p \log \frac{p}{L} + (1-p) \log \frac{1-p}{L} - \frac{1-p}{L} \sum_i \log r_i. \quad (7)$$

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Since the joint probability of error and any input combined is $(1-p)/L$, the total probability of error is $1 - p = P(e)$. Substitution of $P(e)$ in Eq. 7 yields

$$H(XY) = \log L + H(e) - \frac{P(e)}{L} \sum_i \log r_i. \quad (8)$$

The information transfer can be expressed as

$$I(X;Y) = H(X) + H(Y) - H(XY). \quad (9)$$

Substituting (2), (6), and (8) in (9), we obtain

$$I(X;Y) \leq \log L - H(e) - \frac{P(e)}{L} \sum_i \log r_i. \quad (10)$$

The equality sign holds when $H(Y) = \log L$. We know that $H(Y)$ does not vary rapidly as $p(y)$ departs from a uniform distribution, and we also know that $p(y)$ is reasonably uniform because of the dominance of the major diagonal terms. Thus we should expect that the actual transmitted information will normally be rather close to this upper bound. If we consider the special case $r_i = L - 1$, then the equality sign holds in (10), and we note also that the upper and lower bounds are equal.

4. Comparison of an Actual Sensory-Aid Channel with the Model Channel

Human subjects were presented with input stimuli drawn with uniform probability from a population of 64. The actual equipment used has been described in detail in the author's thesis.² To compare this channel with the proposed channel, a series of tests was analyzed with the aid of the TX-0 computer. A major premise for the model channel is that all of the major diagonal terms are equal. The manner in which the actual channel differs from the model is illustrated by Fig. XXIX-2 in which the ordinate represents

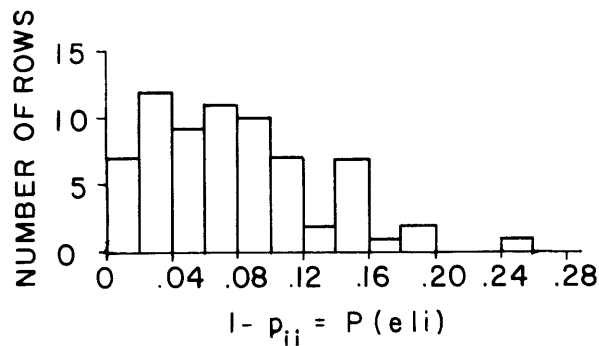


Fig. XXIX-2. Histogram of row $P(e)$.

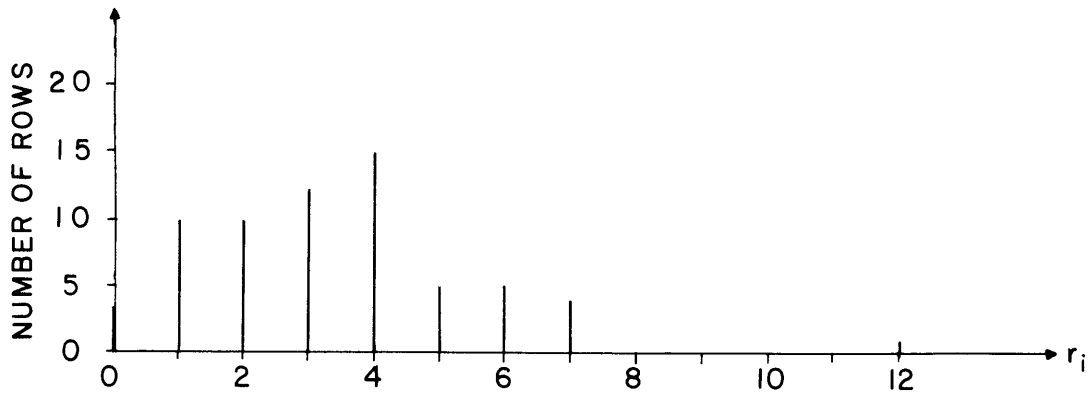
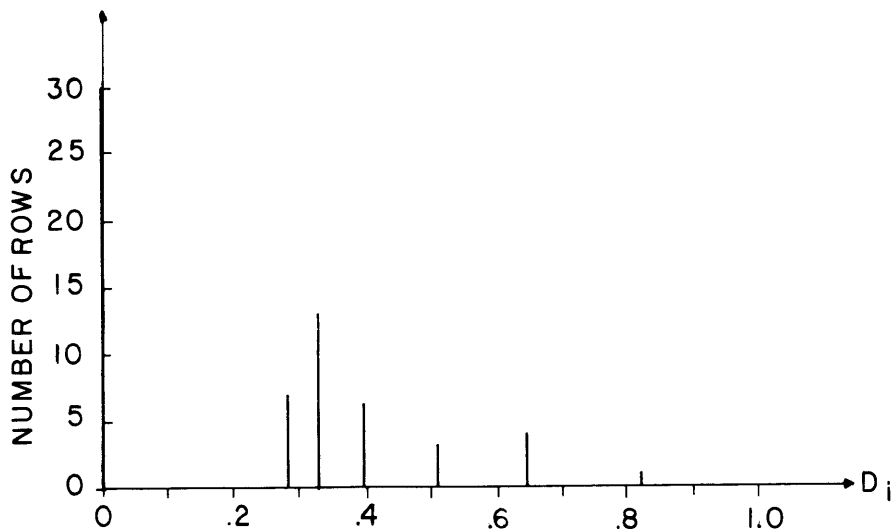
Fig. XXIX-3. Distribution of r_i .

