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# Team and Noncooperative Solutions to Access Control with Priorities

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**Abstract**—We consider decentralized medium-access control in which many pairwise interactions occur between randomly selected users that belong to a large population. In each local interaction, the users involved compete over an access opportunity. A given user has a fixed number of access attempts and a fixed budget for buying different priority levels. In each time-slot, the access is attributed to the user with the largest priority level. We analyze this problem under both cooperative as well as competitive frameworks. We show that unlike many standard team problems, optimal pure policies do not exist in the team framework, but both an optimal solution as well as equilibria exist within the class of mixed policies. We establish structural properties as well as explicit characterization of these: We show that the optimal policy requires only three priority levels, whereas the noncooperative game possesses a unique symmetric equilibrium point that uses at most two priority levels. Our analysis is applied to power control over wireless capture channels, where the budget constraint corresponds to the battery lifetime.

## I. INTRODUCTION

### A. Background and Motivation

Many networking control problems can be formulated as priority assignment for accessing some service. We consider situations in which the choice of priority is done by each user, without knowing in advance the priority choices of other users. If users simultaneously attempt access, then access is granted only to the one with the highest priority. Each user acquires priorities, where the higher the priority, the more it costs. The budget for acquiring priorities is limited, and the performance criterion is the expected number of successful access attempts that a user may obtain within a given budget.

We consider both the team framework, in which all users share the common objective of maximizing the above criterion averaged over the whole population, as well as the non-cooperative framework, in which each user maximizes its own performance measure and where the solution concept is the Nash equilibrium. We restrict attention to a regime of weak interactions in which upon an access attempt, a user is either faced with no other simultaneous attempt or might face a single opponent that attempts to access the network at the same time. This framework is similar to the pairwise interaction paradigm in evolutionary game theory (see, e.g., [7]), and may correspond, for example, to sparse network topologies (such as ad-hoc networks).

As a motivating example, we consider medium access over a shared capture radio-channel. The priority level is

mapped to the transmission power, and the budget constraint is mapped to the energy available in the battery of the mobile. Another possible application of our model could be sequential (multiple-goods) auctions, in which the user that makes the highest bid obtains the good. It is assumed that each (limited-budget) user pays for the right to make a bid (independently of whether the bid is accepted or not), where the cost of a bid is proportional to the priority level that is sought.

In this paper, we choose to formulate and investigate the access problem under the power control framework described above, as we believe that a concrete model makes the exposition more lucid<sup>1</sup>. Our analysis reveals that unlike many standard team problems, optimal pure policies do not exist in the team framework, but both an optimal solution, as well as equilibria exist within the class of mixed policies. Focusing on symmetric working points, we fully characterize both the team solution and the equilibrium point, which turn out to be unique. We show that the optimal policy requires only three priority (or power) levels, where the Nash equilibrium uses only two priority levels. This result is significant from an engineering perspective, as network architectures usually limit the number of priority classes to two or three out of practical concerns (see, e.g., [12], [3], [4]).

### B. Related Work

The control of Quality of Service (QoS) through priorities has become a basic element in networking architectures such as Diffserv [3], and has been studied in various contexts, including both system optimal design and a distributed, non-cooperative framework (see, e.g., [5], [1], [6]). The vast majority of the theoretic work in the area differs from ours by assuming full knowledge of the system state and/or involving queueing networks [2], [6], [10], [14], [13]. Priority queues are usually different from our framework, as low-priority jobs may not be lost, but just deferred in their processing time, whereas in our model a “transmission” with an inferior priority results in an attempt loss.

The specific motivating example of power control under energy constraints has been an active research area (see [9] for a recent survey on the game-theoretic perspective of the problem). Most of the existing work focuses on time-average power constraint or on a maximal power constraint.

<sup>1</sup>Nonetheless, we address in Section V some features that are specific for other networking applications.

We consider a different constraint in the form of a limited energy budget to be spent during the battery lifetime. The latter setup may correspond, for example, to sensor networks where the battery of the sensor is limited and can be charged only occasionally (e.g., by solar energy).

In a previous paper [8], we have considered a related problem of power control with energy budget constraint over a CDMA-like interference channel, which enables multiple-packet reception. Interestingly, not only the solution methodology for that case is completely different, but also the structure of the optimal and equilibria policies are fundamentally unlike. Whereas the use of the same priority over time is system-wide optimal and an equilibrium in CDMA systems, carrying over the same policy to the current framework might result in the worst possible performance.

### C. Paper Structure

The structure of this paper is as follows. The network model and problem formulation are presented in Section II. The analysis of the cooperative-team framework is provided in Section III. In Section IV we study the noncooperative game that arises when users are free to adjust their assignment policies. Section V briefly considers more general “reception” rules, which may encompass a variety of network applications.

## II. MODEL AND PROBLEM STATEMENT

### A. General Setting

We consider a large population of mobiles. Each has a battery with  $K$  energy units. Time is discrete. At each time unit a mobile has a transmission opportunity. If it has  $k \leq K$  energy units left then it can transmit with any *integer* energy level  $1 \leq l \leq k$ . If  $k = 0$  then it cannot transmit. Every  $N$  time units the battery is replaced with a new one with energy level  $K$ . Assume that there are pairwise interactions: when a mobile attempts transmission, the receiver is with probability  $(1 - \delta)$  in the range of yet another mobile which is randomly selected from the whole population. At each transmission opportunity the interaction occurs with another randomly selected mobile. The time slots are common to all mobiles but when a mobile is at the  $i$ th stage in his battery lifetime, it interacts with a mobile that is at a random stage  $j$ , uniformly distributed between 1 to  $N^2$ .

**User Policy.** Due to the above assumptions, a general transmission policy  $u$  may be characterized by the number of times each power level is used, since the specific times in which each level is applied are insignificant. Hence, a (pure) policy  $u$  will be described by a  $K+1$  vector  $u = (n_0, n_1, \dots, n_K)$ , where  $n_i$  represents the number of slots during the lifetime of the battery in which a power of  $i$  is used for transmission ( $n_0$  stands for the number of slots in which there is no transmission). The

following constraints must obviously be met for every feasible user policy:

$$\sum_{i=0}^K n_i = N \quad (1)$$

$$\sum_{i=1}^K i n_i \leq K. \quad (2)$$

Let  $x_i \equiv x_i(N) := n_i/N$  denote the fraction of time that power  $i$  is employed for a given policy  $u$ . Throughout the paper, we shall alternatively use the vector  $x = (x_0, x_1, \dots, x_K)$  for representing a policy.

Our model may allow for mixed policies as well. A mixed policy  $\sigma$  is a collection of pure policies  $(u(1), \dots, u(m))$  chosen with probabilities  $(q_1, \dots, q_m)$ ,  $\sum_m q_m = 1$ .

**Reception Rule.** At any given time, a transmission attempt with power level  $i > 0$  is successful, if and only if (i) there is no simultaneous transmission, or (ii) the interfering transmission uses a power level strictly lower than  $i$ .

