# IX. GASEOUS ELECTRONICS

## Academic and Research Staff



# A. ELECTRON-ATOM COLLISION FREQUENCY IN THE CESIUM AFTERGLOW

In a previous Quarterly Progress Report  $^1$  experimental measurements of the electron cyclotron absorption line shape were reported. These measurements were used to deduce the approximate variation of the electron-atom collision frequency as a function of electron velocity and the result was found not to agree, except over limited ranges of electron velocity, with the collision frequency measurements of others. This was not necessarily unexpected, since the other experiments do not agree very well among themselves.

Subsequently, measurements made with the use of the same technique have yielded collision frequencies in agreement with the originally reported values. A computer analysis has been developed in which the collision frequency as a function of velocity, **v** (v), is determined from the temperature dependence of the half-width at half maximum,  $\Delta(T)$ , of the electron cyclotron absorption coefficient. Previous to the development of the computer program, an approximate collision frequency had been determined by setting it equal to  $\Delta(T)$ . This gives the exact answer only for cases in which  $\Delta(T)$  is independent of temperature.

#### **1.** Computer Analysis

The electron cyclotron absorption coefficient is

$$
a(\omega - \omega_{\mathbf{b}}, \mathbf{T}) = \text{const} \left( \int_0^\infty \frac{\mathbf{v}^4 v_{\mathbf{c}} e^{-\mathbf{m} \mathbf{v}^2 / 2 \mathbf{k} \mathbf{T}} \mathbf{dv}}{v_{\mathbf{c}}^2 + (\omega - \omega_{\mathbf{b}})^2} \right), \tag{1}
$$

where  $v_c$ , m, v, T, and  $\omega_b$  are the electron collision frequency, mass, velocity, temperature, and radian cyclotron frequency,  $\omega$  is the radian frequency of the probing signal, and k is Boltzmann's constant. The ratio

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$$
r = \frac{a(\Delta, T)}{a(0, T)} = \frac{1}{2} = \frac{\int_0^\infty \frac{v^4 v_c e^{-mv^2/2kT} dv}{v_c^2 + \Delta^2}}{\int_0^\infty \frac{v^4 e^{-mv^2/2kT} dv}{v_c}} \tag{2}
$$

defines the half-width,  $\Delta(T)$ , at the half-maximum of a.

The computer analysis developed initially used  $\Delta(T)$  as input data and determined the polynomial form for  $v_c(v)$  best satisfying Eq. 2. This method was found to be extremely sensitive to small variations in the input data  $\Delta(T)$  and therefore unworkable.

A second approach that employs a computer program to calculate  $\Delta(T)$ , given a trial form for  $v_c(v)$ , has been found to be much more workable. The best form of  $v_c(v)$  is then found by trial and error. This computer program also evaluates the averages of  $v_c(v)$ ,

$$
\overline{(v_{\rm c}(v))} = \frac{\int_0^\infty v_{\rm c}(v) v^4 e^{-mv^2/2kT} dv}{\int_0^\infty v^4 e^{-mv^2/2kT} dv}
$$
\n(3)

and

$$
\left(\frac{1}{v_{c}(v)}\right)^{-1} = \left(\frac{\int_{0}^{\infty} \frac{1}{v_{c}(v)} v^{4} e^{-mv^{2}/2kT} dv}{\int_{0}^{\infty} v^{4} e^{-mv^{2}/2kT} dv}\right)^{-1}.
$$
\n(4)

For purposes of evaluation of the program and comparison of the variation with temperature of these various averages of  $v_c(v)$ , the electron-atom collision frequency in argon as determined by Frost and Phelps<sup>2</sup> has been used as the "trial"  $v_c(v)$  in the above-mentioned computer analysis. (The argon  $v_c(v)$  was chosen because an experiment is now under way in which  $\Delta(T)$  for argon is being measured and will be compared with the values calculated above as a check on the experimental method.)

