GENERAL PHYSICS

I. MOLECULAR BEAMS[*]

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RESEARCH OBJECTIVES AND SUMMARY OF RESEARCH

1. Helium Beam Experiments

Continued progress has been made in our program of low temperature atomic beam research. We expect during the next year to complete the following experiments.

a. Measurement of the cross section of ^{4}helium atoms for collision with both ^{4}helium and ^{3}helium in the temperature range 1.2-4.2°K (D. E. Oates).

b. Measurement of the velocity distribution of atoms evaporating from liquid helium II at temperatures around 0.6°K (J. W. McWane).

c. Measurement of the total evaporation rate from liquid helium II as a function of temperature down to 0.3°K (R. F. Tinker).

In another generation of experiments we shall use atomic beam techniques to study the diffusion of ^{3}helium in ^{4}helium (J. R. Clow and G. A. Herzlinger) and the Brown-Twiss effect in evaporating helium (J. R. Clow and M. Scully). We are also collaborating with C. Chase (Francis Bitter National Magnet Laboratory) in some low counting level ion experiments (W. B. Davis).

Dr. R. C. Pandorf is constructing a dilution refrigerator and aiding in the development of our new low-temperature facilities.

2. Superconductor Studies

Some preliminary results have been obtained by T. R. Brown in an experiment to examine the periodic magnetic fields near the surface of a type II superconductor in the vortex phase. This is done by passing a beam of neutral atoms in a given hyperfine state near the surface and searching for transitions by the usual method of atomic beam magnetic resonance. This experiment should be completed this year, but it is likely that many other applications will be found in investigating microscopic magnetic systems.

3. Molecular Microscope

A first model of a "molecular microscope" was constructed by R. Koolish and W. Bigas and the spatial variation of evaporation of K_2O from potassium feldspar

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containing alumina grains was studied. The evaporation is found to correlate reasonably well with the physical features of the sample. Although these results illustrate the principles, the results and further studies are principally of interest to ceramic engineers. Accordingly, we have designed an instrument in which a biological sample will be diffusely illuminated with water molecules and the reevaporation of these molecules will be studied with approximately 6000 Å resolution. One of the possible samples for initial study would be a freeze-dried sample of cells containing ice damage. R. Koolish and T. Postol, as well as several undergraduates, are doing preliminary studies for this instrument. Dr. J. Weaver will be joining us from Yale University as a postdoctoral fellow to continue this work. D. Ofsevit has been studying electrostatic electron lenses containing a conducting septum which will allow correction of aberrations. Such a lens would be very valuable in electron microscopy in general and may also find application as part of the molecular microscope.

J. G. King, J. R. Zacharias

A. HELIUM EVAPORATION AT LOW TEMPERATURES

An attempt is being made here to predict the features of evaporation from superfluid helium, using the kinetic theory of excitations. The assumption that the thermal energy of superfluid helium resides in the bulk excitations[1] accounts for many of the observed properties of helium. Hence, it is reasonable to explore the possibilities of excitation-evaporating atom conversion and interactions. The effects of surface modes will be ignored in this report. Atkins[2] predicts that these modes do not exist at the relatively high energies required for evaporation (~7 °K), or if they do, they do not obey the usual dispersion law, $\epsilon \sim p^{2/3}$.

A crucial question in establishing a model of evaporation based on elementary excitations is whether or not the possible conversion processes at or near the surface are momentum-conserving. The most relevant experimental evidence has been reported by Cunsolo.[3] He observes that a negatively charged vortex ring can eject an electron at the surface, giving all of its energy to the electron. In their range of experimental conditions, Rayfield and Reif's[4] dispersion relation for ring vortices predicts at least an order of magnitude mismatch in momentum. Also, first and second sound pulses reflect internally off the free surface without conserving momentum. Thus it seems that the free surface is able to adsorb or supply momentum as required by other dynamic considerations. If the surface adsorbs momentum parallel to itself, however, it could attenuate superfluid flow. The experiments of Kukolich[5] put an upper limit of 10^{-6} for the conversion coefficient for such processes.

These considerations indicate that the question of momentum conservation at the surface is far from settled. Simple models of both momentum-conserving and nonconserving evaporation will be explored in order to cover either eventuality, and indeed, possibly to aid in distinguishing between the two.

1. Momentum Nonconserving Evaporation

To make a tractable model, assume that there is a one-to-one correspondence between an evaporating atom and a particular excitation incident onto the He vacuum interface. Such an excitation must have at least 7.18°K energy, the latent heat per atom of He at absolute zero (ℓ_O).[6] Any extra energy would be given to the free particle.

