III. MICROWAVE SPECTROSCOPY*

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A. WORK COMPLETED

1. THE MAGNETIC FIELD DEPENDENCE OF THE TEMPERATURE COEFFICIENT OF InSb's HALL CONSTANT

This report summarizes work completed by Theodore A. Postol and submitted as a thesis to the Department of Physics, M.I.T., June 1966, in partial fulfillment of the requirements for the degree of Bachelor of Science.

The magnetic field dependence of the temperature coefficient of the Hall constant of InSb was studied from 300°K-400°K and from 100-4000 Gauss. A low field dependence of the form $R_H \propto [\text{cons}/TB^{0.20}]$ was found for one sample orientation, and no field dependence was observed for a second orientation. A magneto resistance experiment was also performed in order to further study this field dependence. No dependence was observed. This directional dependence does seem to exist, and does not seem to be explainable in terms of geometrical effects. It is still not understood.

M. W. P. Strandberg

2. SUPERCONDUCTIVE TUNNELING

This work has been completed by Stuart C. Schaffner and submitted as a thesis to the Department of Physics, M.I.T., June 1966, in partial fulfillment of the requirements for the degree of Bachelor of Science. A summary of the thesis research follows.

The theory of superconductivity is developed in sufficient detail to provide a background for understanding the phenomenon of superconductive tunneling. The formalism

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developed is then used to obtain expressions for the tunneling current. After a brief review of excited-state tunneling, Josephson tunneling is developed in detail. The electrodynamic equations obtained are then applied to Josephson junctions. Special attention is given to the DC characteristics of Josephson junctions, and the effect of magnetic fields. The possibility of using one- and two-junction devices as sensitive magnetometers is developed. Problems of noise, drift, and size are investigated as limiting factors on the ultimate sensitivity of such a device.

M. W. P. Strandberg

B. GREEN'S FUNCTION SOLUTION OF THE BOLTZMANN EQUATION IN THE ANOMALOUS SKIN-DEPTH REGION

The classical transport properties of a metal may be determined with the Boltzmann equation. In particular, it is possible to understand the experimental phenomena of the "radio-frequency size effect."[1] For this case consider a single metal crystal characterized by a spherical Fermi surface and a scalar mean-free time, t_o. The real space geometry of the problem may be taken as follows: The metal is infinite, and we shall be interested only in points measured along the positive Z-axis. A uniform magnetic field points along the X-axis, and a time-variant electric field in the Y-direction is impressed at $Z = 0$. The momentum space coordinates are the energy $-\epsilon$, the momentum parallel to the magnetic field $-P_H$, and τ, the dimensionless time that the electron spends in its orbit. τ may be thought of as the angular coordinate of the electron's motion around its orbit. The angle may exceed 2π, thereby indicating that the electron has completed more than one revolution. The linearized equation for the change in the distribution function caused by the electric field is

$$V_z \frac{\partial f_1}{\partial z} + \Omega_o \frac{\partial f_1}{\partial \tau} + i\omega f_1 + \frac{f_1}{t_o} = eV_y E(z) \frac{\partial f_o}{\partial \epsilon}, \tag{1}$$

where Ω_o is 2π times the cyclotron frequency.

With the following definitions: θ is the angle between the magnetic field and the momentum; $f_1(z,\tau,\theta) = \phi(z,\tau,\theta) \frac{\partial f_o}{\partial \epsilon} \epsilon_F$; $\frac{\partial f_o}{\partial \epsilon} = \delta(\epsilon-\epsilon_F)$; and $\xi = Z/D_o$, where D_o is the diameter of the largest orbit, the equation becomes

$$\frac{\sin\theta}{2} \cos\tau \frac{\partial \phi}{\partial \xi}(\xi,\tau,\theta) + \left[\frac{\partial}{\partial T} + \gamma_o\right] \phi(\xi,\tau,\theta) = \frac{\epsilon D_o}{\epsilon_F} \frac{\sin\theta \sin\tau}{2} E(\xi). \tag{2}$$