### B. Team Problem Formulation

We denote by  $g_{N,K}(\sigma)$  the expected number of successful transmissions per battery lifetime of a mobile when all mobiles use the same mixed policy  $\sigma$  for given parameters  $N$  and  $K$ . Accordingly,  $g_{N,K}(\sigma)$  would be regarded as the utility of the mobile. The objective in the team problem is to set a unified policy which maximizes the utility  $g_{N,K}(\sigma)$  over  $\sigma$ . The chosen  $\sigma$  can be regarded as a fixed access protocol that all mobiles must obey.

In order to be able to compare strategies for different parameters  $N, K$  we introduce the Throughput Per Slot (TPS) criterion which divides the former criterion by number of slots  $N$ , i.e.,  $TPS(\sigma) = \frac{g_{N,K}(\sigma)}{N}$ . Obviously, maximizing  $TPS(\sigma)$  is an equivalent problem to maximizing  $g_{N,K}(\sigma)$ .

When restricting ourselves to pure policies  $u$ , the team-objective becomes to maximize  $g_{N,K}(u)$  over  $u$ , where

$$g_{N,K}(u) = \delta(N - n_0) + (1 - \delta) \frac{1}{N} \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j. \quad (3)$$

Indeed, when there is no interference, all non-zero power levels lead to a successful transmission, whereas in the presence of interference, the probability that a transmission with power level  $i$  is successful is given by  $\sum_{j=0}^{i-1} n_j/N$ .

### C. Noncooperative Game Formulation

In a noncooperative framework, users are self-optimizing and are free to determine their own policy in order to maximize their expected number of successful transmissions (or alternatively their expected TPS). A Nash equilibrium point (NEP) is a collection of user strategies for which no user can obtain a higher number of expected successful transmissions by unilaterally modifying its transmission strategy. In the current paper, we shall focus on *symmetric* Nash equilibria. A symmetric Nash equilibrium is a working point where all

<sup>2</sup>More generally, we may consider any distribution for the random stage of the opponent. In that case, the results in this paper will remain the same, if we plausibly assume that users choose the number of times that each power level will be used, and then uniformly randomize over the possible permutation for a given choice.

mobiles use the same strategy  $\sigma$ , and furthermore, for all other strategies  $\tilde{\sigma}$ ,

$$g_{N,K}(\sigma) \geq g_{N,K}(\tilde{\sigma}, \sigma), \quad (4)$$

where  $g_{N,K}(\tilde{\sigma}, \sigma)$  is the utility of a user who deviates to the policy  $\tilde{\sigma}$ , while the rest of the population uses  $\sigma$ .

For simplicity, we shall restrict our attention in the bulk of this work to the case where  $N = K$ . A feasible policy under this setting is to use a power level of one at all time slots. Obviously, such policy would result in zero TPS when  $\delta = 0$ , hence it is not an optimum nor an equilibrium for this value of  $\delta$ . However, for the other extreme of  $\delta = 1$ , the same policy becomes an optimal solution as well as an equilibrium point.

#### D. Numeric Examples

We provide below some numeric examples and derive some interesting properties. For simplicity, we consider cases where  $N = K$  and  $\delta = 0$ , which corresponds to the case where a user interacts with probability one with another user in each of its stages. In addition, we focus below on pure strategies. We use the format  $(n_0, n_1, \dots)$  to describe a policy.

a) *The case of  $N = 3$ :* The feasible policies that use all energy are  $(0, 3, 0, 0)$ ,  $(1, 1, 1, 0)$ ,  $(2, 0, 0, 1)$ . The expected number of packets transmitted successfully in a cycle of duration 3, if all use the same policy is  $g_{3,3}(0, 3, 0, 0) = 0$ ,  $g_{3,3}(1, 1, 1, 0) = 1$ ,  $g_{3,3}(2, 0, 0, 1) = 2/3$ . The policy  $(1, 1, 1, 0)$  is seen to be the best pure strategy<sup>3</sup>. It is an equilibrium (in pure strategies) as well; a deviation to  $(0, 3, 0, 0)$  or to  $(2, 0, 0, 1)$  decreases the utility from 1 to  $1/3$ .  $(0, 3, 0, 0)$  is not an equilibrium as a deviation of a player to  $(1, 1, 1, 0)$  or to  $(2, 0, 0, 1)$  increases its utility to  $1/3$ .  $(2, 0, 0, 1)$  is not an equilibrium since a player deviating to  $(0, 3, 0, 0)$  increases its utility from  $2/3$  to 2.

b) *The case of  $N = 4$ :* The feasible policies that use all energy are  $(0, 4, 0, 0, 0)$ ,  $(1, 2, 1, 0, 0)$ ,  $(2, 0, 2, 0, 0)$ ,  $(2, 1, 0, 1, 0)$ ,  $(3, 0, 0, 0, 1)$ . The policies  $(1, 2, 1, 0, 0)$  and  $(2, 1, 0, 1, 0)$  are both optimal pure policies for the team problem, and obtain  $g_{4,4}(1, 2, 1, 0, 0) = g_{4,4}(2, 0, 2, 0, 0) = 5/4$ . None of the above policies is an equilibrium: any deviation from  $(0, 4, 0, 0, 0)$  strictly increases the utility of the deviator. By deviating from  $(1, 2, 1, 0, 0)$  to  $(2, 0, 2, 0, 0)$  the utility of the deviator increases to  $6/4$ . A deviation from  $(2, 0, 2, 0, 0)$  or from  $(2, 1, 0, 1, 0)$  to  $(0, 4, 0, 0, 0)$  increases the utility to 2. Finally, deviating from  $(3, 0, 0, 0, 1)$  to  $(0, 4, 0, 0, 0)$  increases the utility to 3.

In the list below we provide the optimal pure policies for the team problem and the associated TPS up to  $N = 10$ .

- $N = 2$ :  $TPS = 0.25$
- $N = 3$ :  $TPS = 0.333$
- $N = 4$ :  $TPS = 0.313$
- $N = 5$ :  $(2, 1, 2, 0, 0, 0)$ ,  $(2, 2, 0, 1, 0, 0)$ ,  $TPS = 0.32$
- $N = 6$ :  $(2, 2, 2, 0, \dots, 0)$ ,  $(3, 1, 1, 1, 0, \dots, 0)$ ,  $TPS = 0.333$

<sup>3</sup>It can be easily shown that there always exists an optimal policy that uses all the available energy. Indeed, given a policy that does not use all energy, we may always construct a policy that does use all energy and obtains the same TPS (by assigning the access energy to the highest used power level).

- $N = 7$ :  $(3, 2, 1, 1, 0, \dots, 0)$ ,  $TPS = 0.347$
- $N = 8$ :  $(3, 3, 1, 1, 0, \dots, 0)$ ,  $TPS = 0.344$
- $N = 9$ :  $(3, 3, 3, 0, \dots, 0)$ ,  $(4, 2, 2, 1, 0, \dots, 0)$ ,  $TPS = 0.346$
- $N = 10$ :  $(4, 3, 2, 1, 0, \dots, 0)$ ,  $TPS = 0.35$

We observe the following properties from our numerical study.

