VII. NOISE IN ELECTRON DEVICES

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RESEARCH OBJECTIVES

The purpose of the work on the noise performance of multiterminal-pair networks (Quarterly Progress Report No. 68, pages 53-59) has been to ascertain the limiting noise performance, as well as to determine appropriate measures of noise performance for amplifiers with more than two terminal pairs. This phase of the work has been completed and the results were submitted to the Department of Electrical Engineering, M.I.T., June 1963, as part of an Sc.D. thesis by W. D. Rummler. This thesis is to be published as Technical Report 417 of the Research Laboratory of Electronics, M.I.T.

Our future effort will be concentrated in three areas. The first concerns the noise performance of devices in which quantum effects are expected to be detectable or, in fact, dominant. Because amplitude noise measurements on the c.w. gaseous maser oscillator are relatively convenient to make, the theoretical and experimental work will be concentrated around this device.

(a) A quantum-mechanical analysis of the noise in optical maser oscillators will be undertaken. The purpose of the analysis is to determine the limitations of the semiclassical analysis, briefly summarized in this report.

(b) Measurements on the amplitude noise in optical maser oscillators will continue. Our intent is to determine the noise level of "quiet" optical maser oscillators by refinement of the existing apparatus.

(c) The sensitivities of various experimental methods for the determination of amplitude noise of optical waves will be compared experimentally.

The second area of interest is noise in frequency multipliers, especially those made from varactors. We hope to develop simple parameters to indicate the noisiness of a signal and of a multiplier through which this signal passes, and then to compare the noise performamce of various types of multiplier chains.

The third area of interest is the noise contribution of double-sideband degenerate parametric amplifiers in a variety of applications.

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A. ANALYSIS OF NOISE IN OPTICAL MASER OSCILLATOR

With the aid of a semiclassical analysis, Lamb[1] has shown that a gaseous laser with a Doppler-broadened line obeys equations similar to those of the van der Pol oscillator. Because of these results, it is reasonable to attempt to relate the existing work on noise in van der Pol oscillators to fluctuations in the optical maser. If one considers a maser oscillating in a single mode and if one disregards the coupling to other modes of the optical cavity, one may represent the dynamic behavior of the amplitude of the oscillation by the equivalent circuit of Fig. VII-1. The nonlinear negative conductance of the maser material has the assumed voltage dependence

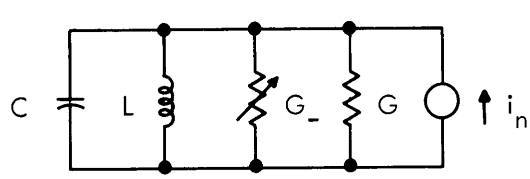

Fig. VII-1. Equivalent circuit.

$$G_- = G_m(1-a\langle V^2\rangle), \tag{1}$$

where the angular brackets represent a (short-)time average over a few cycles of the optical frequency, and a is a constant. The conductance G represents the losses and external loading of the optical cavity. The inductance L and capacitance C of the circuit resonate at the optical-cavity frequency ω_o; their magnitude with respect to G is determined by the "cold" cavity Q. If one intends to represent microphonics as caused by the vibration of the mirrors, an important contribution to the frequency modulation of the optical maser output, one would have to vary L and C in time, keeping their ratio fixed if the Q remains unaffected by these variations. Here we shall disregard such effects in the belief that they may not contribute to the amplitude modulation of the maser output. The noise current generator incorporates the fluctuations associated with thermal effects (negligible in the case of an optical maser) and with the emission of the active material. If one disregards changes in i_n that are due to the nonlinearities of G_-, one may use for it the well-known value[2-4] associated with the spontaneous emission in a linear, active material. The mean-square fluctuations within the frequency increment $\Delta\nu$ are

$$\overline{i_n^2} = \alpha 4G_m h\nu\Delta\nu. \tag{2}$$

Here, $\alpha(\geq 1)$ is a factor accounting for the incomplete inversion in the maser medium.

$$\alpha = 1 + \frac{1}{\exp\left(\frac{h\nu}{k|T_m|}\right) - 1}, \tag{3}$$

where T_m is the (equivalent) negative temperature of the maser medium. To the equivalent circuit of Fig. VII-1 one may apply the theory of noise in a van der Pol oscillator.[5] One assumes the time dependence of the voltage

$$V(t) = R(t)\cos[\omega_o t-\theta(t)]. \tag{4}$$

Furthermore, one notes that R(t) and θ(t) are relatively slow functions of time. Finally, one treats the case of low noise, for which

$$R(t) = R_o + R_1(t). \tag{5}$$

Here,

$$R_1(t) \ll R_o$$

$$\theta(t) = \theta_o + \theta_1(t), \tag{6}$$

where $\theta_1(t)$ is a slow function. Under these assumptions, for the steady-state amplitude R_o one finds

$$R_o = \sqrt{\frac{G_m - G}{aG_m}}, \tag{7}$$

and for steady-state phase θ_o

$$\theta_o = \text{const.} \tag{8}$$

The noise terms R_1 and θ_1 satisfy the (approximate) differential equations

$$\dot{R}_1 + \frac{\omega_o}{Q'} R_1 = -\frac{1}{\omega_o C}\left\langle \frac{di_n}{dt} \sin(\omega_o t - \theta_o)\right\rangle \tag{9}$$

$$R_o \dot{\theta}_1 = -\frac{1}{\omega_o C}\left\langle \frac{di_n}{dt} \cos(\omega_o t - \theta_o)\right\rangle, \tag{10}$$

where

$$Q' = \frac{\omega_o C}{G_m - G}. \tag{11}$$

Here, Q' is the "hot Q" of the oscillator. Near threshold it varies between a value much greater than the "cold Q" to a value comparable with it. The spectra of R_1 and θ_1 can be obtained from Eqs. 2, 9, and 10. Defining the spectral density of the driving noise current generator, $W_o(\nu)$, by

$$W_o(\nu) = \frac{\overline{i_n^2}}{2\Delta \nu}, \tag{12}$$

one has

$$W_{R_1}(\nu) = \frac{\frac{1}{2} W_o}{C^2\left[\omega^2 + \omega_o^2/Q'^2\right]}, \tag{13}$$

with $\omega = 2\pi\nu$, and

$$W_{\theta_1}(\nu) = \frac{\frac{1}{2} W_o}{R_o^2 C^2} \frac{1}{\omega^2}. \tag{14}$$