Figure I-1 shows the free particle "excitation" curve $P_O^2/2m + \ell_O$ displaced ℓ_O above the zero of the quasi-particle excitation curve determined by neutron scattering.[1] Note that excitations with momenta in the range 0.5-$1.0 \, \overset{\circ}{A}^{-1}$ have insufficient momentum
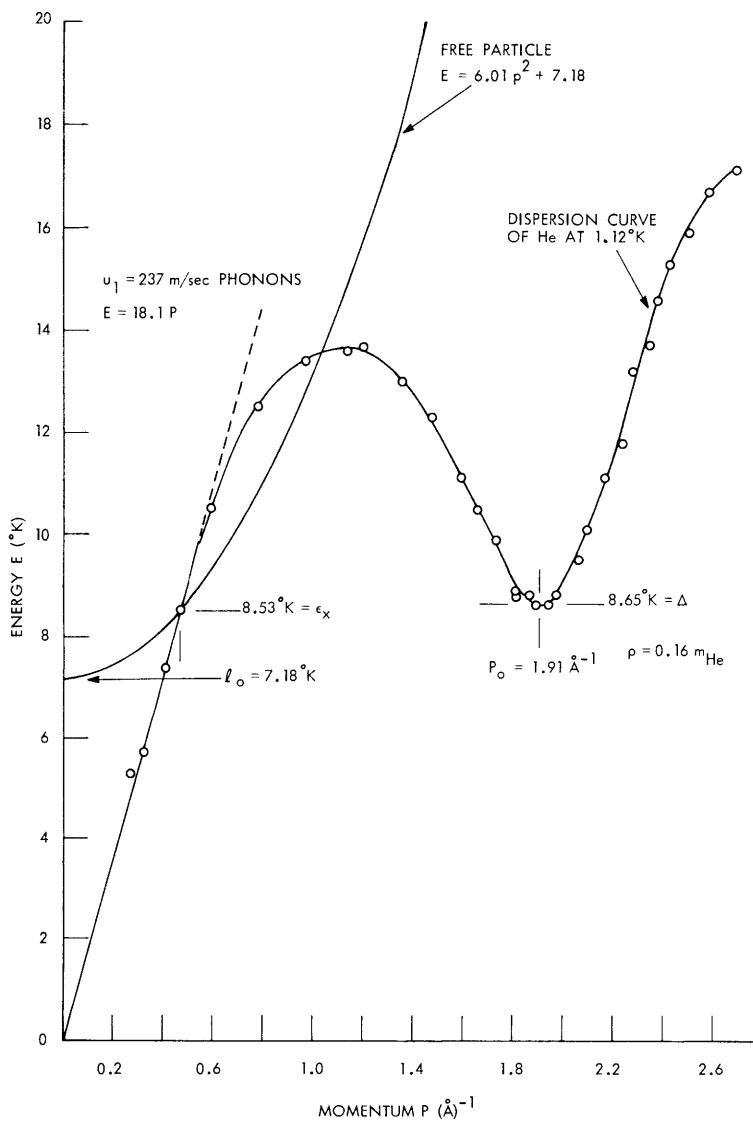

Fig. I-1. Helium quasi-particle dispersion curve with the free-particle curve displaced ℓ_O above zero.

to create a free particle of the required energy. The lower intersection of the curves occurs at

$$P_x = 0.468 \ \text{Å}^{-1} \qquad \epsilon_x = 8.53 \ °K.$$

This momentum corresponds to an evaporating atom with velocity $v_x = 74.2$ m/sec. All other excitations, particularly the rotons, have excess momenta which must be removed. The large roton minimum only 1.54 °K above ℓ_o but at large momentum is bound to significantly affect the distribution of evaporating atoms because of its low energy, large momentum, and large phase-space volume. The actual distribution depends on the assumed dynamical process that adjusts the momentum excess or deficit. Several ad hoc assumptions based on a one-to-one model will be examined.

The current (J) in atoms per second, per unit area, per solid angle, per velocity range dv_o seen in the evaporating beam is

$$J = \frac{d^2 n}{d\Omega dp} \ v \cos \theta \ \frac{dp}{d\epsilon} \ \frac{d\epsilon}{dv_o} \ \frac{dn_o}{dn} \ \frac{d\Omega}{d\Omega_o}. \tag{1}$$

These terms are clarified below. In general, a subscript o refers to a quantity outside the helium.