- 1) There need not be a (symmetric) equilibrium point in pure strategies.
- 2) A power greater than three is not used for the team problem.
- 3) The optimal TPS under pure strategies is not monotone in  $N$ .

The potential of using mixed policies is highlighted in the next example. Let  $N = 5$ , and consider the mixed policy of using with probability of  $1/2$  each of the two policies  $(2, 1, 2, 0, 0, 0)$ ,  $(2, 2, 0, 1, 0, 0)$ . Note that the TPS in this case is equivalent to the one obtained for  $N = 10$  and  $(4, 3, 2, 1, 0, \dots, 0)$ , which is also the optimal (pure) policy for  $N = 10$ . The latter policy thus obtains  $TPS = 0.35$ , which is a strictly higher value than the one obtained while restricting the mobiles to pure strategies.

In the next section we show that a TPS of 0.35 is a tight upper bound on *any* policy (pure or mixed). We further show that it can be obtained for any  $N = K$  by the use of mixed policies. The in-existence of an equilibrium in pure policies motivates the study of mixed policies for the noncooperative framework as well, which is covered in Section IV.

### III. THE TEAM PROBLEM

In this section we consider the team problem, in which a central authority assigns a unified policy to all users, who must obey it. The policy can be thus be viewed as a *protocol*. The natural objective is to find a protocol that maximizes the average number of successful transmissions (or the TPS) across users. In Section III-A we consider this optimization problem under pure policies, and obtain some structural properties of the best policy. In Section III-B we derive an upper bound on the TPS for any  $N$ . In Section III-C we show that the upper bound is always achievable when mixed policies are allowed. Implications of these results are discussed in Section III-D.

#### A. Pure Strategies

In this subsection we restrict attention to the set of pure policies, and analyze the optimal policy among this set. From a practical-engineering viewpoint, the underlying complexity in implementing pure strategies can be lower compared to mixed policies, which require randomization between several pure policies.

We start our analysis with a lemma that provides an alternative expression for  $g_{N,K}$ , which will be central in our subsequent analysis of the problem.

*Lemma 1:* Let  $u = (n_0, n_1, \dots, n_K)$  be a unified transmission policy. Then

$$g_{N,K}(u) = \delta(N - n_0) + (1 - \delta) \frac{1}{2N} \left( N^2 - \sum_{i=0}^K n_i^2 \right). \quad (5)$$



*Proof:* Note first that  $N^2 = (n_0 + n_1 + \dots + n_K)^2 = 2 \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j + \sum_{i=0}^K n_i^2$ . Hence,

$$\sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j = \frac{N^2 - \sum_{i=0}^K n_i^2}{2}. \quad (6)$$

Substituting (6) into (3) gives (5).  $\square$

The following result is a direct consequence of Lemma 1.

*Proposition 1:* There always exists an optimal unified policy which satisfies the following relation

$$n_K \leq n_{K-1} \leq \dots \leq n_1. \quad (7)$$

*Proof:* Let  $u = (n_0, \dots, n_K)$  be an optimal unified policy. Assume that  $n_i > n_j$  for some indexes  $i$  and  $j$  such that  $i > j$ . Consider now the modified policy  $\tilde{u} = (n_0, \dots, \tilde{n}_N)$ , where  $\tilde{n}_k = n_k$ , for every  $k \neq i, j$ ,  $\tilde{n}_i = n_j$ ,  $\tilde{n}_j = n_i$ . Then  $\tilde{u}$  obviously obeys the constraints (1)–(2). Moreover, noting (5),  $\tilde{u}$  achieves the same throughput as  $u$ , hence it is an optimal policy as well.  $\square$

The above monotonicity result suggests that there is no benefit in using higher power levels more frequently than lower power levels are used. Note that for the case of  $\delta = 0$  it can be further shown that  $n_K \leq n_{K-1} \leq \dots \leq n_1 \leq n_0$ , i.e., the number of no-transmissions is higher than the number of transmission at any power level. However, this inequality need not hold for general  $\delta$ .

In the remaining of this subsection, we consider the case of  $N \geq K$ , which may be relevant, for example, in ad-hoc or sensor wireless networks, in which energy is relatively limited. Our main result for that case suggests that a power level greater than 3 would not be used in *any* optimal unified policy (regardless of how large  $N$  and  $K$  are). Formally,

*Theorem 2:* Assume that  $N \geq K$ . Let  $u$  be an optimal unified policy. Then  $n_i = 0$  for  $i > 3$ .

For the proof of the theorem we require four lemmas.

*Lemma 2:* For every policy  $u$

$$n_0 \geq n_2 + 2n_3 + 3n_4. \quad (8)$$

*Proof:* Combining (1) and (2) and recalling that  $N \geq K$  we get that  $n_0 + n_1 + \dots + n_K \geq n_1 + 2n_2 + 3n_3 + \dots$ . Thus

$$\begin{aligned} n_0 &\geq n_2 + 2n_3 + 3n_4 + \dots + (k-1)n_k + \dots \\ &\geq n_2 + 2n_3 + 3n_4. \end{aligned}$$

$\square$

*Lemma 3:* Assume that  $N \geq K$ . Further assume that  $u$  is an optimal unified policy with  $n_4 > 0$  then  $n_1 > 0$ .

*Proof:* Note first that  $n_4 > 0$  implies that  $n_0 \geq 3$  by (8). Assume by contradiction that  $n_1 = 0$  and consider the modified policy  $\tilde{n}_4 = n_4 - 1$ ,  $\tilde{n}_1 = 2$ ,  $\tilde{n}_0 = n_0 - 1$ , and  $\tilde{n}_k = n_k$  for  $k \neq 1, 2, 4$ . Note that this policy obeys the constraints (1)–(2). We next show that  $2(g_{N,K}(\tilde{u}) - g_{N,K}(u)) > 0$  which contradicts the optimality of  $u$ . Using (5),  $2(g_{N,K}(\tilde{u}) - g_{N,K}(u)) = 2\delta + (1-\delta)(n_4^2 + n_1^2 + n_0^2 - (n_4-1)^2 - (n_1+2)^2 - (n_0-1)^2) = 2\delta + (1-\delta)(2n_4 + 2n_0 - 6)$ , which is obviously strictly positive since  $n_4 \geq 1$  and  $n_0 \geq 3$ .  $\square$

