A NUMERICAL STUDY OF FRONTOGENESIS

by

STEPHEN EDWARD MUDRICK

B.Sc., University of Maryland, 1966

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

March, 1973

Signature of Author
Department of Meteorology, March 16, 1973

Certified by
Thesis Supervisor

Accepted by
Chairman, Departmental Committee on Graduate Students

A NUMERICAL STUDY OF FRONTOGENESIS

by

Stephen Edward Mudrick

Submitted to the Department of Meteorology on March 16th, 1973 in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

Fine resolution, dry, inviscid, Boussinesq, quasigeostrophic and primitive equations models are used in a study of frontogenesis. These three-dimensional models employ horizontal and vertical resolution on the order of 100 km and 1 km, respectively; an integration uses about 40 gridpoints in each horizontal direction and 20 in the vertical.

The initial states consist of two baroclinic basic currents upon which are superimposed quasi-geostrophically balanced, small amplitude perturbations corresponding to the most unstable mode in each case. In the second case the wave grows by barotropic as well as by baroclinic processes.

The most rapid surface frontogenesis occurs where the synoptic scale, quasi-geostrophic convergence contributes significantly to the pure deformational increase of the horizontal temperature gradient. In these integrations this distribution favors formation of warm fronts.

The horizontal deformation, as well as the "indirect" vertical circulation, is important in producing upper level frontogenesis. The two models generate similar patterns of vertical motion. A feedback mechanism relating the action of the horizontal deformation and the indirect circulation and leading to upper level frontogenesis is postulated.

Thesis Supervisors: Norman A. Phillips, Frederick Sanders

Title: Professors of Meteorology

ACKNOWLEDGEMENT

The author extends his sincere appreciation to his thesis advisors, Professors Norman Phillips and Frederick Sanders, for their advice and guidance throughout the course of this research. Many fruitful discussions with them and with other Professors and graduate students at M.I.T. are remembered. The staff of the Information Processing Center at M.I.T. is cited for its assistance in the preparation of the numerical programs. Financial support for the author during his stay at M.I.T. came in part from a NASA Traineeship and from a Research Assistantship supported by NSF grant number A28724X.

The National Center for Atmospheric Research, which is sponsored by the National Science Foundation, provided computer time and facilities for the major computations. The assistance of the computing facility and the library staff is most appreciated. The interest of Drs. Edwin Danielsen and Melvyn Shapiro of NCAR is also appreciated.

Thanks are extended to Miss Isabelle Kole for assistance with the figures and to Mrs. Marie L. Gabbe for typing the manuscript. Eugenia de Rivas programmed the original version of the primitive equations model.

The love, moral support and patience provided by the author's wife during the period of this study cannot be acknowledged satisfactorily. Finally, thanks go to the author's parents for making this all possible.

TABLE OF CONTENTS

ABSTRACT	2
ACKNOWLEDGEMENT	3
TABLE OF CONTENTS	4
LIST OF FIGURES	6
LIST OF TABLES	11
1. INTRODUCTION	12
1.1 A description of frontogenesis.	12
1.2 Theoretical studies of frontogenesis.	14
1.21 Two-dimensional models of frontogenesis.	15
1.22 Three-dimensional models of frontogenesis.	19
1.3 The present investigation.	21
2. PROCEDURES USED IN THE RESEARCH	23
2.11 The dynamical models used. The "primitive" equations.	23
2.12 The quasigeostrophic equations.	26
2.2 Procedures used in the numerical integrations.	28
2.21 Determination of the basic state.	30
2.22 Determination of the perturbation streamfunction.	35
3. RESULTS OF THE NUMERICAL INTEGRATIONS	47
3.1 Vertical finite-differencing in the models.	49
3.2 Experiment I - the shallow baroclinic wave: PE versus QG high resolution integration.	49
3.21 Surface evolution of the integrations.	62
3.22 Upper level development.	66
3.23 Kinematics of experiment I.	68
3.24 Summary of experiment I.	78
3.25 Realism of the integrations.	78
3.3 Experiment II - the deep barotropic-baroclinic wave: PE versus QG high resolution integrations.	81
3.31 Surface evolution of the integrations.	97
3.32 Upper level development.	102
3.33 Kinematics of experiment II.	106
3.34 Summary of experiment II.	117
3.35 Realism of experiment II.	119
4. SURFACE FRONTOGENESIS	122
4.1 Horizontal deformation and surface frontogenesis.	122

4.2	Quasigeostrophic initiation of surface frontogenesis.	131
4.3	The structure and movement of numerical frontal zones.	137
4.4	The effects of friction on PE frontogenesis.	144
4.5	Summary.	151
5.	ON THE DYNAMICS OF UPPER LEVEL FRONTOGENESIS	152
5.1	Upper level frontogenesis in the numerical model.	152
5.2	Deformation and upper level frontogenesis. Part I.	172
5.3	Quasigeostrophic control of upper level frontogenesis.	176
5.4	Deformation and upper level frontogenesis. Part II.	184
5.5	Ageostrophic effects and upper level frontogenesis.	195
5.6	Summary.	205
6.	CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH	209
6.1	Suggestions for future research.	211
APPENDICES		
	Appendix A. Determination of the basic state.	213
	Appendix B. The primitive equations (PE) model.	218
	Appendix C. The quasigeostrophic (QG) model.	228
	Appendix D. The initialization procedure for the PE model.	231
	Appendix E. Energetics for the PE model.	236
	Appendix F. Mechanism to produce NE-SW tilted trough.	241
REFERENCES		
243		
BIOGRAPHICAL SKETCH		
248		

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
2-1	The channel model geometry.	24
2-2	The basic state.	32
2-3	Basic state I.	33
2-4	Basic state II.	34
2-5	Perturbation phase speed C_r and growth rate KC_i versus channel length L ; basic states I and I-N.	38
2-6	Perturbation phase speed C_r and growth rate KC_i versus channel length L ; basic states II and II-N.	39
2-7	Amplitude and zonal phase for most unstable perturbation for basic state I.	41
2-8	Energy conversions for basic state I perturbation.	42
2-9	Amplitude and zonal phase for most unstable perturbation for basic state II.	43
2-10	Energy conversions for basic state II perturbation.	44
3-1	Vertical finite-differencing for PE and QG models.	50
3-2	1PE surface pressure, level $K=1$, $Z=.025$.	52
3-3	1PE surface buoyancy, level $K=1$, $Z=.025$.	53
3-4	1PE pressure and buoyancy, level $K=9$, $Z=.425$.	54
3-5	1PE north-south cross sections. 0 Hours.	55
3-6	1PE north-south cross sections. 108 Hours.	56
3-7	1QG surface pressure, level $K=1$, $Z=.025$.	57
3-8	1QG surface buoyancy, level $K=1$, $Z=.050$.	58

<u>Figure</u>	<u>Page</u>
3-9 1QG pressure and buoyancy, level K=9, Z=.450.	59
3-10 1QG north-south cross sections. 0 Hours.	60
3-11 1QG north-south cross sections. 108 Hours.	61
3-12 Surface pressure evolution. 1PE versus 1QG.	63
3-13a Growth rate of 1PE versus 1QG.	69
3-13b 1PE XY averaged buoyancy versus time, levels K=1,3,5.	69
3-14 X averaged vertical motion. 1PE, 1QG, 0 Hours; 1QG 108 Hours.	74
3-15 X averaged vertical motion. 1PE, 102, 108 Hours.	75
3-16 1PE energy variation versus time.	77
3-17 Synoptic pattern 30 June 1969, 12Z.	80
3-18 2PE surface pressure, level K=1, Z=.025.	82
3-19 2PE surface buoyancy, level K=1, Z=.025.	83
3-20 2PE pressure, level K=9, Z=.425.	84
3-21 2PE buoyancy, level K=9, Z=.425.	85
3-22 2PE north-south cross sections. 0 Hours.	86
3-23 2PE north-south cross sections. 108 Hours.	87
3-24 2PE north-south cross sections. 144 Hours.	88
3-25 2QG surface pressure, level K=1, Z=.025.	89
3-26 2QG surface buoyancy, level K=1, Z=.050.	90
3-27 2QG pressure, level K=9, Z=.425.	91
3-28 2QG buoyancy, level K=9, Z=.450.	92
3-29 2QG north-south cross sections. 0 Hours.	93

<u>Figure</u>		<u>Page</u>
3-30	2QG north-south cross sections. 108 Hours.	94
3-31	2QG north-south cross sections. 144 Hours.	95
3-32	Comparison of 2PE, 2QG buoyancy at level K=9, 180 Hours.	96
3-33	Surface pressure evolution. 2PE versus 2QG.	98,99
3-34a	Growth rate of 2PE versus 2QG.	107
3-34b	2PE XY averaged buoyancy versus time, levels K=1,3,5.	107
3-35	X averaged vertical motion. 2PE, 2QG, 0 Hours.	113
3-36	X averaged vertical motion. 2PE, 2QG, 144 Hours.	114
3-37	2PE energy variation versus time.	116
3-38	Synoptic pattern 31 August 1972, 12Z.	120
4-1	1PE surface divergence, deformation and dS_1/dt , level K=1, Z=.025. 0 Hours.	126
4-2	1PE surface divergence, deformation and dS_1/dt , level K=1, Z=.025. 96 Hours.	129
4-3	1QG surface buoyancy, vertical motion; 1PE surface winds and pressure, level K=1. 96 Hours.	130
4-4	1PE geostrophic potential vorticity, level K=2, Z=.075. 0 and 96 Hours.	134
4-5	1PE absolute vorticity and stability, level K=2. 96 Hours.	135
4-6	Transverse structure of surface fronts. 2PE, 2QG. 144 Hours.	139
4-7	2PE frontal cross sections.	141
4-8	Location of 2PE and 2QG frontal zones compared to low, level K=1.	143

<u>Figure</u>		<u>Page</u>
4-9	Surface winds and pressure, level K=1, Z=.025. 120 Hours. Runs C-2, C-2F.	146
4-10	Surface buoyancy, level K=1, Z=.025. 120 Hours. Runs C-2, C-2F.	147
4-11	East-west buoyancy cross sections, J=16, Y=.4833. 120 Hours. Runs C-2, C-2F.	150
5-1	Evolution of atmospheric upper level frontal zone. October 28-30 1963. 500 mb.	153
5-2	Vertical cross section along east coast of United States, October 30 1963, 00Z.	154
5-3	2PE buoyancy, vertical motion, level K=9. 144 Hours.	161
5-4	Cross section AA intersecting upper front. 2PE. 144 Hours.	162
5-5	2PE north-south cross section. I=15. 144 Hours. Jet trough.	164
5-6	2PE north-south cross sections. Buoyancy and potential vorticity at jet trough. 0 and 144 Hours.	165
5-7	Run C-1 pressure, buoyancy and vertical motion, level K=10. 144 Hours.	169
5-8	Run C-1 buoyancy. 0 and 144 Hours.	170
5-9	2PE horizontal deformation plus buoyancy. 108 Hours.	174
5-10	2PE horizontal deformation plus buoyancy. 144 Hours.	175
5-11	Angle between axis of dilatation and buoyancy isolines for selected region of fig. 5-10b.	177
5-12	Evidence of near-geostrophic balance of upper front. 2PE, 144 Hours.	179

<u>Figure</u>	<u>Page</u>
5-13 2PE, 2QG vertical motion, level K=9, Z=.450. 144 Hours.	182
5-14 2PE, 2QG cross sections BB.	183
5-15 2PE, 2QG cross sections AA.	185
5-16 2PE absolute vorticity, pressure and vertical motion, level K=9. 144 Hours.	188
5-17 2PE vorticity and buoyancy advection. 144 Hours.	189
5-18 2PE, 2QG cross sections AA, BB, levels K=6-10. 144 Hours.	191
5-19 2PE vertical motion and stability, 144 Hours.	198
5-20 After Hoskins (1971).	203
B-1 PE half and full timestep grids.	221
B-2 W(21,16,2) versus time, steps 2140-2159. Run 2PE.	221
D-1 Y-Z grid for variables.	233
F-1 Mechanism to produce "NE-SW" tilted trough.	242

LIST OF TABLES

<u>Table</u>		<u>Page</u>
2-1	Basic states investigated by the instability study.	36
3-1	Integrations performed on PE and QG models.	48
3-2	Energy transformation terms versus time, Run 2PE.	112
4-1	Nondimensional values of $d\zeta/dt$ and horizontal deformation for selected initial data points. 1PE versus 1QG, level K=1.	128
5-1	Maximum dimensional values of $ \nabla_n \zeta $ in trough for levels 6 through 10 for various times, Run 2PE.	159
5-2	Cross-isobar flow $-\vec{n} \cdot \nabla_n p$ versus height for vertical column directly beneath the jet core in 2PE cross section BB.	194
5-3	Terms from equations (5-5) and (5-7) evaluated at point M in 2PE cross section AA (fig. 5-19a).	194

1. INTRODUCTION

This chapter will present a brief review of the subject of frontogenesis, leading to a rationale for this investigation. The methods used in the study will then be outlined.

1.1 A description of frontogenesis.

Frontogenesis is the process by which frontal zones are formed in the atmosphere. Various papers have presented historical reviews of the subject (see for example Sanders, 1954; Eliassen, 1959, 1966; Palmen and Newton, 1969 and Phillips, 1970 for different perspectives). While frontal zones may appear on the scale of the local sea breeze or the larger "coastal front" phenomenon, we are interested here in frontogenesis as it occurs on the "synoptic" scale, usually within the context of a growing baroclinic wave. It is such zones, on the order of 10^3 km in length and 10^2 km or less in width, that are of most interest in the evolution of mid-latitude weather.

Frontogenesis on such scales is generally observed to occur in two modes, those fronts forming and becoming most intense at the earth's surface and those developing in the middle and upper troposphere. These modes often proceed independent of one another. Both types are often observed to develop without latent heat release and on time scales too short for radiative effects to be of significance (see for example Faller, 1956 for a demonstration of surface frontogenesis within a rotating tank experiment). Surface effects of

topography, friction and heating appear to be of secondary importance in frontogenesis; at least models neglecting these processes are able to produce the phenomenon as will be seen.

Surface frontogenesis is associated with regions of convergence and upward motion. There is usually a maximum of cyclonic vorticity present within the frontal zone, along with the increased horizontal temperature gradient and vertical stability. The zones are most intense at the earth's surface and often become diffuse at a height of a few km (Sanders, 1955) and they usually occur in conjunction with a growing baroclinic wave. Horizontal deformation fields have been shown to be important at intensifying frontal zones (see for example Sawyer, 1956; Bergh, 1967, showed that in a meso-scale study the absolute magnitude of the total horizontal deformation was more systematically characteristic of frontogenesis than were the divergence or vorticity fields).

Mid- or upper level frontogenesis, on the other hand, is usually associated with descending motion and relatively dry air; development is often maximum in the middle or upper troposphere and is found with regions of strong winds ("jet streams"). Like the surface zones the vertical component of vorticity is usually large within the zone. In addition the following conditions are commonly noted:

a) upper level frontogenesis often is associated with amplifying upper level systems; intense cases of frontogenesis occur within strongly developing waves (Reed, 1955; Shapiro, 1970),

- b) air within the more intense frontal zones can often be traced back to the lower stratosphere; hence the tropopause must have been discontinuous or must have "folded" into the troposphere (Reed and Sanders, 1953; Danielson, 1964),
- c) two mechanisms have been found that separately or together work toward frontogenesis — horizontal confluence (the action of the horizontal deformation field) and an "indirect" circulation within the frontal zone such that the strongest descent and warming occurs on the warm side of the zone, thus strengthening the horizontal temperature gradient (Reed and Sanders, 1953; Elliott and Brown, 1956; Campana, 1967; Bosart, 1970).

"Wet" frontal zones are sometimes found in the mid-troposphere; these are usually extensions of surface fronts and are not considered here as "upper level" frontal zones.

1.2 Theoretical studies of frontogenesis.

Bergeron (1928) and Petterssen (1935) showed that the deformation of the horizontal wind could tighten pre-existing temperature gradients. Petterssen and Austin (1942) pointed out that the increased vorticity within surface frontal zones was also an important frontal characteristic, in addition to the temperature gradient. While a conservative quantity like the potential temperature (assuming adiabatic motion) could be collected by a nondivergent deformation field, the vorticity had to be generated by the convergence observed at the front. Thus vertical as well as horizontal motions were

important in frontogenesis and had to be explained.

Assuming hydrostatic balance and the continuity of pressure across the frontal zone, Petterssen and Austin showed that a sloping frontal zone required the surface isobars to possess a cyclonic kink within the zone. The presence of vorticity is thus required if the front is to be in geostrophic balance. In addition they presented evidence that frontal zones are in near-geostrophic balance to the extent that the Margules (1906) formula for the frontal slope is approximately correct.

1.21 Two-dimensional models of frontogenesis.

Based upon the observed near-geostrophic balance of frontal zones at and above the surface, quasigeostrophic reasoning has often been used to relate the horizontal and vertical wind fields within intensifying frontal zones. It is generally accepted that the synoptic scale horizontal winds work through the deformation process to tighten temperature gradients in selected regions. If the flow within these regions is to remain in near-geostrophic balance the vertical wind shear must also increase; this means a direct circulation with warm air rising and cold air sinking must be set up around the evolving frontal zone. In order to handle the mathematical complexities, such studies have been two-dimensional, assuming changes along the front to be negligible compared to changes across the zone. Many have concentrated upon deriving a diagnostic relation between the transverse, ageostrophic circulation and the "observable"

fields of temperature and geostrophic wind across the frontal zone.

The most successful of these was by Eliassen (1962) who introduced a transformation replacing the cross-front geometric coordinate y by the "absolute momentum" $m = U - fy$ (where U is the geostrophic wind component parallel to the front and f , the Coriolis parameter is constant). Then $-\frac{\partial m}{\partial y}$ is the vertical component of absolute geostrophic vorticity. Where the vorticity is large, as within a frontal zone, m will change rapidly compared to y ; m serves as a "stretched" coordinate.

Eliassen's "forcing function" successfully predicts the direct, transverse circulations usually observed with intensifying surface frontal zones. Difficulties arise for upper level zones where "indirect" circulations are present; usually schemes relating the downstream advection of temperature or vorticity to the nearly unobservable cross-front geostrophic wind component are needed to resolve the problem (see for example Danielsen, 1964 and Bosart, 1970).

In addition to the diagnostic studies, time dependent, two-dimensional, analytic models of surface frontogenesis utilizing the quasigeostrophic equations have been developed (Stone, 1966; Williams and Plotkin, 1968; Williams, 1968). Exact, steady solutions for the large time limit can be obtained this way with a time independent, nondivergent, "stretching" horizontal deformation field forcing frontogenesis, as Bergeron had postulated. The frontal zones produced are unrealistic, however; there is no slope with height and

there is no vorticity nor stability maximum where the frontal zone intersects the earth's surface.

A third avenue of research on surface frontogenesis was pursued by Williams. He extended the two-dimensional models to the primitive equations and used numerical methods to obtain solutions. Assuming hydrostatic, adiabatic, inviscid motion and making the Boussinesq approximation (as had been done in the quasigeostrophic studies), Williams (1967) modeled the growing baroclinic wave and showed a realistic "cold" front structure could be produced. The horizontal deformation was not initially present in the basic state and this mode of frontogenesis has become known as the "shearing" mechanism. Earlier Arakawa (1962), using similar initial conditions, showed the so called "balance" equations could also produce this frontal structure. These frontal zones sloped with height and did contain increased vorticity and stability; they developed within a finite time whereas the quasigeostrophic models required in effect an infinite length of time. Williams (1972) also repeated the "stretching" deformation studies using the primitive equations in Boussinesq form and obtained realistic frontal zones.

The above three directions of research were elegantly synthesized by Hoskins (1971) and by Hoskins and Bretherton (1972). Starting with the primitive equations and assuming hydrostatic, adiabatic, inviscid motion, then requiring cross-front geostrophic balance and utilizing a transformation similar to that of Eliassen (1962), they obtained analytic solutions for the "stretching" and

"shearing" deformation studies (the Boussinesq approximation was also used). The solutions produce frontal zones similar to Williams' numerical models. Hoskins' and Bretherton's work shows how the effect of the divergent wind, missing in the quasigeostrophic models, works to rotate the front in the transverse plane and to increase the convergence and vorticity within the frontal zone.

In addition, Hoskins and Bretherton extended their model for a study of upper level frontogenesis. Assuming the tropopause separates two regions of uniform static stability, they showed that the action of a large scale deformation field (confluence) could initiate tropopause folding and upper level frontogenesis, without requiring advection of temperature or vorticity along the front. They again utilized the Boussinesq approximation but the problem had to be integrated numerically; unlike the surface front studies no understanding of the dynamics involved in the process is readily apparent from their work.

The two-dimensional studies have produced some understanding of the process of surface frontogenesis, at least in the final stages of development. The action of the horizontal deformation field is important in both the "stretching" and "shearing" models; in the latter case the deformation field, while not effective initially, becomes more important with time. The presence of the bottom surface keeps the direct vertical circulation from reducing the strength of the increasingly tight temperature gradient; discontinuities in temperature would result except for the low Richardson

numbers and breakdown to turbulence that occurs in the atmosphere, limiting the zone to a greater than zero thickness.

In the case of upper level frontogenesis the processes involved are less well understood; the circulation is more complicated due to the lack of a nearby rigid boundary. The apparent conflict between Eliassen's (1962) work, indicating the need for downstream advection in order to get the indirect circulation, and that of Hoskins and Bretherton (1972), not requiring those advections in order to initiate upper level frontogenesis, is resolved by the fact that Eliassen neglects some nonlinear terms included in the later work.

1.22 Three-dimensional models of frontogenesis.

The two-dimensional studies attempt to model the synoptic scale forcing that drives frontogenesis; the "stretching" deformation model includes the effect of the deformation fields while the "shearing" model contains the effect of cold air advected from the north and warm air advected from the south by a growing wave. Only with a fully three-dimensional model can the synoptic scale evolution be included realistically; the problem is then too difficult for analytic studies so numerical methods must be used. A fantastic amount of gridpoints (or spectral components) would be required for such models to approach the resolution achievable by two-dimensional models so in order to be feasible three-dimensional studies of frontogenesis usually employ channel models and cyclic boundary conditions

to limit the horizontal domain plus reduced horizontal and vertical resolution. Many such studies have been done, integrating the dry, primitive equations in one form or another (Edelmann, 1963; Økland, 1969; Hadfield, 1970; Eliassen and Raustein, 1968, 1970 for example).

All the above models utilized basic states independent of the east-west direction. All possessed a zonal wind increasing with height and some included horizontal shear. A small amplitude sinusoidal perturbation, independent of height, was superimposed upon the basic state and the initial conditions were then integrated for varying lengths of time.

In general the models produced realistic cyclogenesis and frontogenesis, but with two noticeable characteristics. A tongue of warm air was advected into and trapped within the occluding cyclone at the surface (this occurred in Edelmann's model with and without friction). In addition, the warm fronts produced were generally stronger and more pronounced than the cold fronts, in particular, the surface convergence being significantly stronger at the warm front. None of the three-dimensional models have produced upper level frontogenesis; this is reasonable since the vertical resolution used has been no less than 4 km between levels.

In summary, the dynamics of surface frontogenesis have been elucidated, particularly by the two-dimensional models of Hoskins and Bretherton. The formation of frontal zones within larger scale cyclogenesis has been observed in three-dimensional models but has not been studied systematically. Upper level frontogenesis has only

been simulated in a two-dimensional study; no agreement exists upon the manner in which the observed indirect circulation develops within the upper level frontal zone.

1.3 The present investigation.

The purpose of this study is to learn more about frontogenesis on the cyclone scale, both at the surface and at upper tropospheric levels, by the use of fine resolution, three-dimensional atmospheric models. In order to resolve developing fronts we require a horizontal and vertical resolution of about 10^2 km and 1 km, respectively; in order to model adequately the growing wave we need a domain on the order of 5000 km on a side and about 15 km in the vertical. Such a fine resolution, finite-difference model will thus require nearly 38,000 gridpoints as no "nested grid" techniques will be attempted.

The models will be restricted to hydrostatic, adiabatic, frictionless motion and the Boussinesq approximation will be made. We use the mid-latitude "beta plane" approximation and an east-west re-entrant channel with rigid horizontal and vertical boundaries. The artificial effect of the walls can be removed by selecting initial conditions such that nothing happens near the walls.

In the case of surface frontogenesis we want to pursue the question of how the zones evolve within the growing baroclinic wave. The problem of how a two-dimensional field of temperature is transformed into elongated zones where one dimension predominates is quite

nonlinear and difficult; we will approach the study of frontogenesis from a perspective similar to the two-dimensional studies where the action of effects not included in the quasigeostrophic equations has been emphasized. Accordingly we will integrate the same initial conditions with both fine resolution primitive equations and quasigeostrophic models and will concentrate on the differences in the integrations.

Such a comparison of the two models will yield insight into the dynamics of upper level frontogenesis, should such an upper level frontal zone be produced. We want to determine how an indirect circulation becomes established and we will investigate the role of this circulation versus the action of horizontal deformation in the frontogenesis.

Chapter 2 discusses the procedures used to prepare an integration. The initial conditions are similar to other three-dimensional integrations except that an explicit, sloping tropopause is included and the initial perturbation is allowed to possess more realistic horizontal and vertical structure; this is done by actually finding the most unstable perturbation for a given basic state.

Chapter 3 discusses the synoptic scale evolution of the integrations, two of which were carried out with both fine resolution models. Chapters 4 and 5 then concentrate upon the surface and upper level frontogenesis, respectively, and Chapter 6 presents conclusions and suggestions for future research.

2. PROCEDURES USED IN THE RESEARCH

For the purpose of our experiments the atmosphere will be simulated by an east-west re-entrant channel possessing rigid horizontal and vertical surface boundaries and located on a mid-latitude beta-plane. Boundary conditions require only that there be no flow through these surfaces. The atmosphere will be considered to be inviscid, adiabatic and hydrostatic and we will further apply the Boussinesq approximation which assumes that the distribution of pressure and density is always close to the distribution of pressure and density in an adiabatically stratified atmosphere, that the maximum frequencies allowed are on the order of the Brunt-Väisälä frequency, and that the vertical depth of the channel is small compared to the scale height of an adiabatically stratified atmosphere. For some integrations friction will be added.

2.11 The dynamical models used. The "primitive" equations.

The above assumptions yield the hydrostatic or so-called "primitive" equations and are the basis for the "PE" model. Cartesian geometry is employed with the x-axis pointing eastward (figure 2-1). The basic unit of length will be the channel width D , and the basic unit of time will be f_0^{-1} where f_0 is the mid-latitude value of the Coriolis parameter. H is the depth of the channel. The beta plane approximation assumes the latitudinal or y variation of the Coriolis force is constant and equal to $\beta = D/a$ where a is the radius of the earth.

Fig. 2-1. The channel model geometry. D , H and L are the dimensional width, depth and length. Nondimensional length $\ell = L/D$. $w = 0$ at $Z = 0, 1$; $w = 0$ at $Y = 0, 1$; all quantities are cyclic in X .

The nondimensional set of equations thus obtained is:

$$\begin{aligned}
 \frac{du}{dt} - fv &= -\frac{\partial p}{\partial x} \\
 \frac{dv}{dt} + fu &= -\frac{\partial p}{\partial y} \\
 b &= \frac{\partial \varphi}{\partial z} \\
 \frac{db}{dt} &= 0 \\
 \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0
 \end{aligned} \tag{2.1}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$; $f = 1 + \beta(\gamma - 1/2)$.

Boundary conditions:

$$w(x, y, z=0) = w(x, y, z=1) = v(x, y=0, z) = v(x, y=1, z) = 0 ,$$

$$\alpha(x+l) = \alpha(x)$$

where α is any dependent variable and l is the channel length.

The variables b and p are the deviations of the temperature and pressure, respectively, from the temperature and pressure of an adiabatically stratified atmosphere; b is called the "buoyancy" and p the "pressure". The variables u , v , and w have their usual meaning. A formal derivation of a similar set of equations excluding rotation and not utilizing the hydrostatic assumption is presented by Ogura and Phillips (1962).

The variables are dimensionalized as follows, where the capital letters denote dimensional quantities. P and T are the total pressure and temperature.

$$U, V = f_0 D u, f_0 D v$$

$$W = f_0 H w$$

(2.2)

$$P = P_{00} \left[\left(1 - \frac{gH}{C_p T_{00}} z \right)^{C_p/R} + \frac{f_0^2 D^2}{R T_{00}} p \right]$$

$$T = T_{00} \left[1 - \frac{gH}{C_p T_{00}} z + \frac{f_0^2 D^2}{gH} b \right],$$

where P_{00} and T_{00} are the surface pressure and temperature; g , C_p and R have their usual meaning and $f_0 = 10^{-4} \text{ sec}^{-1}$.

Equations (2.1) show b is conserved following a parcel.

Another such conserved quantity for these equations is the (Ertel, 1942) potential vorticity η :

$$\frac{d\eta}{dt} = 0; \quad \eta \equiv \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f \right) \frac{\partial b}{\partial z} + \frac{\partial u}{\partial z} \frac{\partial b}{\partial y} - \frac{\partial v}{\partial z} \frac{\partial b}{\partial x} \quad (2.3)$$

This relation is useful in studying the dynamics of equations (2.1) but cannot be used to integrate the system in time as there exists no unique relation between η and the variables u , v and b .

2.12 The quasigeostrophic equations.

In addition to the assumptions used to obtain the PE set of

equations, if we further assume the Rossby number $U/f_0 D$ is small compared to unity and the Richardson number is large (see, for example, Charney, 1962) we can derive another related set of equations, the so called quasigeostrophic equations. These form the basis of the "QG" model.

Let $\bar{p}^{xy}(z, t_0)$ be the horizontally averaged PE "pressure" p at the start of the PE integration. Then let $\bar{B}^{xy}(z, t_0) \equiv \frac{d}{dz} \bar{p}^{xy}$. The PE pressure p is thus analogous to the QG quantity $\bar{p}^{xy}(z, t_0) + p'(x, y, z, t)$ where $(\bar{\quad})^{xy} = 0$ and the PE buoyancy b is analogous to the QG quantity $\bar{B}^{xy} + \frac{\partial p'}{\partial z}$. $\bar{B}^{xy}$ is constant in time. With these definitions in mind the QG equations can be written (non-dimensionally) as follows:

$$\frac{dg}{dt} (\nabla_H^2 p') + \beta \frac{\partial p'}{\partial x} = \frac{\partial w}{\partial z} \quad (2.4)$$

$$\frac{dg}{dt} \left(\frac{\partial p'}{\partial z} \right) + N^2 w = 0$$

where $\frac{dg}{dt} \equiv \frac{\partial}{\partial t} - \frac{\partial p'}{\partial y} \frac{\partial}{\partial x} + \frac{\partial p'}{\partial x} \frac{\partial}{\partial y}$, $\nabla_H^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $N^2 \equiv \frac{d\bar{B}^{xy}}{dz}$.