$\frac{d^2 n}{d\Omega dp}$ is the quasi-particle excitation density per unit particle momentum range, per solid angle.

Since only atoms possessing $\epsilon \sim 7$ °K will be considered, at temperatures $T \approx 1$ °K,

$$\frac{d^2 n}{d\Omega dp} = \frac{p^2}{h^3} e^{-\beta \epsilon}.$$

v is the quasi-particle velocity $\frac{d\epsilon}{dp}$.

$\frac{d\epsilon}{dv_o}$ is the relation between an excitation's energy and the velocity v_o of the consequent beam atom obtained through equating the energies

$$\epsilon - \ell_o = \frac{1}{2} m v_o^2.$$

The last two terms depend on the dynamical details:

$\frac{dn_o}{dn}$ is the conversion efficiency.

$\frac{d\Omega}{d\Omega_o}$ gives the spreading of the beam by refraction.

Substituting and collecting these terms gives

$$J = e^{-\beta \ell_o} \frac{mv_o p^2}{h^3} e^{-\beta \frac{mv_o^2}{2}} \frac{dn_o}{dn} \frac{d\Omega}{d\Omega_o}.$$ (2)

Model I

Assume that the momentum of an "evaporating" excitation is conserved parallel to the surface and the conversion efficiency is constant for allowed conversions.

Conservation of momentum parallel to the surface requires

$$p \sin \theta = mv_o \sin \theta_o \qquad \phi = \phi_o.$$

Therefore

$$\frac{d\Omega}{d\Omega_o} = \left(\frac{mv_o}{p}\right)^2 \frac{\cos \theta_o}{\left[1 - \sin^2 \theta_o \left(\frac{mv_o}{p}\right)^2\right]^{1/2}}.$$

The resultant current near the perpendicular is

$$J_I = e^{-\beta \ell_o} \left(\frac{mv_o}{h}\right)^3 e^{-\beta \frac{mv_o^2}{2}} \frac{dn_o}{dn}.$$ (3)

Under the assumption of constant but different conversion efficiencies a for the phonon and roton regions,

$$\frac{dn_o}{dn} = \begin{cases} a_{ph} & \text{for} \quad \ell_o < \epsilon_o < \Delta \\ 2a_r + a_{ph} & \epsilon_o > \Delta \end{cases}$$

Thus, if $a_r \neq 0$ the high velocity part of the spectrum is enhanced. Except for this discontinuous enhancement, the distribution is Maxwellian.

Model II

Assume that momentum can be lost only parallel to the liquid surface. Now, near the perpendicular

$$\frac{d\Omega}{d\Omega_o} = \frac{mv_o}{p}$$

and

$$J_{II} = e^{-\beta\ell_o} \frac{(mv_o)^2 p}{h^3} e^{-\beta\frac{mv_o^2}{2}}.$$

In the phonon region

$$p = p_{ph} = \frac{\epsilon}{u_1} = \frac{1}{u_1}\left(\ell_o + \frac{1}{2}mv_o^2\right),$$

where u_1 is the velocity of first sound.

In the roton region

$$p = p_r = p_o + \sqrt{2\mu\left(\ell_o - \Delta + \frac{1}{2}mv_o^2\right)}.$$

In the regions of interest the dominant terms are

$$p_{ph} = \frac{\ell_o}{u_1} = 0.397\ \mathring{A}^{-1} \qquad p_r = 1.91\ \mathring{A}^{-1}.$$

Then approximately

$$J_{II} = e^{-\beta\ell_o} \frac{(mv_o)^2}{h^3} e^{-\beta\frac{mv_o^2}{2}}\begin{cases}0.4a_{ph} & \text{for} \quad \ell_o < \epsilon_o < \Delta \\ 4a_{ph} + 0.4a_{ph} & \epsilon_o > \Delta\end{cases} \tag{4}$$

Model III

Assume that the excess momentum is lost along the direction of travel. Then $\frac{d\Omega}{d\Omega_o} = 1$. Using the previous assumptions and approximations, we have

$$J_{III} = e^{-\beta\ell_o} \frac{mv_o}{h^3} e^{-\beta\frac{mv_o^2}{2}}\begin{cases}0.16a_{ph} & \text{for} \quad \ell_o < \epsilon_o < \Delta \\ 7.2a_r + 0.16a_{ph} & \Delta < \epsilon_o\end{cases} \tag{5}$$

Common to all of these models is an enhancement of the high-velocity evaporating atoms if roton conversion is allowed. If a_r were chosen properly, beam-beam collisions

could cause any of these models to yield a distribution that is resolved as being Maxwellian of as much as one degree above the actual liquid temperature, in accordance with the results of Johnston and King.[7] Extremely careful angular and velocity measurements could make it possible to differentiate the three models.