*Lemma 4:* Assume that  $N \geq K$ . Let  $u$  be an optimal unified policy with  $n_4 > 0$  then

$$n_0 - n_1 \leq n_3 - n_4 + 2, \quad (9)$$

$$n_1 - n_2 \leq n_3 - n_4 + 2. \quad (10)$$

*Proof:* To prove (9), consider the modified policy  $\tilde{u}$  with  $\tilde{n}_4 = n_4 - 1$ ,  $\tilde{n}_3 = n_3 + 1$ ,  $\tilde{n}_1 = n_1 + 1$ ,  $\tilde{n}_0 = n_0 - 1$  (note that  $n_0 > 0$  from (8) and the lemma's conditions, hence  $\tilde{n}_0 \geq 0$ ), and  $\tilde{n}_k = n_k$  for  $k \neq 4, 3, 1, 0$ . Note that  $\tilde{u}$  is a valid policy, since it obeys (1) and (2) because  $u$  does (the energy investment of both policies is equal). Since  $u$  is an optimal policy we must have  $2(g_{N,K}(\tilde{u}) - g_{N,K}(u)) \leq 0$ . Using (5) this means that

$$\begin{aligned} 2\delta + (1-\delta)[n_4^2 + n_3^2 + n_1^2 + n_0^2 - (n_4-1)^2 \\ - (n_3+1)^2 - (n_1+1)^2 - (n_0-1)^2] \leq 0. \end{aligned}$$

Noting that  $2\delta$  is non-negative and rearranging terms in the inequality above, this inequality holds if  $2n_4 - 2n_3 - 2n_1 + 2n_0 - 4 \leq 0$  which is easily seen to be equivalent to (9). The inequality (10) is proven similarly, yet instead of shifting an energy unit from  $n_0$  to  $n_1$ , we shift an energy unit from  $n_1$  to  $n_2$  (note that such shift is possible by Lemma 3).  $\square$

*Lemma 5:* Let  $u$  be an optimal unified policy for some  $N$  and  $K$  so that  $N \geq K$ . Then  $n_4 = 0$

*Proof:* Assume by contradiction that  $n_4 > 0$ . Then

$$3n_4 + 2n_3 + n_2 - n_1 \leq n_0 - n_1 \leq n_3 - n_4 + 2, \quad (11)$$

where the first inequality follows from (8) and the second one from (9). Hence,  $4n_4 + n_3 - 2 \leq n_1 - n_2 \leq n_3 - n_4 + 2$ , where the first inequality follows from (11) and the second one from (10). The last set of inequalities suggests that  $5n_4 \leq 4$  which contradicts the assumption that  $n_4 > 0$ .  $\square$

We are now ready to prove the theorem. Note first that  $n_4 = 0$  for every optimal unified policy by Lemma 5. Assume by contradiction that there exists an optimal policy with  $n_i > 0$  for some  $i > 4$ . Then as in the proof of Proposition 1, the policy  $\tilde{u}$ , with  $\tilde{n}_k = n_k$ ,  $k \neq i, 4$ ,  $\tilde{n}_4 = n_i > 0$ ,  $\tilde{n}_i = n_4 = 0$  is optimal as well. But this contradicts Lemma 5.  $\square$

## B. Asymptotic analysis

We henceforth restrict attention to the case  $K = N$ . In the remaining of this section, we use the vector  $x = (x_0, x_1, \dots, x_N)$  for representing a policy, where  $x_i \equiv n_i/N$ . With this representation, (5) can be written as

$$TPS(x) = \delta(1 - x_0) + \frac{1}{2}(1 - \delta) \left( 1 - \sum_{i=0}^K x_i^2 \right). \quad (12)$$

The battery lifetime constraint (1) is

$$\sum_{i=0}^{\infty} x_i = 1, \quad (13)$$

while the energy constraint (2) is

$$\sum_{i=0}^{\infty} i x_i \leq 1. \quad (14)$$

In addition there is an "integrity" constraint: the  $x_i$ 's are restricted to multiples of  $N^{-1}$ .

We now consider the problem with  $N$  very large.  $x_i$  is then interpreted as the long-run fraction of time (or *frequency*) that a power of  $i$  units is used. The integrity constraint disappears, and we are left with an optimization problem, which is easily seen to be a strictly convex one.

**Lemma 6:** The problem of maximizing  $TPS(x)$  in (12) subject to (13)–(14) is a strictly convex optimization problem.

**Proof:** Since  $TPS(x)$  is quadratic in  $x_i$  with a negative multiplicative term  $-(1-\delta)$ , and the constraints are affine, the optimization problem is (strictly) convex. Note that in the case of  $\delta = 1$  the trivial unique solution of this problem is  $x_1 = 1$ .  $\square$

The optimal TPS in the asymptotic case is of course an upper bound to the maximal TPS that can be obtained for every  $N$  (with the integrity constraint present). We emphasize that the last statement is valid not only for pure strategies, but also for mixed strategies, as the solution for the case of  $N \rightarrow \infty$  may be viewed as the frequency in which each power level should be used, regardless if the frequencies are obtained under pure or mixed policies. A complete characterization of the optimal policy for the asymptotic case is provided below.

**Theorem 3:** Assume  $N = K$  and let  $N \rightarrow \infty$ . The optimal frequencies  $x_i$  as a function of  $\delta$  and the corresponding TPS are given by:

- $0 \leq \delta \leq \frac{1}{3}$ :  $x_0 = \frac{4-7\delta}{10(1-\delta)}$ ;  $x_i = \frac{(3-2i)\delta+4-i}{10(1-\delta)}$ ,  $i = 1, 2, 3$ ;  
 $TPS = \frac{7-2(\delta+\delta^2)}{20(1-\delta)}$ .
- $\frac{1}{3} < \delta \leq \frac{2}{3}$ :  $x_0 = \frac{2-3\delta}{6(1-\delta)}$ ;  $x_i = \frac{2+3\delta(1-i)}{6(1-\delta)}$ ,  $i = 1, 2$ ;  
 $TPS = \frac{1}{12} \frac{4-3\delta^2}{1-\delta}$ .
- $\delta > \frac{2}{3}$ :  $x_1 = 1$ ;  $TPS = \delta$ .

**Proof:** Noting that

$$TPS(x) = \frac{1}{2} \left( 1 - \sum_{i=0}^K x_i^2 \right) + \delta \left( \frac{1}{2} - x_0 + \frac{1}{2} \sum_{i=0}^K x_i^2 \right),$$

we introduce the Lagrangian

$$\begin{aligned} \mathcal{L}(\mathbf{x}) = & \frac{1}{2}(\delta+1) + \frac{1}{2}(\delta-1) \sum_{i=0}^K x_i^2 - \delta x_0 \\ & + \lambda \left( \sum_{i=0}^K x_i - 1 \right) + \mu \left( \sum_{i=0}^K i x_i - 1 \right), \end{aligned} \quad (15)$$

where  $\lambda$  is the Lagrange multiplier associated with the number of time slots, and  $\mu$  with the power constraint. We ignore in (15) the positivity constraints for each  $x_i$ , assuming that  $x_i$  involved are all positive, yet directly consider this constraint in our analysis below.