Boundary conditions: Let $p' = \widetilde{p'}^x + p'^x$ where $(\widetilde{\quad})^x$ denotes an average over x and $(\quad)^x$ denotes the deviation. We require

$$p'^x = 0, \quad \frac{\partial}{\partial t} \left(\frac{\partial}{\partial y} \widetilde{p'}^x \right) = 0 \quad \text{AT } y = 0, 1$$

$$w = 0 \quad \text{AT } z = 0, 1$$

all quantities cyclic as before.

For this set of equations we define $p \equiv \bar{p}^{xy} + p'$, $b \equiv \bar{B}^{xy} + \frac{\partial p'}{\partial z}$.

The non-divergent components of the wind are derivable from ρ' :

$u_o = -\frac{\partial \rho'}{\partial y}, v_o = \frac{\partial \rho'}{\partial x}$. The variables u_o, v_o, w, b and p are then dimensionalized by equations (2.2).

The quasigeostrophic equations possess a certain symmetry, as Phillips (1956) points out. If ρ' and w are initially such that

$\rho'(x, y, z, t_o) = -\rho'(x - \frac{1}{2}, 1 - y, z, t_o)$, etc., they will remain this way. Such behavior is not possible for the primitive equations; the vorticity equation derived from that set contains a non-linear term

$(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial y} + f) \frac{\partial w}{\partial z}$ which destroys any initial symmetry. This symmetry, or lack of it in the QG solutions will be discussed later.

We can combine equations (2.4) to form a "potential vorticity" equation; the quantity η_g is conserved following horizontal, non-divergent advection:

$$\frac{d_g \eta_g}{dt} = 0; \quad \eta_g = \nabla_H^2 \rho' + \beta(y - 1/2) + \frac{\partial}{\partial z} \left[\left(\frac{dB^{xy}}{dz} \right)^{-1} \frac{\partial \rho'}{\partial z} \right] \quad (2.5)$$

Since η_g can be inverted to obtain ρ' this equation not only describes the dynamics of the quasigeostrophic system but also may be used to integrate the system forward in time. Details of the finite difference schemes used to integrate the PE and QG equations are found in the appendix, sections B and C.

2.2 Procedures used in the numerical integrations.

This section describes the steps taken to prepare the initial

fields used by the models. We begin with a three-dimensional stream-function consisting of a zonally independent basic state upon which is superimposed a three-dimensional disturbance of small amplitude. As the integration proceeds the disturbance grows in amplitude, drawing energy from the basic state in some manner. The basic state is chosen to model a mid-latitude westerly jet and an associated sloping tropopause; this means the zonal wind contains both horizontal and vertical shear.

Having chosen a basic state that is presumably unstable to small disturbances, we can pick a particular channel length and use linear perturbation theory to find the structure of the most rapidly growing mode for that wavelength. This is readily done with the quasigeostrophic equations using a procedure presented by J. Brown (1969). By doing this for a range of channel lengths we can find one such mode that grows more quickly than all others. This would presumably be the disturbance to appear if we used an infinitely long channel, added an infinitesimally small perturbation of random shape to the basic state and integrated for a long time. In this sense choosing the basic state also determines the disturbance structure and the channel length.

This method leaves us free to choose the magnitude of the perturbation relative to the basic state. We have chosen this magnitude so that the maximum value of the perturbation meridional wind

$\left| \frac{\partial \psi}{\partial x} \right|_{\text{MAX}}$ will be 5% of the maximum value of the basic state wind.

This is done for all runs unless otherwise stated. The basic state

and perturbation are added together; we then add a constant times z to the total streamfunction so that the initial three-dimensional buoyancy field will be non-negative. This addition has no effect upon the dynamics of either the PE or QG models.

For the QG model this total streamfunction is the initial pressure field. The buoyancy and vertical motion fields are derivable from the pressure field as described in section C of the appendix. For the PE model small corrections are added for the initial wind and buoyancy fields, based on the next higher-order terms in the Rossby number expansion used in deriving the quasigeostrophic theory (Phillips, 1960). The "initialization" program used for this purpose utilizes the total streamfunction described above as input and is described in the appendix, section D. The resulting initial pressure field for the PE model will thus be slightly different than the QG initial pressure field.

With the QG pressure field and the PE wind and buoyancy fields specified, the models are ready for integration.

2.21 Determination of the basic state.

The basic state varies only in the meridional (y, z) plane. A jet possessing linearly increasing westerly wind with height (z) is centered within the channel; on either side of the jet the winds are zero. The jet varies latitudinally (in y) as a sine squared. Above the tropopause, which is an intersection (higher in the south sloping to lower values in the north) dividing the channel into a

lower, stable region where the buoyancy decreases to the north and an upper, more stable region where the buoyancy increases to the north, the westerly jet decreases with height. The stability for the "troposphere" and "stratosphere" is taken as constant in each region; values are obtained by assuming a tropospheric lapse rate of -6.5°A per km with a standard atmosphere temperature of 265°A at 650 mb and an isothermal stratosphere at 216°A . These values are used for all numerical integrations and produce dimensional values of the squared buoyancy frequency, N^2 equal to $4.6 \times 10^{-4} \text{ sec}^{-2}$ and $1.3 \times 10^{-4} \text{ sec}^{-2}$ for the stratosphere and troposphere, respectively.

The (nondimensional) zonal wind field $U(y, z)$ and buoyancy field $B(y, z)$ are both derivatives of a zonal streamfunction $P(y, z)$ where $U = -\frac{\partial P}{\partial y}$ and $B = \frac{\partial P}{\partial z}$. This function P is the Boussinesq "pressure", really the deviation of the pressure from that in an adiabatically stratified atmosphere as described in section 2.11. P increases with height.

Figure 2-2 summarizes the above; section A of the appendix presents analytic expressions for U, B and $H_T(y)$, the tropopause height. Figures 2-3a and 2-4a present schematics of the basic states used for the two major experiments to be described in Chapter 3. The remainder of figures 2-3 and 2-4 display the analytic U and B fields for these two basic states.

The sine squared horizontal variation allows the zonal wind to be barotropically as well as baroclinically unstable to small disturbances. The type of instability present will depend upon the details

Basic state distribution of B, U and $H_T(y)$.

Buoyancy isolines — · — · — B

Zonal wind isolines ——— U

Tropopause height ———— $H_T(y)$

$N_T^2 = 1.3 \times 10^{-4} \text{ sec}^{-2}$, $N_S^2 = 4.6 \times 10^{-4} \text{ sec}^{-2}$.

Latitudinal (y) variation of U in
troposphere at any level z_0 .

$$U(y, z_0) \sim \sin^2 \left[\pi \left(\frac{y - y_0}{y_1 - y_0} \right) \right] ; y_1 - y_0 = \frac{1}{2} - y_0$$

Fig. 2-2. The basic state.

FIG. 2-3. BASIC STATE I. COMPUTATIONS MADE WITH 20 GRIDPOINTS IN VERTICAL, 62 IN HORIZONTAL. LEVEL K=1 IS AT Z=.025; K=20 IS AT Z=.975.

FIG. 2-4. BASIC STATE II. COMPUTATIONS MADE WITH 20 GRIDPOINTS IN VERTICAL, 42 IN HORIZONTAL. LEVEL K=1 IS AT $Z=.025$; K=20 IS AT $Z=.975$.

of the basic state and is discussed in the next section.

2.22 Determination of the perturbation streamfunction.

The procedure used to find the "most unstable" perturbation for a given basic state has been sketched above. We assume the dynamics of the perturbation are adequately described by linear, quasigeostrophic theory and we vary the channel length until the most rapidly growing mode for the basic state is found. A two-dimensional numerical technique developed by Brown (1969) is used to do this.

Several basic states were studied using this technique; results from four of them are now discussed. Table 2-1 describes the characteristics of the basic states. A zonal jet having a speed of zero at the bottom, 40 m sec^{-1} at the tropopause and 30 m sec^{-1} at the top of the channel is studied for the two jet widths of 3000 and 4000 km. The channel width is 6000 km for these cases, which are referred to in table 2-1 as cases I-N and I, respectively. Next, a zonal jet having a speed of 12 m sec^{-1} at the bottom, 48 m sec^{-1} at the tropopause and 36 m sec^{-1} at the top is studied for two widths of 2400 and 3000 km. For these cases the channel width is 4800 km; they appear as cases II-N and II in table 2-1. In all cases the channel depth is 15 km and the tropospheric and stratospheric squared buoyancy frequencies or "stabilities" are $1.3 \times 10^{-4} \text{ sec}^{-2}$ and $4.6 \times 10^{-4} \text{ sec}^{-2}$, respectively.

Basic State	Channel width D $\times 10^3 \text{ km}$	Jet width D_J $\times 10^3 \text{ km}$	Wind speed at channel center			Tropopause height			Minimum Richardson number		Meridional temperature gradient		Trop. vert. wind shear	Strat. vert. wind shear	Horiz. wind shear
			U_0	U_M	U_T	$H_T(y_0)$	$H_T(y_2)$	$H_T(y_1)$	$R_{i \text{ MIN}}$ trop.	$R_{is \text{ MIN}}$ strat.	$^{\circ}\text{C}$ trop.	$^{\circ}\text{C}$ strat.	$\times 10^{-3} \text{ sec}^{-1}$	$\times 10^{-3} \text{ sec}^{-1}$	$\times 10^{-6} \text{ sec}^{-1}$
<u>I</u>	6	4	0	40	30	13.1	10.5	8.9	9.0	93.2	23	14	3.81	-2.22	20
I-N	6	3	0	40	30	12.2	10.5	9.3	9.0	93.2	17	12	3.81	-2.22	26.7
<u>II</u>	4.8	3	12	48	36	10.6	9.0	7.8	8.1	115.0	18	9	4.00	-2.00	32
II-N	4.8	2.4	12	48	36	10.2	9.0	8.0	8.1	115.0	15	8	4.00	-2.00	40

Table 2-1. Basic states investigated by the instability study. Underlined cases are used for the two major experiments. For all states the channel depth $H = 15 \text{ km}$, $f_0 = 10^{-4} \text{ sec}^{-1}$, $N_S^2 = 4.6 \times 10^{-4} \text{ sec}^{-2}$, $N_T^2 = 1.3 \times 10^{-4} \text{ sec}^{-2}$ and $\mathcal{R} = .75$ (see Appendix A). $U_0 = U(Y=1/2, Z=0)$, $U_M = U[1/2, H_T(1/2)]$, $U_T = U(1/2, 1)$, $R_{i \text{ MIN}} = N_T^2 H_T(1/2)^2 / (U_M - U_0)^2$, $R_{is \text{ MIN}} = N_S^2 [1 - H_T(1/2)]^2 / U_M^2 (1 - \mathcal{R})^2$.

Cases I-N and I were chosen to model a fairly broad, moderately strong jet with weak surface winds; cases II-N and II model a more narrow, stronger jet with fairly strong surface westerlies. All cases were chosen so that the minimum values of the Richardson number would be around 10 for the troposphere and around 100 for the stratosphere.

Plots of perturbation phase speed and growth rate versus channel length for these cases are displayed in figures 2-5 and 2-6. The following is apparent from these plots:

- 1) The second class of states has a longer channel length for the maximum growth rate (4000 km for II-N, 4800 km for II) than the first class (3500 km for I-N, 3600 km for I), and a higher perturbation phase speed than the first class.

- 2) As the jet is widened within each class the channel length for the maximum growth rate increases, the value of the maximum growth rate decreases and the perturbation phase speed increases.

These findings are consistent with a similar, more complete study by Brown (1969). He has found cases with two wavelengths of maximum growth rate; our study found no such double maximum but more complete exploration of the channel length was not carried out in regions where the numerical technique failed to converge within what was considered to be a reasonable amount of computer time.

Fig. 2-5. Perturbation phase speed C_r and growth rate KC_i versus channel length L ; basic states I and I-N. Error limits of .3% exist due to convergence criterion.

$K = 2\pi/\lambda$.

Fig. 2-6. Perturbation phase speed C_r and growth rate KC_i versus channel length L ; basic states II and II-N. Error limits of .3% exist due to convergence criterion.
 $K = 2\pi/\ell$.

At a given time the linear quasigeostrophic perturbation determined by Brown's technique can be written as $p(x, y, z) = \text{constant} \times A(y, z) \cos\{K[x - \delta(y, z)]\}$ where ℓ , the channel length is incorporated into the zonal wave number $K = 2\pi/\ell$ and the constant is arbitrary. An examination of the amplitude $A(y, z)$ and the zonal phase $\delta(y, z)$ reveals the structure of the perturbation. As x varies from 0 to ℓ the "trough" occurs where p is a minimum, i.e., where $\cos\{K[x - \delta]\} = -1$ and the "ridge" occurs where p is a maximum. The horizontal and vertical slope of the perturbation trough and ridge can thus be seen from a plot of $\delta(y, z)$.

Figures 2-7 and 2-9 display the perturbation amplitude $A(y, z)$ and zonal phase $\delta(y, z)$ for the wavelength of maximum growth rate for cases I and II. These were the cases chosen for integration with the three-dimensional models; the basic states for these cases appeared in figures 2-3 and 2-4. Figures 2-8 and 2-10 display the meridional structure of the energy transformation terms

$$-(\overline{u^x v^x}) \frac{\partial \tilde{\phi}}{\partial y} \quad \text{and} \quad (\overline{v^x b^x}) \frac{\partial \tilde{B}}{\partial y} \quad \text{which indicate conversion}$$

of zonal kinetic to eddy kinetic energy and conversion of eddy potential to zonal potential energy, respectively. These figures were computed using 20 grid points in z and 62 in y for case I and using 20 grid points in z and 42 in y for case II; all numbers in figures 2-7 through 2-10 are non-dimensional.

We see from the figures that the most unstable perturbation for case I turns out to be a shallow, baroclinically growing, baro-

a. Nondimensional perturbation amplitude $A(y,z)$.
Largest labeled value is .0020; max. at bottom is .0036.

b. Perturbation zonal phase $\delta(y,z)$.
Largest labeled value is 3.40 radians.

Fig. 2-7. Amplitude and zonal phase for most unstable perturbation for basic state I. Numbers indicate relative magnitudes; actual values are meaningless.

a. Nondimensional perturbation amplitude $A(y,z)$.
Largest labeled value is 76.1.

a. Nondimensional conversion of zonal kinetic to eddy kinetic energy. Largest labeled value is 1.76×10^4 .

b. Nondimensional conversion of eddy potential to zonal potential energy. Largest labeled value is -9.00×10^2 .

Fig. 2-10. Energy conversions for basic state II perturbation. Disturbance gains $\sim 4/9$ units of energy via baroclinic growth for every one unit gained via barotropic growth.

tropically damped wave; the channel length is 3600 km. The maximum perturbation amplitude is located at the center of the channel at level 1 and decreases rapidly with height in the lower troposphere; there is a further decrease above the tropopause (figure 2-7a). The amplitude is centered beneath the jet maximum and decreases latitudinally away from the channel center. The zonal phase diagram (figure 2-7b) shows the trough and ridge surfaces slope westward with height and with increasing distance from the center of the channel. The maximum conversion of eddy kinetic to zonal kinetic energy (figure 2-8a) occurs on either side of the jet maximum at tropopause level.

The situation is different for case II; this perturbation turns out to be growing by both baroclinic and barotropic processes and the channel length is 4800 km. The maximum perturbation amplitude is centered within the jet at tropopause level, decreasing latitudinally away from the channel center as in case I. There is a slight double maximum in the amplitude at the lowest level (figure 2-9a). The zonal phase diagram (figure 2-9b) shows the ridge and trough surfaces slope westward with height up to the tropopause and slightly eastward with height in the stratosphere. There is an eastward slope as we move latitudinally away from the perturbation center. The conversion of zonal potential to eddy potential energy (figure 2-10b) is strongest at the lowest level, decreasing with height to the tropopause; a second maximum occurs just above the tropopause. Figure 2-10a shows zonal kinetic energy is being

converted to eddy kinetic energy in regions on either side of the jet maximum.

For both cases I and II we note a near symmetry about the center line $(\gamma = 1/2, z)$ of the channel, particularly below the tropopause. Since the basic flow U for both cases is also symmetric below the tropopause we may expect the quasigeostrophic solution for the three dimensional equations to display the symmetric development discussed earlier. As the initial symmetry is not perfect the developing solutions are not constrained to remain completely symmetrical.

The most unstable modes for basic states I-N and II-N are similar to those of states I and II. The wider zonal jets of states I and II allow for more north-south resolution for a given horizontal grid spacing and these states therefore were chosen for the model integrations.

3. RESULTS OF THE NUMERICAL INTEGRATIONS

Table 3-1 presents a description of all the integrations made with the primitive equations (PE) and quasigeostrophic (QG) models. All but one run use the basic states labeled I and II in Chapter 2 (see table 2-1). The high resolution runs of the PE and QG models will be referred to as the two major experiments and are discussed in this chapter; they are underlined in table 3-1.

We will present horizontal pressure and buoyancy for selected times and vertical levels from these integrations; the evolution of the three-dimensional flow can thus be discerned. Corresponding PE and QG plots will be shown so direct comparisons can be made. Finally the realism of the integrations compared to developing atmospheric disturbances will be discussed.

A channel depth of 15 km is used for all integrations; this depth includes all of the troposphere and some region of the lower stratosphere (for the standard atmosphere 15 km reaches to an upper pressure of about 120 mb). This placement of the rigid upper lid, where no vertical motion occurs, should leave sufficient room for realistic tropopause movement. We still make the Boussinesq approximation which formally requires the depth of the channel to be small compared to the scale height of an adiabatically stratified atmosphere, i.e. around 30 km. In our case this ratio is about 1/2 which is not small, hence we are "stretching" the Boussinesq approximation somewhat.

Run	Model	Basic state	Horizontal resolution				Horizontal		Time- step min	Frcst period days	Dscrptn
			X (E - W)		Y (N - S)		domain	km			
			grdpts	rsltn km	grdpts	rsltn km	(E - W)	(N - S)			
OPE	PE	I	38	100	62	100	3600	6000	5	4	NF *
OQG	QG	I	38	100	62	100	3600	6000	20	4	NF *
<u>1PE</u>	PE	I	38	100	62	100	3600	6000	5	4½	NF
<u>1QG</u>	QG	I	38	100	62	100	3600	6000	20	4½	NF
C-1	PE	I	20	200	32	200	3600	6000	10	6	NF
C-1F	PE	I	20	200	32	200	3600	6000	10	6	F
<u>2PE</u>	PE	II	42	120	42	120	4800	4800	5	7½	NF
<u>2QG</u>	QG	II	42	120	42	120	4800	4800	20	7½	NF
C-2	PE	II	20	266 2/3	32	160	4800	4800	10	6	NF
C-2F	PE	II	20	266 2/3	32	160	4800	4800	10	6	F
C-3	PE	II-N	20	222 2/9	32	160	4000	4800	10/8	8	NF

Table 3-1. Integrations performed on PE and QG models. F = friction, NF = no friction. All runs use 20 vertical levels for 3/4 km vertical resolution. All runs except * have initial N-S perturbation wind set to 5% of max. value of basic state wind; * has initial value of 20%. In addition OPE uses a slightly different initialization procedure than that used by other PE runs (see Appendix D).

The price we pay for the use of the approximation over such a depth becomes apparent when the variable ρ is dimensionalized to obtain the total pressure using equation (2.2). Near and above the tropopause the pressure for an adiabatically stratified atmosphere and the deviation of pressure from that value turn out to be of the same magnitude and this distorts the total dimensional value of the pressure. This vertical distortion is more than compensated by the simplification in the dynamics effected by the Boussinesq approximation. Studies utilizing this assumption have yielded apparently realistic frontogenesis at both the surface and upper tropospheric levels (see Chapter 1). We will be interested mainly in the comparison of the horizontal variation of quantities in the integrations and in the atmosphere; no such distortion exists in this direction.

3.1 Vertical finite-differencing in the models.

Figure 3-1 shows the relative vertical positions of p , b and w for the QG model versus the positions of u , v , w , b and p for the PE model. The pressure p and vertical motion w are located similarly for the two models but the buoyancy b varies. Nevertheless we will refer to p and b at the lowest level ($k = 1$) as the "surface" pressure and buoyancy, for both models.

3.2 Experiment I - the shallow baroclinic wave: PE versus QG high-resolution integration.

The integrations discussed below are from model calculations

Fig. 3-1. Vertical finite-differencing for PE and QG models.

$$\bar{V}_K = (U_K, V_K). \quad \Delta z = .05.$$

utilizing 38 gridpoints in the east-west direction, 62 in the north-south direction and 20 in the vertical, or 47,120 gridpoints in all. The channel dimensions are 3600 km long, 6000 km wide and 15 km deep, so we have a horizontal resolution Δ (distance between gridpoints) of 100 km and a vertical resolution of $3/4$ km. Basic state I was integrated with this resolution on the PE and QG models out to 108 hours or $4\frac{1}{2}$ days of forecast time; these runs will be called 1PE and 1QG, respectively.

Figures 3-2 through 3-11 display the integration results. Horizontal plots of the PE surface pressure (vertical level 1) for days 0, 3 and $4\frac{1}{2}$ are displayed in figure 3-2. The corresponding PE surface buoyancy plots are shown in figure 3-3. The QG surface pressure and buoyancy plots for the same times are shown in figures 3-7 and 3-8. (Note that the PE and QG buoyancies are at slightly different heights as shown in figure 3-1.) Horizontal plots of the PE pressure and buoyancy at level 9 (corresponding crudely to the 450 mb level) for days 0 and $4\frac{1}{2}$ appear in figure 3-4; the QG plots at the same level and for the same times appear in figure 3-9.

Figure 3-5 shows north-south vertical cross sections for day 0 from run 1PE. Isotachs of the east-west wind component u are displayed, along with isentropes of the buoyancy and circulation vectors of the v , w wind components in the (y,z) cross section plane. The vectors point downwind away from the asterisks marking selected gridpoints. The cross sections are taken at the westerly jet ridge and trough so the jet will be perpendicular to the cross sections. These

A. 0 HOURS. $L = -.0219$. 3 MB BETWEEN H AND L.

B. 72 HOURS. $L = -.0235$. 12 MB BETWEEN H AND L.

C. 108 HOURS. $L = -.0266$. 26 MB BETWEEN H AND L.

FIG. 3-2. 1PE SURFACE PRESSURE, LEVEL K=1, Z=.025.

A. 0 HOURS. H=.0319.

B. 72 HOURS. H=.0319.

C. 108 HOURS. H=.0319.

FIG. 3-3. 1PE SURFACE BUOYANCY, LEVEL K=1, Z=.025.
CONTOUR INTERVAL CORRESPONDS TO .72°C.

A. PRESSURE. 0 HOURS. MIN CONTOUR = -.0152. CONTOUR INTERVAL CORRESPONDS TO $3 \frac{1}{3}$ MB.

B. PRESSURE. 108 HOURS. MIN CONTOUR = -.0152. CONTOUR INTERVAL AS IN A.

C. BUOYANCY. 0 HOURS. MAX CONTOUR = .0640. CONTOUR INTERVAL CORRESPONDS TO $.72^{\circ}\text{C}$.

D. BUOYANCY. 108 HOURS. MAX CONTOUR = .0640. CONTOUR INTERVAL AS IN C.

FIG. 3-4. 1PE PRESSURE AND BUOYANCY, LEVEL K=9, Z=.425.
 $13 \leq J \leq 52$ IS DISPLAYED.

A. JET RIDGE, I=14. MAX U=.0673, MAX LABELED B=.144.

B. JET TROUGH, I=32. MAX U=.0669, MAX LABELED B=.144.

FIG. 3-5. 1PE NORTH-SOUTH CROSS SECTIONS. 0 HOURS. SEE TEXT FOR EXPLANATION. MULTIPLY U BY 600 M SEC⁻¹ FOR DIMENSIONAL VALUE.

A. JET RIDGE, I=19. MAX U=.0647, MAX LABELED B=.144.

B. JET TROUGH, I=37. MAX U=.0682, MAX LABELED B=.144.

FIG. 3-6. 1PE NORTH-SOUTH CROSS SECTIONS. 108 HGURS.

A. 0 HOURS. $L = -.0219$. 3 MB BETWEEN H AND L.

B. 72 HOURS. $L = -.0229$. 10 MB BETWEEN H AND L.

C. 108 HOURS. $L = -.0242$. 20 MB BETWEEN H AND L.

FIG. 3-7. 1QG SURFACE PRESSURE, LEVEL $K=1$, $Z=.025$.

A. 0 HOURS. H=.0318.

B. 72 HOURS. H=.0318.

C. 108 HOURS. H=.0318.

FIG. 3-8. 1QG SURFACE BUOYANCY, LEVEL K=1, Z=.050.

CONTOUR INTERVAL CORRESPONDS TO $.72^{\circ}\text{C}$.

A. PRESSURE. 0 HOURS. MIN CONTOUR = -.0144.

B. PRESSURE. 108 HOURS. MIN CONTOUR = -.0152.

C. BUOYANCY. 0 HOURS. MAX CONTOUR = .0640.

D. BUOYANCY. 108 HOURS. MAX CONTOUR = .0640.

FIG. 3-9. 1QG PRESSURE AND BUOYANCY, LEVEL K=9, Z=.450.

CONTOUR INTERVALS AS IN FIG. 3-4. $13 \leq J \leq 52$ IS DISPLAYED.

A. JET RIDGE, I=13. MAX U=.0670, MAX LABELED B=.126.

B. JET TROUGH, I=32. MAX U=.0672, MAX LABELED B=.126.

FIG. 3-10. 1QG NORTH-SOUTH CROSS SECTIONS. 0 HOURS. SEE TEXT FOR EXPLANATION. MULTIPLY U BY 600 M SEC^{-1} FOR DIMENSIONAL VALUE.

A. JET RIDGE, I=20. MAX U=.0672, MAX LABELED B=.126.

B. JET TROUGH, I=37. MAX U=.0668, MAX LABELED B=.126.

FIG. 3-11. 1QG NORTH-SOUTH CROSS SECTIONS. 108 HOURS.

sections intersect the horizontal plots of PE pressure and buoyancy in figures 3-2a, 3-3a, 3-4a and 3-4c along lines RR and TT.

Figure 3-6 shows similar PE cross sections for day $4\frac{1}{2}$; figures 3-10 and 3-11 display corresponding cross sections from run 1QG for days 0 and $4\frac{1}{2}$. The QG cross sections display isotachs of the east-west geostrophic wind $-\frac{\partial p'}{\partial y}$ rather than the total east-west wind component as in the PE cross sections. Also, the circulation vectors in the QG cross sections are based upon the north-south geostrophic wind component $\frac{\partial p'}{\partial x}$ and the vertical motion w . The intersection of these cross sections with the appropriate horizontal plots are marked as indicated above.

3.21 Surface evolution of the integrations.

The surface evolution of the PE and QG integrations is seen from figures 3-2, 3-3, 3-7 and 3-8. Solutions of the two models evolve in a similar manner. A wave embedded in weak surface westerly flow moves eastward and gradually amplifies; during its evolution the pattern moves off the eastern edge of the plots and reappears on the western edge. In both models high and low pressure centers strengthen with time and the buoyancy field, virtually independent of longitude (x) at the start, becomes progressively distorted. As seen in figure 3-12a, the pressure centers developed in the two solutions move eastward in a similar manner after an initial difference in location disappears by 36 hours. The initial pressure difference of about 3 mb increases by $4\frac{1}{2}$ days to 26 mb in the PE solution and to 20 mb in the

FIG. 3-12. SURFACE PRESSURE EVOLUTION. 1PE ——— VERSUS 1QG - - - -

QG solution.

The QG surface pressure and buoyancy patterns retain the "symmetric" character discussed previously. The PE solutions do not; as time progresses the PE isobars become increasingly more packed in the low and spread out in the high. (Note that the contour interval in figures 3-2 and 3-7 changes with time.) The developing asymmetry can be seen as a result of the non-linear interaction of the vertical component of the vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and the divergence $-\frac{\partial \omega}{\partial z}$ found in the PE vorticity equation but not found in the QG vorticity equation. Horizontal convergence increases the vorticity through the action of vortex tube stretching; divergence decreases it. The asymmetrical development of the PE pressure centers compared to the QG centers is also seen in figure 3-12b where the nondimensional pressure of the surface low and high is plotted versus time for both solutions. Plotting the total surface pressure difference (high minus low) versus time as in figure 3-12c, we see the rate of increase of the difference is nearly the same for the two models out to about 24 hours, then the PE rate increases with time over the QG rate.

The most striking difference between the PE and QG surface evolution is that at $4\frac{1}{2}$ days the PE buoyancy has developed sharp "frontal zones" while the QG buoyancy has not. (Compare figures 3-3c and 3-8c.) The QG buoyancy never loses its symmetric character while the PE buoyancy becomes more and more distorted with time, finally resulting in the "warm" and "cold" frontal zones. Both PE and QG fields display their strongest gradients north of the advancing

southern warm air and south of the advancing northern cold air but the PE solution has formed a narrow warm tongue and broad cold region while the QG solution has kept the warm and cold tongues equally broad. A discussion of the structure of the frontal zones appears in Chapter 4.

The initial QG nondivergent streamfunction and the initial PE pressure consisting of the streamfunction plus the divergent circulation (added by the PE initialization procedure) are displayed for level 1 in figures 3-7a and 3-2a respectively; the differences in location of the initial PE and QG pressure centers in figure 3-12a arise because the streamfunction is very weak at level 1 - no closed low pressure area exists at this time. The pressure centers of the two solutions exhibit more similar movement after 36 hours when the QG centers have become better defined.

We now compare the movement of the pressure centers to the perturbation phase speed predicted for this experiment by the two-dimensional, linear instability analysis in Chapter 2. From that analysis (figure 2-5a) the centers are predicted to move eastward at a speed of $10.6 \pm 0.1 \text{ m sec}^{-1}$. (The error limits arise from the convergence criterion used in determination of the phase speed.) Using figure 3-12a to obtain an average speed for the PE and QG centers we find that for the period from 12 to 24 hours the PE centers move eastward at an average speed of $9.3 \pm 2.3 \text{ m sec}^{-1}$ and the QG centers move at an average speed of $11.6 \pm 2.3 \text{ m sec}^{-1}$. (The error limits arise here because locations of the centers are

known only to the closest gridpoint.) Agreement exists between the actual and predicted movement within the error of measurement.

3.22 Upper level development.

An examination of the vertical cross sections taken through the westerly jet ridge and trough for the two solutions (figures 3-5 and 3-6 for the PE solution at times 0 and $4\frac{1}{2}$ days, respectively and figures 3-10 and 3-11 for the corresponding QG sections) plus the horizontal plots of pressure and buoyancy at level 9 for the two solutions (figures 3-4 and 3-9) reveals the major details of the synoptic development. Initially (figures 3-5 and 3-10) both solutions exhibit rising motion in the troposphere beneath the jet in the ridge and sinking motion in the troposphere beneath the jet in the trough. There is little vertical motion indicated above the jet. The speed of the zonal wind component is nearly the same for both solutions at the start, about $.067$ or 40 m sec^{-1} in the ridge and in the trough.