2. Momentum-Conserving Evaporation

If momentum is assumed to be conserved in the evaporation process, the various interactions between phonons (P) and rotons (R) yielding evaporating atoms (E) must be examined. The possible two- and three-body interactions are summarized in Table I-1. Note that those involving rotons are either impossible or restricted to such high minimum energies that they would be insignificant at low temperatures. This is true also of all four-body interactions in which rotons participate. The remaining simple candidates, interactions 1), 3), and 6) can be easily distinguished by the effects of geometry and momentum conservation on their velocity distributions.

Table I-1. Summary of two- and three-body interactions yielding evaporating atoms.

Interaction	Evaporating atom energy (°K)		Comment
	min	max	
1) $P \rightarrow E$	8.5	8.5	monoenergetic, isotropic
2) $R \rightarrow E$			impossible
3) $P \rightarrow E + P'$	8.5	13.5	
4) $P \rightarrow E + R$			impossible
5) $R \rightarrow E +$ (anything)			impossible
6) $P + P' \rightarrow E$	7.2	8.5	
7) $P + R \rightarrow E$	18	30	
8) $R + R' \rightarrow E$	17.3	~20	sharply peaked near 17.3 °K

It is useful to use Fig. I-1 to see the effects of geometry in various combined states. If an excitation of energy and momentum (ϵ_1, p_1) adsorbs a phonon with energy less than 9.0 °K, the resulting state is within a cone in the ϵ, p plane bounded by $\epsilon = u_1(p_1 \pm p) + \epsilon_1$ above the initial state. If the angle θ between the momenta of the excitation and the phonon is zero, the resultant state is on the right; if $\theta \sim \frac{\pi}{2}$, it is in the center; if $\theta \sim \pi$, it is on the left and the two excitations must be directed parallel to the liquid surface.

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Corresponding observations can be made for emission of a phonon. Multiple phonon adsorption or emission must also result in states within the cone.

Reaction 1) is quite distinctive. It would be essentially monoenergetic, isotropic, and extremely unlikely at low temperatures because of the long lifetime of the phonon states.

The features of evaporation attributable to reactions 3) and 6) individually can be understood qualitatively by using the considerations outlined above. For the $P \rightarrow P' + E$ reaction the following can be seen.

1. There will be no evaporating atoms slower than $v_x = 74.2$ m/sec corresponding to the curve intersection at $P_x = 0.468 \text{ Å}^{-1}$.

2. Since states yielding evaporating atoms near but above v_x must have nearly opposite phonon directions, these velocities will have low probability, particularly normal to the surface.

3. The location of the maximum of the velocity distribution will be rather temperature-insensitive, and at constant temperature lower away from the normal.

4. The evaporation intensity will have an exponential temperature dependence $e^{-\theta/T}$ with a characteristic temperature $\theta \sim 9\,°K$.

For the $P + P' \rightarrow E$ reaction the following are the major features.

1. There will be no evaporating atoms faster than v_x.

2. Since near zero velocity evaporating atoms are created only by phonons of almost equal and opposite momenta, they will be suppressed, particularly away from the normal.

3. The velocity distribution will be roughly Maxwellian of an enhanced temperature.

If the interaction Hamiltonian is assumed to be constant, these features can be quantitatively predicted.

For the reaction $P \rightarrow P' + E$, the evaporating atom differential velocity distribution as a function of θ_o, the angle from the normal can be expressed as an integral over the possible incident phonon directions (θ, ϕ)

$$J_{IV} = A v_o^2 \int p^2 e^{-\beta u_1 p} \, d(\cos \theta) \, d\phi. \tag{6}$$

Here, A is a constant, including the interaction Hamiltonian, p is the necessary incident phonon momentum at (θ, ϕ) which conserves energy and momentum.

Straightforward dynamical considerations require that if p obeys

$$p < \frac{p_o \cos \theta_o}{\cos \theta},$$

then

$$p = \frac{1}{2} \cdot \frac{p_m^2 - p_o^2}{p_m - p_o \cos \eta},$$

where

$$p_m = \frac{p_o^2/2m + \ell_o}{u_1}$$

and

$$\cos \eta = \sin \theta \sin \theta_o \cos \phi + \cos \theta \cos \theta_o.$$

These terms are clarified in Fig. I-2.