We recall from Proposition 1 that the optimal solution satisfies  $x_1 \geq x_2 \geq x_3$ . Depending on  $\delta$ , the largest  $i$  for which  $x_i > 0$  is either 3, 2, or 1. This is a direct consequence of Theorem 2, which holds for every  $N$  (and also in the limit  $N \rightarrow \infty$ ). We shall denote this largest  $i$  by  $i^*$ . Assume that  $i^* > 1$  (the case  $i^* = 1$  is treated separately below). In this case, the extremum of the Lagrangian corresponds to

an interior point. Indeed, since for  $1 \leq i \leq i^*$ , we focus on optimal solutions that satisfy  $x_i > 0$  and we are thus away from the boundary  $n_i = 0$  for these indices; additionally  $x_0 > 0$ , since a power level larger than one is being used. The optimal solution is thus obtained by equating the gradient of the Lagrangian to zero, which leads to the following equations  $\frac{\partial \mathcal{L}}{\partial x_0} = (\delta-1)x_0 - \delta + \lambda = 0$ ,  $\frac{\partial \mathcal{L}}{\partial x_i} = (\delta-1)x_i + \lambda + \mu i = 0$  for  $i = 1, \dots, i^*$ , or equivalently

$$x_0 = \frac{\delta - \lambda}{\delta - 1}, \quad x_i = -\frac{\lambda + \mu i}{\delta - 1}. \quad (16)$$

We now consider the different alternatives for  $i^*$ . Assume  $i^* = 3$ . Substituting (16) in the constraint equations (13)–(14) (recall that the inequality (14) is active in the optimum, see Footnote 3) and taking into account that  $x_i = 0$  for  $i \geq 4$ , we obtain that  $\mu = -\frac{2\delta+1}{10}$  and  $\lambda = \frac{3\delta+4}{10}$ . Substituting these quantities back in (16) yields  $x_0 = \frac{4-7\delta}{10(1-\delta)}$  and

$$x_i = \frac{(3-2i)\delta+4-i}{10(1-\delta)}, \quad (17)$$

resulting in  $TPS = \frac{7-2(\delta+\delta^2)}{20(1-\delta)}$ . It can be seen from (17) that  $x_3$  decreases with  $\delta$ . Since the non-negativity constraints for the  $x_i$ 's have not been explicitly considered in the formulation of the problem, the  $\delta$  threshold from which a power level 3 will no longer be used is obtained by equalizing  $x_3$  in (17) to zero. It is obtained that for  $\delta > \frac{1}{3}$  only two levels are used.

By proceeding analogously for  $i^* = 2$ , we get  $\mu = -\frac{\delta}{2}$  and  $\lambda = \frac{1}{3} + \frac{\delta}{2}$ , which lead to  $x_0 = \frac{2-3\delta}{6(1-\delta)}$  and

$$x_i = \frac{2+3\delta(1-i)}{6(1-\delta)}, \quad (18)$$

with a resulting TPS of  $\frac{1}{12} \frac{4-3\delta^2}{1-\delta}$ . Substituting  $x_2 = 0$  in (18), we obtain a threshold value of  $\delta = \frac{2}{3}$ , above which a power level beyond 1 is a suboptimal choice. For  $\delta > \frac{2}{3}$ , it follows immediately from (12) that the optimal policy is to transmit at every slot with a power level of 1. In this case the TPS is nothing but the probability  $\delta$  of having no interference.  $\square$

The evolution of the optimal power allocation as a function of  $\delta$  is summarized in Fig. 1, and the corresponding TPS is given in Fig. 2.

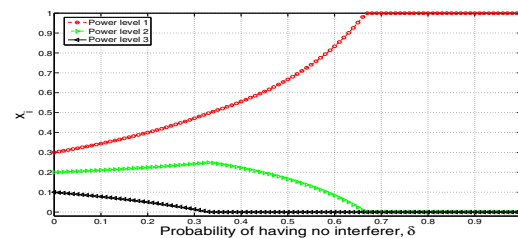


Fig. 1. Optimal distribution of power levels as a function of the probability of having no interferer.

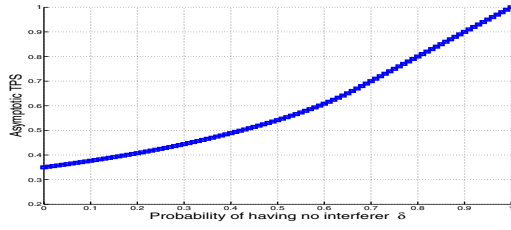


Fig. 2. Asymptotic TPS as a function of the probability of having no interferer.

### C. Optimal Policy in Mixed Policies

As shown in Section II-D, the use of mixed strategies may increase the TPS. The upper bound on performance obtained in Section III-B, leads to the objective of achieving this bound via mixed strategies. We next establish that the upper-bound is indeed achievable for every  $N$ , and explicitly derive the mixed policy that leads to the corresponding optimal performance.

With some abuse of notations, we use the notation  $u = (n_0, n_1, n_2, n_3)$  for a policy which uses a maximal power level of 3. Consider the following three pure policies:

$u(1) = (0, N, 0, 0)$ ,  
 $u(2) = (N - \lfloor N/2 \rfloor - \text{mod}(N/2), \text{mod}(N/2), \lfloor N/2 \rfloor, 0)$ ,  
 $u(3) = (N - \lfloor N/3 \rfloor - \text{mod}(N/3), \text{mod}(N/3), 0, \lfloor N/3 \rfloor)$   
 (where  $\lfloor y \rfloor$  stands for the largest integer smaller than  $y$ , and  $\text{mod}(y/z)$  is the remainder in dividing two integer numbers  $y$  and  $z$ ). We show below that any required frequency vector  $(x_0, x_1, x_2, x_3)$  can be obtained by a mixed policy that uses the above three pure policies.

**Theorem 4:** Any required frequency vector  $(x_0, x_1, x_2, x_3)$  is attained by a mixed policy that uses the pure policies  $u(1), u(2), u(3)$  with probabilities  $p_3 = x_3 \frac{N}{\lfloor N/3 \rfloor}$ ,  $p_2 = x_2 \frac{N}{\lfloor N/2 \rfloor}$ , and  $p_1 = 1 - p_2 - p_3$ .

*Proof:* Note first that the battery lifetime constraint (13) is obeyed since  $\sum_{i=0}^3 n_i = N$  for each of the three pure policies. Observe next that a power level of 3 is used only in  $u(3)$ . Hence, the probability of transmitting at this power level is  $\frac{p_3 \lfloor N/3 \rfloor}{N} = x_3$ . Similarly, a power level of 2 is used only in  $u(2)$ . Hence, the probability of transmitting at this power level is  $\frac{p_2 \lfloor N/2 \rfloor}{N} = x_2$ . In order to obey the total energy constraint (14), it remains to be shown that  $x_1 = 1 - 3x_3 - 2x_2$ . To that end, we examine the probability for using a power level of 1 in each pure policy, and multiply it by the probability of using that policy. This gives

$$\begin{aligned} & (1 - p_2 - p_3) + \frac{\text{mod}(N/2)}{N} p_2 + \frac{\text{mod}(N/3)}{N} p_3 \\ &= 1 - p_2 \left( 1 - \frac{\text{mod}(N/2)}{N} \right) - p_3 \left( 1 - \frac{\text{mod}(N/3)}{N} \right) \\ &= 1 - x_2 \frac{N - \text{mod}(N/2)}{\lfloor N/2 \rfloor} - x_3 \frac{N - \text{mod}(N/3)}{\lfloor N/3 \rfloor} \\ &= 1 - x_2 \frac{N - \text{mod}(N/2)}{\frac{1}{2}(N - \text{mod}(N/2))} - x_3 \frac{N - \text{mod}(N/3)}{\frac{1}{3}(N - \text{mod}(N/3))}, \end{aligned}$$

which means that  $x_1 = 1 - 3x_3 - 2x_2$ .  $\square$

The significance of Theorem 4 is that the upper bound TPS can be obtained for every  $N$  by implementing the optimal frequencies obtained in Theorem 3 via the mixed policy derived above.