In Fig. IX-1 are plotted  $(\overline{v_{\circ}(v)})$ ,  $(\frac{1}{v_{\circ}(v)})$ , and  $\Delta(T)$ , with the  $v_{\circ}(v)$  values given by Frost and Phelps used in the computer program. Also plotted for comparison is the actual  $v_{\alpha}(v)$ , but evaluated with  $v = \left(\frac{3kT}{m}\right)^{\alpha}$ 

The various averages of  $v_c(v)$  are seen to smear-out the sharper velocity dependence of the actual  $v_c(v)$ . Also,  $(\overline{v_c(v)})$  is always greater than  $\left(\frac{1}{v_c(v)}\right)^{-1}$ . This will always be true because, as can be seen, the integral for  $(\overline{v_{c}(v)})$ , Eq. 3, emphasizes the values of

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Fig. IX-1. Plotted as a function of temperature for electron-atom collisions in argon, three thermal averages of  $v_c(v)$ ,  $(\overline{v_c(v)})$ ,  $(\overline{v_c(v)})^{-1}$ , and  $\Delta(T)$ (the half-width at half-maximum of the electron cyclotron absorption curve). Shown for comparison is  $v_c(v)$  plotted with  $v = \left(\frac{3kT}{m}\right)^{1/2}$ .

 $v_c(v)$  where  $v_c(v)$  is largest, and the integral for  $\left(\frac{1}{v_c(v)}\right)^{-1}$ , Eq. 4, emphasizes the values of  $v_c(v)$  where  $v_c(v)$  is smallest. It also appears to be generally true that  $\Delta(T)$  is less than either of the two other averages,  $\overline{(v_C(v))}$  and  $\left(\frac{1}{v_C(v)}\right)^{-1}$ . This has also been demonstrated with several other trial forms for  $v_c(v)$ . As is seen from Fig. IX-1 these averages at a given temperature may differ by more than an order of magnitude! Experimentally, the average  $(\overline{v_{c}(v)})$  is measured whenever an absorption coefficient is measured with  $|\omega - \omega_b| \gg v_c$  (see Eq. 1), as in the wings of the cyclotron absorption curve or in an absorption experiment with  $\omega_b = 0$  and  $\omega \gg \nu_c$ . The average  $\left(\frac{1}{\nu_c(v)}\right)$  is measured at the peak of the cyclotron absorption curve (Eq. 4), or in an experimental measurement of the DC mobility of the electrons. From Eq. **1** and Fig. IX-1, it can be seen that the ratio

$$
\frac{(\overline{v_{\rm c}(v)})\left(\frac{1}{v_{\rm c}(v)}\right)^{-1}}{\Delta^2(\rm T)} \approx \frac{(\omega - \omega_{\rm b})^2 a\left[(\omega - \omega_{\rm b}), \rm T\right]}{a(0, \rm T) \Delta^2(\rm T)},\tag{5}
$$

where  $|\omega-\omega_b| \gg v_c$ , will deviate from one only when  $v_c(v)$  has a velocity dependence, being equal to one when  $v_c$  is constant.

# 2. Computer Fitting of  $v_c(v)$  to Cesium Data

Figure IX-2 shows the experimentally measured half-width of the electron cyclotron resonance in cesium plotted as a function of  $(4kT/m)^{1/2}$ . Also plotted is a half-width,  $\Delta_{\text{trial}}$ , which was calculated by using an assumed form for  $v_c$ . This assumed form for



Fig. IX-2.  $v_c$  versus velocity and  $\Delta$  versus (4kT/m)<sup>1/2</sup>, showing a comparison between the experimental and trial  $\Delta$  and the form of the trial  $v_c$ .

 $v_c$  gave the best fit to the experimental data out of four trials and will be improved upon. This trial  $v_c$  is also plotted on Fig. IX-2 versus velocity. For large values of v the form of  $v_c$  was assumed to be given by Brode's electron beam data. To obtain a reasonable fitting, it was found necessary to ignore the values of  $v_c$  obtained by Brode at low

electron velocities, as is indicated on the graph. This computer-determined form of  $v_c$ (v) is still in strong disagreement with other experimental measurements and work continues in order to resolve this.

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## References

- 1. J. C. Ingraham, Quarterly Progress Report No. 77, Research Laboratory of Electronics, M.I.T., April 15, 1965, p. 112.
- 2. L. S. Frost and A. V. Phelps, Scientific Paper 64-928-113-P6, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, June 18, 1964.