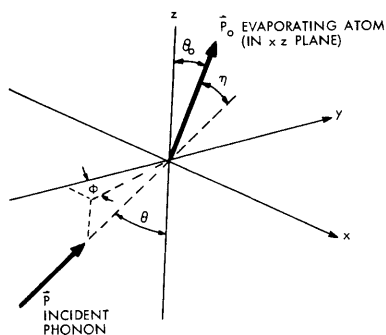For the $P + P' \rightarrow E$ reaction the result is similar:
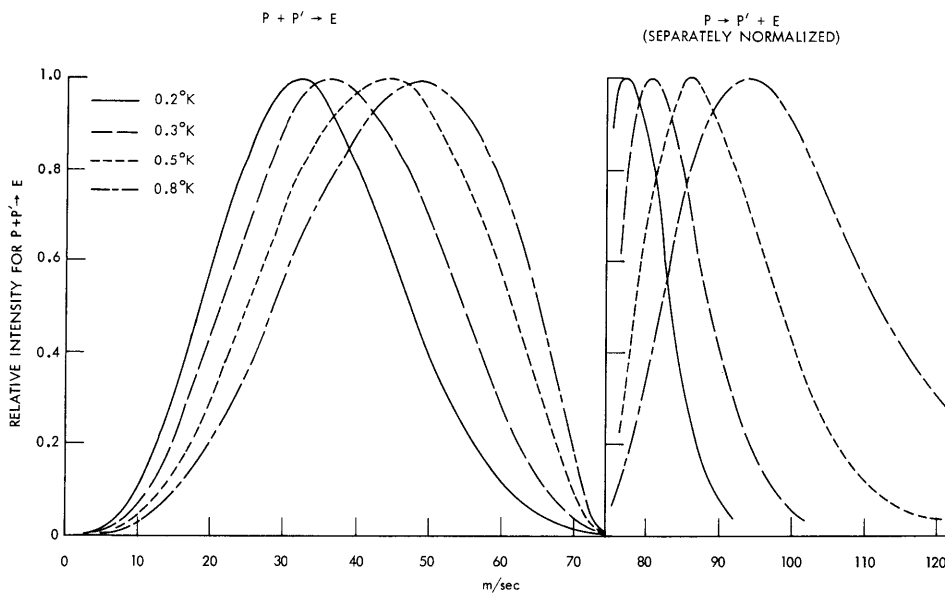

Fig. I-2. Angular conventions.

Fig. I-3. Normalized reaction velocity distributions at $\theta_o = 0$.

$$J_V = A' v_o^2 \int \frac{p^2 d (\cos \theta) \, d\phi}{\left[e^{\beta u_1 (p_m - p)} - 1 \right] \cdot \left[e^{\beta u_1 p} - 1 \right]}.$$

A' contains the same factors as A, except that the Hamiltonian term is the one appropriate to the reaction P + P' → E.

These two evaporation rates have been calculated for a variety of temperatures, and are plotted, each separately normalized, in Fig. I-3. The relative magnitudes of each curve above and below 74.2 m/sec, corresponding to J_{IV} and J_V, respectively, are arbitrary and depend on the relative strengths of the two interaction Hamiltonians in A and A'.

If it is assumed that the two-phonon, one-evaporating atom Hamiltonian is the same, regardless of whether the second phonon is adsorbed or emitted, then A = A'. In this case J_V dominates and causes a non-Maxwellian beam with an excess of low-velocity atoms. This is essentially because J_V is dominated by a Boltzmann factor with a characteristic energy of the total energy of the evaporating atom, while J_{IV} is dominated by a similar term involving the incident phonon energy, which must be greater than the energy of the evaporating atom.

If P + P' → E is not allowed, J_{IV} alone leads to a non-Maxwellian beam with no atoms slower than 74.2 m/sec and an excess of fast atoms. Such a beam might be detected as approximately Maxwellian of enhanced temperature. This apparent enhancement is ~0.7-0.6°K for beams from liquid helium between 0.2°K and 0.8°K.

As with the momentum nonconserving models, a temperature enhancement results. The actual size of this effect depends on the various interaction Hamiltonians. Because of the distinctive temperature and angular dependences predicted, with more detailed experimental measurements the various models used in this report can be distinguished.