### D. Discussion

The combination of Theorems 3 and 4 leads to a globally optimal (mixed) policy that achieves the upper bound on performance and hence can be set as a unified protocol. It is important to emphasize that the number of pure policies that are used in the optimal mixed policy remains a constant (three), and does not grow with  $N$ . In addition, the complexity in computing the optimal mixed policy relates to calculating expressions such as  $N/2$  and  $N/3$ , which do not become much more complex for a large  $N$ . Hence, the optimal policy is appealingly implementable.

At a higher perspective, we note that the approach used in Theorems 3–4 can be applied in more general contexts, besides throughput optimality. For example, assume that half of the population should be given some priority in terms of the obtained TPS, compared to the other half. The precise definition of the Quality of Service (QoS) differentiation between the two sub-populations can be casted as a (continuous) optimization problem. After solving the problem and obtaining the frequencies for each subset of the population, Theorem 4 can be invoked in order to implement the corresponding protocol.

## IV. THE NONCOOPERATIVE GAME

This section is dedicated to the study of the noncooperative framework and the underlying Nash equilibria. Our main focus is on symmetric equilibria (23), which may be regarded as protocols, from which no user has an incentive to unilaterally deviate. In Section IV-A we prove the uniqueness of the symmetric equilibrium point, and further provide a complete characterization thereof. Using the characterization, Section IV-B compares the performance of the optimal policy obtained in Section to the unified equilibrium policy via the so-called price-of-anarchy (PoA) performance measure. We conclude this section by showing that asymmetric equilibria exist in general, yet leave their full analysis for future work. Throughout this section, we shall focus on the case of  $N = K$ , which enables us to provide a concrete comparison between optimal and equilibrium performance.

### A. Symmetric Equilibria

We start our analysis by showing that in any symmetric equilibrium point (23), power levels equal or greater than three would never be used.

**Theorem 5:** Let  $u$  be a unified equilibrium point. Then  $n_i = 0$  for every  $i \geq 3$ .

*Proof:* The idea behind the proof is to establish first that a power level of three would not be used in any best response. The theorem's claim would then follow by induction on  $n_i$ . The proof proceeds in the following steps.

Step 1: When considering policies with  $n_i = 0$ ,  $i > 3$ , there is no best-response with  $n_3 > 0$ : Consider a policy  $u' = (n'_0, n'_1, n'_2, n'_3)$ ,  $n'_3 > 0$  for all players (where  $n'_i = 0$  for every  $i \geq 3$ ). Let  $u = (n_0, n_1, n_2, n_3)$  be a best response to  $u'$ . Note that the energy constraint (2) is met with equality for a best-response, hence  $\sum_{i=1}^3 in_i = N$ . Assume by contradiction that  $n_3 > 0$ . Introduce also the policy  $\hat{u} = (\hat{n}_0, \hat{n}_1, \hat{n}_2, 0)$  where  $\hat{n}_2 = n_2 + n_3$ ,  $\hat{n}_0 = n_0 - n_3$ ,  $\hat{n}_1 = n_1 + n_3$  (note that  $\hat{u}$  obeys the energy constraint (2)). We show below that  $\hat{u}$  obtains a larger value compared to  $u$  contradicting the optimality of the latter.

$$\begin{aligned} g(u, u') &= \delta(N - n_0) + \frac{1 - \delta}{N} (n_1 n'_0 + n_2(n'_0 + n'_1) + n_3 \sum_{i=0}^2 n'_i) \\ &= \delta(N - n_0) + \frac{1 - \delta}{N} (N n'_0 + n_2(n'_1 - n'_0) + n_3(n'_1 + n'_2 - 2n'_0)) \\ &= \delta(N - n_0) + \frac{1 - \delta}{N} (N n'_0 + (n_2 + n_3)(n'_1 - n'_0) + n_3(n'_2 - n'_0)) \\ &< \delta(N - \hat{n}_0) + (1 - \delta) \frac{1}{N} (N n'_0 + \hat{n}_2(n'_1 - n'_0)) = g(\hat{u}, u'), \end{aligned}$$

where the first equality follows from the energy constraint (written as  $n_1 = N - 2n_2 - 3n_3$ ). The inequality follows from  $\hat{n}_0 < n_0$  and also from  $n'_0 > n'_2$  (which holds since  $n'_3 > 0$ ). Step 2:  $n_3 = 0$  in any best-response. Consider now a general policy  $u' = (n'_0, n'_1, n'_2, n'_3, \dots)$  employed by all users, and a best-response of a deviator  $u = (n_0, n_1, n_2, n_3, \dots)$ . The utility for the deviating user can be decomposed as:

$$\begin{aligned} g(u, u') &= g(u_{(0-3)}, u'_{(0-3)}) + g(u_{(0-3)}, u'_{(\geq 4)}) \\ &\quad + g(u_{(\geq 4)}, u'_{(0-3)}) + g(u_{(\geq 4)}, u'_{(\geq 4)}), \end{aligned} \quad (19)$$

where for every  $I \subset \mathbb{N}$ , the notation  $u_{(I)}$  stands for the sub-vector  $\{n_i\}_{i \in I}$  (thus, for example,  $g(u_{(\geq 4)}, u'_{(0-3)})$  is the number of successful transmissions obtained in interactions where all users use power levels 0 – 3 and the deviator uses power levels greater or equal to 4). Obviously,  $g(u_{(0-3)}, u'_{(\geq 4)}) = 0$ . As before, assume by contradiction that  $n_3 > 0$  and consider an alternative policy for the deviating user  $\hat{u} = (\hat{n}_0, \hat{n}_1, \hat{n}_2, 0, \dots)$  where  $\hat{n}_2 = n_2 + n_3$ ,  $\hat{n}_0 = n_0 - n_3$ ,  $\hat{n}_1 = n_1 + n_3$ ,  $\hat{n}_i = n_i$  for  $i \geq 4$ . It follows from Step 1 that  $g(\hat{u}_{(0-3)}, u'_{(0-3)}) > g(u_{(0-3)}, u'_{(0-3)})$ . Since the other three terms in (19) are not affected by the transition from  $u$  to  $\hat{u}$ , we conclude that  $g(\hat{u}, u') > g(u, u')$ . Hence  $n_3 = 0$  in any best response.