Consider first the evolution of the QG solution. An examination of the pressure and buoyancy at level 9 for day $4\frac{1}{2}$ (figures 3-9b and 3-9d) shows that little change has occurred from the start except for an increase in perturbation amplitude. The ridge and trough cross sections for day $4\frac{1}{2}$ (figure 3-11) show the westerly jet is basically unchanged except for an increase in speed below level 5. The horizontal gradients of buoyancy developing at the surface turn out to be quite shallow vertically, as seen in the cross sections. The rising motion in the ridge and sinking motion in the trough remains basically

unchanged from the start.

Consider now the evolution of the PE integration. The pressure and buoyancy at level 9 for day $4\frac{1}{2}$ (figures 3-4b and 3-4d) show that there has been some tightening of the pressure and buoyancy gradients in the trough as the perturbation has amplified. Comparing the PE ridge and trough cross sections for day $4\frac{1}{2}$ (figure 3-6) we see the sinking motion in the trough is stronger than the rising motion in the ridge. The jet has narrowed considerably at all levels in the trough as opposed to the ridge; the buoyancy gradient has done likewise. As in the QG solution the surface buoyancy gradients virtually disappear above level 3.

Experiment I was halted at $4\frac{1}{2}$ days of forecast time just as the PE surface fronts formed. Beyond this time the lack of sub-grid scale diffusion causes the onset of numerical noise near the fronts and within the low center where the warm tongue is trapped between the frontal zones. Halting the experiment at this time allows us to examine the evolution of the surface fronts with a minimum of numerical noise present. A detailed discussion of the surface frontal zones will be found in Chapter 4.

The above is not to say that the numerical procedure breaks down as soon as gradients appear having a thickness on the order of the distance between gridpoints. It will be seen later in this chapter that an integration can be carried out for days after the initial formation of surface frontal zones without significant corruption of the fields away from the surface.

3.23 Kinematics of experiment I.

In this section we will compare the growth rates for the model solutions with that predicted for our "most unstable perturbation" and we will look at the energetics of the solutions. In addition we will compare mean meridional circulations for the two models.

For fixed gridpoint values in the y and z direction let Δp be the difference between the maximum and minimum pressures in the x direction. This quantity plotted against time yields the perturbation growth rate since the basic state is independent of x. Figure 3-13a shows this Δp versus time for $y = .50833$, or the row of gridpoints just north of the east-west center line of the channel, and for vertical levels 1,2,3 and 10 for both QG and PE solutions. Δp is plotted on a log scale; a straight line indicates exponential growth. All four QG curves are close to straight lines for the first 48 hours; the doubling times for these curves and for others not plotted are all about 35 hours during this period. After this time the QG perturbation growth rate decreases with time; this decrease is more noticeable at the lower levels than at level 10. The PE curves show a slightly greater growth rate for the period from one to two days, the doubling time being about 34 hours for this period. A similar plot of the eddy kinetic energy versus time (not shown) for the PE solution implies a perturbation amplitude doubling time of 33 hours for the period from one to two days. Near the end of the forecast period the growth rates for the PE solution at levels 1,2 and 3 decrease significantly while at level 10 there is virtually no

decrease. The formation of frontal gradients at the lower levels in the PE solution and to a lesser extent in the QG solution perhaps is related to the larger decrease in the growth rate with time at these levels.

Our results thus show that the nongeostrophic effects in the PE model work to increase slightly the perturbation growth rate. In an analytic study utilizing the same system of equations and the same boundary conditions but including only vertical shear in the basic state, Derome and Dolph (1970) found that the change in the perturbation growth rate c_i due to nongeostrophic effects is of order Rossby number squared and works to reduce c_i . We have found an increase in c_i of about 4% due to nongeostrophic effects; calculating a Rossby number R_0 , we have a half wavelength of 1800 km, a wind speed of 40 m sec^{-1} and a mean value of the Coriolis parameter of 10^{-4} sec^{-1} , resulting in a value of $R_0 = .22$. The results of Derome and Dolph would thus suggest a reduction in growth rate of 4% should be observed. As no horizontal shear was included in their basic state the comparison appears to be inconclusive.

The perturbation growth rate predicted in Chapter 2 corresponds to a doubling time of 28.3 ± 0.1 hours (the error limits being due to the convergence criterion used in the analysis) while runs 1PE and 1QG yield doubling times closer to 35 hours. The instability study in Chapter 2 utilized 20 gridpoints in the horizontal and 10 in the vertical; a repeat calculation with 32 in the horizontal and 20 in the vertical yielded a longer doubling time of 32.0 ± 0.1 hours

for the perturbation, closer to the observed PE and QG doubling times. The growth rate curve corresponding to a doubling time of 32 hours is plotted as the "predicted growth rate" in figure 3-13a.

We will now consider the evolution of the integrations from an energetics viewpoint. The initial plots of the pressure (figures 3-2a and 3-4a for the PE integration and figures 3-7a and 3-9a for the QG integration) show that the trough and ridge lines slope westward with height and tilt westward as we proceed latitudinally away from the channel center and that the perturbation amplitude decreases from level 1 to nearly zero at level 9. This behavior was indicated in the perturbation analysis presented in Chapter 2 where figures 2-7a and 2-7b display the perturbation amplitude and zonal phase, respectively. That analysis revealed that the perturbation grows in a baroclinically amplifying, barotropically damped manner (figures 2-8a and 2-8b).

The disturbance continues to amplify in this manner after $4\frac{1}{2}$ days, for both models. The same slope of the ridge and trough lines occurs at day $4\frac{1}{2}$ as at the start (figures 3-2c and 3-4b for the PE integration; figures 3-7c and 3-9b for the QG integration). We can get a direct check upon the energetics by computing the amount of energy in various forms versus time; this has been done for the PE integration and will be presented later.

Returning to the north-south vertical cross sections let us look at the circulation in the cross section plane. Initially (fig-

ures 3-5 and 3-10 for the PE and QG ridge and trough cross sections, respectively) air moving northward in the lower troposphere in the ridge cross sections for the two models is rising at an angle less steep than the angle of the buoyancy surfaces with respect to the horizontal. (Remember that the asterisks mark the tail of the vectors.) Air moving southward and downward in the lowest several levels in the trough cross sections is descending at an angle less steep than the inclination of the buoyancy surfaces. This behavior is also apparent in the cross sections at the end of the run (figures 3-6 and 3-11). This motion allows relatively warm air to rise and relatively cool air to sink, thus releasing potential energy, yet the system is gravitationally stable. This type of circulation is at the heart of the baroclinic instability process; a good description can be found in Kuo (1956), for example.

The QG equations used here require the horizontally averaged buoyancy to remain constant in time. Since the buoyancy is the deviation in temperature from that found in an adiabatically stratified atmosphere, this means the QG model cannot change the horizontally averaged temperature at any level as the integration proceeds. There is no such limitation on the PE model. A plot of the x-y averaged buoyancy versus time for levels 1, 3 and 5 is displayed in figure 3-13b for the PE solution. The average buoyancy generally decreases with time at levels 1 and 3 while it increases with time at level 5 so the stability of the lower troposphere is increasing with time. Over the period of integration the average dimensional

temperature decreases by one-half degree absolute at the lowest level; a comparison of figures 3-3a and 3-3c shows that over the period of integration the surface area occupied by cold air has increased while that occupied by warm air has decreased. The developing PE disturbance is transporting relatively warm air upward and northward and relatively cold air southward and downward. In a pioneering integration of a multi-level primitive equations model, Hinkelmann (1959) found this stabilization of the lower troposphere and this increase in area occupied by the cold air at the bottom.

Figure 3-14a shows the x averaged vertical motion for both PE and QG solutions at the start of the integration. Note the three cell circulation characteristic of a baroclinically growing disturbance - the strong indirect cell flanked by weaker direct circulations. (A circulation with relatively warm air rising and relatively cold air sinking is referred to as "direct", etc.) The maximum circulation strength is in the lower troposphere near levels 4 and 5; above the tropopause the circulation is very weak.

Figures 3-14b and 3-15b show the x averaged vertical motion for the QG and PE solutions, respectively, at the end of the integration period at $4\frac{1}{2}$ days. Figure 3-14b was computed by hand and less points were used than for figure 3-15b which was machine computed and plotted. The QG meridional circulation has changed very little in character but has increased in strength by a factor of fifty or so. The PE circulation after $4\frac{1}{2}$ days also displays the strong indirect cell flanked by weaker, direct cells. The entire pattern is stronger and is displayed

A. 1PE, 1QG. 0 HOURS. MAX VALUE = 1.59×10^{-5} OR .0024 CM SEC⁻¹.

B. 1QG. 108 HOURS. MAX VALUE = 9.0×10^{-4} OR .14 CM SEC⁻¹.

FIG. 3-14. X AVERAGED VERTICAL MOTION.

A. 1PE. 102 HOURS. MAX VALUE $= 2.18 \times 10^{-3}$ OR .33 CM SEC⁻¹.

B. 1PE. 108 HOURS. MAX VALUE $= 2.21 \times 10^{-3}$ OR .33 CM SEC⁻¹.

FIG. 3-15. X AVERAGED VERTICAL MOTION.

northward compared to the QG circulation.

The effect of apparent inertio-gravity wave activity appears in the southern part of the channel in figure 3-15b. Figure 3-15a shows the x-averaged vertical motion for the PE integration 6 hours previous to figure 3-15b; similar wave-like activity is present at this time. This activity began to appear about 36 hours before the end of the run. The persistence of such a pattern in time should preclude the possibility of the activity being "numerical noise"; an attempt to determine the period of oscillation of the activity by "eyeball" was inconclusive.

We have seen from the slope of the trough and ridge lines that both PE and QG disturbances amplify by the baroclinic conversion of potential to kinetic energy. Both solutions display mean meridional circulations consistent with this mode of energy conversion. Figure 3-16 presents a direct check of the PE energetics; the zonal and eddy kinetic energy plus the total potential energy are plotted there versus time. (Similar information is not available for the QG solution.) An arbitrary constant has been added to the total potential energy so that the initial value is zero in figure 3-16. The various forms of energy for the PE model are defined in the appendix, section E.

The change in total energy (total kinetic plus total potential) is small over the period of integration, being about 0.44 per cent of the initial total kinetic energy.

FIG. 3-16. 1PE ENERGY VARIATION VERSUS TIME.

Figure 3-16 confirms the conversion of potential to kinetic energy to be the energy source for the PE disturbance; except for the first twelve hours there is a decrease in the potential energy and a general increase in both the eddy kinetic and zonal kinetic energy over the period of integration.

3.24 Summary of experiment I.

Experiment I consists of a shallow, baroclinically growing, barotropically damped perturbation developing beneath a rather broad, zonal jet. After $4\frac{1}{2}$ days the PE solution has developed a surface cyclone and anticyclone with associated shallow warm and cold frontal zones; the QG solution has undergone similar development but without the sharp frontal formation. The PE solution displays other realistic synoptic developments such as an increase in stability in the lower troposphere with time and a tightening of the pressure gradient in the developing cyclone accompanied by a weakening of the pressure gradient within the anticyclone. Both models move the developing cyclone and anticyclone in a similar manner.

3.25 Realism of the integrations.

The basic states used for our two major experiments are believed to be possible states of the atmosphere although the mid-latitude westerlies never exist in a state independent of longitude. Table 2-1 summarizes various parameters of the basic states; the values of stability, Richardson number, maximum wind speed and

meridional temperature gradient are not unreasonable atmosphere values for a mid-latitude jet zonally averaged over a region the size of North America.

The models neglect diabatic effects of friction, radiation and latent heat release. The basic cyclone scale energy sources, that is, the potential and kinetic energy of the zonal flow, are present. We will consider the integrations to be "realistic" if similar examples of atmospheric development can be found. We can then conclude the diabatic effects were of secondary importance in the development of the atmospheric disturbances.

A cyclone that developed in Colorado on June 28, 1969 and moved to south central Canada by June 30 appears somewhat similar to the numerical disturbance produced by run 1PE. Figures 3-17a and 3-17b display the surface and 500 mb maps for 12Z on June 30, 1969 (compare the surface map of figure 3-17a to figures 3-2c and 3-3c, the PE pressure and buoyancy patterns at level 1 for day 4 $\frac{1}{2}$). Both atmospheric and numerical disturbances possess a zonal wavelength of approximately 3600 km and the meridional temperature variation at mid-tropospheric levels is about 20°C for both disturbances.

The width of the jet for the atmospheric case appears to be less than 3000 km compared to 4000 km for the experiment I basic state, although the atmospheric disturbance appears to have evolved in a barotropically damped manner, similar to the experiment I disturbance.

3.3 Experiment II - the deep barotropic-baroclinic wave:

PE versus QG high resolution integrations.

The integrations for experiment II were performed with 35,280 gridpoints; 42 in both the east-west and north-south directions and 20 gridpoints in the vertical, corresponding to a horizontal resolution (Δ) of 120 km and a vertical resolution of 3/4 km. (The channel is 4800 km long and wide and is 15 km deep.) Basic state II was integrated with this resolution on the PE and QG models out to 180 hours, or $7\frac{1}{2}$ days of forecast time. These runs will be named 2PE and 2QG, respectively.

The results are displayed in figures 3-18 through 3-31. Horizontal plots of the non-dimensional PE pressure for vertical level 1 and for days 0, 3, $4\frac{1}{2}$ and 6 appear in figure 3-18. The corresponding nondimensional PE buoyancy plots appear in figure 3-19. Figure 3-20 displays the PE pressure at level 9, corresponding crudely to about 450 mb, for these same times. The corresponding PE buoyancy plots are displayed in figure 3-21. Plots of the non-dimensional QG pressure and buoyancy for the same times and levels appear in figures 3-25 to 3-28. The PE and QG buoyancies for a given level k occur at slightly different heights, as shown in figure 3-1.

Figures 3-22 to 3-24 display north-south vertical cross sections for days 0, $4\frac{1}{2}$ and 6 from the PE integration. As in experiment I the sections are taken at the westerly jet ridge and trough where the jet is perpendicular to the sections. Figures 3-29 to

FIG. 3-18. 2PE SURFACE PRESSURE, LEVEL K=1, Z=.025.

A. 0 HOURS. $H=.0398$.

B. 72 HOURS. $H=.0397$.

C. 108 HOURS. $H=.0407$.

D. 144 HOURS. $H=.0488$. CONTOUR INTERVAL HAS INCREASED TO 1.38°C ; LINES INTERSECTING FRONTS ARE DISCUSSED IN CH. 4.

FIG. 3-19. 2PE SURFACE BUOYANCY, LEVEL $K=1$, $Z=.025$.
CONTOUR INTERVAL CORRESPONDS TO $.92^{\circ}\text{C}$.

A. 0 HOURS. $L = -.0273$.

B. 72 HOURS. $L = -.0285$.

C. 108 HOURS. $L = -.0298$.

D. 144 HOURS. $L = -.0316$.

FIG. 3-20. 2PE PRESSURE, LEVEL K=9, $Z = .425$.

CONTOUR INTERVAL CORRESPONDS TO $2 \frac{2}{3}$ MB.

A. 0 HOURS. H=.0906.

B. 72 HOURS. H=.0917.

C. 108 HOURS. H=.0920.

D. 144 HOURS. H=.0915.

FIG. 3-21. 2PE BUOYANCY, LEVEL K=9, Z=.425.
CONTOUR INTERVAL CORRESPONDS TO .92°C.

A. JET RIDGE, I=32. MAX U=.0999, MAX LABELED B=.250.

B. JET TROUGH, I=12. MAX U=.0990, MAX LABELED B=.250.

FIG. 3-22. 2PE NORTH-SOUTH CROSS SECTIONS. 0 HOURS. SEE TEXT FOR EXPLANATION. MULTIPLY U BY 480 M SEC^{-1} FOR DIMENSIONAL VALUE.

A. JET RIDGE, I=25. MAX U=.0860, MAX LABELED B=.240.

B. JET TROUGH, I=6. MAX U=.0991, MAX LABELED B=.250.

FIG. 3-23. 2PE NORTH-SOUTH CROSS SECTIONS. 108 HOURS.

A. JET RIDGE, I=34. MAX U=.0711, MAX LABELED B=.240.

B. JET TROUGH, I=15. MAX U=.1013, MAX LABELED B=.250.

FIG. 3-24. 2PE NORTH-SOUTH CROSS SECTIONS. 144 HOURS.

A. 0 HOURS. $L = -.0375$.
21 MB BETWEEN H AND L.

B. 72 HOURS. $L = -.0386$.
26 MB BETWEEN H AND L.

A. 0 HOURS. $H=.0391$.

B. 72 HOURS. $H=.0392$.

C. 108 HOURS. $H=.0391$.

D. 144 HOURS. $H=.0391$. LINES INTERSECTING FRONTS ARE DISCUSSED IN CH. 4.

FIG. 3-26. 2QG SURFACE BUOYANCY, LEVEL $K=1$, $Z=.050$.
CONTOUR INTERVAL CORRESPONDS TO $.92^{\circ}\text{C}$.

A. 0 HOURS. $L=-.0270$.

B. 72 HOURS. $L=-.0280$.

C. 108 HOURS. $L=-.0288$.

D. 144 HOURS. $L=-.0293$. LINE SEGMENTS AA, BB ARE DISCUSSED IN CH. 5.

FIG. 3-27. 2QG PRESSURE, LEVEL $K=9$, $Z=.425$.
CONTOUR INTERVAL CORRESPONDS TO $2\frac{2}{3}$ MB.

A. 0 HOURS. H=.0900.

B. 72 HOURS. H=.0907.

C. 108 HOURS. H=.0908.

D. 144 HOURS. H=.0905. LINE SEGMENTS
AA, BB ARE DISCUSSED IN CH. 5.

FIG. 3-28. 2QG BUOYANCY, LEVEL K=9, Z=.450.
CONTOUR INTERVAL CORRESPONDS TO .92°C.

A. JET RIDGE, I=32. MAX U=.0995, MAX LABELED B=.220.

B. JET TROUGH, I=12. MAX U=.0993, MAX LABELED B=.220.

FIG. 3-29. 2QG NORTH-SOUTH CROSS SECTIONS. 0 HOURS. SEE TEXT FOR EXPLANATION. MULTIPLY U BY 480 M SEC^{-1} FOR DIMENSIONAL VALUE.

A. JET RIDGE, I=24. MAX U=.0912, MAX LABELED B=.220.

B. JET TROUGH, I=4. MAX U=.0924, MAX LABELED B=.230.

FIG. 3-30. 2QG NORTH-SOUTH CROSS SECTIONS. 108 HOURS.

A. JET RIDGE, I=33. MAX U=.0788, MAX LABELED B=.220.

B. JET TROUGH, I=13. MAX U=.0846, MAX LABELED B=.230.

FIG. 3-31. 2QG NORTH-SOUTH CROSS SECTIONS. 144 HOURS.

a. 2PE buoyancy. Max contour = .0900,
contour interval corresponds to 1.38°C .

b. 2QG buoyancy. Max contour = .0880,
contour interval corresponds to $.92^{\circ}\text{C}$.

Fig. 3-32. Comparison of 2PE, 2QG buoyancy at level K=9, 180 Hours.

3-31 display the corresponding QG cross sections.

3.31 Surface Evolution of the integrations.

The surface evolution of the integrations is displayed in figures 3-18 and 3-19 for the PE model and in figures 3-25 and 3-26 for the QG model. Both solutions develop in a similar fashion. A low pressure region develops on the northern side of the surface westerly flow; this low is elongated east-west and develops a cyclonic pressure trough extending east-southeast into the downstream ridge. A high pressure area develops at the southern end of this ridge. As time progresses the pressure trough strengthens and cuts further eastward into the ridge and an associated buoyancy gradient or "warm front" develops, stretching southeast from the low toward the high. A "cold front" trough also develops south and southwest of the low; the cold front sweeps forward as the low appears to occlude. As neither model contains any mixing processes the warm air at the surface trapped between the two fronts cannot be dissipated and is swept back around the cyclone in a narrow tongue. In both solutions the buoyancy gradients decrease in width to a limiting thickness of 3Δ ; the structure of these gradients will be discussed in Chapter 4.

During its development, the surface pattern moves off the eastern edge of the plots and reappears on the western edge. In both models after 4 days the surface low moves north northeast and the high moves south southeast, as shown in figure 3-33a. There is an initial dimensional pressure difference at the surface of about 21 mb, due

A. Position of high and low centers versus time.

Fig. 3-33. Surface pressure evolution. 2PE versus 2QG.

FIG. 3-33 CONTINUED. SURFACE PRESSURE EVOLUTION. 2PE VERSUS 2QG.

2PE ——— 2QG - - -

mainly to the presence of the surface westerly flow; by day 6 this has increased to about 40 mb in the QG and to about 45 mb in the PE model.

The QG solution displays the symmetric behavior observed in experiment I while the PE solution does not. The QG high and low pressure centers possess equally strong pressure gradients. The QG cyclone appears to "occlude" as the northern end of the cold front approaches the warm front; the QG high also appears to "occlude" as the southeast end of the warm front advances toward the southwest end of the cold front (figures 3-25c-d, 3-26c-d). As in experiment I the PE low possesses a stronger pressure gradient than does the PE high (compare figures 3-18c-d to figures 3-2b-c). The occlusion process develops more quickly for the PE cyclone than for the QG cyclone although the PE surface buoyancy contains more numerical noise than does the QG buoyancy; the PE high does not occlude (compare figures 3-19c-d to figures 3-26c-d). The PE warm front develops earlier and remains more intense than the PE cold front (figures 3-19b-d) while in the QG solution the two fronts develop simultaneously and are of equal strength (figures 3-26b-d). The PE cold front sweeps eastward faster and is more curved than the QG cold front (compare figure 3-19d to 3-26d). Finally, the PE cyclone becomes much broader in area than the PE anticyclone (figure 3-18d) and by the end of the run at day $7\frac{1}{2}$ the cyclone is actually filling at the surface, as shown by figure 3-33b where the PE and QG surface pressures are plotted against time. Figure 3-33c displays the total

surface pressure difference for the models versus time; the QG pressure difference appears to increase slightly faster for the first two days or so after which the PE pressure difference increases more rapidly.

The surface high and low pressure centers show very similar movement for the two models. As seen from figure 3-33a, only after 144 hours of forecast time are the PE and QG low pressure centers more than 3Δ apart.

Figure 3-33a can also be used to compare the average speed of the PE and QG surface pressure centers to the perturbation phase speed predicted for this experiment by the two-dimensional instability analysis in Chapter 2. Figure 2-6a shows the phase speed of the most unstable perturbation to be $9.7 \pm 0.1 \text{ m sec}^{-1}$. Calculations from figure 3-33a show that for the first 36 hours of the forecast the QG high and low pressure centers move eastward at an average speed of $10.2 \pm 0.9 \text{ m sec}^{-1}$, in good agreement with the predicted speed. (The error limits arise because the exact location of a pressure center is known only to the nearest gridpoint.) During this same period the PE pressure centers travel a greater distance and hence move at a higher average speed than do the QG centers; evidently the adjustment between the initial wind and buoyancy fields taking place within the PE integration affects the movement of the surface pressure centers during this time. During the following 24 hour period from 36 to 60 hours both PE and QG low

centers move eastward at $11.1 \pm 1.4 \text{ m sec}^{-1}$ and both high centers move eastward at $9.7 \pm 1.4 \text{ m sec}^{-1}$.

3.32 Upper level development.

Figure 3-22a is a north-south vertical cross section through the westerly jet ridge at the start of the PE integration. This cross section cuts through the horizontal plots of pressure and buoyancy displayed in figures 3-18a, 3-19a, 3-20a and 3-21a for this time on the north-south line RR. Similarly the north-south cross section through the jet trough (figure 3-22b) intersects the horizontal plots on the line TT. The intersections of the other PE and QG cross sections are likewise indicated on the appropriate horizontal plots.

Initially (figures 3-22 and 3-29) both solutions show rising motion in the ridge and sinking in the trough for the lower troposphere with relatively little vertical motion occurring at jet level. (Actual values of the vertical motion are not obtainable from these cross sections; a more detailed discussion of vertical motion appears in Chapters 4 and 5.) The wind speed at the jet center is about the same for both runs and is about the same in the ridge and the trough, being around .100 or 48 m sec^{-1} .

As time proceeds the development of the surface frontal zones can be seen in the cross sections (compare figures 3-23 to 3-30 for the PE versus QG development at $4\frac{1}{2}$ days and compare figures 3-24 to 3-31 for development at 6 days). Although these cross sections do

not intersect the surface fronts perpendicular to their length (see figures 3-19d and 3-26d) they do reveal that these horizontal buoyancy gradients decrease in intensity with height above the surface for both PE and QG solutions.

In the ridge cross sections for both solutions at days $4\frac{1}{2}$ and 6 (figures 3-23a, 3-24a, 3-30a and 3-31a) the air moving northward is rising south of and beneath the jet maximum and there is a thermally direct circulation associated with the developing surface buoyancy gradient. Both solutions show the jet widening and weakening with time in the ridge, the PE jet maximum at 6 days ($.0711$ or 34.1 m sec^{-1}) being less than the QG maximum ($.0788$ or 37.8 m sec^{-1}). The PE solution at $4\frac{1}{2}$ and 6 days shows air moving southward and downward in a region of the lower stratosphere and upper troposphere north of the jet maximum. The corresponding ridge cross sections for the QG solution do not show descent in this region.

The situation is different for the trough cross sections (figures 3-23b and 3-24b for the PE trough at $4\frac{1}{2}$ and 6 days, respectively; figures 3-30b and 3-31b for the corresponding QG trough). Both PE and QG solutions show the jet to be tighter and stronger in the trough than in the ridge at 6 days ($.0846$ or 40.6 m sec^{-1} versus $.0788$ or 37.8 m sec^{-1} for the QG; $.1013$ or 48.6 m sec^{-1} versus $.0711$ or 34.1 m sec^{-1} for the PE). For this same time both solutions show air beneath the jet maximum to be moving southward and downward along the isentropes of buoyancy and show the horizontal buoyancy gradient beneath the jet to be stronger and more narrow in the trough

than in the ridge. In addition there is rising motion to the south of and sinking motion to the north of the surface cold front intersected by the trough cross section at day 6 for both solutions.

The evolution of the pressure and buoyancy at vertical level 9 can be seen for the PE run in figures 3-20 and 3-21 and for the QG run in figures 3-27 and 3-28. As the PE pressure wave amplifies with time we note a progressive strengthening of the pressure gradient in the trough as compared to the ridge (figure 3-20d); this jet decreases in width with time. The QG pressure does not develop such a strong gradient in the trough compared to the ridge but the strongest regions of geostrophic wind are found midway between the ridge and trough lines (figure 3-27d). The geostrophic wind in the trough is seen to be slightly stronger than that in the ridge for the QG solution at 6 days from a comparison of figures 3-31a and 3-31b.

In conjunction with the developing pressure gradient in the PE solution, a remarkable tightening of the horizontal buoyancy gradient begins to appear at level 9 at 3 days and by day 6 this gradient has narrowed to a width of 3Δ just as the surface fronts have done (figures 3-21a-d). Examination of the developing buoyancy gradient at levels between the surface and level 9 (not shown) and examination of the PE trough cross section for day 6 (figure 3-24b) shows that the horizontal buoyancy gradient is weaker at levels 4 to 7 than at level 1 or at level 9, indicating that this upper level development is not an extension of the surface front. Figure 3-24b shows the strong, narrow jet with the isotachs of east-west wind

lying nearly parallel to the isentropes of buoyancy within the upper tropospheric, sloping, stable zone beneath the jet. A comparison of figures 3-24b and 3-31b reveals this orientation to be lacking beneath the jet in the QG solution. In addition, figure 3-24b shows that the strongest region of sinking motion is concentrated in and on the southern or warm edge of the sloping stable zone beneath the jet. These are characteristics of upper level frontal zones; they will be discussed in Chapter 5.

The QG buoyancy gradient at level 9 also strengthens and narrows with time (figures 3-28a-d) but never attains the intensity of the PE buoyancy gradient (compare figure 3-21d to 3-28d). Figure 3-28d does show the QG buoyancy gradient to be more narrow in the trough than in the ridge at 6 days as we have previously noted from a comparison of figures 3-31a and b.

A final comparison between the evolution of the PE and QG buoyancy gradients at level 9 is given by figures 3-32a and 3-32b which show the respective buoyancy plots for day $7\frac{1}{2}$, at the end of the integrations. (Note the contour interval for figure 3-32a is 50% larger than the interval for figure 3-32b; had the contour intervals been the same the difference would have been even more pronounced.) The PE gradient has collected most of the north-south buoyancy variation within the narrow frontal zone. The QG gradient shows a continued strengthening in the trough relative to the ridge (compare figure 3-28d to 3-32b) but in no way attains the intensity nor the

narrowness of the PE buoyancy gradient. We will discuss the evolution of these frontal zones in Chapter 5.

3.33 Kinematics of experiment II.

In this section we will first compare the growth rates for the PE and QG solutions to the predicted growth rate from the instability computation of Chapter 2. We will then discuss the amplification of the PE and QG disturbances from an energetics viewpoint followed by a comparison of the PE and QG mean meridional circulations.

Figure 3-34a shows the perturbation pressure difference in x (with y and z held constant) versus time for the row of gridpoints just north of the east-west center line of the channel ($J = 22$) and for vertical level $k = 1, 5$ and 10 for both the PE and QG solutions. The QG curves are close to straight lines on the semi-logarithmic plot for the first three days and have doubling times of about 28 hours at one day into the forecast. The PE curve for level 1 has a negative growth rate for the first 12 hours and then has a doubling time of 18 hours until the 60th hour or so; the other two PE curves show much less irregularity at the start and possess doubling times of 25 and 30 hours at one day into the forecast. The growth of eddy kinetic energy for the PE solution (not shown) implies a doubling time of 28 hours at one day into the forecast. Apparently the PE perturbation possesses a doubling time of around 28 hours at one day of forecast time, about the same as the QG doubling time. The irregularity of the PE curves versus the QG curves is presumably due

FIG. 3-34B. 2PE XY AVERAGED BUOYANCY VS. TIME, LEVELS K=1,3,5.

to the adjustment occurring between the wind and buoyancy fields within the PE solution.

The instability study in Chapter 2 predicted a doubling time of 27.8 ± 0.1 hours for the perturbation used in experiment II. That study utilized 20 gridpoints in the horizontal and 10 in the vertical; an increase in resolution to 42 gridpoints in the horizontal and 20 in the vertical changed the predicted perturbation doubling time from 27.8 ± 0.1 to 26.8 ± 0.1 hours, still close to the observed PE and QG doubling times of 28 hours. This is the "predicted growth rate" line plotted on figure 3-34a.