Fig. XXIX-4. Deviation from uniformity of off-diagonal terms.

the number of rows falling within the indicated range of probability of error. The data of Fig. XXIX-2 are based on the analysis of responses to 3936 stimuli. The average, $P(e)$, was calculated to be 0.075.

The most significant feature of the model, however, is the assumption that all of the off-diagonal terms in a row are either zero or $\frac{1 - p_{ii}}{r_i}$. The test data indicate that there is, indeed, a large number of zeros, as indicated by Fig. XXIX-3 in which the ordinate represents the number of rows having the indicated number of nonzero terms (r_i).

Since each nonzero off-diagonal term in the i^{th} row should equal $\frac{1 - p_{ii}}{r_i}$, the following term can be taken as a measure of the difference between the actual and model channels.

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$$D_i = \frac{\sum_{j, i \neq j} \left| \text{nonzero } p_{ij} - \frac{1 - p_{ii}}{r_i} \right|}{1 - p_{ii}} \quad (11)$$

A plot of the frequency of occurrence of values of this measure is given in Fig. XXIX-4.

5. Comparison of an Experimental Measurement of $I(X;Y)$ with the Upper and Lower Bounds

The appropriate comparisons for a channel with $L = 64$ are illustrated by Fig. XXIX-5.

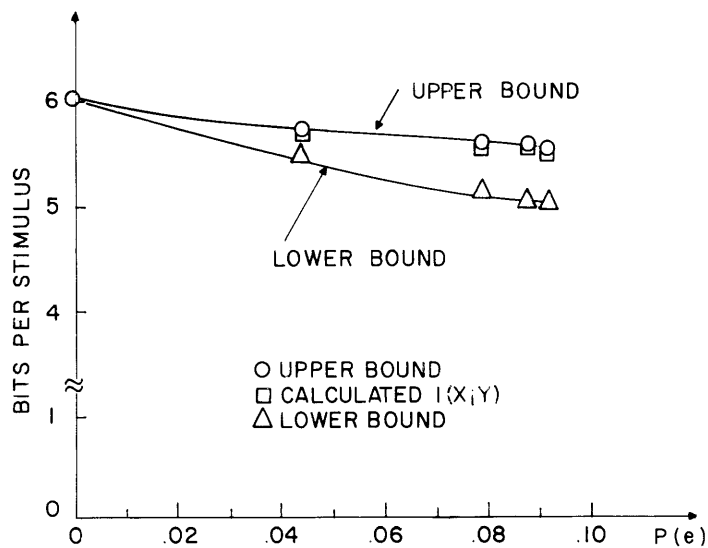


Fig. XXIX-5. Comparison of derived bounds and computed information transfers.

The upper bound that is plotted is given by (10), and the lower bound by (4). As is expected, the actual information transfer is very close to the upper bound.

D. E. Troxel

References

1. R. M. Fano, Transmission of Information (The M. I. T. Press, Cambridge, Mass., and John Wiley and Sons, Inc., New York, 1961), p. 187.
2. D. E. Troxel, Tactile Stimulation of the Fingers as an Information Input to the Blind, S. M. Thesis, Department of Electrical Engineering, M. I. T., September 1960.