Step 3: In any best-response  $n_i = 0$  for  $i > 2$ . Assume by induction on  $k$  that  $n_k = 0$ ,  $n'_k = 0$ . It is readily seen that the policy  $(n_0, n_1, \dots, n_k = 0, n_{k+1}, \dots)$  with  $n_{k+1} > 0$  is suboptimal, since  $\hat{u} = (n_0 - n_{k+1}, n_1 + n_{k+1}, \dots, \hat{n}_k = n_{k+1}, \hat{n}_{k+1} = 0, \dots)$  obviously obtains strictly higher TPS. Indeed, the deviating user benefits from the use of power level  $k$  as it did from power level  $k + 1$  (due to the induction assumption that  $n'_k = 0$ ), and in addition it obtains a strictly positive benefit from additional power-1 transmissions. Hence,  $n_{k+1} = 0$ . Since  $n_i = 0$ ,  $i > 2$  for any best response, there is no equilibrium point in which mobiles use power levels above two.  $\square$

Taking into account that  $n_3 = 0$ , it follows from the energy constraint (2) (which is met with equality) that  $n_0 = n_2$  for any user policy. We next express the utility of a “deviating” user with such policy  $u = (n_0 = n_2, n_1, n_2)$ , where all others use a policy  $u' = (n'_0, n'_1, n'_2)$ .

$$\begin{aligned} g(u, u') &= \delta(N - n_0) + \frac{1 - \delta}{N} (n_2(n'_1 + n'_0) + n_1 n'_0) \quad (20) \\ &= \delta(N - n_2) + \frac{1 - \delta}{N} (n_2(n'_1 + n'_0) + (N - 2n_2)n'_0) \\ &= \delta N + (1 - \delta)n'_0 + n_2 \frac{1 - \delta}{N} \left( n'_1 - \left( n'_0 + \frac{\delta N}{1 - \delta} \right) \right). \end{aligned}$$

Define

$$A(n'_1, n'_0) = \left( n'_1 - \left( n'_0 + \frac{\delta N}{1 - \delta} \right) \right). \quad (21)$$

Clearly, the sign of  $A(n'_1, n'_0)$  would determine the best-response (BR) of the deviating user, as we summarize below.

$$\begin{cases} A(n'_1, n'_0) = 0 : & \text{Any strategy } (n_0, n_1, n_2) \text{ is BR} \\ A(n'_1, n'_0) > 0 : & n_0 = n_2 = \frac{N}{2}, n_1 = 0 \\ A(n'_1, n'_0) < 0 : & n_0 = n_2 = 0, n_1 = N \end{cases} \quad (22)$$

Using (22), we may explicitly characterize the symmetric equilibrium point, as we summarize in the next theorem. When the policies below result in non-integer numbers, mixed policies are used in the spirit of Theorem 4.

**Theorem 6:** (i) The symmetric equilibrium point exists and is unique. It is given by

$$\begin{cases} \delta \leq \frac{1}{2} : & n_0 = n_2 = \frac{1-2\delta}{3(1-\delta)}N, n_1 = \frac{1+\delta}{3(1-\delta)}N \\ \delta > \frac{1}{2} : & n_0 = n_2 = 0, n_1 = N. \end{cases} \quad (23)$$

(ii) The corresponding TPS are given by

$$\begin{cases} \delta \leq \frac{1}{2} : & TPS = \frac{1}{3}(1 + \delta), \\ \delta > \frac{1}{2} : & TPS = \delta. \end{cases} \quad (24)$$

**Proof:** (i) Consider first the case where  $\delta > \frac{1}{2}$ . For that case,  $A(n'_1, n'_0) = \left( n'_1 - \left( n'_0 + \frac{\delta N}{1 - \delta} \right) \right) < (n'_1 - (n'_0 + N)) \leq 0$ , which immediately leads to the best response policy of  $n_0 = n_2 = 0, n_1 = N$ .

Consider next the case of  $\delta \leq \frac{1}{2}$  and the possible values for  $A(n'_1, n'_0)$ . Assume that  $A(n'_1, n'_0) > 0$ ; then obviously  $n'_1 \geq n'_0$ ; however, the best-response (22) in this case is such that  $0 = n_1 < n_0 = N/2$ . Hence  $A(n'_1, n'_0) > 0$  does not lead to a symmetric equilibrium. Similarly, assume that  $A(n'_1, n'_0) < 0$ . This implies that  $n'_1 < n'_0 + \frac{\delta N}{1 - \delta} \leq n'_0 + N$ ; however the best-response in this case is such that  $N = n_1 \geq n_0 + N = N$ . Hence  $A(n'_1, n'_0) < 0$  does not lead to a symmetric equilibrium as well. The remaining case is  $A(n'_1, n'_0) = 0$ . Since the deviating user is indifferent about its policy (as long as it uses power levels not greater than two), a symmetric equilibrium is obtained for  $n_1 = n_0 + \frac{\delta N}{1 - \delta}$ . Using the energy constraint, the last equation immediately implies that  $n_0 = \frac{1-2\delta}{3(1-\delta)}N$ ,  $n_1 = \frac{1+\delta}{3(1-\delta)}N$ .



(ii) For  $\delta > \frac{1}{2}$ , it is immediate that the TPS is  $\delta$ , since users always transmit with a power level of one; the TPS in this case is thus equivalent to the probability of not facing an interferer. For the case of  $\delta \leq \frac{1}{2}$ , we substitute  $A(n'_1, n'_0) = 0$  and the allocation rule (23) in (20) and obtain that  $TPS = \delta + \frac{1-2\delta}{3}$  which establishes the result.  $\square$

The evolution of the power allocation at the symmetric equilibrium as a function of  $\delta$  is summarized in Fig. 3, and the corresponding TPS is given in Fig. 4.

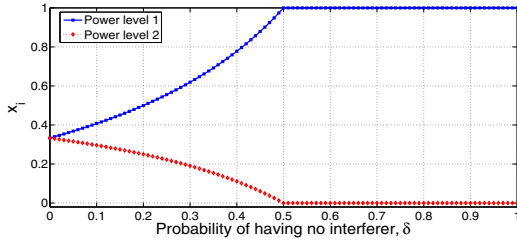


Fig. 3. The distribution of power levels at the symmetric equilibrium as a function of the probability of having no interferer.

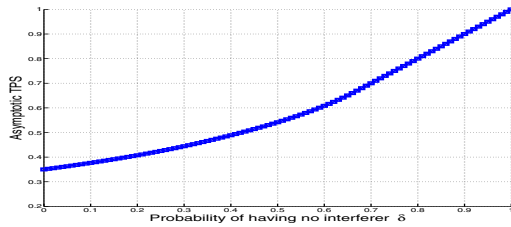


Fig. 4. The TPS at the symmetric equilibrium as a function of the probability of having no interferer.

### B. Efficiency Loss

Equipped with a complete characterization of both the symmetric optimal solution and the symmetric equilibrium point, we may compare the performance at both frameworks. A popular measure for comparison is the price of anarchy (PoA) [11], which corresponds in our case to the ratio between the TPS obtained in the team problem and the TPS at the symmetric equilibrium<sup>4</sup>. We emphasize that we do not consider here asymmetric working points.