One may have noticed that increasing the resolution in the instability study resulted in a decrease in the predicted growth rate for experiment I and an increase in the predicted growth rate for experiment II. These two experiments were also integrated with the PE model using a coarser horizontal resolution (runs C-1 and C-2 in table 3-1); for basic state I the coarse resolution eddy kinetic energy grew slightly faster than the fine resolution eddy kinetic energy so increasing the horizontal resolution decreased the perturbation growth rate. For basic state II, however, the coarse resolution eddy kinetic energy grew slightly slower than did the fine resolution eddy kinetic energy; increasing the horizontal resolution thus increased the growth rate. The change in horizontal resolution in the PE integrations changed the perturbation growth rates in the same sense as did the change in horizontal and vertical resolution in the instability computations.

Let us now review the development of the PE and QG integrations of experiment II, placing the emphasis upon the energy sources for the growing perturbation. The PE and QG pressure plots for day zero (figures 3-18a and 3-20a plus figures 3-25a and 3-27a) show the same slope of the trough and ridge lines in the horizontal and vertical as indicated by the perturbation zonal phase in figure 2-9b. These slopes are consistent with the results of the instability study showing the perturbation to grow via a "baroclinic" process in the lower troposphere (figure 2-10b) and a "barotropic" process at higher levels (figure 2-10a).

Consider again the north-south vertical cross sections displayed in figures 3-22 to 3-24 and figures 3-29 to 3-31. Initially and as the integrations proceed and for both PE and QG solutions, the northward moving air in the lower troposphere tends to rise at an angle less steep than the isentropes of buoyancy and the southward moving air tends to descend at an angle less steep than the isentropes. The baroclinic conversion of potential energy to eddy kinetic energy is associated with just such movement as discussed earlier in this chapter.

An examination of the horizontal and vertical slope of the ridge and trough lines shows that as they amplify, both the PE and QG disturbances continue to gain energy from the potential and kinetic energy of the basic state (compare figure 3-18 to 3-20 and figure 3-25 to 3-27). By day 6 both solutions show the lessening importance of the latter energy source; the horizontal slope or

"tilt" of the troughs at level 9 has become more north-south as opposed to the orientation of northeast north of the jet and southeast south of the jet at day 3, for example. A presentation later in this section of the energetics of the PE integration will directly confirm this decrease of "barotropic" energy conversion toward the end of the run.

An examination of the surface buoyancy evolution displayed in figures 3-19 and 3-26 shows that for the PE integration the area occupied by the cold air increases with time relative to the area occupied by the warm air but that for the QG integration the areas remain about the same. We have mentioned earlier that the QG model cannot change the large scale atmospheric stability but the PE model is free to do so. The release of potential energy in the latter model is associated with a lowering of the center of gravity of the system; this is consistent with the net cooling of the lowest layers that results in the PE integration. Figure 3-34b displays the horizontally averaged buoyancy versus time for levels 1, 3 and 5 for the PE solution. As in experiment I the average buoyancy generally decreases with time for levels 1 and 3 and increases with time for level 5 so that the lower tropospheric stability increases with time. After $4\frac{1}{2}$ days the average dimensional temperature decrease for the lowest level is 0.79°C compared to 0.50°C for experiment I (compare figure 3-34b to 3-13b); after $7\frac{1}{2}$ days the decrease has amounted to a large value of 5.11°C .

Let us now discuss the development of the mean meridional circulation in the PE and QG integrations. Figure 3-35 displays the zonally (x) averaged vertical motion for both PE and QG solutions at the start of the integrations. The tropospheric meridional circulation is dominated by a thermally direct cell with weak indirect cells alongside. The evolving PE solution generally maintains the direct circulation beneath the jet maximum out to four days of forecast time although there are fluctuations in direction (see table 3-2). After five days the tropospheric circulation becomes more similar to that of experiment I - an indirect center cell flanked by weaker direct cells. By day six the QG solution has also developed this three cell circulation. Figures 3-36a and 3-36b exhibit the QG and PE zonally averaged vertical motion fields at this time. As before the QG zonally averaged vertical motion was computed by hand so less gridpoints were used than for the computer plotted PE zonally averaged vertical motion. Both display similar features such as the strong, northward sloping region of strong descent in the indirect cell. The maximum dimensional values of x averaged descent occurring within this region reach $-.15 \text{ cm sec}^{-1}$ for the QG solution and $-.57 \text{ cm sec}^{-1}$ for the PE solution, the latter being due to the developing upper level frontal zone. By this time in the forecast both models have formed strong surface buoyancy gradients and the effect of truncation error appears in the x averaged vertical motion at the lower levels. In contrast to the results of experiment I the PE x averaged vertical motion

TIME HOURS	DIRECTION OF CIRC. OF CELL BELOW JET	{ P → ZKE } X 10 ⁻⁶	{ P → EKE } X 10 ⁻⁶	{ EKE → ZKE } X 10 ⁻⁶
0	DIRECT	.006	.034	-.25
12	DIRECT	1.0	-.021	-.47
36	WEAK INDIRECT	-.07	.073	-1.5
60	DIRECT	.91	.24	-4.1
72	DISORGANIZED	-.38	.57	-5.9
84	INDIRECT	-.08	1.4	-7.8
96	DIRECT	.26	2.6	-8.7
108	DIRECT	.32	4.2	-8.2
120	INDIRECT	.13	6.1	-6.5
132	INDIRECT	-.76	8.0	-4.4
144	INDIRECT	-.19	8.6	-2.8
156	INDIRECT	-.34	8.5	-1.2
168	INDIRECT	-.24	7.6	.21
180	INDIRECT	-.74	5.8	1.1

Table 3-2. Energy transformation terms versus time, Run 2PE.

{ P → ZKE } represents conversion of potential to zonal kinetic energy, etc.

Fig. 3-35. X averaged vertical motion, 2PE, 2QG. 0 Hours.

Max value is 1.09×10^{-5} or $.0016 \text{ cm sec}^{-1}$.

A. 2QG. MAX VALUE IN DESCENDING CELL IS -1.05×10^{-3} OR $-.15 \text{ CM SEC}^{-1}$.

B. 2PE. MAX VALUE IN DESCENDING CELL IS -3.76×10^{-3} OR $-.57$ CM SEC $^{-1}$.

FIG. 3-36. X AVERAGED VERTICAL MOTION. 144 HOURS.

during its development displayed no wavelike oscillations in the southern part of the channel. The evolving meridional circulation in experiment II occupied much more of the channel volume than did the circulation in experiment I and hence may have masked any such small scale, small amplitude activity.

Figure 3-37 shows the eddy kinetic, zonal kinetic and total potential energy versus time for the PE solution; similar information is not available for the QG solution. (The various forms of energy for the PE model are defined in the appendix, section E.) An arbitrary constant was added to the total potential energy so that the initial value would be zero for figure 3-37. The change in total energy over the course of the PE integration is small, being about 5.4 per cent of the initial total kinetic energy. We see that for the first four days the major source of eddy kinetic energy is the zonal kinetic energy whereas after this time the potential energy becomes the major source. At day six the PE perturbation is growing mainly from the potential energy when the three-cell indirect meridional circulation is present (figure 3-36b). From day $6\frac{1}{2}$ on the disturbance is actually growing in a barotropically damped manner, feeding kinetic energy into the zonal flow similar to the experiment I disturbance. By this time the disturbance growth rate is decreasing; even though the eddy kinetic energy continues to increase figure 3-33b shows that the central surface pressure of the cyclone actually increases after day $6\frac{1}{2}$.

FIG. 3-37. 2PE ENERGY VARIATION VERSUS TIME.

An evaluation of the energy transformation terms for the PE solution indicates the direction of the mean meridional circulation cell located beneath the mean zonal jet is well correlated to the direction of conversion of potential to zonal kinetic energy. (A thermally direct cell converts potential to zonal kinetic energy, etc.) This loss or gain of zonal kinetic energy is small compared to the exchange of zonal and eddy kinetic energy as can be seen from table 3-2 where the nondimensional transformation terms are displayed for the PE solution along with the direction of the major circulation cell located beneath the mean zonal jet. (The various transformation terms appearing in the table are derived in the appendix, section E.)

3.34 Summary of experiment II.

Experiment II utilizes a perturbation with maximum amplitude at the tropopause level drawing energy from both the basic state kinetic and potential energy, the former source initially dominating. The basic state consists of a zonal jet of moderate strength (48 m sec^{-1} maximum) with a significant surface westerly wind (12 m sec^{-1} maximum). After $4\frac{1}{2}$ days both PE and QG integrations have produced a surface cyclone and anticyclone with associated "frontal zones". As the integrations proceed in time both solutions continue to evolve in an apparently realistic manner with the PE solution developing an upper level frontal zone. The PE solution generally appears to be the more realistic synoptically

as was the case in experiment I.

We have seen from the results of experiments I and II that the QG model does a rather good job of forecasting disturbance development and particularly movement when compared to the PE model. This suggests that, while certain aspects of a quasigeostrophic forecast must be inherently non-meteorological, a high resolution quasigeostrophic model may do well in competition against present primitive equations forecasting models.

The time integration of experiment I was halted after $4\frac{1}{2}$ days when the width of the evolving surface frontal zones began to approach the grid resolution. The major purpose of that experiment was to study the evolution of these surface zones before the lack of horizontal resolution became significant. The major purpose of experiment II was to study the evolution of the upper tropospheric flow which developed after the surface frontal zones had formed. In experiment II truncation errors associated with the narrow surface gradients of wind and buoyancy become more noticeable with time in both solutions, especially the PE (figures 3-19c-d, also figures 3-26c-d), but the vertical cross sections and upper level pressure and buoyancy fields show that these errors seem to be confined mainly to the lowest horizontal levels. Eventually these errors begin to corrupt even the mid- and upper level tropospheric evolution but by day $7\frac{1}{2}$ when the runs were terminated, figures 3-32a and 3-32b show the buoyancy fields at level 9 to be largely unaffected by small scale noise.

Studies of the general circulation of the atmosphere have shown that within the troposphere kinetic energy is being fed from the eddys into the zonally averaged flow and that the eddys draw their energy from the meridional temperature gradient. A perturbation amplifying in a baroclinically growing, barotropically damped manner would be converting energy in the same direction while a perturbation growing barotropically would be working backward compared to the observed general circulation. One might conclude, then, that studies of perturbations amplifying primarily by barotropic instability, such as the experiment II integration, will not lead us to a more complete understanding of the atmosphere. We have seen, however, that the experiment II disturbance developed to maturity in a baroclinically growing, barotropically damped manner. If it should be true that atmospheric disturbances likewise tend to become barotropically damped as they evolve then our results suggest that cyclogenesis, especially at upper tropospheric levels, may be as often initiated by barotropic instability as by baroclinic instability.

3.35 Realism of experiment II.

A large cyclone that developed in south central Canada on August 30, 1972 appears quite similar to our experiment II disturbance. Figures 3-38a and 3-38b display the surface and 500 mb maps for 12Z August 31, 1972. The surface synoptic map is similar to the run 2PE level 1 development at 4 $\frac{1}{2}$ days (figures 3-18c and 3-19c).

The zonal wavelength of the atmospheric disturbance is approximately 4800 km, the wavelength of the numerical disturbance. In addition the mid-tropospheric meridional temperature gradient is about 18°C and the width of the jet is about 3000 km for both disturbances.

The August 30 storm developed a strong surface warm front and weaker cold front as indicated by both the appearance of the frontal zone pressure troughs and the horizontal temperature contrasts in figure 3-38a. The warm front developed toward the southeast, cutting into the downstream ridge as did the warm front in run 2PE. In addition, evidence of mid-tropospheric frontogenesis was found at 500 mb; from 00Z to 12Z the flow between the troughs and upstream ridges contained tightening temperature gradients. During this time dew point depressions increased in these regions, indicating sinking motion. Finally, an examination of the horizontal and vertical tilt of the developing trough and ridge lines from the 850 mb and 300 mb maps (not shown) suggested the storm may have developed by barotropic as well as by baroclinic instability of the initial flow pattern.

4. SURFACE FRONTOGENESIS

The previous chapter discussed the evolution of disturbances that grew from small amplitude perturbations of the basic state. Here we are interested in the surface frontal zones that evolved within the disturbances. Development of these zones in the PE and QG models (runs 1PE, 1QG and 2PE, 2QG) will be compared.

The PE model allows horizontal advection by the divergent wind; this is lacking in the QG model. In the first section this advection is seen to enhance the action of the horizontal deformation field in PE frontogenesis. The synoptic scale divergence, included in the QG model, is then shown to exert control over effects that lead to frontogenesis in the PE model.

Section three compares fully developed fronts, which are seen to possess structure similar to that predicted by the two-dimensional frontogenesis models. The final section discusses the effects of adding friction to the PE model; with friction present the developing frontal zones appear more realistic.

4.1 Horizontal deformation and surface frontogenesis.

Two mechanisms leading to realistic surface frontogenesis have been studied with two-dimensional models, the classical non-divergent stretching deformation process (Williams, 1972; Hoskins and Bretherton, 1972) and the so called "shearing deformation" process, that is, the effect of cold winds from the north and warm

winds from the south associated with cold front formation in a growing, baroclinic wave (Williams, 1967, Hoskins and Bretherton, 1972). In the latter studies the only deformation field is that associated with an unstable perturbation; it is initially ineffective because the dilatation axis is oriented 45 degrees from the isotherms. The developing perturbation temperature field, however, changes this orientation into one more favorable for frontogenesis. This idealized process in which all velocity fields are independent of y appears to be much less effective in producing fronts than the former models which begin with an impressed two-dimensional field of deformation.

Rather than attempt to analyze our completely three-dimensional results into one or the other of these idealized models of surface frontogenesis, we will describe the general relation of deformation and divergence to frontogenesis in the growing baroclinic wave. The more realistic character of the classical process suggests that is the more fruitful model to consider if one is so desired. The present integrations demonstrate the natural evolution of the initial deformation and temperature fields which are assumed in these models.

Data presented in this section will be from runs 1PE and 1QG; this PE integration was terminated just as the fronts were forming and truncation error associated with strong frontal gradients was thus minimized.

If $S_3 \equiv (\nabla b)^2$ and $\frac{db}{dt} = 0$, then it can be shown that

$$\frac{dS_3}{dt} = -2 \left[e_{11} \left(\frac{\partial b}{\partial x_1} \right)^2 + e_{22} \left(\frac{\partial b}{\partial x_2} \right)^2 + e_{33} \left(\frac{\partial b}{\partial x_3} \right)^2 \right] \quad (4-1)$$

where x_1, x_2, x_3 are Cartesian coordinates in the direction of the principal axes of the deformation tensor

$$\begin{pmatrix} e_{11} & 0 & 0 \\ 0 & e_{22} & 0 \\ 0 & 0 & e_{33} \end{pmatrix}.$$

The components e_{ij} are defined as $\frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$ where v_i is the velocity component along the x_i axis. If $\nabla \cdot \vec{v} = 0$, that is, if $e_{11} + e_{22} + e_{33} = 0$ then $\frac{dS_3}{dt}$ is a maximum with respect to constant $(\nabla b)^2$ if ∇b is oriented parallel to the axis corresponding to the most negative of e_{11}, e_{22}, e_{33} .

For conditions on a level surface where $w = 0$, we can specialize equation (4-1) by considering $S_2 = \left(\frac{\partial b}{\partial x} \right)^2 + \left(\frac{\partial b}{\partial y} \right)^2$.

If we define

$$\alpha = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y},$$

$$\beta = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \quad |D| = (\alpha^2 + \beta^2)^{1/2},$$

$$\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y},$$

the principal axes x', y' are rotated by an angle Θ from x, y , where

$\sin 2\Theta = \alpha |D|^{-1}$ and the quadrant of Θ is selected so that x' is the dilatation axis $\left(\frac{\partial u'}{\partial x'} < \frac{\partial v'}{\partial y'} \right)$; the time rate of

change of S_2 is then

$$\begin{aligned} \frac{dS_2}{dt} &= -2 \left[\frac{\partial u}{\partial x} \left(\frac{\partial b}{\partial x} \right)^2 + \frac{\partial v}{\partial y} \left(\frac{\partial b}{\partial y} \right)^2 + \alpha \frac{\partial b}{\partial x} \frac{\partial b}{\partial y} \right] \\ &= -2 \left[\frac{\partial u'}{\partial x'} \left(\frac{\partial b}{\partial x'} \right)^2 + \frac{\partial v'}{\partial y'} \left(\frac{\partial b}{\partial y'} \right)^2 \right] \\ &= -[|D| + \delta] \left(\frac{\partial b}{\partial x'} \right)^2 + [|D| - \delta] \left(\frac{\partial b}{\partial y'} \right)^2. \quad (4-2) \end{aligned}$$

Equation (4-2) contains the action of both the horizontal deformation and the divergence δ . The relationship between the divergence, deformation (meaning from now on horizontal deformation) and frontogenesis function as expressed in equation (4-2) can be seen nicely from figure 4-1 taken from run 1PE at day zero and at level $k = 1$; the divergence (figure 4-1a), deformation plus isolines of buoyancy (figure 4-1b)¹ and deformation plus frontogenesis function (figure 4-1c) are displayed. The regions of maximum and minimum $\frac{dS_2}{dt}$ are well correlated to the alignment of the local dilatation axis with respect to the isolines of buoyancy; west of the buoyancy trough (the region of the future warm front) the divergence field contributes positively to $\frac{dS_2}{dt}$, but east of the buoyancy trough (the region of the future cold front) it has an opposite effect. (With respect to equation(4-2), $|D| \max = .046 \times 10^{-4} \text{ sec}^{-1}$, $\delta \min = -.015 \times 10^{-4} \text{ sec}^{-1}$.)

As the QG model neglects advection by the divergent wind the action of convergence upon frontogenesis will be lacking. The frontogenesis function $\frac{dS_2}{dt}$ for the QG model ($u = -\frac{\partial p}{\partial y}$, $v = \frac{\partial p}{\partial x}$) should reflect only the action of $|D|$, δ vanishing in equation 4-2).

¹ The buoyancy field should be located one grid distance further south with respect to the deformation field in figures 4-1 and 4-2b. The divergence (and vertical motion fields appearing later) are staggered with respect to the boundaries in order to locate correctly the vertical motion in the staggered grid system.

A. DIVERGENCE. $L = -.0154$ OR $-1.54 \times 10^{-6} \text{ SEC}^{-1}$.

B. HORIZONTAL DEFORMATION PLUS
SURFACE BOUYANCY.

CONTOUR INTERVAL CORRESPONDS TO $.72^\circ \text{ C}$.

C. HORIZONTAL DEFORMATION PLUS $\frac{d}{dt} S_1$.
 $H = 4.45, 2.59$; $L = -3.30, -1.89$, ALL $\times 10^{-4}$.

FIG. 4-1. 1PE SURFACE DIVERGENCE, DEFORMATION AND $\frac{d}{dt} S_1$,
LEVEL $K=1$, $Z=.025$. 0 HOURS. $1/8 \leq Y \leq 7/8$ IS DISPLAYED.

Table 4-1 shows $\frac{dS_2}{dt}$ for the PE and QG models for the four points in figure 4-1c labeled by "H" and "L" where the PE $\frac{dS_2}{dt}$ reaches maximum and minimum values. The initial pressure and buoyancy values differ only slightly in the two models so the values of deformation should be similar. The QG and PE frontogenesis functions compare in the manner expected. (The QG values were hand computed from print-outs of ρ , which unfortunately introduced some round-off error in table 4-1.)

Figure 4-2 displays the same fields as figure 4-1 but at day 4; a strong warm front but a weaker cold front are forming². The deformation is working toward frontogenesis in both warm and cold frontal zones. Convergence contributes to $\frac{dS_2}{dt}$ in the warm front formation ($|D| \sim .32 \times 10^{-4} \text{ sec}^{-1}$; $\delta \sim -.13 \times 10^{-4} \text{ sec}^{-1}$) but does little for the cold front. Gilchrist (1971) has found a similar orientation of fronts and dilatation axes in a hemispheric general circulation model.

Figure 4-3a, the four day QG b field at level 1 shows a definitely weaker warm front than does the PE model (figure 4-2b) but its cold front has about the same strength. We therefore have independent corroboration in a fully three-dimensional context of the

²Figure 4-3c presents a plot of ρ at level 1 for this time; vectors of the horizontal wind are shown for selected gridpoints (asterisks mark vector tails). Equation (4-2) is for parcels; this figure allows one to see where the parcels are going.

I	J	$\frac{dS_2}{dt} _{PE}$ $\times 10^{-4}$	$ D _{PE}$	θ_{PE}	SIGN OF δ IN 1PE	$\frac{dS_2}{dt} _{QG}$ $(\pm 0.8) \times 10^{-4}$	$ D _{QG}$ $(\pm .018)$	θ_{QG} $(\pm 45^\circ)$
11	35	4.5	.032	-4°	-	3.1	.036	0°
30	27	2.6	.037	7°	+	4.2	.054	0°
15	27	-1.9	.035	-85°	-	-2.5	.036	-90°
33	35	-3.3	.034	-87°	+	-2.5	.036	-90°

Table 4-1. Nondimensional values of $\frac{dS_2}{dt}$ and horizontal deformation for selected initial data points. 1PE versus 1QG, level K=1. Error limits for 1QG quantities are in parentheses. Angles are deformation dilatation axis with respect to X (east-west) axis.

A. DIVERGENCE. $L = -.162$ OR $-1.62 \times 10^{-5} \text{ SEC}^{-1}$.

B. HORIZONTAL DEFORMATION PLUS
SURFACE BUOYANCY.
CONTOUR INTERVAL CORRESPONDS TO $.72^{\circ}\text{C}$.

C. HORIZONTAL DEFORMATION PLUS $\frac{d}{dt} S_2$.
VALUES AT H ARE 1.95×10^{-2} AND 0.57×10^{-2} ,
VALUE AT L IS -1.32×10^{-2} .

FIG. 4-2. 1PE SURFACE DIVERGENCE, DEFORMATION AND $\frac{d}{dt} S_2$,
LEVEL K=1, $Z=.025$. 96 HOURS. $1/8 \leq Y \leq 7/8$ IS DISPLAYED.

basic difference between the original quasigeostrophic model of frontogenesis by Stone (1966) and the recent nongeostrophic model by Hoskins and Bretherton (1972); the latter allows horizontal advection by the divergent wind field and this effect is obviously crucial in our numerical experiment.

4.2 Quasigeostrophic initiation of surface frontogenesis.

The two-dimensional models of surface frontogenesis picture the process as being initiated by quasigeostrophic effects. When ageostrophic terms, initially small, become significant compared to geostrophic terms, the models produce realistic frontal zones in a short time. The "geostrophic" effects are modeled by the nondivergent deformation field in the "stretching deformation" studies and by the presence of vertical wind shear in the "shearing deformation" studies, in which the basic state wind is again nondivergent.

The more realistic initial conditions of our fully three-dimensional integrations include the "synoptic" scale divergence δ ignored in the two-dimensional studies. We will see that it is δ , a "geostrophic" effect (included in the QG model), that acts as a control over "ageostrophic" effects (terms in the PE but missing in the QG model) responsible for frontogenesis in the PE model.

Examples of δ have already been presented for the PE model. At vertical level 1 this quantity is the vertical motion w multiplied by $-1/\Delta z = -20$. In the PE model the vertical motion is calculated directly at each timestep from the continuity equation since it is

needed to evaluate the advective terms; in the QG model no such need exists and w is calculated only when desired for data purposes. This is done by inverting the QG thermodynamic equation (2-4) directly for w (see Appendix section C).

Both PE and QG vertical motion fields can contain "noise" presumably due to truncation error associated with strong frontal gradients, for example. (We are unable to state whether small scale irregularities in the PE model may correspond to phenomena such as internal gravity waves.) In runs 1PE and 1QG the fields of w are fairly free of such noise. Figure 4-2a shows the PE δ at level 1 for day 4 while figure 4-3b shows the corresponding QG δ (if we multiply w by $-1/\Delta z = -20$). The strongest dimensional values of convergence and divergence from these figures are $-.162 \times 10^{-4} \text{ sec}^{-1}$ and $.111 \times 10^{-4} \text{ sec}^{-1}$ for the PE; $-.095 \times 10^{-4} \text{ sec}^{-1}$ and $.097 \times 10^{-4} \text{ sec}^{-1}$ for the QG model.

The action of the "ageostrophic" effects in the PE model can be observed directly by studying the change in time of the geostrophic potential vorticity η_g defined in equation (2-5). This quantity is conserved by the QG but not by the PE model. Since η_g is basically the vertical component of absolute vorticity plus the time variable part of the stability multiplied by a constant (minus another constant), and since both the vorticity and the stability become large in atmospheric frontal zones relative to regions outside the zones, only the PE model is able to generate meteorologically realistic frontal zones if the initial field of η_g is everywhere small.

η_G can thus serve in the PE model as a possible indicator of the presence of fronts (Williams, 1967; Phillips, 1970).

The PE pressure is used to compute η_G for the PE model; due to the second derivative in z , $k = 2$ is the lowest level available. Figures 4-4a and 4-4b display η_G at days 0 and 4, respectively, along with the level 2 buoyancy (dashed lines) for run 1PE. Initially only the earth's vorticity is significant. After 4 days two regions of increased η_G have appeared, one of very large values (maximum nondimensional value of .832) at the center of the cyclone and elongated slightly toward the warmfront (the pressure for level 1, day 4 appears in figure 4-3c) and one less strong region (maximum value of .329) located in the southern part of the anticyclone. At day 4 the cyclone and warmfront contain large values of vorticity (figure 4-5a) and the "high" contains relatively large values of stability (figure 4-5b); the large vorticity and stability must explain the increases in η_G .

The increase in η_G suggests "ageostrophic" effects are responsible for bringing about the increases in vorticity and stability. At the ground the change in relative vorticity for a parcel is given by

$$\frac{dS}{dt} \approx -(S+f)S \quad (4-3)$$

where $S = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$. Initially $S \ll f$ so $\frac{dS}{dt} \sim -S$ as in the QG model ($f = 1 + \frac{df}{dy} [\gamma^{-1/2}]$; since $\gamma \sim 1/2$ in the region of interest and since $\frac{df}{dy} < 1$ for these runs we can consider $f \sim 1$

a. 0 Hours. Max contour is .300.

b. 96 Hours. H values are .832 and .329.

Fig. 4-4. 1PE Geostrophic potential vorticity, $K=2$, $Z=.075$.

Dashed lines are buoyancy at $K=2$, $1/8 \leq y \leq 7/8$ is displayed.

a. Absolute vorticity. $Z=.075$.
 $H=2.01$ or $2.01 \times 10^{-4} \text{ sec}^{-1}$.

b. Stability. $Z=.100$. $H=.132$ or $\frac{\partial T}{\partial z} = -3.66^\circ \text{C km}^{-1}$, $L=.034$ or $\frac{\partial T}{\partial z} = -8.37^\circ \text{C km}^{-1}$.

Fig. 4-5. 1PE Absolute vorticity and stability, $K=2$, 96 Hours.

$1/8 \leq y \leq 7/8$ is displayed.

for this argument). Thus the "geostrophic" effect causes ζ to be positive in regions of convergence ($\delta < 0$) and causes ζ to be negative in regions of divergence ($\delta > 0$) by the time the "ageostrophic" $-\zeta\delta$ term becomes significant. In regions of convergence $-\zeta\delta$ is then positive and works with $-\delta$ to allow ζ to become large and positive; in regions of divergence the two terms work opposite each other to restrict the magnitude of ζ . This explains the observed change in vorticity. The change in stability $\frac{\partial k}{\partial z}$ is also controlled by δ as long as the ageostrophic terms remain small.

Figures 4-3a and b show that the QG warm front is forming in a region of convergence while the cold front forms in a region of divergence. Figures 4-2a and b show that the PE warmfront also forms in a region of convergence while the cold front straddles a line of zero divergence. This explains why the warm front has larger values of vorticity than does the cold front (figure 4-5a).

The increased sharpness of the warm front compared to the cold front in the PE model is seen in run 2PE (see figure 3-19, for example) as well as run 1PE. It is also characteristic of the original computations by Edelmann (1963) and later results by Økland (1969) and Hadfield (1970). Our model differs from theirs most notably in resolution (especially in the vertical) and in our more natural choice of the initial perturbation. The more rapid increase in $\nabla_n k$ in the warm front has been explained via the help of δ in equation (4-2). δ is also important in producing the frontal vorticity as shown by equation (4-3). The model of frontal formation by Hoskins and

Bretherton (1972) also obeys equation (4-3) at the ground, but their model is too idealized to distinguish between warm and cold fronts. Our results show the importance of the divergence field on synoptic scale processes as well as on the frontal scale.

It is difficult to avoid the conclusion that there is a tendency for warm front formation to be favored over cold front formation in idealized unstable cyclone waves, and that if in certain regions (such as eastern North America) sharp cold fronts are more common than sharp warm fronts, the explanation must be sought in geographical factors or effects not included in our PE model³.

The horizontal resolution used in our integrations precludes the development of gradients as intense as those obtainable in two-dimensional models of frontogenesis, yet the next section will show that the strong frontal zones formed in run 2PE are similar in structure to frontal zones produced in those models.

4.3 The structure and movement of numerical frontal zones.

Runs 1PE and 1QG were terminated before the formation of sharp frontal zones. In contrast, integration of runs 2PE and 2QG was continued for several days after sharp surface frontal zones appeared (figures 3-19 and 3-26 present the evolution of the surface buoyancy

³ Note an atmospheric development similar to run 2PE, shown in figure 3-38a, that produced a stronger warm front than cold front.

fields). Once these zones have attained a limiting thickness of 3Δ , the numerical scheme with its implicit damping of short waves is able to maintain the structure and limit the occurrence of truncation error, although small scale noise becomes more noticeable with time, especially in the PE model.

In this section we examine the transverse structure of these sharp frontal zones. It will be seen that they are similar to the frontal zones produced by two-dimensional models of frontogenesis. In addition a comparison of the location of the zones with respect to the cyclone will be made for the PE and QG models.

Figures 3-19d and 3-26d show the surface frontal zones for day 6 of integration time for runs 2PE and 2QG. Vertical cross sections have been made through the frontal zones for both models, along the lines indicated in these figures. Numerical effects have had ample opportunity to modify the zones as they had already formed by day $4\frac{1}{2}$ (figures 3-19c and 3-26c). The cross sections are displayed in figure 4-6; isolines of buoyancy and geostrophic wind normal to the sections are shown. The QG warm and cold front sections (figures 4-6a and 4-6b) are quite similar to the "pseudofronts" originally studied by Stone (1966). The horizontal buoyancy gradients are strongest nearest the surface but there is little or no slope with height of the gradients. In addition the relative vorticity (judged from the contours of geostrophic wind) is positive ahead of the fronts where the stability is small and is negative behind the fronts where the stability is greater. There are some differences

FIG. 4-6. TRANSVERSE STRUCTURE OF SURFACE FRONTS. 2PE, 2QG. 144 HOURS. TICS MARK LOCATION OF BUOYANCY GRIDPOINTS. FIGS. 3-19D AND 3-26D SHOW LOCATION OF CROSS SECTIONS THROUGH FRONTS.

between the QG warm and cold fronts; the surface jet is slightly stronger for the warm front and the region ahead of the warm front is slightly unstable ($\frac{\partial \theta}{\partial z} < 0$) while this does not occur for the cold front.