This equation can be solved by introducing a Green's function. The first step, however, is to notice that θ does not appear as a differential operator. We shall therefore treat Eq. 2 as a partial differential equation in only two variables, and θ will appear as a parameter in the solution. If we let

$$\frac{\sin\theta}{2} = a_o$$

and

$$\frac{i\omega}{\Omega_o} + \frac{1}{t_o\Omega_o} = \gamma_o,$$

the equations may then be written

$$\left[a_o \cos\tau \frac{\partial}{\partial\xi} + \frac{\partial}{\partial\tau} + \gamma_o\right] \phi(\xi,\tau) = \frac{eD_o}{\epsilon_F} a_o \sin\tau E(\xi). \tag{3}$$

The boundary conditions are $\phi(\xi,\tau) = \phi(\xi,\tau+2\pi)$ and $\phi_{z=\infty} = 0$ (ϕ is obviously periodic in θ). The solution to Eq. 3 is

$$\phi(\xi,\tau) = \int G_\tau(\xi,\xi') \frac{eD_o}{\epsilon_F} E(\xi')\, d\xi',$$

where $G_\tau(\xi,\xi')$ is the solution to the equation

$$\left[a_o \cos\tau \frac{\partial}{\partial\tau} + \gamma_o\right] G_\tau(\xi,\xi') = \sin\tau a_o \delta(\xi-\xi'). \tag{4}$$

As long as the metal is infinite in both directions, G is only a function of $\xi - \xi'$.

$$\left[a_o \cos\tau \frac{\partial}{\partial\xi} + \frac{\partial}{\partial\tau} + \gamma_o\right] G(\xi,\tau) = \sin\tau a_o \delta(\xi).$$

At all points in phase space, except the "line" $\xi = 0$, this is a first-order, quasi-linear, homogeneous, partial differential equation in two variables.

$$\left[a_o \cos\tau \frac{\partial}{\partial\xi} + \frac{\partial}{\partial\tau} + \gamma_o\right] G(\xi,\tau) = 0. \tag{5}$$

The function has a discontinuity across the $\xi = 0$ line.

$$G(0_+,\tau) - G(0_-,\tau) = \frac{\sin\tau}{\cos\tau}.$$

Solutions to quasi-linear, partial differential equations can be obtained as follows: The equation

$$P(\tau,\xi,G)\frac{\partial G}{\partial \tau} + Q(\tau,\xi,G)\frac{\partial G}{\partial \xi} = R(\tau,\xi,G)$$

has solutions of the form

$$f[U(\tau,\xi,G), V(\tau,\xi,G)] = 0,$$

where f is a perfectly arbitrary function. U + V are the two independent solutions of

$$\frac{d\tau}{P} = \frac{d\xi}{Q} = \frac{dG}{R}.$$

From Eq. 5, this becomes

$$\frac{d\xi}{\alpha_o \cos \tau} = \frac{d\tau}{1} = \frac{dG}{-\gamma_o G}. \tag{6}$$

The independent solutions are $U = Z - \alpha_o \sin \tau$ and $V = \gamma_o \tau + \ln G$. Therefore, the solutions to Eq. 5 are of the form

$$f(Z-\alpha_o \sin \tau, \gamma_o \tau + \ln G) = 0. \tag{7}$$

Two types of functional forms for f can be readily handled. The first type is $f(U, V) = g(U)\,h(V)$. This function implies $G = e^{-\gamma_o \tau}$, independent of ξ. This solution will never fit the boundary conditions. The second type of functional form is $f(U, V) = g(U) + h(V)$. This implies

$$G = A(Z-\alpha_o \sin \tau)\, e^{-\gamma_o \tau}, \tag{8}$$

where A is an arbitrary function. We demand that G be periodic in τ and that it have a discontinuity at $Z = 0$, thereby determining the form A must have.

This approach can be made to yield solutions that solve the equations and fit the boundary conditions. These solutions can be generalized to take care of the effect of an interface for a semi-infinite metal. The question remains, however, whether there are other solutions (corresponding to other forms of f) which also solve the problem. The answer is that our solution is not mathematically unique. But it is asserted without proof that our solution will contain a correct and complete description of the physical, nontransient phenomena.