A measurement with a photocell or photomultiplier on the light output of a single maser detects only $W_{R_1}(\nu)$. The mean-square fluctuation $\overline{R_1^2}$ may be obtained from the integral over all frequencies of $W_{R_1}(\nu)$. For the modulation ratio m^2 one finds

$$m^2 = \frac{\overline{R_1^2}}{R_o^2} = \frac{a h \nu_o}{\frac{1}{2} R_o^2 C} \frac{G_m}{G_m - G}. \tag{15}$$

The output power may be expressed in terms of an external Q, Q_e:

$$P = \frac{1}{2} R_o^2 \frac{\omega_o C}{Q_e}. \tag{16}$$

One may assign a bandwidth $\Delta\omega_e$ to $1/Q_e$ by

$$\Delta\omega_e = \omega_o/Q_e. \tag{17}$$

Then the modulation ratio becomes

$$m^2 = \frac{a\hbar\nu_o\Delta\omega_e}{P}\frac{G_m}{G_m - G}. \tag{18}$$

One can make estimates as to the magnitude of m^2. Taking the He-Ne visible maser line at 6328 Å, $\Delta\omega_e = 10^7$ mc, and an output power of 1 μw, one has

$$m^2 = a\frac{G_m}{G_m - G}\, 1.9 \times 10^{-5}. \tag{19}$$

The factor $aG_m/(G_m-G)$ is greater than unity. We believe that a value of m^2 as obtained here is within reach of achieved experimental sensitivities.

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References

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4. H. A. Haus and J. A. Mullen, Noise in optical maser amplifiers, Presented at the Polytechnic Institute of Brooklyn Symposium on Optical Masers, New York, April 1963.

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B. OPTIMUM NOISE PERFORMANCE OF MULTITERMINAL AMPLIFIERS

This report is an abstract of an Sc.D. thesis, Department of Electrical Engineering, M.I.T., August 19, 1963, which will appear as Technical Report 417, in January 1964.

The problem is to find the optimum noise performance that can be achieved from a linear multiterminal-pair signal source used in the most general linear way with a linear multiterminal-pair amplifier; this optimum performance is achieved at a single-output terminal-pair in a narrow band of frequencies. The most general linear way in which to use such a source and amplifier is to imbed both networks in an arbitrary linear lossless network and take the output through this network. The optimum noise performance of such a system is defined as the maximum signal-to-noise ratio that can be achieved at large exchangeable signal power. It is shown that this is a meaningful criterion for any system that is to deliver ultimately an amount of signal power considerably greater than $kT_o\Delta f$, the noise power available from a room-temperature resistor in the same bandwidth. The techniques described can be used, however, to determine the maximum signal-to-noise ratio that can be achieved at any value of exchangeable (or available) signal power.

The discussions are simplified by presenting the results in a plot with noise-to-signal ratio and the reciprocal of exchangeable signal power as coordinates – the noise-performance plane. This plot is useful for examining the noise performance of any linear system. The noise performance of a conventional two-terminal-pair amplifier is examined on this plane. Such a system defines two points in the noise-performance plane: one by the noise-to-signal ratio and exchangeable signal power of the source; and another by the noise-to-signal ratio and exchangeable signal power at the output. In terms of these two points, noise figure, exchangeable-power gain, and noise measure are given a geometrical interpretation in the noise-performance plane.

It is shown that the noise performance – signal-to-noise ratio and exchangeable signal power – that can be achieved by imbedding a multiterminal-pair source and a multiterminal-pair amplifier in an arbitrary lossless network can also be achieved by imbedding the source in an arbitrary lossless network to reduce it to a one-terminal-pair source and using this one-terminal-pair source to drive an arbitrary lossless reduction of the amplifier. Thus the optimization problem can be solved by considering separately the noise performance of the source and the noise performance of the amplifier. For this reason, the problem is attacked by considering a set of problems of increasing difficulty. First, the noise performance that can be achieved with a noisy (positive or negative) resistance used in conjunction with a one-terminal-pair source is examined. The points in the noise-performance plane that can be achieved at the output of such a system all lie on a straight line through the source point with a slope equal to the exchangeable noise power of the resistance. Subsequently, the problem of the noise performance of a multiterminal-pair amplifier used with a one-terminal-pair source is considered. After the noise performance of a multiterminal-pair source imbedded in an arbitrary lossless network has been examined, the general noise-performance problem can be solved by inspection.

With this work used as a basis, definitions are given for the noise figure, exchangeable-power gain, and noise measure of a multiterminal-pair amplifier that is used with a multiterminal-pair source. These quantities are derived and/or interpreted by comparing a system with its noiseless and "gainless" equivalent – its equivalent source network.

The optimization procedure is extended to multifrequency networks coupled in an arbitrary manner by a lossless nonlinear device that obeys the Manley-Rowe formulas. In this context, the optimum noise performance of an ideal (lossless) double-sideband parametric amplifier is examined for both the cases in which the noises in the two sidebands are uncorrelated and partially correlated.

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