The PE warm and cold front sections (figures 4-6c and 4-6d) are similar to those produced by the primitive equations studies of Williams (1967, 1972) and the "cross-front geostrophic" models of Hoskins and Bretherton (1972). Both frontal zones possess horizontal buoyancy gradients strongest at the bottom level and these gradients slope with height over the cold air. The relative vorticity, again judged from the geostrophic wind contours, appears to have maximum positive values where the stability is also relatively large, although the vorticity in the cold front is obviously much weaker than in the warm front. Due to the "noise" present on the warm side of the warm front, the stability is negative just ahead of the front.

The actual distribution of divergence and absolute vorticity is presented for the PE fronts in figure 4-7. In both cases the vorticity is a maximum at the leading edge of the frontal zone and is located in a region of convergence (negative divergence); the vorticity and convergence are seen to be much stronger for the warm front than for the cold front, similar to our findings in the previous section concerning run 1PE. The numerical noise associated with the frontogenesis is seen to be confined mainly to the lowest horizontal level and does not destroy the frontal structure.

FIG. 4-7. 2PE FRONTAL CROSS SECTIONS. BUOYANCY, DIVERGENCE AND ABSOLUTE VORTICITY $\times 10^{-4} \text{ SEC}^{-1}$.

SECTIONS SAME AS FIGS. 4-6C AND 4-6D. BUOYANCY _____

Both PE and QG models are thus capable of generating and maintaining strong surface gradients of buoyancy although the vertical structure of the QG frontal zones is meteorologically unrealistic. Consider now the relative movement of these zones with respect to the low pressure center for the two models. Figure 4-8 presents the 2PE and 2QG frontal zones located with respect to the gridpoint of minimum pressure, for days $4\frac{1}{2}$ and 6. The PE cold front has become more curved and has moved further into the warm air than has the QG cold front, presumably due to the horizontal advection by the divergent component of the wind, missing in the QG model. This same effect advects both PE fronts toward the warm air (figures 4-7a and 4-7c indicate a circulation from cold toward warm air for both fronts) with respect to the QG fronts. The QG model must conserve the average temperature at each level so the occlusion process is retarded; the PE model is able to reduce the area of warm air with respect to cold at the ground.

The QG model does well in locating the fronts with respect to the low center, when compared to the PE model, for runs 2PE and 2QG. Chapter 3 showed that for these runs movement of the surface low was quite similar for the two models out to 6 days. A comparison of runs 1PE and 1QG showed that the QG model also did well with respect to movement of the low center but at day $4\frac{1}{2}$ the developing QG fronts are poorly located, compared to the PE fronts, with respect to the low center (figures 3-2c and 3-3c show the PE surface pressure and buoyancy for this time; figures 3-7c and 3-8c show

----- QG FRONTAL BOUNDARIES
 ————— PE FRONTAL BOUNDARIES

L MARKS LOW PRESSURE CENTER

Fig. 4-8. Location of 2PE and 2QG frontal zones compared to low. K=1.

the QG fields). The basic state used for runs 2PE and 2QG contained a significant nondivergent wind at level 1 while the basic state used for runs 1PE and 1QG had little wind at that level. Advection by the divergent wind component is probably of more relative importance in the latter cases; this may explain why the QG model did more poorly with respect to the PE.

4.4 The effects of friction on PE frontogenesis.

In this section modification of the frontogenesis process by surface friction will be discussed. While friction is not necessary for frontogenesis, its presence should modify developing frontal zones since they become strongest at the surface where frictional effects are most important. As the horizontal resolution of the models is already low compared to the thickness of developed frontal zones, this section will concentrate on the modification by friction of the developing "synoptic" and frontal scale gradients. It will be seen that the crude representation of friction used here produces the expected result of reducing the growth rate of the synoptic scale disturbance and of the associated frontogenesis. In addition, friction causes the cold front to slope more, the warm front to slope less and the vertical wind shear within the cold front to be reduced well below the geostrophic value, as is observed.

Friction was incorporated into the PE model by adding stress terms in the momentum equations only; no change was made in the thermodynamic equation. The terms were of the form $\frac{\partial}{\partial z} \tau_x$ where

$\mathcal{T}_x = A(z) \frac{\partial \vec{v}}{\partial z}$; in finite differences $\frac{\partial \mathcal{T}_x}{\partial z} \rightarrow \frac{1}{\Delta z} (\mathcal{T}_k - \mathcal{T}_{k-1})$
 where $\mathcal{T}_k = \frac{A_k}{\Delta z} (\vec{v}_{k+1} - \vec{v}_k)$. Skin friction was included by
 setting $\mathcal{T}_0 = \mathcal{K} |\vec{v}_1| \vec{v}_1$; after a series of experiments $\mathcal{K}$ was
 taken as .0015. The coefficients A_k were chosen such that A_1
 had a nondimensional value corresponding to a mean coefficient of
 eddy viscosity of $270 \text{ gm cm}^{-1} \text{ sec}^{-1}$, a value suggested by Palmen
 (1955); A_2 was then taken as $\frac{1}{2} A_1$, A_3 as $\frac{1}{5} A_1$ and A_4 through A_{19}
 as $\frac{1}{10} A_1$ and $A_{20} = 0$.

The PE model was run, with and without friction, for basic
 states I and II, using a reduced number of gridpoints per level but
 retaining 20 levels in the vertical. All runs were integrated to
 6 days. For both basic states friction did little to alter the
 movement of the surface patterns but the total eddy kinetic energy
 of the basic state I friction run was reduced after 6 days to 54%
 of the non-friction value whereas the basic state II friction run
 total eddy kinetic energy amounted to 86% of the non-friction value
 after 6 days. Since friction changed the characteristics of the
 cyclone scale evolution less for the basic state II experiments
 these runs will now be discussed. These runs are labelled C-2F and
 C-2 and are described in table 3-1. They possess a horizontal re-
 solution ΔX of $266 \frac{2}{3} \text{ km}$ and ΔY of 160 km with a vertical resolution
 of $\frac{3}{4} \text{ km}$.

Figures 4-9 and 4-10 display the surface pressure and buoy-
 ancy for these runs for day 5; both runs have developed in a similar

a. Run C-2, no friction. 36 mb between H and L.

b. Run C-2F, friction. 26 mb between H and L.

Fig. 4-9. Surface winds and pressure, level K=1, Z=.025. 120 Hours.

Asterisks mark vector tails.

a. Run C-2, no friction. $H=.0454$.
Contour interval corresponds to 1.38°C .

b. Run C-2F, friction. $H=.0466$.
Contour interval corresponds to $.92^{\circ}\text{C}$.

Fig. 4-10. Surface buoyancy, level $K=1$, $Z=.025$. 120 Hours.

manner. The friction has reduced the "noise" behind the cold front while making little change in the warm sector (note the different contour intervals for each of the figures); the extent of the tongue of warm air behind the cyclone center has been greatly reduced. We expect that cross-isobar flow toward lower pressure should increase as a result of friction; this can be seen from figures 4-9a and b where vectors of horizontal wind are plotted for selected gridpoints (the asterisks mark vector tails). The reduction in growth rate of the disturbance due to friction is evident from the reduced surface pressure difference (high minus low) for the friction run; for run C-2F this quantity is about 74% of its value for run C-2.

Friction has also reduced the frontogenesis; this is evident in figures 4-10a and b for the cold front but is not so apparent for the warm front. Plots of the magnitude of the buoyancy gradient for these figures (not shown) bear this out; in the cold front the maximum value is reduced from .290 to .190 and in the warm front the maximum is reduced from .320 to .250. Hoskins and Bretherton (1972) also found that friction brought about a reduction in frontogenesis. They argued that "in the surface boundary layer at a front there is increased convergence but above there is a compensating divergence whose effect is frontolytic". At level 1 run C-2F was found to have higher values of convergence ahead of the fronts (especially the warm front) than for run C-2 while at level 3 the friction run had significantly less convergence ahead of the fronts, compared to the non-friction run.

In addition to reducing the tongue of warm air transported through the cyclone center, friction has increased the realism of the integration by changing the relative slopes of the warm and cold fronts. This can be seen from figures 4-11a and b which display east-west vertical cross sections along $J = 16$, for both runs, at day 5. Since the relative intersection of the section with the frontal zones is about the same for both runs (see figure 4-10), it is apparent that friction has reduced the warm frontal slope while increasing the slope of the cold front (the reduction in strength of the cold front is quite noticeable).

Sanders (1955) found that, near the surface, the vertical wind shear within strong frontal zones was much weaker than the corresponding geostrophic vertical shear while above several thousand feet the values of actual and geostrophic shear were nearly in agreement. The ratio of the magnitude of vertical shear to the horizontal buoyancy gradient was computed at the strongest point of the warm and cold frontal zones, for both runs C-2F and C-2 at day 5, for level 1. In the non-friction case the ratio was approximately unity for both frontal zones. In the friction case the ratio remained at unity for the warm front but fell to less than $1/2$ for the cold front. The question of why the cold front shear should be reduced but not the warm front shear will not be pursued here; the results do show that frictional effects can account for Sanders' finding.

A. RUN C-2, NO FRICTION. MAX LABELED VALUE IS .250.

B. RUN C-2F, FRICTION. MAX LABELED VALUE IS .240.

FIG. 4-11. EAST-WEST BUOYANCY CROSS SECTIONS, J=16, Y=.4833. 120 HOURS.

This section has shown that the addition of a crude representation of friction tends to reduce cyclogenesis and frontogenesis, as expected, and that in certain respects the frontal development becomes more realistic. Friction, at least as modeled here, is unable to reduce significantly the onset of numerical "noise" once strong frontal gradients develop.

4.5 Summary.

This chapter has considered the relationship of quasigeostrophic and non-geostrophic processes important in surface frontogenesis. Attention was focused upon the interaction of these effects on the synoptic scale before sharp frontal zones appear. The evolving field of divergence was seen to exert control over the action of the non-geostrophic processes that ultimately produce realistic frontogenesis. Horizontal advection by the divergent component of the wind was seen to enhance the action of the horizontal deformation field as frontogenesis proceeds.

In addition to concluding that quasigeostrophic processes exert control over frontogenesis, a result indicated by the two-dimensional studies, it was seen that under certain conditions even the quasigeostrophic model itself, given enough horizontal resolution, can generate and advect "frontal zones", producing a reasonable "forecast" of the more complex primitive equations model.

5. ON THE DYNAMICS OF UPPER LEVEL FRONTOGENESIS

This chapter will be concerned with the development of upper tropospheric frontal zones. Theories of upper level frontogenesis were discussed in Chapter 1. This chapter begins with a synoptic example of an upper level frontal zone; similar developments within numerical runs based upon the two major basic states discussed earlier are then presented. The upstream ends of the fronts are seen to be located in regions where horizontal deformation works toward frontogenesis.

Attention is then focused upon one of these integrations. A comparison of corresponding PE and QG model integrations reveals both produce similar fields of vertical motion in the vicinity of the upper level jet. An argument is presented to relate the observed strong sinking motion beneath the jet to the advection of anti-cyclonic relative vorticity, increasing in magnitude with height; based upon this a theory relating the effects of the horizontal deformation to the developing vertical circulation within the upper level frontal zone is presented.

5.1 Upper level frontogenesis in the numerical model.

Figures 5-1 and 5-2 appear through the courtesy of Professor F. Sanders (Sanders, 1967). They show the development of an intense upper level frontal zone associated with an amplifying upper trough

Fig. 5-1. Evolution of atmospheric upper level frontal zone. October 28-30, 1963.

Fig. 5-2. Vertical cross section along east coast of United States.
October 30, 1963. 00Z. See text for explanation.

for the period October 28-30, 1963. Figures 5-1a through 5-1d display contours (dashed lines at 120 m intervals) and isentropes of potential temperature (solid lines at 4°C intervals) on the 500 mb surface at 12 hour intervals. Stippled areas and hatched areas mark horizontal gradients of potential temperature of at least 4°C in 40 km and 8°C in 40 km, respectively. At the time of figure 5-1a a zone of large potential temperature gradient existed from 300 mb to 500 mb with little development below. As time progressed the front weakened at 300 mb, then at 400 mb but developed to great intensity at 500 and 600 mb as it either propagated or was advected downward as low as the 800 mb level. Sanders estimated this speed of descent at about 5 cm sec^{-1} over the 36 hour period; this estimate agreed with diagnostic calculations of particle vertical velocity for the frontal region received from the National Meteorological Center facsimile circuit. The maximum computed downward vertical velocity was located southwest of the front itself. As Sanders describes this situation, "... the region of strong contrast, though moving much more slowly than the wind flow at higher levels, has overtaken and wrapped around the trough downstream from the region of initial formation ... the elongated appearance of the frontal zone on the constant pressure charts probably reflects a horizontal streaming out from a relatively restricted and slow moving locus of frontogenesis."

Figure 5-2 displays a vertical cross section along the east coast of the United States at the time of figure 5-1d. The narrow

zone of large stability is seen to slope southward and downward from the tropopause to the 800 mb level (Solid lines of constant potential temperature are drawn at 4°C intervals). The stippled area marks a relatively dry region, supporting the idea of subsidence being associated with the frontal zone. The letters "T" marking tropopause levels show a distinct jump just north of the jet stream, which is shown by dashed isotachs of speed of the observed east-west wind at 20 kt intervals. The isolines of wind speed and of potential temperature are seen to lie parallel to one another within the frontal zone; this point is discussed below.

The development of the upper level frontal zone presented above is seen to be associated with a region of strong temperature gradient (on constant pressure surfaces) that forms upstream from a trough and advances slowly around the trough; with a region of descent, the maximum descent values being southwest of the frontal zone, and with a "break" or "fold" in the tropopause height and a tendency for isotachs and isentropic surfaces to lie parallel within the frontal zone. Similar findings were reported by other investigators (Chapter 1).

Since fronts are characterized by large vorticity (see figure 5-2) and since potential vorticity is the dot product of the (absolute) vorticity and the entropy gradient, the appearance of large vorticity without unexplainably large values of potential vorticity requires that the large vorticity vector lie more or less parallel to the isentropic surface. In other words, the horizontal

velocity must vary markedly in the direction normal to isentropic surfaces but hardly at all along the isentropic surfaces. This argument applies with less force to stratospheric air which begins, so to speak, with relatively large values of potential vorticity before it might become part of a frontal zone.

The tropopause is marked by a strong gradient of potential vorticity and stability. An examination of these gradients in a vertical cross section can reveal the position of the tropopause relative to a developing jet stream or frontal zone.

Let us now return to data presented in Chapter 3 revealing the numerical development of a phenomenon similar to the upper level frontal zone. Figure 3-21 shows the evolution of the buoyancy field at horizontal level 9, corresponding crudely to the 450 mb level, for the PE integration in experiment II, called 2PE. The region of strongest horizontal buoyancy gradient is seen to first form about midway between the ridge and downstream trough at three days into the forecast (figure 3-21b); as time progresses the region strengthens its gradient and lengthens as the upstream end remains in about the same location relative to the ridge while the downstream end propagates downwind around the trough (figures 3-21c, 3-21d). The maximum dimensional wind speed in the trough at level 9 at $4\frac{1}{2}$ days (see figure 3-23b for the nondimensional wind; quantities are dimensionalized by equations (2-2)) is about 40 m sec^{-1} so a parcel moving at this speed would move from the ridge to the trough, a distance of about 3200 km, in one day, much faster than the downwind propaga-

tion speed of the buoyancy gradient. Our numerical buoyancy gradient evolves and moves horizontally in a manner similar to the synoptic example.

The dimensional temperature difference between the buoyancy isolines in figure 3-21 is 0.92°C (assuming an average surface temperature T_{00} of 300°A in equation(2-2)). The strongest gradient at day 6 (figure 3-21d) in the zone is about 8.3°C in 360 km as compared to over 8°C in 40 km in the synoptic example. Buoyancy gradients in the model do not decrease in width to much less than 3Δ (or 360 km for this integration); for run 2PE the entire north-south tropospheric temperature decrease is 18°C so we should never see a gradient stronger than 18°C in 360 km.

Sanders estimated that the upper level zone he observed was descending at an average speed of 5 cm sec^{-1} over a 36 hour period. Evidence indicating descent of the numerical zone is now presented. In the trough of the developing upper wave for vertical levels 6 through 10 we record the maximum value of $|\nabla_{\text{H}} b| = \left[\left(\frac{\partial b}{\partial x} \right)^2 + \left(\frac{\partial b}{\partial y} \right)^2 \right]^{1/2}$ as we move northward (increase J) through the frontal zone, for selected times (table 5-1). At day $4\frac{1}{2}$ the maximum value of $|\nabla_{\text{H}} b|$ was located at level 8 but at days $5\frac{1}{2}$ and 6 it was located at level 7 (data for day 5 were unavailable). Descent of the zone of one level in 24 hours, or descent at an average speed of 0.9 cm sec^{-1} is thus suggested. Table 5-1 also shows that, in the trough, the zone slopes northward (toward increasing J) with height below level 8; at day 6 the northward slope extends to level 10.

DAYS LEVEL K	4 $\frac{1}{2}$		5 $\frac{1}{2}$		6	
	$ \nabla_H b $	J	$ \nabla_H b $	J	$ \nabla_H b $	J
10	5.076	17	4.752	14	4.536	16
9	6.660	18	7.560	16	7.956	15
8	$\rightarrow$ 7.092	19	8.352	16	8.640	14
7	6.804	18	$\rightarrow$ 8.532	15	$\rightarrow$ 8.820	14
6	6.624	18	8.172	15	8.352	14
	I = 6		I = 12		I = 15	

Table 5-1. Maximum dimensional values of $|\nabla_H b|$ in trough for levels 6 through 10 for various times, Run 2PE. Arrow marks level of maximum $|\nabla_H b|$; units in $^{\circ}\text{C} (360 \text{ km})^{-1}$.

The numerical integration provides values of the instantaneous vertical motion $w \equiv \frac{dz}{dt}$ at every gridpoint at 12 hour intervals. Figure 5-3 displays the buoyancy at level 9 and the vertical motion at the same level (located $\Delta z/2$ or 3/8 km above the buoyancy), for day 6. The region of the frontal zone from the ridge downstream to the trough lies in an area of downward motion; the maximum value of descent occurs in figure 5-3b at $I = 10$, $J = 15$ and has a value of $-.024$ or -3.6 cm sec^{-1} . This location is marked by an x in figure 5-3a; the strongest descent is located on the southwest edge of the zone. The minimum value of downward motion in the region of descent we are discussing has a dimensional value of -1.2 cm sec^{-1} .

Consider now a vertical cross section constructed perpendicular to the frontal zone along line segment AA in figure 5-3a. Isotachs of wind speed $(u^2 + v^2)^{1/2}$ and isentropes of the buoyancy are displayed in figure 5-4a; the vertical motion and values of $|\nabla_h b|$ (for levels 6-10) appear in figure 5-4b.

The largest values of $|\nabla_h b|$ occur in the frontal zone, which is seen to slope northeastward with height. Values of $|\nabla_h b|$ decrease below level 8, an indication that this upper level frontal zone has developed independently of any surface zones. The isotachs of wind speed and the isolines of buoyancy tend to line up in the area of the frontal zone where $|\nabla_h b|$ is largest (figure 5-4b) located just beneath the jet core. The maximum dimensional wind speed at the jet core is 54.3 m sec^{-1} .

a. Buoyancy. $Z=.425$, $H=.0915$. Contour interval corresponds to $.92^{\circ}\text{C}$. X marks strongest descent in Fig. 5-3b.

b. Vertical motion. $Z=.450$. Max $H=.0178$ or 2.7 cm sec^{-1} . Min $L=-.0240$ or -3.6 cm sec^{-1} .

Fig. 5-3. 2PE Buoyancy, Vertical motion, Level K=9. 144 Hours.

FIG. 5-4. CROSS SECTION AA INTERSECTING UPPER FRONT. 2PE. 144 HOURS. $1 \leq K \leq 14$ IS DISPLAYED.

TIC MARKS LOCATE BUOYANCY GRIDPOINTS.

The vertical motion displayed in figure 5-3b contains small scale "noise". Figure 5-4b shows, however, that the vertical motion field is quite coherent in the vertical and any small scale noise, at least in this cross section, is insignificant compared to frontal scale motion. The strongest values of descent in figure 5-4b (corresponding to a dimensional value of -2 cm sec^{-1}) occur below the jet core (marked by a "J") but displaced by almost 2 grid increments (Δ) to the southwest of the point of maximum $|\nabla_{\eta} b|$. (The distance between gridpoints in this NE-SW cross section is $\sqrt{2} \Delta$.)

We will now examine a north-south cross section (again at 6 days) through the trough (marked by TT on figure 5-3a). Figure 5-5 displays the isotachs of east west wind, isentropes of buoyancy and circulation vectors of wind components in the cross section plane. Maximum dimensional value of wind in the jet core is about 97 kts. The upper level zone lies beneath the jet core and the strongest descent again occurs on the warm side of the zone. Also on the warm edge of the zone air is descending at an angle steeper than the isentropes so the buoyancy surfaces are being carried downward in that region. The tendency for the isentropes and isotachs to run parallel to one another in the frontal zone is not as marked as in cross section AA.

Figure 5-6b displays isentropes of potential vorticity as well as isentropes of buoyancy, for the same cross section TT. Figure 5-6a shows a similar cross section taken in the trough at

Fig. 5-5. 2PE North-South cross section. I=15. 144 Hours, Jet trough.

A. 0 HOURS. I=12. MAX PV VALUE =.698.

B. 144 HOURS. I=15. MAX PV VALUE =.778.

FIG. 5-6. 2PE NORTH-SOUTH CROSS SECTIONS. BUOYANCY AND POTENTIAL VORTICITY AT JET TROUGH.

the start of the integration 6 days earlier (figure 3-22b displays isotachs of the wind for this section). From figure 5-6 we see the tropopause is defined both by an increase in stability $\frac{\partial \theta}{\partial z}$ with height and by the gradient of potential vorticity (the region of gradient of potential vorticity clearly located in the stratosphere, from $J = 14$ to $J = 26$ and above $K = 12$ in figure 5-6a, is associated with the variation of relative vorticity of air near the jet). In general, potential vorticity values of 200 can be considered to lie at the base of the stratosphere except where this contour cuts upward into the stratosphere ($J = 10$ to $J = 18$ in figure 5-6a).

A comparison of figures 5-6a and 5-6b reveals the change that has occurred within the trough over the six day period. The tropopause, which originally descended from level $K = 14$ to $K = 10$ in a gentle slope, has become nearly vertical in the region of the jet core (see figure 5-5). The 200 isoline of potential vorticity has moved downward from $K = 9^{1/4}$ (figure 5-6a) to $K = 8$ (figure 5-6b); the tropopause is seen to have "folded" slightly and to have descended slightly in the vicinity of the high level end of the frontal zone. The numerical integration thus lends support to the concept of a folded tropopause discussed in Chapter 1.

Comparing figures 5-2 and 5-5, the atmospheric and the model trough cross sections, the major differences noticeable in the vicinity of the upper level frontal zone are the lack of packing together of the buoyancy surfaces as opposed to the atmospheric frontal zone and the weaker parallelism of the wind and buoyancy

isolines in the numerical experiment. Truncation error due to the lack of sufficient horizontal and vertical resolution may be responsible for the differences, also the time integration scheme tends to damp out small scale features (see appendix). We can only speculate that increased resolution would produce more packing of the buoyancy isolines into the frontal zone, i.e., a more stable zone. This would lead to a stronger parallelism of the wind and buoyancy isolines within the zone as previously discussed.

The evolution of the upper level trough in the synoptic example, from a small amplitude perturbation to an apparently fully deepened "cut off" low, took about 4 days while the eddy kinetic energy was still increasing in the integration after $7\frac{1}{2}$ days. (Charts from the U.S. Weather Bureau's "daily weather map" series, not shown here, were used to determine the growth of the atmospheric example; figure 3-37 shows the eddy kinetic energy versus time for the integration.) The horizontal width of the nearly zonal jet which existed before the atmospheric development began was about 1500 km compared to the 3000 km width of the basic state II jet; this difference may account for the more rapid development of the atmospheric system since the north-south temperature gradient and the jet winds at 500 mb were similar to analogous quantities in basic state II. We know that reduction in width of the jet increases the growth rate of the most unstable perturbation (see section 2.22).

We have compared the upper level frontogenesis produced in run 2PE to an atmospheric example; the numerical and atmospheric phenomena are similar in many respects. The horizontal buoyancy and potential temperature gradients appear to evolve and move in a similar manner. The numerical frontal zone, like the atmospheric example, may be descending, at any rate it is associated with sinking motion which is strongest on the warm edge of the zone; the model has thus reproduced the "indirect circulation" of upper level frontal zones (Chapter 1). We can conclude that this numerical integration, in spite of the limited gridpoint resolution, has well reproduced the phenomenon of upper level frontogenesis.

One naturally wonders how the variation of initial conditions would affect the numerical development of upper level frontogenesis. Fine resolution run 1PE was discussed in Chapter 3; that run, utilizing basic state I, was integrated to $4\frac{1}{2}$ days. No pronounced upper level development was present at that time. Run C-1, a coarse resolution version of run 1PE, was integrated to 6 days of forecast time; this run employed a horizontal resolution of 200 km. There were 20 gridpoints east-west, 32 gridpoints north-south and 20 levels in the vertical. Figure 5-8b displays the buoyancy at level $K = 10$ at the start of the integration; figures 5-7a through c show the pressure, buoyancy and vertical motion at level 10 for day 6. A weak upper level frontal zone has developed in the region southwest of the "cut off" low in the pressure field. Figure 5-7c shows the strongest descent again is located on the southern or warm edge

A. PRESSURE. 4.2 MB BETWEEN CONTOURS.

B. BUOYANCY. $.72^{\circ}\text{C}$ BETWEEN CONTOURS.

C. VERTICAL MOTION. MAX DESCENT $= -3.8 \text{ CM SEC}^{-1}$.
MAX ASCENT $= 3.5 \text{ CM SEC}^{-1}$.

FIG. 5-7. RUN C-1 PRESSURE, BUOYANCY AND VERTICAL MOTION, LEVEL K=10.

144 HOURS. $8 \leq J \leq 26$ IS DISPLAYED.

of the buoyancy gradient. Figure 5-8a is a north-south vertical cross section of the buoyancy taken for this time in the pressure trough; this section shows the sloping, stable zone extending southward from the base of the stratosphere. Unfortunately, potential vorticity values are not available for this cross section.

We thus have examples of upper level frontogenesis in numerical integrations utilizing two different basic states, one of which (basic state I) was barotropically stable to small disturbances while the other (basic state II) allowed perturbations to amplify barotropically as well as baroclinically. One run was initiated with a perturbation possessing an amplitude maximum at the bottom of the channel; for the other run the initial perturbation possessed an amplitude maximum at tropopause level. In both of these runs the initial north-south perturbation wind maximum was set to 5% of the maximum basic state wind. The wavelength of the disturbances varied from 3600 to 4800 km; the width of the basic state jet varied from 3000 to 4000 km. Perhaps we may conclude that the onset of upper level frontogenesis is associated with particular developments in the structure of a given amplifying wave rather than with the particular energy source or with the particular "length" or "width" of the wave. We will pursue this line of reasoning in the next section of this chapter.

5.2 Deformation and upper level frontogenesis. Part I.

Let us review the horizontal pressure and buoyancy plots presented in the previous section. Both of the numerical runs show confluence in the pressure field between the ridge and downstream trough; in addition the upstream end of the region of strengthened buoyancy gradient is located in the same area (see figures 3-20d and 3-21d for run 2PE; figures 5-7a and 5-7b for run C-1). Returning to the synoptic example presented at the beginning of the chapter the height contours in figure 5-1 show confluence downstream from the ridge; in that region the upstream end of the tightest potential temperature gradients appear.

Confluence in the horizontal pressure pattern (or in the height pattern on constant pressure surfaces) is seen to provide a link between the synoptic example and the numerical results. In addition other numerical runs (not shown) utilizing basic state II-N (run C-3) and basic state I with a larger initial perturbation amplitude (run OPE; see table 3-1) exhibited upper level frontogenesis within a region of confluence downstream from the ridge.

In chapter 1 we discussed the role of deformation in theories of frontogenesis. We saw in Hoskins' model (Hoskins, 1971, 1972; Hoskins and Bretherton, 1972) that apparently realistic upper level frontogenesis could be driven by horizontal deformation in the basic state wind field. In that two-dimensional model confluence and horizontal deformation are in effect the same thing. Based

upon Hoskins' results our findings here concerning confluence suggest horizontal deformation may be working to tighten the upper level buoyancy gradient.

The direct action of deformation with respect to frontogenesis was described in precise terms in Chapter 4; we saw that for horizontal flow frontogenesis would be a maximum if the isolines of b were located parallel to the axis of dilatation. Figures 5-9 and 5-10 display the horizontal deformation calculated from the horizontal wind field (not the geostrophic wind) superimposed upon the corresponding buoyancy plot for levels 6 and 9 and for days $4\frac{1}{2}$ and 6 from run 2PE. The length of the tics provides an estimate of the relative magnitude of the deformation field; the orientation indicates the direction of the dilatation axis. Unfortunately the buoyancy pattern is displaced a value one Δ too far to the north but this does not affect the conclusion to be drawn: the total deformation is working to create frontogenesis in the region of the buoyancy ridge and the upstream end of the upper level frontal zone. At the downstream end of the zone the deformation acts to weaken the buoyancy gradient and in between the deformation is either weak, as in the trough, or is oriented at nearly 45 degrees to the buoyancy isolines. The pattern is largely unchanged with height or with time as seen from figures 5-9 and 5-10; the alignment between the dilatation axis and the buoyancy isolines in the upstream region of the buoyancy gradient actually improves from day $4\frac{1}{2}$ to day 6.

a. Level K=6.

b. Level K=9.

Fig. 5-9. 2PE Horizontal Deformation plus Buoyancy. 108 Hours.
 .92°C between contours. B should be plotted one Δy further south.

a. Level K=6.

b. Level K=9.

Fig. 5-10. 2PE Horizontal Deformation plus Buoyancy. 144 Hours.
.92°C between contours. B should be plotted one $\Delta\gamma$ further south.

Figure 5-11 was constructed in order to show better the angle between the buoyancy isolines and the direction of the dilatation axis in the region of the upstream end of the buoyancy gradient in figure 5-10b. The dashed lines are isolines of buoyancy; the solid lines are isolines of the difference in degrees between the dilatation axis and the normal to the buoyancy gradient (which is tangent to the buoyancy isolines). The difference is seen to be less than 45° everywhere within the region of enhanced buoyancy gradient shown in figure 5-11.

Our work, along with that of Hoskins and Bretherton, tends to re-establish the importance of horizontal deformation in the development of upper level frontal zones. The way in which the deformation may interact dynamically with frontogenesis will be considered in section 5.4.