The form A must take is determined in the following way: $Z - \alpha_o \sin \tau$ is the trajectory of the electron in phase space (see Fig. III-1). Each time the trajectory crosses the $\xi = 0$ line, A suffers a discontinuous jump of magnitude $\sin \tau/\cos \tau$. In an interval of 2π

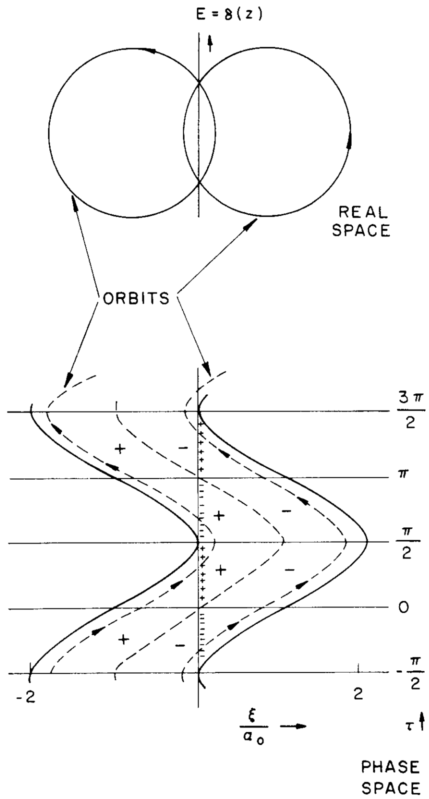

Fig. III-1. Electron orbits for delta-function fields in real and phase space.

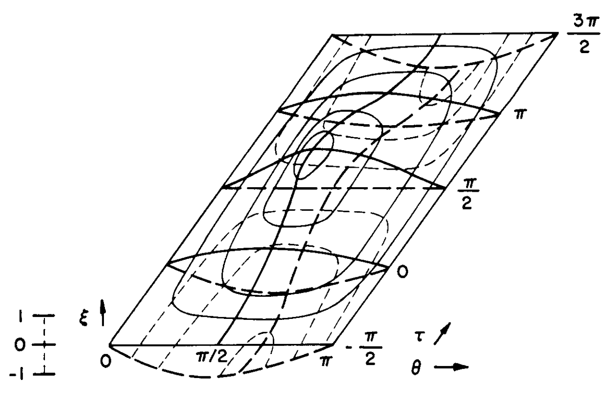

Fig. III-2. Area of phase space inside which $G(\xi,\tau,\theta)$ is nonzero.

Table III-1. Expression for $G(\xi, \tau, \theta)$.

$$G(\xi,\tau,\theta) = 0 \qquad \text{if} \left| \frac{\xi}{\alpha_o} - \sin\tau \right| > 1$$

$$G(\xi,\tau,\theta) = \frac{-1}{1-e^{-\gamma_o 2\pi}} \, \frac{(\xi/\alpha_o - \sin\tau)}{\sqrt{1-(\xi/\alpha_o - \sin\tau)^2}} \, e^{-\gamma_o\tau}\left[e^{-\gamma_o\left[2\pi - \overset{-}{\sin}^{-1}(-\xi/\alpha_o + \sin\tau)\right]} + e^{-\gamma_o\left[2\pi - \overset{+}{\sin}^{-1}(-\xi/\alpha_o + \sin\tau)\right]}\right]$$

if $\xi < 0$
$-\frac{\pi}{2} \leq \tau < \frac{\pi}{2}$

$$G(\xi,\tau,\theta) = \frac{-1}{1-e^{-\gamma_o 2\pi}} \, \frac{(\xi/\alpha_o - \sin\tau)}{\sqrt{1-(\xi/\alpha_o - \sin\tau)^2}} \, e^{-\gamma_o\tau}\left[e^{-\gamma_o\left[-\overset{-}{\sin}^{-1}(-\xi/\alpha_o + \sin\tau)\right]} + e^{-\gamma_o\left[2\pi - \overset{+}{\sin}^{-1}(-\xi/\alpha_o + \sin\tau)\right]}\right]$$