The PoA as a function of  $\delta$  is depicted in Figure 5. It is seen that the efficiency loss is always smaller than 9 percent. An interesting direction for future work is to study the efficiency loss in cases where the energy available is larger (i.e.,  $K > N$ ) and examine whether users misuse the access energy.

### C. Existence of Asymmetric Equilibria

We focused in preceding subsections on symmetric Nash equilibria. In this subsection we show that asymmetric equilibria exist in general. In view of (21), any set of policies

<sup>4</sup>In general, the PoA corresponds to the ratio between the optimal solution and the *worst* Nash equilibrium. However, in our case, both the symmetric optimal solution and the symmetric equilibrium are unique.

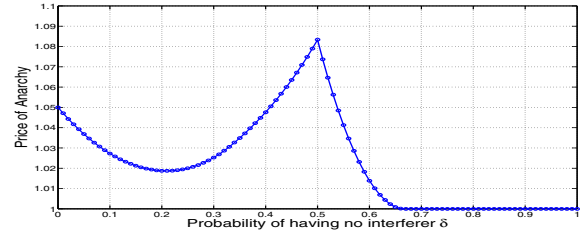


Fig. 5. Efficiency loss as a function of  $\delta$ . The y-axis is the ratio between the symmetric-optimal TPS and the symmetric equilibrium.

(that use power levels less than 3), for which the *average* distribution of power levels among the user-population satisfies  $n_1 = n_0 + \frac{\delta N}{1-\delta}$ , leads to an equilibrium. Indeed, no user will benefit from deviating, as all  $(n_0, n_1, n_2)$  policies are in fact best responses. A particular case of the above are the symmetric equilibria obtained in Theorem 6. Based on this observation, it is possible to construct asymmetric equilibria as, for instance,

- A fraction  $\frac{1+\delta}{3(1-\delta)}$  of the population use  $n_1 = N$ ,  $n_0 = n_2 = 0$ . The remaining fraction  $\frac{2-4\delta}{3(1-\delta)}$  use  $n_0 = n_2 = \frac{N}{2}$ ,  $n_1 = 0$ .

In the present paper, we do not focus on asymmetric equilibria, yet point to their existence. The numeric example above strongly relies on our characterization of the symmetric equilibrium. It remains to be verified whether additional asymmetric equilibria (which may lead to different TPS) do exist. The comprehensive analysis of asymmetric equilibria remains a challenging direction for future work.

## V. EXTENSIONS AND CONCLUSION

We briefly mention how to adapt the analysis to variations on the initial model, and present some conclusions and future research directions.

### A. “Soft-Capture” Wireless Network

Assume that if two stations transmit at the same power level then a given packet is successfully received with probability  $a \leq 1/2$ . Let  $\bar{a} = 1 - a$ . If powers are different then, as before, the packet transmitted with larger power is successful and the other is not. The objective to maximize is given by

$$\begin{aligned} g^{cap} &= \delta(N - n_0) + (1 - \delta) \frac{\bar{a}}{N} \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j + (1 - \delta) \frac{a}{N} \sum_{i=1}^K \sum_{j=0}^i n_i n_j \\ &= \delta(N - n_0) + (1 - \delta) \frac{1}{N} \left( \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j + a \sum_{i=1}^K n_i^2 \right) \\ &= \delta(N - n_0) - (1 - \delta) n_0^2 a / N \end{aligned} \quad (25)$$

$$\begin{aligned} &+ (1 - \delta) \left[ \frac{1 - 2a}{N} \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j + \frac{2a}{N} \left( \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j + \frac{1}{2} \sum_{i=0}^K n_i^2 \right) \right] \\ &= \delta(N - n_0) - (1 - \delta) \frac{a n_0^2}{N} + (1 - \delta) \left[ \frac{1 - 2a}{N} \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j + a N \right], \end{aligned} \quad (26)$$

where we used (6). Consider  $a = 1/2$ . In this case,  $g^{cap}$  equals  $-\delta n_0 - (1 - \delta)n_0^2/(2N)$  plus some constant that does not depend on  $u$ . For any  $\delta$ , this utility is maximized at  $n_0 = 0$  which means  $n_1 = N$  and  $n_i = 0$  for all  $i \neq 1$ . The case  $a < 1/2$  remains to be investigated in future work.

### B. General Access Problems (Zero May Win Too)

Through the model we used for the power control problem, we intended to introduce a methodology that can be useful for general control of priority access. We briefly comment on some specific variations that may be needed in other network applications. In a general priority assignment context, one may again enumerate priority levels using the integers  $\{0, 1, 2, \dots\}$ ; an access request with priority  $i \geq 1$  would prevail if it is the only request, or if all other requests are with lower priority. It may even be granted access (with some positive probability  $a$ ) in the case that another request is made with the same priority level (as in the “soft-capture” model above). Yet, there may be a difference in the way that priority level 0 is treated, compared to the way that power level 0 is modeled in the power control problem. In the power control framework, when the transmission power is zero then transmission fails, in particular, for the following two cases: (i) there is no interference, or (ii) there is interference with another mobile that “transmits” with a power of zero. This need not be the situation in other priority assignment models. For example, the lowest priority (i.e., zero) can be interpreted as “best-effort” service in QoS-supporting network architectures.

To concretize our discussion, assume that a request with zero priority will be successful w.p.1 in case (i) above, and with positive probability  $a$  in case (ii) (i.e., the “soft-capture” rule includes priority zero as well). The expected utility is given by  $\delta N$  plus the third term of (26), which yields

$$g^{general} = \delta N + (1 - \delta) \left[ \frac{1 - 2a}{N} \sum_{i=1}^K \sum_{j=0}^{i-1} n_i n_j + aN \right].$$

Interestingly, the optimal and equilibria policies for any  $\delta$  coincide with those obtained for the original power control problem with  $\delta = 0$ . Note that for  $\delta = 1$  or for  $a = 0.5$ , the performance does not depend on the policy anymore (all policies are thus optimal).

### C. Concluding Remarks

We have considered in this paper the priority assignment problem that corresponds to “sparse” multiple access networks, in which pairwise interactions occur. We have provided an explicit solution for the team problem, and a complete characterization of the symmetric equilibrium in a noncooperative framework. Interestingly, the number of power levels that is used in the competitive setup is smaller than the corresponding number for the team problem (this holds for every  $\delta$ ). This phenomenon is counter-intuitive perhaps, as in many noncooperative networking scenarios, the users consume the network resources in a more aggressive way, compared to the socially-optimal point (e.g., in queuing networks, see [6]).

The framework and results of this paper may be extended in several ways. One direction is to consider general battery life-time  $N$  and general budget  $K$  and obtain complete characterization of both the team and game problems. Another challenging extension is to relax the assumption on sparsity and to consider interactions of more than two users and their consequences. We also leave for future study the heterogeneous node case, where nodes may have different budgets which naturally lead to asymmetric solutions.

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