5.3 Quasigeostrophic control of upper level frontogenesis.

Horizontal deformation, or loosely speaking, confluence, cannot provide the entire explanation for upper level frontogenesis. The region of strongest buoyancy gradient in figure 5-10b is located in the general vicinity of the trough but deformation is doing little there. The pressure shows little if any confluence near the trough at that time and level (figure 3-20d). The strongest values of descent, however, do occur near this region (figure 5-3b); the maximum descent is located on the warm edge of the frontal zone (figures 5-3a and 5-3b). This configuration of vertical motion

Fig. 5-11. Angle between axis of dilatation and buoyancy isolines for selected region of fig. 5-10b. Dashed lines are buoyancy isolines.

across the zone contributes to frontogenesis on parcels moving through the zone.

Further insight into the dynamics of upper level frontogenesis depends upon some understanding of the relationship of the vertical motion to the evolving horizontal wind and buoyancy fields. Quasi-geostrophic theory tells us the field of instantaneous vertical motion is that required by a system in order to remain in hydrostatic and near-geostrophic balance. If we can show that the wind field within which the numerical frontal zone is located is close to geostrophic balance (the PE model already assumes hydrostatic balance) then an examination of the comparison integration 2QG, which utilized the quasigeostrophic model, would seem to be profitable.

Figure 5-12a is a horizontal plot of pressure corresponding to the time and level of figure 5-10b. Vectors of the total horizontal wind are drawn for selected gridpoints; the asterisks mark the gridpoints and serve as vector tails. By and large the wind field is well aligned to the isobars. Cross-isobar flow toward lower pressure exists in the region of geostrophic confluence and a flow toward higher pressure is present further downstream toward the trough. Figure 5-12b is drawn for vertical cross section AA (see figure 5-3a) for levels 6-10. It shows the magnitude of the buoyancy gradient $|\nabla_h b|$ in solid lines as well as the magnitude of the "thermal wind imbalance" $|\nabla_h b| - f \left| \frac{\partial \vec{u}}{\partial z} \right|$. Within the

a. Wind and pressure at level K=9.
Asterisks mark vector tails.

b. $|\nabla_h b|$ — and "geostrophic imbalance" $|\nabla_h b| - f \left| \frac{\partial \vec{V}}{\partial z} \right|$ - - -
Cross section AA (shown in fig. 5-12a), levels $6 \leq K \leq 10$.

Fig. 5-12. Evidence of near-geostrophic balance of upper front.
2PE, 144 Hours.

frontal zone (large values of $|\nabla_h \theta|$) the buoyancy gradient exceeds the vertical shear by about 15% while there is a region southwest of the frontal zone where the vertical shear exceeds the buoyancy gradient by about 40%. We will consider figures 5-12a and b to provide evidence of "near-geostrophic balance" within the region of the frontal zone.

Let us now review the comparison of runs 2PE and 2QG made in Chapter 3. Figures 3-20 through 3-24 display the evolution at level 9 of run 2PE while figures 3-27 through 3-31 present the same information for run 2QG. The disturbance evolved and moved similarly in both runs, with some differences. The PE solution developed a single jet (figure 3-20d or 5-12a) while the QG solution developed a jet on either side of the trough (figure 3-27d). The major difference was the development of the upper level frontal zone by only the PE solution (compare figure 3-21d to 3-28d). By day 6 the thickness of the PE buoyancy gradient at level 9 has decreased to 3Δ (figure 3-21d); even by day $7\frac{1}{2}$ the level 9 QG buoyancy gradient has failed to achieve that tightness (figure 3-32b).

From this point comparison of the PE and QG solutions at day 6 will be discussed. That both runs are at similar states of development at this time can be seen from figure 3-34a which shows the perturbation pressure amplitude versus time for both runs. Note the curves for level 10 are nearly identical.

The vertical motion from the QG model was discussed in Chapter

4; a section in the appendix describes the procedure used to obtain this quantity. As seen in Chapter 4 both PE and QG derived vertical motion can contain "noise" due to truncation error. Figure 5-13 presents horizontal plots of PE and QG vertical motion for level 9 at 6 days of integration time. The PE centers of upward and downward motion are stronger than their QG counterparts, yet on the whole the patterns are quite similar with both possessing two centers of descent and one center of ascent.

Next consider the relationship of the vertical motion and the horizontal winds in the vicinity of the upper level jet. Cross sections BB were taken at day 6 for each model more or less parallel to the buoyancy gradient vector and in the region of confluence downstream from the ridge; the locations are indicated for the QG run in Figures 3-27d and 3-28d and in figures 5-3a and 5-12a for the PE run. Cross sections AA, introduced earlier for the PE model, were taken downstream from sections BB and were located in each model about where the confluence is zero and the upper level jet achieves its maximum speed. These locations are also shown in the above figures.

Consider first cross sections BB. Figures 5-14a and b display isolines of vertical motion and isotachs of the geostrophic wind normal to the sections for the PE and QG models, respectively. The pattern of descent relative to the jet is quite similar although the PE values are somewhat stronger in the vicinity of the jet core. In both models the region of strongest descent in the mid- and upper

a. 2PE. Max descent $= -.0240$ or -3.6 cm sec^{-1} .

b. 2QG. Max descent $= -.0125$ or -1.9 cm sec^{-1} .

Fig. 5-13. 2PE, 2QG Vertical motion, Level K=9, Z=.450. 144 Hours.

A. 2PE CROSS SECTION BB.

B. 2QG CROSS SECTION BB.

FIG. 5-14. 2PE, 2QG CROSS SECTIONS BB. VERTICAL MOTION $\times 10^{-3}$ _____
 GEOSTROPHIC WIND NORMAL TO SECTION----- $4 \leq K \leq 15$ IS DISPLAYED.
 TICS MARK W GRIDPOINTS.

troposphere tends to be located beneath the jet.

Figures 5-15a and b display similar data from the two models for vertical cross sections AA, except that the isotachs are for total wind, not geostrophic wind, for the PE section in figure 5-15a. While the PE downward motion is nearly twice as strong as the QG downward motion, both models produce the strongest descent directly beneath the jet core.

Despite the presence of noise in both the PE and QG vertical motion, figures 5-13 through 5-15 show that both models produce generally similar patterns of descending motion for the time discussed above. We conclude that the developing vertical motion pattern is controlled to a large extent by physical processes included within the quasigeostrophic model. This is a significant finding for it says that the "indirect" vertical circulation within upper level frontal zones may be initiated by the processes that lead to vertical motion in the QG model.

5.4 Deformation and upper level frontogenesis. Part II.

Section 5.2 showed that horizontal deformation (confluence) was working toward frontogenesis in the region where the upstream end of the upper level frontal zone was located. Here we try to clarify the role of the deformation in the dynamic process of frontogenesis. The fact that the PE frontal zone is in "near-geostrophic balance" allows us to proceed using quasigeostrophic theory. We will

A. 2PE CROSS SECTION AA. DASHED LINES ARE ISOTACHS OF TOTAL WIND SPEED.

B. 2QG CROSS SECTION AA. DASHED LINES ARE ISOTACHS OF GEOSTROPHIC WIND NORMAL TO SECTION.

FIG 5-15. 2PE, 2QG CROSS SECTIONS AA. VERTICAL MOTION $\times 10^{-3}$ _____

$4 < K \leq 15$ IS DISPLAYED. TICS MARK W GRIDPOINTS.

show that the region of descent beneath the jet core is associated with negative vorticity advection, increasing with height. Quasi-geostrophic reasoning suggests this is the source of the descent and suggests that horizontal deformation, in addition to the kinematic effect of tightening the horizontal temperature gradient, works toward frontogenesis by inducing cross-isobar flow toward lower pressure; this produces increased negative vorticity advection, increasing with height, below the jet core.

Quasigeostrophic reasoning tells us horizontal confluence should be associated with cross-isobar flow toward lower pressure; this should induce a direct circulation (relatively warm air rising, relatively cold air sinking) below the level of the jet core due to continuity considerations (Sawyer, 1956; Eliassen, 1962). The PE integration shows this cross-isobar flow in the region of confluence (figure 5-12a); both PE and QG integrations show the direct circulation (figures 5-14a, 5-14b). These latter two figures show the strong descent directly beneath the jet core superimposed upon the direct circulation.

If deformation is playing a dynamically active role in the frontogenesis then it must be related to this strong descent beneath the jet core. The connection is not obvious; we will first try to determine the origin of this sinking motion. Since both the PE and QG models display similar patterns of descent we again turn to quasigeostrophic theory.

The quasigeostrophic "omega" equation (see Phillips, 1963 section 3c, for example) relates the instantaneous field of vertical motion to the advection of vorticity and thickness (buoyancy in our model) by the geostrophic wind. This suggests we look at vorticity and buoyancy advection in the region of strong sinking beneath the jet for the PE results. Figure 5-16 presents isolines of pressure and absolute vorticity (vertical component of relative vorticity plus earth's vorticity) for level 9 at day 6 for the PE run; a region of strong negative vorticity advection (NVA) by the geostrophic wind is present all the way from the ridge to the trough. This same region is the location of the elongated zone of sinking motion, also shown in the figure. We are interested here in the region of confluence from upstream of cross section BB downstream to about cross section AA (both shown in figure 5-16).

Figure 5-17a presents the vorticity advection

$$-\vec{v} \cdot \nabla_h \left(\frac{\partial \zeta}{\partial x} - \frac{\partial u}{\partial y} + f \right) \quad \text{and thermal advection } -\vec{v} \cdot \nabla_h b \quad \text{for}$$

cross section BB. Vorticity advection is seen to be somewhat "noisy" on the anticyclonic side of the jet (the jet core location is marked by a "J") but there is a definite region of NVA, increasing with height, below the jet core. There is little thermal advection below the jet. Above the core there is a maximum in the horizontal of cold advection. A comparison of figures 5-17a and 5-14a shows the downward vertical motion is generally located where the above mentioned advective effects occur.

Fig. 5-16. 2PE Absolute vorticity, pressure and vertical motion
Level K=9, 144 Hours.

A. CROSS SECTION BB.

B. CROSS SECTION AA.

FIG. 5-17. 2PE VORTICITY AND BUOYANCY ADVECTION. 144 HOURS. $5 \leq K \leq 15$ IS DISPLAYED.

$$-\vec{U} \cdot \nabla_h \left(\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} + f \right) \times 10^{-3} \quad \text{—————} \quad -\vec{U} \cdot \nabla_h b \times 10^{-3} \quad \text{-----}$$

Next observe figure 5-17b which presents the same advectations for cross section AA. A very similar pattern is apparent; a comparison with figure 5-15a shows the downward motion and the advective effects mentioned above are again located in a similar manner.

It is thus reasonable to assume there exists a region of NVA, the magnitude of which increases with height below the jet core, at least for the region between cross sections AA and BB. For this same region there also appears to exist a maximum of cold advection above the jet core with little thermal advection below the jet. In addition a cross section taken in the region of confluence downstream from the ridge at day $4\frac{1}{2}$ (not shown) reveals the same arrangement of vorticity and thermal advection and downward motion relative to the jet core; this arrangement apparently changes little between days $4\frac{1}{2}$ and 6.

The quasigeostrophic "omega" equation relates sinking motion to regions of maximum cold advection and to regions of $\frac{\partial}{\partial z} NVA > 0$. We are interested in the region below the jet core; we want to relate $\frac{\partial}{\partial z} NVA > 0$ to $w < 0$ in that region for the PE model. We first present additional evidence of the "near-geostrophic balance" in the PE frontal zone. Figures 5-18a and 5-18b present the vertical component of relative vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ and the Laplacian of pressure $\nabla_h^2 p$, respectively, for cross section BB for levels 6 through 10, from the PE model. The two patterns

A. 2PE SECTION BB. $\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$

B. 2PE SECTION BB. $\nabla_H^2 \rho$

C. 2QG SECTION BB. $\nabla_H^2 \rho$

FIG. 5-18. 2PE, 2QG CROSS SECTIONS AA, BB,
LEVELS K = 6 - 10. 144 HOURS.

D. 2PE SECTION AA. $|\nabla_H b|$

E. 2QG SECTION AA. $|\nabla_H b|$

are quite similar¹. Based upon this similarity and upon the relative absence of thermal advection beneath the jet core we present an argument relating $\frac{\partial}{\partial z} \text{NVA} > 0$ to $w < 0$. This argument is a variation of one presented by Professor N. Phillips.

We assume there is little thermal advection and that the relative vorticity can be expressed as $\nabla_h^2 \rho$ on a horizontal surface; let this surface be at the level of the strongest winds. In a region of negative vorticity advection $\frac{\partial}{\partial t} \nabla_h^2 \rho < 0$ if the advection is the dominant effect. At the location of strongest NVA $\frac{\partial}{\partial t} \nabla_h^2 \rho$ will be the most negative, hence $\frac{\partial}{\partial t} \rho$ should be the most positive. Consider the pressure pattern at a level below this surface to be similar to the pattern on this surface; this is consistent with our assumption that $-\vec{v} \cdot \nabla_h b$ (thermal advection) is small. At this lower surface we again obtain $\frac{\partial}{\partial t} \rho > 0$ at the location of maximum NVA but if the winds are less strong the magnitude of $\frac{\partial}{\partial t} \rho$ should be less and $\frac{\partial}{\partial z} \frac{\partial \rho}{\partial t} > 0$ in between levels. This requires $\frac{\partial b}{\partial t} > 0$ since $b = \frac{\partial \rho}{\partial z}$. Now if $-\vec{v} \cdot \nabla_h b$ is small b can increase only by the $-w \frac{\partial b}{\partial z}$ term in the thermodynamic equation (since $\frac{db}{dt} = 0$); hence $w < 0$ is required to maintain the near geostrophic balance.

¹ The similarity even extends to the 2 grid interval "noise" appearing at level 10 in both figures. This is reasonable since the wind field is used to derive the pressure field (see appendix section B).

We can now relate the cross-isobar flow toward lower pressure induced by the horizontal deformation to the strong sinking beneath the jet core. The cross-isobar flow increases the negative vorticity advection in the center of the jet; if the cross-isobar flow increases with height below the jet core then the increase in height of NVA due to the cross-isobar flow will be positive and the sinking motion will be enhanced.

A quantitative measurement of cross-isobar flow is given by $-\vec{\omega} \cdot \nabla_{\perp} \rho$; a positive value means flow toward lower pressure. Table 5-2 presents values of $-\vec{\omega} \cdot \nabla_{\perp} \rho$ for a column taken directly beneath the jet core in cross section BB ($I = 36$, $J = 32$, K varies from 6 to 12). The cross-isobar flow does increase in magnitude with height. Since the gradient of vorticity across the jet also increases with height below the jet core (figures 5-18a) we expect that in this region the NVA associated with the cross-isobar flow increases significantly with height.

We thus have a rationale whereby horizontal deformation plays a dynamically active role in upper level frontogenesis. If we can show that the strong sinking motion works to increase $\frac{\partial}{\partial z}$ NVA beneath the jet (which we have tried to argue drives the sinking motion) we would have a "positive feedback mechanism" to drive the frontogenesis. The last section of this chapter considers the role of the vertical motion and does establish such a feedback.

K	$-\vec{v} \cdot \nabla_h p \times 10^{-4}$
6	0.236
7	0.551
8	1.080
9	2.201
10	3.757
11	5.511
12	5.626

Table 5-2. Cross-isobar flow $-\vec{v} \cdot \nabla_h p$ versus height for vertical column directly beneath the jet core in 2PE cross section BB.

I = 36, J = 32, K = 6 - 12.

Terms from equation (5-5):

$$\begin{array}{rcl}
 \frac{\partial u}{\partial x} \left(\frac{\partial b}{\partial x} \right)^2 + \frac{\partial w}{\partial y} \left(\frac{\partial b}{\partial y} \right)^2 & +.000399 & \\
 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{\partial b}{\partial x} \frac{\partial b}{\partial y} & +.000747 & \\
 \left(\frac{\partial w}{\partial x} \frac{\partial b}{\partial x} + \frac{\partial w}{\partial y} \frac{\partial b}{\partial y} \right) \frac{\partial b}{\partial z} & & -.002045
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} +.001146$$

Terms from equation (5-7):

$$\begin{array}{rcl}
 \frac{\partial u}{\partial x} \left(\frac{\partial u}{\partial z} \right)^2 + \frac{\partial w}{\partial y} \left(\frac{\partial w}{\partial z} \right)^2 & +.000885 & \\
 \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{\partial u}{\partial z} \frac{\partial w}{\partial z} & -.000561 & \\
 \frac{\partial u}{\partial z} \frac{\partial b}{\partial x} + \frac{\partial w}{\partial z} \frac{\partial b}{\partial y} & +.000838 & \\
 \frac{\partial w}{\partial z} \left(\frac{\partial v}{\partial z} \right)^2 & & -.001276
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} +.001162$$

Table 5-3. Terms from equations (5-5) and (5-7) evaluated at point M in 2PE cross section AA (fig. 5-19a).

5.5 Ageostrophic effects and upper level frontogenesis.

The QG model reproduces the major features of the PE vertical motion field in the vicinity of the upper frontal zone, namely the overall descent between the ridge and downstream trough and the location of the maximum descent beneath the jet core in the region of confluence downstream from the ridge. There are significant differences; the PE descent in figure 5-15a is nearly twice as strong as the corresponding QG sinking motion (figure 5-15b). The horizontal plots of vertical motion in figure 5-13 show the PE zone of descent to be stronger and more narrow than the QG zone. The PE vertical motion no doubt is enhanced by processes not included within the QG model. In this section we discuss these "ageostrophic" effects, showing how they allow a feedback mechanism to develop within the PE model that can ultimately lead to sharp fronts.

The vorticity and thermodynamic equations can be written as follows for the PE model:

$$\frac{d}{dt} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f \right) = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f \right) \frac{\partial w}{\partial z} + \frac{\partial u}{\partial z} \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \frac{\partial w}{\partial x} \quad (5-1)$$

$$\frac{d}{dt} b = 0 \quad (5-2)$$

and for the QG model (see Chapter 2 for definitions of the operators, etc.):

$$\frac{dg}{dt} \left(\nabla_h^2 p' + f \right) = \frac{\partial w}{\partial z} \quad (5-3)$$

$$\frac{d}{dt} \left(\frac{\partial \rho'}{\partial z} \right) + w N^2 = 0 \quad (5-4)$$

A comparison of these equations reveals that several effects are missing in the QG formulation:

- 1) The QG thermodynamic equation (5-4) neglects the vertical advection of the time and space variable part of the buoyancy $\frac{\partial \rho'}{\partial z}$ (the vertical advection of the "large scale" buoyancy $B(z)$ is included since $N^2 \equiv \frac{d}{dz} B(z)$),
- 2) Advection by the divergent component of the horizontal wind as well as vertical advection is neglected in the $\frac{d}{dt}$ operator compared to the $\frac{d}{dt}$ operator,
- 3) The QG absolute vorticity $\nabla_h^2 \rho' + f$ can be changed only by the divergence $-\frac{\partial w}{\partial z}$; also the latitudinal variation of f is neglected in this process.

Given the vertical motion produced by the QG model, suppose we could "turn on" the "ageostrophic" effects. What would happen with respect to the evolving buoyancy gradient? Consider first point 1). We can derive from equation(5-2) two equations for the change in magnitude of the horizontal buoyancy gradient

$$|\nabla_h b| = \left[\left(\frac{\partial b}{\partial x} \right)^2 + \left(\frac{\partial b}{\partial y} \right)^2 \right]^{1/2} \quad \text{and stability} \quad \frac{\partial b}{\partial z} \quad \text{felt by}$$

a parcel for the PE model:

$$\begin{aligned} \frac{d}{dt} |\nabla_h b| = -|\nabla_h b|^{-1} \left\{ \frac{\partial u}{\partial x} \left(\frac{\partial b}{\partial x} \right)^2 + \frac{\partial v}{\partial y} \left(\frac{\partial b}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \frac{\partial b}{\partial x} \frac{\partial b}{\partial y} \right. \\ \left. + \left(\frac{\partial w}{\partial x} \frac{\partial b}{\partial x} + \frac{\partial w}{\partial y} \frac{\partial b}{\partial y} \right) \frac{\partial b}{\partial z} \right\} \end{aligned} \quad (5-5)$$

$$\frac{d}{dt} \frac{\partial b}{\partial z} = - \frac{\partial u}{\partial z} \frac{\partial b}{\partial x} - \frac{\partial v}{\partial z} \frac{\partial b}{\partial y} - \frac{\partial w}{\partial z} \frac{\partial b}{\partial z} \quad (5-6)$$

Equation (5-5) is essentially the equation for 2-dimensional frontogenesis presented by Miller (1948). We are interested here only in the effect of the terms involving vertical motion. Equations similar to the above for the QG model would include these "vertical" terms but $\frac{\partial b}{\partial z}$ would be replaced by N^2 which is constant in time. The PE model allows the effect of terms multiplied by $\frac{\partial b}{\partial z}$ in equation (5-5) to increase with time.

The buoyancy gradient in the region of interest between the ridge and downstream trough is oriented such that $\frac{\partial b}{\partial x}$, $\frac{\partial b}{\partial y} < 0$, also $\frac{\partial b}{\partial z} > 0$. Figures 5-14b and 5-15b show that the QG vertical motion, at least in the region between QG cross sections AA and BB (shown in figure 3-27d) is such that on the cyclonic side of the jet $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y} > 0$. Thus on the cyclonic side of the jet $|\nabla_{\perp} b|$ would be increased by the orientation of the QG vertical motion.

Figures 5-14b and 5-15b also show that below the region of maximum downward motion located just beneath the jet core $\frac{\partial w}{\partial z} < 0$. Therefore in this region $-\frac{\partial w}{\partial z} \frac{\partial b}{\partial z} > 0$ so by equation (5-6) $\frac{\partial b}{\partial z}$ is increased. "Turning on" the PE effect of the increasing stability would further increase frontogenesis over the QG rate in this region.

These effects are working in the PE model. Figures 5-19a and b

a. Cross section AA. Point M discussed in text.

b. Cross section BB.

Fig. 5-19. 2PE Vertical motion and stability. 144 Hours.

$6 \leq K \leq 12$ is displayed. Tics mark W gridpoints.

$W \times 10^{-3}$ ——— $\frac{\partial b}{\partial z}$ ———

show $\frac{\partial b}{\partial z}$ and w for cross sections AA and BB, respectively, for levels 6-12. Section BB in the region of confluence shows that beneath the jet core (levels 6-9) $\frac{\partial w}{\partial z} < 0$ where $\frac{\partial b}{\partial z}$ has a relative maximum. Further downstream in section AA the same pattern is observed; in addition the region of larger stability is also the region where w is working via the "indirect circulation" to increase $|\nabla_h b|$. Figure 5-18d shows $|\nabla_h b|$ for section AA and figure 5-18e presents a similar plot for the QG section AA; the PE buoyancy gradient is seen to be stronger and less wide.

A parcel thus experiences an increase in $|\nabla_h b|$ as it moves through the region where this indirect circulation operates, other effects notwithstanding. If the flow surrounding the parcel is to remain in near-geostrophic balance the parcel must also undergo an increase in vertical wind shear; quasigeostrophic reasoning shows this can be done by the vertical variation of the horizontal acceleration induced by cross-isobar flow. Within the confluent region table 5-2 showed that the cross-isobar flow toward lower pressure did increase with height as would be required for this effect to work.

There appears to be little cross-isobar flow in the region of cross section AA (see figure 5-12a for level 9; other levels are similar) where we saw the vertical motion is working to increase $|\nabla_h b|$. The vertical wind shear for parcels flowing through this region can still be increased by the "vertical convergence" of the shear expressed by the term $\frac{\partial w}{\partial z} \left(\frac{\partial V}{\partial z} \right)^2$ in the following equa-

tion (derived from the PE momentum equations):

$$\begin{aligned} \frac{d}{dt} \left| \frac{\partial V}{\partial z} \right| = - \left| \frac{\partial V}{\partial z} \right|^{-1} \left\{ \left(\frac{\partial u}{\partial z} \right)^2 \frac{\partial u}{\partial x} + \left(\frac{\partial v}{\partial z} \right)^2 \frac{\partial v}{\partial y} + \frac{\partial u}{\partial z} \frac{\partial v}{\partial z} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right. \\ \left. + \frac{\partial w}{\partial z} \left(\frac{\partial V}{\partial z} \right)^2 + \frac{\partial u}{\partial z} \frac{\partial b}{\partial x} + \frac{\partial v}{\partial z} \frac{\partial b}{\partial y} \right\} \quad (5-7) \end{aligned}$$

where $\left| \frac{\partial V}{\partial z} \right| = \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right]^{1/2}.$

Below the jet core $\frac{\partial w}{\partial z} < 0$ in cross section AA (figure 5-19a); this allows a parcel to increase both $|\nabla_H b|$ and $\left| \frac{\partial V}{\partial z} \right|$ without significant cross-isobar flow. Table 5-3 shows nondimensional values of the terms in equations (5-5) and (5-7) evaluated at location M within the frontal zone in figure 5-19a. No attempt was made to evaluate the left hand side of these equations; the significant point is that in both equations only the vertical motion effects are working to increase $|\nabla_H b|$ and $\left| \frac{\partial V}{\partial z} \right|$ for a parcel at point M.

Consider next point 2), the advection by the horizontal, divergent wind missing in the QG model. This is the cross-isobar flow discussed in the last section; we saw it allows for increased anti-cyclonic vorticity advection (increasing with height) across the jet that may further enhance the sinking beneath the jet.

We turn now to point 3). Between the ridge and downstream trough the upper level jet is oriented NW-SE so that $\frac{\partial u}{\partial z} > 0$, $\frac{\partial v}{\partial z} < 0$ below the jet core. Given the QG vertical motion in

figures 5-14b and 5-15b the "twisting terms" in the PE vorticity equation (5-1) would act to generate negative relative vorticity (the vertical component) on the anticyclonic side and positive relative vorticity on the cyclonic side of the jet, thus enhancing the vorticity gradient across the jet and tightening up the jet. The stronger PE vertical motion (figures 5-14a and 5-15a) operates in this manner. A comparison of the PE and QG vertical components of relative vorticity (figures 5-18a and 5-18c) in cross section BB within the confluent region does show the PE gradient of vorticity across the jet to be the stronger, an indication that the above effect is working in the PE model.

Figure 5-4b shows both w and $|\nabla_{\perp} \zeta|$ for cross section AA from the PE run. The region of large negative $\frac{\partial w}{\partial z}$ (beneath the jet core) which would decrease the vorticity by action of the "divergence" term in equation (5-1) is located more toward the anticyclonic side of the jet; the frontal zone (region of large $|\nabla_{\perp} \zeta|$) thus would not lose vorticity. Reed and Sanders (1953) found that the "twisting terms" of the vorticity equation had increased the relative vorticity while the "divergence" term had only slightly decreased the relative vorticity of a parcel whose trajectory terminated within an upper level frontal zone at 500 mb. Shapiro (1970) obtained similar results.

Having considered the role of these "ageostrophic" effects contained in the PE but not in the QG model, we can see a "positive feedback mechanism" that allows the PE model to effect upper level

frontogenesis:

a) In the region of the jet where the horizontal confluence is present on a large scale the cross-isobar flow toward lower pressure produces an advection of negative vorticity, increasing with height ($\frac{\partial}{\partial z} NVA > 0$) across the jet.

b) This enhances the sinking motion which is strongest beneath the jet core (already present from the downstream $\frac{\partial}{\partial z}(NVA) > 0$).

c) This pattern of sinking motion increases the vorticity gradient across the jet, thus enhancing the $\frac{\partial}{\partial z}(NVA) > 0$ across the jet.

Frontogenesis in the PE model is thus driven both by the horizontal confluence or deformation effect and by the increasingly strong gradient of vertical motion on the cyclonic side of the jet. The confluence makes the feedback possible by providing the cross-isobar flow across the jet.

Consider the results of Hoskins' (1971) "2 region" deformation model of upper level frontogenesis. In this model a velocity field of pure horizontal deformation given by $u = -C y$, $v = C x$ (where C is constant) acts upon a potential temperature field independent of y . All components of velocity which arise in addition to the deformation field are also required to be independent of y . Hoskins' figure 9, reproduced as our figure 5-20 shows the initiation of tropopause folding and upper level frontogenesis.

Hoskins' model assumes the wind component parallel to the front to be in geostrophic balance with the pressure gradient across

Fig. 5-20. After Hoskins (1971).

the front. The cross-isobar flow toward lower pressure associated with the deformation should cause a direct circulation beneath the level of the jet core. This is seen to be present as indicated in figure 5-20 by the particle motion vectors. In addition there is sinking at and below the jet core; this sinking may be associated with the cross-jet advection of anticyclonic vorticity that would be strongest at jet core level.

In our fully three-dimensional models curvature effects provide downstream $\frac{\partial}{\partial z}(NVA) > 0$ and this allows for additional sinking to be present beneath the jet. Curvature effects can thus enhance the action of sinking motion in the two-dimensional Hoskins' model.

Earlier in this section we presented an argument tying the sinking motion to the observed $\frac{\partial}{\partial z}(NVA) > 0$; this argument required that in the region of the frontal zone there be insignificant thermal advection and that the zone be in near-geostrophic balance. Both conditions are met within the PE model frontal zone. The near-geostrophic balance of the zone may be seen as being a dynamical requirement of the conservation of potential vorticity, as follows. As the vertical motion increases the three-dimensional stability across a parcel flowing through the region, the component of the parcel's vorticity parallel to the stability vector must decrease in order to conserve the parcel's potential vorticity. The vertical motion also increases the parcel's vertical component of vorticity by the action of the vorticity equation "twisting terms"; this would work to increase the component of vorticity in the direction of the stability

vector. Only if the parcel's vertical wind shear increases at the same time can the parcel's horizontal component of vorticity increase, working to decrease the component of vorticity in the direction of the stability vector. An increase in vertical wind shear occurring simultaneously with an increase in horizontal temperature gradient works in the direction of thermal wind balance.

5.6 Summary.

In this chapter evidence of upper level frontogenesis within a numerical integration of the "primitive" equations was presented. Horizontal deformation was shown to be working toward frontogenesis in the upstream region of the zone and the distribution of vertical motion in the vicinity of the associated upper level jet was seen to be similar to that produced by a simpler quasigeostrophic model. The region of strongest descent was also the region of anticyclonic vorticity advection, increasing in magnitude with height. An argument relating the two was presented; based upon the assumed relation a mechanism for upper level frontogenesis was outlined.

The following picture of the development of an upper level frontal zone is suggested. With the amplifying upper level disturbance is associated a region of confluence where the horizontal deformation is working to tighten the flow. This region probably lies somewhere from near the ridge downstream toward the trough of the amplifying wave. Parcels moving through this region experience frontogenesis by the horizontal deformation; at the same time the

the downstream advection of anticyclonic vorticity, increasing with height, sets up a zone of sinking motion which is strongest beneath the jet core. This increases the action of frontogenesis on parcels located on the cyclonic side of the jet by the "indirect circulation" effect. This arrangement of vertical motion also enhances the cross-jet vorticity gradient via the "twisting terms" in the vorticity equation.