if $\xi > 0$
$-\frac{\pi}{2} \leq \tau \; \frac{3\pi}{2}$

$$G(\xi,\tau,\theta) = \frac{-1}{1-e^{-\gamma_o 2\pi}} \, \frac{(\xi/\alpha_o \sin\tau)}{\sqrt{1-(\xi/\alpha_o - \sin\tau)^2}} \, e^{-\gamma_o\tau}\left[e^{-\gamma_o\left[-\overset{-}{\sin}^{-1}(-\xi/\alpha_o + \sin\tau)\right]} + e^{-\gamma_o\left[-\overset{+}{\sin}^{-1}(-\xi/\alpha_o + \sin\tau)\right]}\right]$$

if $\xi < 0$
$\frac{\pi}{2} \leq \tau < \frac{3\pi}{2}$

Note. $\overset{\pm}{\sin}^{-1}(a)$ indicates the larger and smaller values of $\sin^{-1}(a)$ for the range $-\frac{\pi}{2} \leq \sin^{-1}(a) < \frac{3\pi}{2}$. That is, if

$0 < a < 1$, then $\quad \frac{\pi}{2} < \overset{+}{\sin}^{-1}(a) < \pi \quad$ and $\quad 0 < \overset{-}{\sin}^{-1}(a) < \frac{\pi}{2}$;

$1 < a < 0$, then $\quad \pi < \overset{+}{\sin}^{-1}(a) < \frac{3\pi}{2} \quad$ and $\quad \frac{\pi}{2} < \overset{-}{\sin}^{-1}(a) < 0$.

in τ, the trajectory crosses $\xi = 0$ twice. Both jumps appear to the electron to be in the same direction. If the electron were initially located too far from the $\xi = 0$, it would not cross. Therefore, G is taken to be zero outside of the lines in Fig. III-1.

A can best be determined by changing variables. Instead of considering G a function of ξ and τ, we introduce two new coordinates: the position of a volume element in space, a, and its age, t.

$$a = \frac{\xi}{\alpha_o} - \sin \tau, \quad t = \tau, \quad \therefore \; G = A(\alpha_o a)\, e^{-\gamma_o t}. \tag{9}$$

G starts decaying from time $t = 0$. A little volume of phase space moves along its trajectory until at time t it arrives at $\xi = 0$ (see Fig. III-1). $A(\alpha_o a)$ changes discontinuously then and G starts to decay again. This process repeats. Finally, G decays until $t = 2\pi$. A is so determined that the net effect on G of the decay and the two jumps is zero. That is, what was lost by the decay is made up for by the jumps.

Table III-1 gives the representations for G in its various regions. The notation $\overset{\pm}{\sin}{}^{-1}(x)$ implies the larger and smaller of the two values for

$$-\frac{\pi}{2} \leq \sin^{-1}(x) < \frac{3\pi}{2}.$$

Physically, the Green's function describes a very simple physical picture. A group of electrons in the same orbit circulate. If the orbit does not intersect the impulse field, the number of electrons perturbed is zero. This is the outer region in Fig. III-1. For orbits that do intersect, two processes are present. First, as time goes on and the electrons move around the orbit some of them die off. But each time they pass through the impulse (two times per orbit) new electrons are created. These two processes exactly cancel and a steady state is achieved. The "strength" of the impulse is $\sin \tau \times \cos \tau$. The $\cos \tau$ term comes from the fact that different electrons "stay near" the impulse for different times. And the $\sin \tau$ term comes from the fact that the effect of the impulse is to displace the Fermi surface without distortion. The difference between the shifted and unshifted surfaces goes as $\sin \tau$.

Figure III-1 shows the region in which G is nonzero. The horizontal coordinate is ξ/α_o. The maximum excursion of ξ/α_o is ± 2 or $\xi = \pm 1(\sin \theta)$. Figure III-2 is a plot of the region in τ and θ where G is nonzero. Figure III-1 is a cross section in the ξ, τ phase of Fig. III-2.

S. R. Reznek

References

1. A. Fukumoto, "Ultrasonic Attenuation and Size Effect in Gallium," Ph. D. Thesis, Department of Physics, M.I.T., 1966.