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A. HIGHER-ORDER CORRELATION FUNCTIONS OF LIGHT INTENSITY

A measurement of the photoelectron count of a photosurface illuminated by laser light was outlined in Quarterly Progress Report No. 69 (pages 31-33). It was pointed out there that the mean-square deviation of the photoelectron count within a time T is related to the correlation function of the light intensity R(T).[1,2] We shall now show that higher-order moments of the photoelectron count are related to the higher-order correlation functions of the light intensity. In the derivation we assume that the coherence area of the light beam is larger than the photocathode area.

The probability $p_T(K)$ of obtaining exactly K counts in the time interval $0 \leq t \leq T$ from a photocathode illuminated by light of intensity (power) P(t) is

$$p_T(K) = \frac{n_T^K}{K!} e^{-n_T}, \tag{1}$$

where

$$n_T = a \int_0^T P(t)\, dt. \tag{2}$$

Here, a is a proportionality factor incorporating the photoefficiency of the cathode. The falling factorial moment of k^{th} order of the photoelectron count, $\overline{K(K-1) \ldots (K-k+1)}$, is

$$\overline{K(K-1) \ldots (K-k+1)} = \sum_{K=1}^{\infty} \frac{n_T^K}{K!} e^{-n_T} K(K-1) \ldots (K-k+1) = n_T^k. \tag{3}$$

Thus far, we have taken an average with respect to the probability distribution of the Poisson process. If the light intensity itself varies in a statistical manner, an average has to be taken over the statistics of the incident light. This average is denoted by angular brackets.

$$\left\langle \overline{K(K-1) \ldots (K-k+1)} \right\rangle = \left\langle n_T^k \right\rangle = a^k \left\langle \left[\int_0^T P(t)\, dt \right]^k \right\rangle. \tag{4}$$

The right-hand side of Eq. 4 may be written

$$\left\langle n_T^k \right\rangle = k!\, a^k \int_{t_k=0}^{T} \cdots \int_{t_1=0}^{t_2} \left\langle P(t_1) \ldots P(t_k) \right\rangle dt_1 \ldots dt_k. \tag{5}$$

The integrand can be put into the form of a $(k-1)^{th}$-order correlation function by introducing the new variables

$$t_1 = t;\ t_2 = t + \tau_1;\ \ldots\ t_k = t + \tau_{k-1} \tag{6}$$

$$\left\langle n_T^k \right\rangle = k!\ \alpha^k \int_{\tau_{k-1}=0}^{T} \int_{t=0}^{T-\tau_{k-1}} \cdots \int_{\tau_1=0}^{\tau_2} \left\langle P(t)\ldots P(t+\tau_{k-1})\right\rangle d\tau_1 \ldots dt d\tau_{k-1}.$$

If the light is stationary, the expectation value of the integrand does not depend upon t, and the integration over t can be carried out immediately. Furthermore, noting that the integrand is the correlation function of $(k-1)^{th}$ order, $R_{k-1}(\tau_1\ldots\tau_{k-1})$, of the light power, one obtains

$$\left\langle n_T^k \right\rangle = k!\ \alpha^k \int_{\tau_{k-1}=0}^{T} \cdots \int_{\tau_1=0}^{\tau_2} (T-\tau_{k-1})\ R_{k-1}(\tau_1,\ldots\tau_{k-1})\ d\tau_1 \ldots d\tau_{k-1}. \tag{7}$$

Differentiating Eq. 7 with respect to T, one obtains

$$\frac{d}{dT}\left\langle n_T^k \right\rangle = k!\ \alpha^k \int_{\tau_{k-1}=0}^{T} \cdots \int_{\tau_1=0}^{\tau_2} R_{k-1}(\tau_1,\ldots\tau_{k-1})\ d\tau_1 \ldots d\tau_{k-1}. \tag{8}$$

The falling factorial moment of k^{th} order of the photoelectron count within an observation time T is thus simply related to the $(k-1)^{th}$-order correlation function of the light intensity. For the factorial moment of second order, k = 2, one obtains

$$\frac{d^2}{dT^2}\left\langle n_T^2 \right\rangle = 2\alpha^2 R_1(T). \tag{9}$$

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References

1. E. M. Purcell, The question of correlation between photons in coherent light rays, Nature 178, 1449 (1956).

2. L. Mandel, Fluctuations of photon beams and their correlations, Proc. Roy. Soc. (London) 72, 1037-1048 (1958).