In addition, cross-isobar flow toward lower pressure is induced by the confluence. This increases the cross-jet anticyclonic vorticity advection, again increasing in magnitude with height, further strengthening the sinking beneath the jet. This adds to the effect of the "twisting terms", further enhancing the cross-jet vorticity gradient plus increasing the frontogenetical action of the indirect circulation on the cyclonic side of the jet.

Once the confluence is set up this "positive feedback mechanism" should produce a tight frontal zone around a parcel within a short time. Parcels moving through the region of confluence and located on the cyclonic side of the jet below the level of maximum wind would thus experience frontogenesis within the region and then flow downstream, carrying the frontal zone toward the trough. Hence the picture of frontogenesis described here fits Sanders' (1967) concept of parcels "streaming out from a relatively restricted and slow moving locus of frontogenesis" quoted at the beginning of this chapter.

In this picture of upper level frontogenesis the folding and descent of the tropopause appears as a passive result of the process. The more intense the frontogenesis the greater the descent of the tongue of stratospheric air beneath the jet and the greater the probability of finding stratospheric air in the frontal zone. The tightening of the jet and the formation of the front both occur simultaneously as part of the same process.

All the elements of the process discussed above are present to some extent within the PE model solution even though the lack of resolution removes the possibility of seeing gradients as tight as those observed in atmospheric upper level frontal zones. We have not discussed developments within the cyclonically curved flow as parcels move toward and around the trough. Within this region the frontal zone (having formed upstream if our view of frontogenesis is correct) is strengthened or weakened as the parcels carrying the zone downstream interact with the surrounding flow. We would probably learn little about this interaction as the PE model never really formed a tight zone in the first place.

We concluded in Chapter 4 that surface frontogenesis tends to evolve in a manner controlled by quasigeostrophic processes until ageostrophic circulations become significant and effect the development of a realistic frontal zone. The results of this chapter indicate that the same interpretation can be made for upper level frontogenesis.

Finally, the ageostrophic cross-isobar advection induced by the deformation may be of interest for other than frontogenetical reasons; a scheme is presented in the appendix, section F, relating the cross-isobar flow to the observed preponderance of NE-SW tilted troughs in mid-latitude westerly flow.

6. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

Concerning the numerical techniques used here, it is concluded that the primitive equations model with its simplified physics is able to generate both surface and upper level frontal zones from initially broad baroclinic regions. These zones decrease in width to limiting values based upon the model numerics. The model is able to maintain the sharp frontal zones in time, although numerical "noise" tends to develop, particularly at the lowest horizontal level. The addition of an explicit friction process to simulate the atmospheric boundary layer somewhat increases the realism of the surface frontal zones but makes little difference in the generation of the "noise".

Concerning surface frontogenesis:

It is concluded that within the growing baroclinic wave the evolving field of divergence exerts control over the action of processes not found in the quasigeostrophic model, processes that ultimately lead to the formation of strong frontal zones.

It is concluded that as the frontal zones strengthen, the action of the horizontal deformation is enhanced by horizontal advection by the divergent wind component. In this and other simplified primitive equations models the convergence is much stronger in the vicinity of the warm front with the result that warm fronts tend to form earlier and become more pronounced than cold fronts.

Concerning upper level frontogenesis:

It is concluded that upper level frontogenesis is probably a common process in that both initial conditions used here generated upper tropospheric frontal zones. The two amplifying waves were fed by different energy sources (one was barotropically damped; the other was growing barotropically as well as baroclinically) and possessed different structural characteristics such as zonal wavelength and initial height of the perturbation streamfunction amplitude maximum.

It is concluded that in both cases of upper level frontogenesis the horizontal deformation was working to tighten the upstream end of the zone. These findings are in agreement with those of Hoskins and Bretherton (1972).

It is concluded that the "indirect" circulation, that is, the stronger sinking on the warm side of the frontal zone compared to the cold side, is also an important frontogenetic factor. This strong sinking motion occurs beneath the jet in both the quasigeostrophic and primitive equations integrations, indicating its presence can be explained by "quasigeostrophic" processes.

It is concluded that, at least in the one case of numerical upper level frontogenesis studied here, the advection of anticyclonic vorticity, increasing with height, is important in the formation and strengthening of the region of sinking motion. A mechanism relating the horizontal deformation to the intensifying indirect circulation is postulated, based upon cross-isobar flow toward lower pressure.

This mechanism provides a "positive feedback" needed to produce strong upper level frontal zones; it shows the importance of horizontal advection by the divergent wind in upper level as well as in surface frontogenesis.

Concerning the performance of the quasigeostrophic model with respect to the more complex primitive equations model, it is concluded that, at least for the simple initial conditions used here, the quasigeostrophic model is able to "forecast" fairly well the movement of amplifying waves and in one case the formation and movement of surface frontal zones.

6.1 Suggestions for future research.

More work needs to be done on the "prediction" problem, that is, better understanding of where frontal zones will form, given certain initial conditions. First we will have to know more about how an initially two-dimensional surface temperature field gets transformed into an elongated zone where one dimension dominates, or in other words "why do fronts form in lines?" The quasigeostrophic equations probably contain sufficient nonlinearity for such studies.

While the importance of surface frontal zones is obvious in forecasting, the role of upper level frontal development upon subsequent atmospheric evolution is not well known. If the presence of such zones helps to determine the location of mid-level cyclogenesis, for example, they will have to be included in initial data

as well as in the forecasts of the longer range prediction models. Certainly a better understanding of the dynamics of such zones will aid in better predicting their occurrence. Our "positive feedback" mechanism may be only one of many possible.

The manner in which the primitive equations maintain a near-geostrophic balance within the evolving frontal zones, both surface and upper level, is of interest. The "noise" present in the fields of wind and buoyancy may be indicative of the system undergoing geostrophic adjustment by the action of gravity-inertia waves. The way in which this occurs may be discernible from a two-dimensional model of frontogenesis similar to that of Williams (1972), with careful attention given to the action and energetics of the high frequency motions. Continuing advances in the general problem of geostrophic adjustment will shed light on this area.

It is certainly worth investigating further the forecasting skill of a fine resolution quasigeostrophic model. Given a certain amount of computer time in which to produce an "operational" forecast, a simple quasigeostrophic model possessing the maximum possible resolution (for the available computer time) may well produce a forecast as good as a more coarse resolution primitive equations model; in addition the initialization procedure is much simpler for the quasigeostrophic model. Use of real initial data would remove the symmetric appearance of the quasigeostrophic integrations.

Appendix A. Determination of the basic state.

The purpose of this section is to find the basic state functions $u(y,z)$, $b(y,z)$ and $H(y)$, the zonal wind, buoyancy and tropopause height. The channel width and height vary between 0 and 1. In the troposphere we want u to vary as a sine squared in y and to increase linearly in z . In order to find an analytic expression for $H(y)$ we specify $u(y,z=1)$ to vary in y in a similar manner. At the center of the channel ($y = \frac{1}{2}$) we want u to decrease with height above the tropopause.

The following parameters are specified: N_T^2 and N_S^2 , the (constant) tropospheric and stratospheric stabilities ($N_S^2 > N_T^2$); H_0 , the tropopause height at $y = \frac{1}{2}$; $U_0 = u(\frac{1}{2}, 0)$; $U_M = u(\frac{1}{2}, H_0)$ and $S = \frac{U_M - U_0}{H_0}$, the tropospheric vertical wind shear. $\mathcal{K}$ will be defined later. The Coriolis parameter $f = 1$. In addition let Y_0, Y_1 be constants such that $0 \leq Y_0 < \frac{1}{2} < Y_1 \leq 1$; for a symmetrical jet below the tropopause we require $\frac{1}{2} - Y_0 = Y_1 - \frac{1}{2} \equiv \Delta$. For $0 \leq Y \leq Y_0$ and $Y_1 \leq Y \leq 1$ we set $u = 0$ for all z .

The following derivation of $H(y)$ is for $Y_0 \leq Y \leq Y_1$. We set $H(y) = H(Y_0)$ for $0 \leq Y \leq Y_0$ and $H(y) = H(Y_1)$ for $Y_1 \leq Y \leq 1$.

Let $\Theta \equiv \pi \left(\frac{Y - Y_0}{Y_1 - Y_0} \right)$; $0 \leq \Theta \leq \pi$. We define u in the tropopause as

$$u_T(\Theta, z \leq H) = (U_0 + Sz) \sin^2 \Theta \quad (A-1)$$

The thermal wind relationship gives us, where b_T is the buoyancy in the troposphere;

$$\frac{\partial b_T}{\partial y} = - \frac{\partial u_T}{\partial z} = - S \sin^2 \theta$$

Using N_T^2 we can thus write

$$\begin{aligned} b_T(\theta, z \leq H) &= N_T^2 z - S \int \sin^2 \theta dy = N_T^2 z - \frac{2S\Delta}{\pi} \int \sin^2 \theta d\theta \\ &= N_T^2 z - \frac{2\pi S}{\Delta} \left[\frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right] + C. \end{aligned}$$

Let $b_T(1/2, 0) = 0$; this determines C. Next define

$$G(\theta) \equiv \frac{1}{\pi} \left(\theta - \frac{1}{2} \sin 2\theta \right) - 1/2 ; \quad (A-2)$$

$$b_T(\theta, z \leq H) = N_T^2 z - S\Delta G(\theta). \quad (A-3)$$

Now $H(\theta)$ is the tropopause height; b_s, u_s are the stratospheric buoyancy and zonal wind, respectively. $b_s = b_T(H) + (z-H)N_s^2$ or

$$b_s(\theta, z \geq H) = N_T^2 H(\theta) + [z - H(\theta)] N_s^2 - S\Delta G(\theta). \quad (A-4)$$

Again using thermal wind balance

$$\frac{\partial u_s}{\partial z} = - \frac{\partial b_s}{\partial y} = - \frac{\pi}{2\Delta} \frac{\partial b_s}{\partial \theta} = \frac{\pi}{2\Delta} \left[(N_s^2 - N_T^2) \frac{dH}{d\theta} + S\Delta \frac{dG}{d\theta} \right]$$

We can write $u_s = u_T(H) + (z-H) \frac{\partial u_s}{\partial z}$ so that

$$u_s = [u_0 + SH(\theta)] \sin^2 \theta + [z - H(\theta)] \frac{\pi}{2\Delta} \left[(N_s^2 - N_T^2) \frac{dH}{d\theta} + S\Delta \frac{dG}{d\theta} \right]. \quad (A-5)$$

From this we can get

$$\frac{1}{S} u_s(\theta, z=1) = \left(1 + \frac{U_0}{S}\right) \sin^2 \theta - \frac{(N_s^2 - N_T^2) \pi}{4 S \Delta} \frac{d}{d\theta} (1-H)^2.$$

We now set $u_s(\theta, z=1) = \lambda S \sin^2 \theta$; λ is determined below.

Defining $A \equiv 2(1 + \frac{U_0}{S} - \lambda) S \Delta / (N_s^2 - N_T^2) \pi$ we have

$$\frac{d}{d\theta} (1-H)^2 = 2A \sin^2 \theta \quad \text{or} \quad (1-H)^2 = A \left[\theta - \frac{1}{2} \sin 2\theta \right] + C.$$

Since $H_0 = H(\theta = \pi/2) = H(\gamma = 1/2)$, $C = (1-H_0)^2 - \frac{\pi A}{2}$;

$$H(\theta) = 1 - \left[\pi A G(\theta) + (1-H_0)^2 \right]^{1/2}. \quad (\text{A-6})$$

We want $u_s(\gamma, z=1)$ to reach a maximum value equal to $\mathcal{R}$ times the maximum value of u at the tropopause height; that is, $\mathcal{R}$ will be the parameter specified instead of λ . In order to have

$$\frac{\partial u_s}{\partial z} < 0 \quad \text{we require} \quad \mathcal{R} < 1. \quad \text{Hence} \quad u_s(\gamma = 1/2, z=1) =$$

$$\mathcal{R} u_s(\gamma = 1/2, z = H_0), \quad \text{or}$$

$$\lambda S = \mathcal{R} (U_0 + S H_0), \quad \lambda = \frac{\mathcal{R} H_0 U_M}{(U_M - U_0)}.$$

Thus

$$A = \frac{2 \Delta [U_M (1 - \mathcal{R} H_0) - U_0 (1 - H_0)]}{H_0 \pi (N_s^2 - N_T^2)}, \quad \mathcal{R} < 1. \quad (\text{A-7})$$

Next we want to find a streamfunction p' such that $u = -\frac{\partial p'}{\partial \gamma}$,

$$b = \frac{\partial p'}{\partial z}. \quad \text{We have } b \text{ from (A-3), (A-4). Let}$$

$$\bar{b}(z) = \frac{1}{2\Delta} \int_{\gamma_0}^{\gamma_1} b \, d\gamma \quad (\text{A-8})$$

$$\bar{p}(z) = \int_0^z \bar{b} dz \quad (A-9)$$

Then $b_T = \bar{b} + b'_T$ so $b'_T = b_T - \bar{b}$ or $b'_T = N_T^2 z - S\Delta G - \bar{b}$.

Similarly $b'_S = N_T^2 H + (z-H)N_S^2 - S\Delta G - \bar{b}$.

Now $p' = \int_0^z b' dz$ so

$$\begin{aligned} p'_T &= \int_0^z b'_T dz = \int_0^z (N_T^2 z - S\Delta G) dz - \int_0^z \bar{b} dz \\ &= N_T^2 \frac{z^2}{2} - zS\Delta G - \bar{p}(z) + g_1(\theta). \end{aligned}$$

Since $u_T = -\frac{\partial p'_T}{\partial y} = zS\Delta \frac{\partial G}{\partial y} - \frac{\partial g_1}{\partial y} = zS \sin^2 \theta - \frac{\pi}{2\Delta} \frac{dg_1}{d\theta}$

and since $u_T = (U_0 + zS) \sin^2 \theta$ we must have

$$-\frac{\pi}{2\Delta} \frac{dg_1}{d\theta} = U_0 \sin^2 \theta ;$$

$$g_1 = -\frac{2U_0\Delta}{\pi} \int_0^\theta \sin^2 \theta d\theta = -\frac{2U_0\Delta}{\pi} \left(\frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right) ; \text{ using}$$

$$(A-2) \quad g_1 = -\frac{1}{2} U_0 \Delta (2G+1).$$

Thus,

$$p'_T = N_T^2 \frac{z^2}{2} - zS\Delta G - \frac{1}{2} U_0 \Delta (2G+1) - \bar{p}(z). \quad (A-10)$$

Next,

$$\begin{aligned}
 p'_S &= p'_T(z=H) + \int_H^z b'_S dz \\
 &= N_T^2 \frac{H^2}{2} - HS\Delta G - \bar{p}(z=H) + \int_H^z [N_T^2 H + (z-H)N_S^2 - S\Delta G] dz \\
 &\quad - \int_H^z \bar{b} dz \\
 &= N_S^2 \frac{z^2}{2} - zS\Delta G - (N_S^2 - N_T^2)H(z - \frac{H}{2}) - \bar{p}(z) + g_2(\theta) .
 \end{aligned}$$

When $z = H$ we must have $p'_S = p'_T$ so that

$$\begin{aligned}
 p'_S(H) &= N_S^2 \frac{H^2}{2} - HS\Delta G - (N_S^2 - N_T^2) \frac{H^2}{2} - \bar{p}(H) + g_2(\theta) \\
 &= N_T^2 \frac{H^2}{2} - HS\Delta G - \bar{p}(H) + g_2(\theta) .
 \end{aligned}$$

Thus

$$g_2 = -\frac{1}{2} U_0 \Delta (2G+1) :$$

$$\begin{aligned}
 p'_S &= N_S^2 \frac{z^2}{2} - zS\Delta G - (N_S^2 - N_T^2)H(z - \frac{H}{2}) - \frac{1}{2}U_0\Delta(2G+1) \\
 &\quad - \bar{p}(z) .
 \end{aligned}
 \tag{A-11}$$

Appendix B. The primitive equations (PE) model.

The vertical interval $0 \leq z \leq 1$ is divided into K layers;

$\Delta z = 1/K$. Define u, v, b and p at midpoint of each layer;

w at boundaries of each layer. Let $k = 1, 2, \dots, K$.

$$\begin{array}{lcl}
 z=1 & \text{//////} & w_K = 0 \\
 \\
 z=k/K & \text{-----} & w_k \\
 & \text{-----} & u_k, v_k, b_k, p_k \\
 z=k-1/K & \text{-----} & w_{k-1} \\
 & & \left. \vphantom{\begin{array}{c} w_k \\ u_k, v_k, b_k, p_k \\ w_{k-1} \end{array}} \right\} \text{ LAYER } k \\
 \\
 z=0 & \text{//////} & w_0 = 0
 \end{array}$$

Writing the equations in "flux form" we have, where ∇_H is the horizontal gradient operator and $\vec{v}$ is the horizontal velocity, for $k = 1, 2, \dots, K$:

$$\begin{aligned}
 \frac{\partial u_k}{\partial t} &= -\nabla_H \cdot (\vec{v}_k u_k) - \frac{K}{2} [w_k(u_{k+1} + u_k) - w_{k-1}(u_k + u_{k-1})] - \frac{\partial p_k}{\partial x} + f v_k \\
 \frac{\partial v_k}{\partial t} &= -\nabla_H \cdot (\vec{v}_k v_k) - \frac{K}{2} [w_k(v_{k+1} + v_k) - w_{k-1}(v_k + v_{k-1})] - \frac{\partial p_k}{\partial y} - f u_k \\
 p_k &= p_{k-1} + \frac{1}{2K} (b_k + b_{k-1}) \\
 w_k &= w_{k-1} - \frac{1}{K} \nabla_H \cdot \vec{v}_k \quad ; \quad w_0 = 0 \quad \text{and} \quad w_K = 0 \\
 \frac{\partial b_k}{\partial t} &= -\nabla_H \cdot (\vec{v}_k b_k) - \frac{K}{2} [w_k(b_{k+1} + b_k) - w_{k-1}(b_k + b_{k-1})]
 \end{aligned}$$

The continuous set of equations (2-1) conserves the total

energy $E = \frac{1}{\gamma} \int_V \left[\frac{1}{2} (u^2 + v^2) - z b \right] dV$, total buoyancy, total momentum and total potential vorticity. The following finite-difference forms are conserved if one treats $\partial/\partial t$ and ∇_H as continuous operators:

$$E = \frac{1}{\ell} \int_0^\ell \int_0^1 \sum_{k=1}^K \left[\frac{1}{2} (u_k^2 + v_k^2) - \frac{(2k+1)}{2K} b_k \right] \Delta z dy dx ,$$

$$M = \frac{1}{\ell} \int_0^\ell \int_0^1 \sum_{k=1}^K u_k \Delta z dy dx ,$$

$$B = \frac{1}{\ell} \int_0^\ell \int_0^1 \sum_{k=1}^K b_k \Delta z dy dx , \quad \ell = \text{channel length.}$$

No such analogue for potential vorticity has been found. The variation of the above quantities with time for run 2PE will be shown later in this appendix.

The numerical process is essentially a two step Lax-Wendroff procedure using the staggered Eliassen grid representation of the three basic variables u, v and b . It is an extension to three space dimensions of the two-dimensional scheme described by Phillips (1962). Basically, u, v and b are evaluated for the half time step by doing an "uncentered-difference", upstream trajectory computation utilizing u, v and b at the previous full timestep. This half time-step procedure damps all wavelengths, short waves more rapidly than longer waves. The effect of vertical motion is included in the computations at all levels including levels $k=1$ and K . A centered-difference formulation is then used to evaluate u, v and b for the

next full timestep, with the equations in "flux" form. The staggered grids vary at the half and full timestep and are shown in figure B-1.

The coupling between the variables at the half and full timestep eliminates the "computational wave" found in calculations using centered-differencing in time. In addition, the upstream-differencing step, with its implicit damping, the use of the "flux" form of the equations at the full timestep and the variation of the spatial grids at the half and full timesteps apparently are sufficient to eliminate the occurrence of nonlinear instabilities since none are observed (Phillips, 1959, 1962; Bryan, 1966; Gerrity, 1972).

The following criterion for computational stability applies¹:

$$\frac{\Delta T}{\Delta} (|\vec{u}|_{\max} + \sqrt{2} C_{\max}) < 1$$

where $\Delta = \min(\Delta x, \Delta y)$ and $C_{\max}^2 = \left\{ \frac{d\bar{b}}{dz} / (4K^2 \tan^2[\frac{n\pi}{2K}]) \right\}_{\max}$;

$n = 1, 2, \dots, K-1$. C will be a maximum for $n=1$. For run 2PE we have $K = 20$, $\Delta = 1/40$; initially $|\vec{u}|_{\max} \sim 0.1$ and

$\frac{d\bar{b}}{dz} \sim 0.3$ so $\Delta T < \sim .07$ is required. We have used the conservative value of $\Delta T = .03$ for the integration.

¹ This criterion was determined experimentally from research not conducted by the author.

Fig. B-1. Half and full timestep grids.

HALF TIMESTEP $t_0 + \Delta t/2$

FULL TIMESTEP $t_0, t_0 + \Delta t$

w'_k, w_k ARE $\Delta z/2$ ABOVE OTHER VARIABLES.

i RUNS FROM 1 TO I, j FROM 1 TO J.

Fig. B-2. $W(21,16,2)$ versus time, steps 2140 - 2159.

Boundary conditions (see figure B-1). East-west cyclic.

South wall, $y=0$ (north wall $y=1$ similar), half timestep:

u'_{i1k} computed from u'_{i2k} by requiring relative vorticity be same on boundary as at first interior row

$$v'_{i1k} = 0$$

$$w'_{i1k} = w'_{i2k}$$

$$b'_{i1k} \text{ FORECAST}$$

$$\psi'_{i1} = 0.$$

Full timestep:

$$u_{i1k} \text{ forecast}$$

$$v_{i1k} = -v_{i2k}$$

$$w_{i1k} \text{ computed similarly to interior } w's$$

$$b_{i1k} \text{ computed from thermal wind relation } \frac{\partial b}{\partial y} = -f \frac{\partial u}{\partial z}$$

(which is exact at $y=0, 1$ since $v=0$)

$$\psi_{i1} = -\psi_{i2} \text{ } (\psi \text{ explained below}).$$

The procedure used to obtain b_{i1k} is complicated by the fact that b_k and u_k are at the same vertical level. Integrating the finite-difference thermal wind relation for b_k thus yields $K-1$ equations and K unknown b_k 's. An additional constraint is therefore supplied; we utilize a least squares procedure minimizing

$$\sum_{k=2}^K [b_{i1k} - b_{i2k} - (b_{i1k-1} - b_{i2k-1})]^2 ;$$

this allows us to find $b_{k=1}$.

In order to evaluate u and v at time t we need p from the previous time $t - \frac{\Delta t}{2}$. We can get p from b via the hydrostatic equation but we do not know $p_{k=1}$. We get around this as follows. Integrate the hydrostatic equation for a pressure field p^* assuming $p_1 = 0$:

$$p_k^* = p_{k-1}^* + \frac{1}{2K} (b_k + b_{k-1}) , \quad p_1^* = 0 .$$

This differs from the correct p by a function of x and y

$$p_{\text{correct}} = p^* + p_0(x, y)$$

Let $(\frac{\partial \vec{v}_k}{\partial t})^*$ indicate the $\frac{\partial \vec{v}_k}{\partial t}$ values computed from p_k^* , then

$$\frac{\partial \vec{v}_k}{\partial t} = (\frac{\partial \vec{v}_k}{\partial t})^* - \nabla_k p_0(x, y) .$$

The vertically averaged vorticity is correctly predicted in spite of p_0 :

$$\frac{\partial}{\partial t} \left(\frac{\partial \bar{u}}{\partial x} - \frac{\partial \bar{v}}{\partial y} \right) = \frac{\partial}{\partial t} \left(\frac{\partial \bar{u}^*}{\partial x} - \frac{\partial \bar{v}^*}{\partial y} \right) ; \quad (\bar{}) \text{ is an average over } z .$$

Since $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$ we can write a streamfunction for $\vec{\bar{v}}$

$$\bar{u} = - \frac{\partial \psi}{\partial y} , \quad \bar{v} = \frac{\partial \psi}{\partial x} .$$

Given a forecast of $\vec{v}_k^*$ we solve the equation

$$\nabla_H^2 \psi = \frac{\partial}{\partial x}(\bar{v}^*) - \frac{\partial}{\partial y}(\bar{u}^*)$$

where the boundary conditions for ψ are derived from the basic boundary conditions of the model. Then, given ψ , we change u^* and v^* as follows:

$$u_k = u_k^* - \bar{u}^* - \frac{\partial \psi}{\partial y}, \quad v_k = v_k^* - \bar{v}^* + \frac{\partial \psi}{\partial x},$$

(since $\overline{\left(\frac{\partial}{\partial t} \vec{v}_k\right)} = \left(\frac{\partial}{\partial t} \bar{v}_k\right)^* - \nabla_H p_0(x, y)$ we have $\vec{v}_k - \vec{v}_k^* = \bar{v}_k - \bar{v}_k^*$).

We perform a slight variation of this procedure to get the corrected u and v ; we first remove the x independent part of u so $\psi = 0$ at $y = 0, 1$. We must solve a Poisson equation for ψ twice each timestep; this is done by a Fourier decomposition of the forcing function in the x direction and then using a technique found in Richtmyer and Morton (1967), pp 198-201.

In addition, w is needed at each half and full timestep and is obtained from a vertical integration of the continuity equation. Also, at each half and full timestep, v_k is adjusted to satisfy the continuity equation by requiring that $\overline{v}^* = 0$ for all values of y .

The total pressure field is required every so often for data purposes and is calculated in the following manner. Let $p = p_0 + p_a$, where $p_0(x, y) = p(x, y, z=0)$; $p_a(x, y, z=0) = 0$. We know p_a but must now find p_0 . Next we let $p = \bar{p}^x + p'^x$, averaging

over z we have

$$\bar{p}^z = \bar{p}^{xz} + \bar{p}'^z \quad (B-1)$$

Now $\bar{p}^{xz}$ and $\bar{p}'^z$ can be found as shown below. We thus know

$\bar{p}^z$. We can write $p = \bar{p}^z + p'^z$; we thus have an equation for p_0 : $p_0 = \bar{p}^z + p'^z - p_a$. Averaging over z does not change $p_0(x,y)$ so $p_0 = \bar{p}^z - \bar{p}_a^z$; $p = \bar{p}^z - \bar{p}_a^z + p_a$.

It remains to find $\bar{p}^{xz}$ and $\bar{p}'^z$. From the y equation of motion, using the boundary conditions and recalling $\bar{u}^{xz} = 0$ from the continuity equation, we have

$$\frac{\partial}{\partial y} \bar{p}^{xz} = -f \bar{u}^z - \frac{\partial}{\partial y} \bar{v} \bar{v}^z$$

which is integrated over y . Since $v=0$ at $y=0$ we have $\bar{v} \bar{v}^z = 0$ at $y=0$. We can set $\bar{p}^{xz}(y=0)=0$ since p is unknown to an arbitrary constant. Thus

$$\bar{p}^{xz}(y) = - \int_0^y f \bar{u}^z dy - \bar{v} \bar{v}^z(y) \quad (B-2)$$

In order to find $\bar{p}'^z$ we must first find $\nabla_H^2 \bar{p}'^z$ from the "divergence" equation

$$\nabla_H^2 \bar{p}'^z = L^{xz} \quad (B-3)$$

where

$$L = \frac{\partial}{\partial x} f \bar{v}^z - \frac{\partial}{\partial y} f \bar{u}^z - \left[\frac{\partial^2}{\partial x^2} \bar{u} \bar{u}^z + 2 \frac{\partial^2}{\partial x \partial y} \bar{u} \bar{v}^z + \frac{\partial^2}{\partial y^2} \bar{v} \bar{v}^z \right],$$

and $L'^x = L - \bar{L}^x$. At the northern and southern walls

$$\frac{\partial}{\partial y} \overline{\rho'^x z} = -f \overline{u'^x z}$$

Equation (B-3) is inverted to get $\overline{\rho'^x z}$, equation (B-2) provides $\overline{\rho^x z}$ so $\overline{\rho z}$ is known from equation (B-1).

All computations discussed in the thesis were performed at the National Center for Atmospheric Research. Run 2PE will now be discussed; it was run on the Control Data Corporation 7600. The run used 42x42x20 or 35,280 gridpoints and was carried out to 7½ days of forecast time; this required 2160 timesteps (a ΔT of .03 or 5 minutes was used). The total central processing time required for the run, not including the initialization and other preparation, was 7320 seconds, slightly more than two hours, or 3.39 seconds per timestep.

Values of the total energy, total momentum; total buoyancy and total potential vorticity were computed every 30 timesteps. The initial and final values of these quantities are given below to 10 decimal places.

Energy: -.0723210725(t=0); -.0722849385(t=2160); percentage change = +.050%.

Momentum: .0227047469(t=0); .0227047469(t=2160); no change.

Buoyancy: .1064207590(t=0); .1064207590(t=2160); no change.

Potential vorticity: .2614487236(t=0); .2607192226(t=2160); percentage change = -.279%.

Figure (B-2) shows w at gridpoint $I = 21$, $J = 16$, $K = 2$ for the last 20 timesteps of run 2PE. There is no $2\Delta T$ component present as occurs in a "leapfrog" scheme.

Appendix C. The Quasigeostrophic (QG) model.

The QG model is based upon equation (2-5). Given the boundary conditions for equations (2-4) (based upon $v=0$ at $y=0,1$) if we also require $\nabla_H^2 p' = 0$ at $y=0,1$ (where the prime denotes a deviation from the x average) then $\eta'_G = 0$, $\bar{\eta}_G = \text{constant}$ in time at $y=0,1$. This provides boundary conditions for equation (2-5).

Equation (2-5) is integrated in time exactly as the PE model, using the Lax-Wendroff scheme with staggered grids in space (varying at the half and full timestep; see Appendix B). The pressure p and the potential vorticity η_G are at the same level and gridpoint. We have K levels as in the PE model. Let $k=1,2,\dots,K$.

$$\begin{array}{lcl} z=1 & \text{//////} & w_K = 0, N_K^2 = \infty \\ z=k/K & \text{-----} & w_k, b_k, N_k^2 \\ & \text{-----} & p_k, \eta_k \\ z=k-1/K & \text{-----} & w_{k-1}, b_{k-1}, N_{k-1}^2 \end{array} \left. \vphantom{\begin{array}{l} z=k/K \\ z=k-1/K \end{array}} \right\} \text{LAYER } k$$

$$z=0 \quad \text{////////} \quad w_0 = 0, N_0^2 = \infty. \text{ THERE IS NO } b_0.$$

After defining the finite-difference analogues to equations (2-4) and the operators therein, we can obtain the finite-difference analogue to (2-5):

$$\frac{D_k}{Dt} \left[\nabla_H^2 p_k + f - 1 + \frac{K^2}{N_k^2} (p_{k+1} - p_k) - \frac{K^2}{N_{k-1}^2} (p_k - p_{k-1}) \right] = 0,$$

where $\frac{D_k}{Dt} = \frac{\partial}{\partial t} - \frac{\partial p_k}{\partial y} \frac{\partial}{\partial x} + \frac{\partial p_k}{\partial x} \frac{\partial}{\partial y}$; $\frac{D_k}{Dt}$ and ∇_H are continuous.

We then define

$$\eta_1 = \nabla_H^2 p_1 + \frac{K^2}{N_1^2} (p_2 - p_1) + f - 1 ; \quad k=1$$

$$\eta_k = \nabla_H^2 p_k + \frac{K^2}{N_k^2} (p_{k+1} - p_k) - \frac{K^2}{N_{k-1}^2} (p_k - p_{k-1}) + f - 1 ; \quad k=2, \dots, K-1$$

$$\eta_K = \nabla_H^2 p_K - \frac{K^2}{N_{K-1}^2} (p_K - p_{K-1}) + f - 1 ; \quad k=K$$

The vertical finite-difference analogue to equation (2-5) is thus

$$\frac{D_k}{Dt} \eta_k = 0 \quad (C-1)$$

At the half and full timestep we get η_k , we then invert to obtain p_k . This is done by an orthogonal transformation as described in Charney and Phillips (1953) and Yee (1972); the latter reference describes a model very similar to this one.

Whenever such data are desired, b and w are determined from p . Since b_k is staggered between p_k levels we easily determine b_k from p_k using the hydrostatic equation $b_k = \frac{1}{\Delta z} (p_{k+1} - p_k)$, $\Delta z = 1/K$. Determination of w is somewhat more complicated. The boundary conditions from equations (2-4) imply $w^1 = 0$ at $y = 0, 1$. We use this with the thermodynamic equation which we invert to obtain w :

$$w = (N^2)^{-1} \left(-\frac{\partial^2 p}{\partial t \partial z} + \frac{\partial p}{\partial y} \frac{\partial^2 p}{\partial x \partial z} - \frac{\partial p}{\partial x} \frac{\partial^2 p}{\partial y \partial z} \right) \quad (C-2)$$

The right hand side is evaluated using p from full timesteps t and $t-\Delta t$ so that w is actually evaluated for $t - \frac{\Delta t}{2}$, as opposed to the corresponding w from the PE model which is evaluated at t .

A crude computational stability criterion is given by

$$\Delta T < \frac{\Delta x}{|\vec{v}|_{\max}} . \quad \text{For run 2QG we have } \Delta x = 1/40, |\vec{v}| \sim .10$$

so that $\Delta T < 42$ minutes is suggested. We have taken $\Delta T = .12$ or 20 minutes.

Run 2QG was run on the Control Data corporation 7600 at the National Center for Atmospheric Research. The run used $42 \times 42 \times 20$ or 35,280 gridpoints and was carried out to $7\frac{1}{2}$ days of forecast time; this required 540 timesteps. The total central processing time required for the run was 3345 seconds or 6.09 seconds per timestep. (This compares to 3.39 seconds for one step in the PE model; there 2 two-dimensional inversions must be performed per timestep compared to 40 such inversions per timestep here.)

The total potential vorticity $\overline{\eta}_G^{xyz}$ was well conserved over the forecast; the initial and final values are shown below. The boundary conditions are such that $\overline{\eta}_G^{xyz} = 0$.

$$\overline{\eta}_G^{xyz}(t=0) = 3.63808 \times 10^{-13} ; \quad \overline{\eta}_G^{xyz}(t=540) = 3.03998 \times 10^{-13} .$$

Appendix D. The initialization procedure for the PE model.

The initialization program takes the nondivergent streamfunction ψ (consisting of the basic state plus the perturbation streamfunctions) and produces initial fields of u , v and b for the PE model. Using ψ as input, the vertical motion w is computed from the quasi-geostrophic vertical motion equation

$$\frac{\partial^2 w}{\partial z^2} + N^2 \nabla_H^2 w = \frac{\partial}{\partial z} [\vec{u}_0 \cdot \nabla_H (S+f)] - \nabla_H^2 [\vec{u}_0 \cdot \nabla_H b_0] \quad (D-1)$$

where $S = \nabla_H^2 \psi$, $u_0 = -\frac{\partial \psi}{\partial y}$, $v_0 = \frac{\partial \psi}{\partial x}$ and $b_0 = \frac{\partial \psi}{\partial z}$.

The velocity potential χ is then calculated and u and v are then produced. A pressure field p is obtained from ψ via the "balance" equation

$$\nabla_H^2 p = \frac{\partial}{\partial x} \left(f \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left(f \frac{\partial \psi}{\partial y} \right) + 2 \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \left(\frac{\partial^2 \psi}{\partial x \partial y} \right)^2 \right] \quad (D-2)$$

The hydrostatic equation then gives b from p .

The streamfunction $\psi(x, y, z)$ is specified for this program as follows:

$$\psi = A(y, z) \cos \theta + B(y, z) \sin \theta + C(y, z); \quad \theta = \frac{2\pi}{\ell} x$$

where ℓ is the nondimensional channel length. $C(y, z)$ is the basic state streamfunction (Appendix A); $A(y, z)$ and $B(y, z)$ are determined numerically by the procedure that finds the most unstable perturbation (discussed in Chapter 2). We require $\overline{C}^y = 0$, also $A, B = 0$ at $y = 0, 1$. In addition to ψ , $\overline{b}^y(z)$ is specified from

the program described in Appendix A. We define $N^2 \equiv \frac{d\bar{b}^y}{dz}$.

The first step is to compute w from equation (D-1). The right hand side of (D-1) is split into 5 forcing functions

$$\text{R.H.S.} = R_1(y,z) \cos \theta + R_2(y,z) \sin \theta + R_3(y,z) \cos 2\theta + R_4(y,z) \sin 2\theta \\ + R_5(y,z)$$

Likewise we set

$$w = w_1(y,z) \cos \theta + w_2(y,z) \sin \theta + w_3(y,z) \cos 2\theta + w_4(y,z) \sin 2\theta \\ + w_5(y,z)$$

We then integrate in the y,z plane for w_1, w_2, w_3 and w_4 requiring $w=0$ at $y=0,1$ and $z=0,1$. A relaxation procedure is used; this gives us w' where $w' = w - \bar{w}^x$ (and $\bar{w}^x = w_5$ which we do not need). Figure D-1 shows the location of variables ψ and w on the staggered y,z grid system. In order to evaluate the derivatives within equation (D-1) to sufficient accuracy A, B and C were computed for input to this program on grids possessing twice the resolution ($\Delta y, \Delta z$ reduced by 1/2) required for u, v and b .

Now $u = -\frac{\partial \psi}{\partial y} + \frac{\partial \chi}{\partial x}$, $v = \frac{\partial \psi}{\partial x} + \frac{\partial \chi}{\partial y}$. Let $\chi = \bar{\chi}^x + \chi'$.
Since $\frac{\partial \chi}{\partial x} = \frac{\partial \chi'}{\partial x}$ we have $u = -\frac{\partial \psi}{\partial y} + \frac{\partial \chi'}{\partial x}$. Also, since

$\psi = \bar{\psi}^x + \psi'$ and $\frac{\partial \psi}{\partial x} = \frac{\partial \psi'}{\partial x}$ we have

$$v = \bar{v}^x + v' = \frac{\partial \bar{\chi}^x}{\partial y} + \frac{\partial \psi}{\partial x} + \frac{\partial \chi'}{\partial y} \text{ or } v' = \frac{\partial \psi}{\partial x} + \frac{\partial \chi'}{\partial y}.$$

We will determine $\bar{v}^x$ later, we now find χ' from

$$\nabla_H \cdot \vec{v} = \nabla_H^2 \chi = -\frac{\partial w}{\partial z}, \quad (D-3)$$

Fig. D-1. Y - Z grid for variables.

integrated separately for each vertical level k. We require

$$\frac{\partial \chi'}{\partial y} = 0 \quad \text{at } y=0,1 \quad \text{since no flow is allowed through the walls.}$$

Having gotten w_n where $n=1,2,3,4$ we can find $\frac{\partial w_n}{\partial z}$. Let

$$\chi' = \chi'_1(y,z) \cos \theta + \chi'_2(y,z) \sin \theta + \chi'_3(y,z) \cos 2\theta + \chi'_4(y,z) \sin 2\theta;$$

equation (D-3) reduces to 4 second order ordinary differential equations in y. A technique found in Richtmyer and Morton (1967) pp 198-201 is used to integrate such equations.

The meridional circulation is needed to get $\bar{v}^x$. Writing the u and b quasigeostrophic equations $(b'_0 = \frac{\partial p_0}{\partial z}, u'_0 = -\frac{\partial p_0}{\partial y}, v'_0 = \frac{\partial p_0}{\partial x})$ in flux form and averaging over x we have

$$\begin{aligned} \bar{v}_1^x &= -\frac{\partial^2 \bar{p}_0^x}{\partial y \partial t} + \frac{\partial}{\partial y} (\overline{u'_0 v'_0})^x = -\frac{\partial \bar{\Psi}}{\partial z} \\ N^2 \bar{w}^x &= -\frac{\partial^2 \bar{p}_0^x}{\partial z \partial t} - \frac{\partial}{\partial y} (\overline{v'_0 b'_0})^x = N^2 \frac{\partial \bar{\Psi}}{\partial y} \end{aligned}$$

These are combined to give

$$\frac{\partial^2 \bar{\Psi}}{\partial z^2} + N^2 \frac{\partial^2 \bar{\Psi}}{\partial y^2} = -\frac{\partial^2}{\partial z \partial y} (\overline{u'_0 v'_0})^x - \frac{\partial^2}{\partial y^2} (\overline{v'_0 b'_0})^x$$

We require $\bar{\Psi} = 0$ at $y = 0, 1$ and $z = 0, 1$; u'_0 , v'_0 and b'_0 are obtained from ψ' . This is integrated by relaxation using a grid as shown in figure D-1. We then have

$$u = -\frac{\partial \psi}{\partial y} + \frac{\partial \chi'}{\partial x}, \quad v = -\frac{\partial \bar{\Psi}}{\partial z} + \frac{\partial \psi}{\partial x} + \frac{\partial \chi'}{\partial y}.$$

We next obtain the pressure, separately for each level k, from the "balance" equation (D-2); at $y=0,1$ $\frac{\partial p}{\partial y} = -f u = f \left(\frac{\partial \psi}{\partial y} - \frac{\partial \chi'}{\partial x} \right)$.

Equation (D-2) and boundary conditions are integrated again by splitting the right hand side and p into $\cos \theta, \sin \theta, \cos 2\theta, \sin 2\theta$ and x independent terms. Since we have $\bar{b}^{xy}(z)$ as input we can set $\bar{p}^{xy} = 0$ at each level.

The final step is to get the buoyancy from the pressure using $b = \frac{\partial p}{\partial z}$. In the PE model b and p are on the same level; the hydrostatic equation is

$$b_{k+1} + b_k = \frac{2}{\Delta z} (p_{k+1} - p_k) \quad (D-4)$$

Solving for b_k knowing p_k gives us $K-1$ equations for K unknown b_k 's (for K vertical levels). We can require some further constraint be satisfied such as $\sum_{k=1}^K (b_{k+1} - b_k)^2$ be minimized; all such methods used introduced artificial variations in the stability $\frac{\partial b}{\partial z}$. The method finally used to obtain b_k was the centered-difference form of the hydrostatic equation

$$b_k = \frac{1}{2\Delta z} (p_{k+1} - p_{k-1}) \quad \text{for levels } k = 2, 3, \dots, K-1; \text{ equation (D-4) then gave } b_1 \text{ and } b_K.$$

This worked satisfactorily.

Finally b is added to $\bar{b}^{xy}(z)$ to obtain the total b field.

Appendix E. Energetics for the PE model.

The PE equations (2-1) can be written in "flux" form:

$$u_t + (uu)_x + (vu)_y + (wu)_z - \int v = -p_x \quad (E-1)$$

$$v_t + (uv)_x + (vv)_y + (wv)_z + \int u = -p_y \quad (E-2)$$

$$b_t + (ub)_x + (vb)_y + (wb)_z = 0 \quad (E-3)$$

$$u_x + v_y + w_z = 0 \quad (E-4)$$

$$b = p_z \quad (E-5)$$

where $v=0$ at $y=0, 1$; $w=0$ at $z=0, 1$ and all quantities are cyclic in x ; i.e. $()_{x=0} = ()_{x=x+L}$; L is the channel

length. Define $\overline{(\cdot)} \equiv \frac{1}{L} \int_0^L (\cdot) dx$; $(\cdot)' \equiv (\cdot) - \overline{(\cdot)}$. We have

$$\overline{(\cdot)'} = 0. \quad \text{Averaging equations (E-1) through (E-4):}$$

$$\overline{u}_t + (\overline{vu})_y + (\overline{wu})_z - \int \overline{v} = 0 \quad (E-6)$$

$$\overline{v}_t + (\overline{uv})_y + (\overline{wv})_z + \int \overline{u} = -\overline{p}_y \quad (E-7)$$

$$\overline{b}_t + (\overline{vb})_y + (\overline{wb})_z = 0 \quad (E-8)$$

$$\overline{v}_y + \overline{w}_z = 0 \quad (E-9)$$

Rewrite equations (E-6) through (E-8) using the averaging operators and (E-9):

$$\overline{u}_t + \overline{v} \overline{u}_y + \overline{w} \overline{u}_z + (\overline{u'v'})_y + (\overline{u'w'})_z - \int \overline{v} = 0 \quad (E-10)$$

$$\bar{v}_t + \bar{v} \bar{v}_y + \bar{w} \bar{v}_z + (\overline{v'v'})_y + (\overline{v'w'})_z + f \bar{u} = -\bar{p}_y \quad (E-11)$$

$$\bar{b}_t + \bar{v} \bar{b}_y + \bar{w} \bar{b}_z + (\overline{b'v'})_y + (\overline{b'w'})_z = 0 \quad (E-12)$$

Equations (E-10) through (E-12) are the equations of the average quantities. Now subtract equations (E-6) through (E-8) from equations (E-1) through (E-3) and rewrite to obtain

$$\begin{aligned} u'_t + (\bar{u} + u') u'_x + (\bar{v} + v') u'_y + (\bar{w} + w') u'_z + v' \bar{u}_y + w' \bar{u}_z \\ - (\overline{v'v'})_y - (\overline{w'v'})_z - f v' = -p'_x \end{aligned} \quad (E-13)$$

$$\begin{aligned} v'_t + (\bar{u} + u') v'_x + (\bar{v} + v') v'_y + (\bar{w} + w') v'_z + v' \bar{v}_y + w' \bar{v}_z \\ - (\overline{v'v'})_y - (\overline{w'v'})_z + f u' = -p'_y \end{aligned} \quad (E-14)$$

$$\begin{aligned} b'_t + (\bar{u} + u') b'_x + (\bar{v} + v') b'_y + (\bar{w} + w') b'_z + v' \bar{b}_y + w' \bar{b}_z \\ - (\overline{v'b'})_y - (\overline{w'b'})_z = 0 \end{aligned} \quad (E-15)$$

Equations (E-13) through (E-15) are the equations of the deviation quantities. Now multiply (E-10) by $\bar{u}$ and (E-11) by $\bar{v}$ and add; let $\bar{k} = \frac{1}{2}(\bar{u}^2 + \bar{v}^2)$:

$$\begin{aligned} \bar{k}_t + \bar{v} \bar{k}_y + \bar{w} \bar{k}_z + \bar{u} [(\overline{u'v'})_y + (\overline{u'w'})_z] \\ + \bar{v} [(\overline{v'v'})_y + (\overline{v'w'})_z] = -\bar{v} \bar{p}_y \end{aligned} \quad (E-16)$$

Multiply (E-13) by u' and (E-14) by v' and add; let $k' = \frac{1}{2}(u'^2 + v'^2)$:

$$\begin{aligned} k'_t + (\bar{u} + u') k'_x + (\bar{v} + v') k'_y + (\bar{w} + w') k'_z + u' v' \bar{u}_y + u' w' \bar{u}_z \\ + v' v' \bar{v}_y + v' w' \bar{v}_z - u' [(\overline{u' v'})_y + (\overline{u' w'})_z] \\ - v' [(\overline{v' v'})_y + (\overline{v' w'})_z] = -u' p'_x - v' p'_y \end{aligned} \quad (E-17)$$

Multiply (E-3) by $-z$; let $a = -bz$. Use (E-9); we can obtain

$$a_t + (ua)_x + (va)_y + (wa)_z + \bar{w} \bar{b} + w' b' + w' \bar{b} + \bar{w} b' = 0 \quad (E-18)$$

Equation (E-16) becomes, using (E-5) and (E-9)

$$\begin{aligned} \bar{k}_t + (\bar{v} \bar{k})_y + (\bar{w} \bar{k})_z + \bar{u} [(\overline{u' v'})_y + (\overline{u' w'})_z] + \bar{v} [(\overline{v' v'})_y + (\overline{v' w'})_z] \\ - \bar{w} \bar{b} = -(\bar{v} \bar{p})_y - (\bar{w} \bar{p})_z \end{aligned} \quad (E-19)$$

Now define the zonal kinetic energy $\bar{K} \equiv \int_V \bar{k} dV$ where V is the total volume. Integrating (E-19) over V gives us, using the boundary conditions:

$$\begin{aligned} \frac{\partial \bar{K}}{\partial t} = \frac{\partial}{\partial t} \int_V \frac{1}{2} (\bar{u}^2 + \bar{v}^2) dV = - \int_{y,z} \left\{ \bar{u} [(\overline{u' v'})_y + (\overline{u' w'})_z] \right. \\ \left. + \bar{v} [(\overline{v' v'})_y + (\overline{v' w'})_z] - \bar{w} \bar{b} \right\} dy dz \end{aligned} \quad (E-20)$$

Equation (E-17) becomes, writing $u = \bar{u} + u'$, etc.,

$$\begin{aligned}
 k'_t + (uk')_x + (v'k')_y + (w'k')_z + (u'v'\bar{u} + v'v'\bar{v})_y + (u'w'\bar{u} + v'w'\bar{v})_z \\
 - \bar{u}[(u'v')_y + (u'w')_z] - \bar{v}[(v'v')_y + (v'w')_z] - u'[(\overline{u'v'})_y + (\overline{u'w'})_z] \\
 - v'[(\overline{v'v'})_y + (\overline{v'w'})_z] - w'b' = -(u'p')_x - (v'p')_y - (w'p')_z \quad (E-21)
 \end{aligned}$$

Define the eddy kinetic energy $K' \equiv \int_V k' dV$. Note $\bar{k}' \neq 0$ since $k' = \frac{1}{2}(u'u' + v'v')$. Integrating (E-21) over V gives us

$$\begin{aligned}
 \frac{\partial K'}{\partial t} \equiv \frac{\partial}{\partial t} \int_V \frac{1}{2}(u'^2 + v'^2) dV = \int_{y,z} \left\{ \bar{u}[(\overline{u'v'})_y + (\overline{u'w'})_z] \right. \\
 \left. + \bar{v}[(\overline{v'v'})_y + (\overline{v'w'})_z] + \overline{w'b'} \right\} dy dz \quad (E-22)
 \end{aligned}$$

Define the potential energy $P \equiv \int_V a dV$. Integrating (E-18) over V :

$$\frac{\partial P}{\partial t} \equiv \frac{\partial}{\partial t} \int_V a dV = - \int_{y,z} \left\{ \bar{w}b + \overline{w'b'} \right\} dy dz \quad (E-23)$$

Equations (E-20) + (E-22) + (E-23) give us

$$\frac{\partial}{\partial t} (\bar{K} + K' + P) = 0 \quad (E-24)$$

We can thus define the total energy $E \equiv \bar{K} + K' + P$.

The following energy transformation terms appear in (E-20), (E-22) and (E-23):

$$\{EKE \rightarrow ZKE\} = - \int_{y,z} \left\{ \bar{u} [(\bar{u}'v')_y + (\bar{u}'w')_z] + \bar{v} [(\bar{v}'u')_y + (\bar{v}'w')_z] \right\} dy dz$$

$$\{P \rightarrow ZKE\} = + \int_{y,z} \bar{w} \bar{b} dy dz$$

$$\{P \rightarrow EKE\} = + \int_{y,z} \bar{w}' \bar{b}' dy dz$$

Thus we can write

$$\frac{\partial}{\partial t} K' = - \{EKE \rightarrow ZKE\} + \{P \rightarrow EKE\}$$

$$\frac{\partial}{\partial t} \bar{K} = + \{EKE \rightarrow ZKE\} + \{P \rightarrow ZKE\}$$

$$\frac{\partial}{\partial t} P = - \{P \rightarrow ZKE\} - \{P \rightarrow EKE\}$$

Using boundary conditions and integration by parts $\{EKE \rightarrow ZKE\}$

can be rewritten as

$$\int_{y,z} \left\{ (\bar{u}'v') \bar{u}_y + (\bar{u}'w') \bar{u}_z + (\bar{v}'u') \bar{v}_y + (\bar{v}'w') \bar{v}_z \right\} dy dz$$

This form was used for computation purposes.

Appendix F. Mechanism to produce NE-SW tilted trough.

Ageostrophic cross-isobar advection may be of interest for other than frontogenetical reasons. There is a marked predominance of "NE-SW" tilted troughs in mid-latitude westerly flow (northern hemisphere); no one knows why (see Lorenz, 1967, for an excellent discussion on this point). Given a trough with a north-south axis and possessing jet streaks (speed maxima) on either side of the trough, the cross-isobar flow associated with the confluent and diffluent regions may act to advect the jet streaks as shown schematically in figure F-1 (by vorticity advection, perhaps) and hence to distort the trough, causing it to tilt more NE-SW. This effect could work if only one jet streak were present anywhere in the vicinity of the trough or on either side and it could work for both barotropically and baroclinically amplifying waves.

Fig. F-1. Mechanism to produce "NE-SW" tilted trough. Curve PP is an isobar in trough with axis originally oriented N-S. Isotachs indicate areas of speed maximum on either side of trough. Arrows indicate cross-isobar flow in confluent and diffluent regions. Dashed lines represent the distortion of the jet streak axes by cross-isobar advection tending to "tilt" the trough toward a "NE-SW" orientation.

REFERENCES

Those papers marked by a "P" are from the following reference:

Proceedings of the International Symposium on Numerical Weather Prediction, Tokyo, Japan, 1962.

Those papers marked by an "R" are from the following reference:

The Atmosphere and the Sea in Motion; Scientific Contributions to the Rossby Memorial Volume, Rockefeller Institute Press, New York, 1959.

Those papers marked by an "S" are from the following reference:

Sanders, F., Principal Investigator: Frontal Structure and the Dynamics of Frontogenesis. Final Report to NSF. Grant GP-1508, Department of Meteorology, M.I.T., Cambridge, Mass., 1967.

Arakawa, A., 1962: Non-geostrophic effects in the baroclinic prognosis equations. P, 161-175.

Bergeron, T., 1928: Über die Dreidimensional Verknüpfende Wetteranalyse, Geof. Pub. 5, 73-78.

Bergh, R.T., 1967: Analysis of mesoscale frontogenesis and deformation fields. Appendix 3, S, 3.1-3.28.

Bosart, L.F., 1970: Mid-tropospheric frontogenesis. Q.J.R.M.S., 96 442-471.

Bryan, K., 1966: A scheme for the numerical integration of the equations of motion on an irregular grid free of nonlinear instability. M.W.R., 94, 39-40.

Brown, J.A., 1969: A numerical investigation of hydrodynamic instability and energy conversions in the quasi-geostrophic atmosphere: Part I. J.A.S., 26, 352-365.

Campana, K.A., 1967: Frontogenetical processes in the upper troposphere. Appendix 6, S, 6.1-6.47.

Charney, J.G., 1962: Integration of the primitive and balance equations. P, 131-152.

_____, and N.A. Phillips, 1953: Numerical integration of the quasi-geostrophic equations for barotropic and simple baroclinic flows. J. Meteor., 10, 71-99.

- Danielsen, E.F., 1964: Project Springfield Report. Defense Atomic Support Agency, DASA 1517, Washington, D.C., 97 pp.
- Derome, J. and C.L. Dolph, 1970: Three-dimensional non-geostrophic disturbances in a baroclinic zonal flow. Geo. Flu. Dyn., 1, 91-122.
- Edelmann, W., 1963: On the behavior of disturbances in a baroclinic channel. Sum. Report # 2, Research in Objective Weather Forecasting, Part F, Contract AF 61(052)-373, Research Division, Deutscher Wetterdienst, Offenbach/Main, 35 pp.
- Eliassen, A., 1959: On the formation of fronts in the atmosphere. R., 277-287.
- _____, 1962: On the vertical circulation in frontal zones. Geof. Pub., 24, 147-160.
- _____, 1966: Motions of intermediate scale: fronts and cyclones. Advances in Earth Science, M.I.T. Press, Cambridge, Mass., 111-138.
- _____, and E. Raustein, 1968: A numerical integration experiment with a model atmosphere based on isentropic surfaces. Meteo. Ann., 5, 46-63.
- _____, and E. Raustein, 1970: A numerical integration experiment with a 6-level atmospheric model with isentropic information surface. Meteo. Ann., 5, 429-449.
- Elliott, W.P. and H.A. Brown, 1956: A synoptic study of horizontal deformation. Scientific Report # 6, reference 56-8T, The A. and M. College of Texas, 1-16.
- Ertel, H., 1942: Ein Neuer Hydrodynamic Wirbelsatz. Meteo. Zeit., 79, 277-281.
- Faller, A.J., 1956: A demonstration of fronts and frontal waves in atmospheric models. J. Meteor., 13, 1-4.
- Gerrity, J.P., 1972: A note on the computational stability of the two-step Lax-Wendroff from of the advection equation. M.W.R., 100, 72-73.
- Gilchrist, A., 1971: An example of synoptic development in a general circulation model. Q.J.R.M.S., 97, 340-347.

Hadfield, R.R., 1970: A study of atmospheric frontogenesis with the primitive equations. M.S. Thesis, U.S. Naval Postgraduate School, 52 pp.

Hinkelmann, K., 1959: Ein numerisches Experiment mit den primitiven Gleichungen. R, 486-500.

Hoskins, B.J., 1971: Atmospheric frontogenesis: some solutions. Q.J.R.M.S., 97, 139-153.

_____, 1972: Non-Boussinesq effects and further development in a model of upper-tropospheric frontogenesis. Q.J.R.M.S., 98, 532-541.

_____, and F.P. Bretherton, 1972: Atmospheric frontogenesis models: mathematical formulation and solution. J.A.S., 29, 11-37.

Kuo, H.L., 1956: Energy releasing processes and stability of thermally driven motions in a rotating fluid. J. Meteor., 13, 82-101.

Lorenz, E.N., 1967: The nature and theory of the general circulation of the atmosphere. W.M.O., # 218. TP. 115. 161 pp.

Margules, M., 1906: Über temperaturschichtung in stationär bewegter und in ruhender luft, Met. Zeit. Hann-Band, 243.

Miller, J.E., 1948: On the concept of frontogenesis. J. Meteor., 5, 169-171.

Ogura, Y. and N. Phillips, 1962: Scale analysis of deep and shallow convection in the atmosphere. J.A.S., 19, 173-179.

Økland, H., 1969: Experimental integration of a 4-level primitive equation model of the atmosphere. Tellus, 21, 359-366.

Palmen, E., 1955: On the mean meridional drift of the frictional layer of the west wind belts. Q.J.R.M.S., 81, 459-461.

_____, and C.W. Newton, 1969: Atmospheric circulation Systems. Academic Press, New York. 603 pp.

Petterssen, S., 1935: Contribution to the theory of frontogenesis. Geof. Pub. 11, # 6, 27 pp.

_____, and J.M. Austin, 1942: Fronts and frontogenesis in relation to vorticity, M.I.T. and W.H.O.I. Papers in Physical

Oceanography and Meteorology, 7, # 2, 37 pp.

Phillips, N.A., 1956: The general circulation of the atmosphere:
a numerical experiment. Q.J.R.M.S., 82, 123-164.

_____, 1959: An example of nonlinear computational instability. R, 501-504.

_____, 1960: On the problem of initial data for the primitive equations. Tellus, 12, 121-126.

_____, 1962: Numerical integration of the hydrostatic system of equations with a modified version of the Eliassen finite-difference grid. P, 109-120.

_____, 1963: Geostrophic motion. Reviews of Geophysics, 1, 123-176.

_____, 1970: Models for weather prediction. Annual Review of Fluid Mechanics, Vol. 2, Annual Reviews, Inc., Palo Alto, Calif., 251-292.

Reed, R.J., 1955: A study of a characteristic type of upper level frontogenesis. J. Meteor., 12, 226-237.

_____, and F. Sanders, 1953: An investigation of the development of a mid-tropospheric frontal zone and its associated vorticity field. J. Meteor., 10, 338-349.

Richtmyer, R.D., and K.W. Morton, 1967: Difference Methods for Initial Value Problems, Interscience Publishers, New York, 405 pp

Sanders, F., 1954: An investigation of atmospheric frontal zones. Sc.D. Thesis, M.I.T., Department of Meteorology, 132 pp.

_____, 1955: An investigation of the structure and dynamics of an intense surface frontal zone. J. Meteor., 12, 542-552.

_____, 1967: A survey of frontal zones in the middle and upper troposphere. Appendix 5, S, 5.1-5.14.

Sawyer, J., 1956: The vertical circulation at meteorological fronts and its relation to frontogenesis, Proc. Roy. Soc. A, 234, 346-362.

- Shapiro, M.A., 1970: On the applicability of the geostrophic approximation to upper level frontal-scale motions. J.A.S., 27, 408-420.
- Stone, P.H., 1966: Frontogenesis by horizontal wind deformation fields. J.A.S., 23, 455-465.
- Williams, R.T., 1967: Atmospheric frontogenesis: A numerical experiment. J.A.S., 24, 627-641.
- _____, 1968: A note on quasi-geostrophic frontogenesis. J.A.S., 25, 1157-1159.
- _____, 1972: Quasi-geostrophic versus non-geostrophic frontogenesis. J.A.S., 29, 3-10.
- _____, and J. Plotkin, 1968: Quasi-geostrophic frontogenesis. J.A.S., 25, 201-206.
- Yee, S.Y.K., 1972: An efficient numerical scheme for a baroclinic quasi-geostrophic model. J. Comp. Phys., 9, 412-423.

BIOGRAPHICAL SKETCH

The author was born on December 1, 1944 in Washington, D.C. He received his pre-college education in various public schools in the D.C. area and during this time became fascinated with the workings of the atmosphere. He received the Bachelor of Science degree in Physics from the University of Maryland in 1966 and shortly thereafter set off to Boston and M.I.T., where he met and in 1970 married the lovely Miss Joan Van Sinderen.