R. F. Tinker

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B. DIFFUSION OF HELIUM THREE IN SUPERFLUID BACKGROUND

An apparatus is being constructed to measure the diffusion of small concentrations of He^3 atoms in superfluid He^4 using molecular beam techniques. The experiments will consist of injecting a small amount of He^3 into a superfluid sample, and detecting the diffusing He^3 atoms as they evaporate from the superfluid surface. The detection method utilizes a field-ionizing etched tungsten needle,[1] a focussing mass spectrometer, and an electron multiplier.

Most previous work on diffusion in He^3-He^4 mixtures has measured "spin diffusion" and has involved measurement of the dephasing of a set of precessing spins (the He^3 nuclear moments) that diffuse in a nonuniform magnetic field. (It is assumed that spin-spin interactions are negligible and that the dephasing is due entirely to diffusion.) (See, for example, Anderson et al.[2]) These experiments have been done at fairly high He^3 concentration (of the order of 1 per cent) where the diffusion is dominated by He^3-He^3 interactions. These data have proved to be quite useful: the dilute solution approximates a weakly interacting Fermi gas and, by making use of Fermi liquid theory (and the transport coefficient calculations of Hone[3]), Baym, Bardeen and Pines[4] have used diffusion data at various temperatures and concentrations (and hence Fermi energies) to extract the momentum dependence of the effective He^3-He^3 interaction V(k). The data themselves indicate a low temperature dependence similar to that of an ideal Fermi gas; at millidegree temperatures[2] ($T \ll T_F$) D varies as $1/T^2$. The higher temperature dependence has been more complicated, since interaction with the phonon and roton excitations of the superfluid become important. In a recent set of experiments Opfer, Luszczynski, and Norberg,[5] using the spin-echo method, have measured the diffusion coefficient at both high and low concentrations so that concentration-dependent and concentration-independent parts of the diffusion coefficient could be extracted. The temperature dependence of the concentration-independent component was consistent with diffusion via He^3-roton interactions.

The only data for D not involving spin diffusion experiments are the calculations of D from the thermal conductivity measurements of Ptukha.[6] By writing down the equations for heat flow in an He^3-He^4 solution, $Q = STV_n$, $v_n X = D\nabla x$, and the condition for thermodynamic equilibrium,

$$\frac{\partial v_{superfluid}}{\partial t} = 0 = -\nabla u_4 = S_{40}\nabla T - \frac{RT}{M_4}\nabla X,$$

one can derive[7] a relation between K (thermal conductivity) and D (diffusion coefficient) valid at temperatures $T \geqslant 1.2°K$:

$$K = \frac{pS_{40}^2 M_4 D}{RX}$$

p = density,   x = He^3 concentration

S_{40} = He^4 entropy at 0 He^3 concentration.

The resulting data for D are consistent with the more recent lower temperature data of Opfer et al.[5]

Note that the classical Einstein relation D = kTb, where b = mobility, does not seem to be useful here, for if one approximates an He^3 atom as a sphere suspended in a liquid, then using Stokes's law b = $\frac{1}{6\pi\eta a}$ (η = viscosity, a = sphere radius), one gets D $\sim \frac{T}{\eta}$, a relation not substantiated by viscosity data. (The He^3 atom in superfluid He^4 is apparently more complicated than that of a sphere in a viscous fluid.)

In the proposed experiment, physical diffusion, rather than spin diffusion, will be observed. The problem of spin relaxation by paramagnetic impurities in solution and at the walls of the sample container, which plagued some of the earlier spin diffusion measurements (see Low and Rohrschach,[8] for example) will not be present in the proposed experiments, and, also, spin-spin interactions, seemingly very small in the spin diffusion experiments (T_2, transverse relaxation time, is large) will be completely eliminated.

The initial experiments will involve small concentrations of He^3 interacting with the excitations of the superfluid. Since background levels as low as a few counts per minute have been attained in the Molecular Beams group, concentrations far lower than those previous measurements will be used. Eventually, we hope that as low temperatures are achieved, and the excitation densities decrease, the superfluid will approach a condition analogous to that of an ether through which the He^3 atoms will move undeflected.

G. Herzlinger, J. R. Clow, J. G. King

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C. PROPOSED MOLECULAR MICROSCOPE

A simple aperture type of molecular microscope using water molecules with a resolution of 6000 Å is described in this report. Despite its relative simplicity, this design should yield an instrument capable of useful chemical and biological studies.

Consider the device shown in Fig. I-4. What kind of performance can one expect? Let us start by assuming that water will be the molecular species with which the
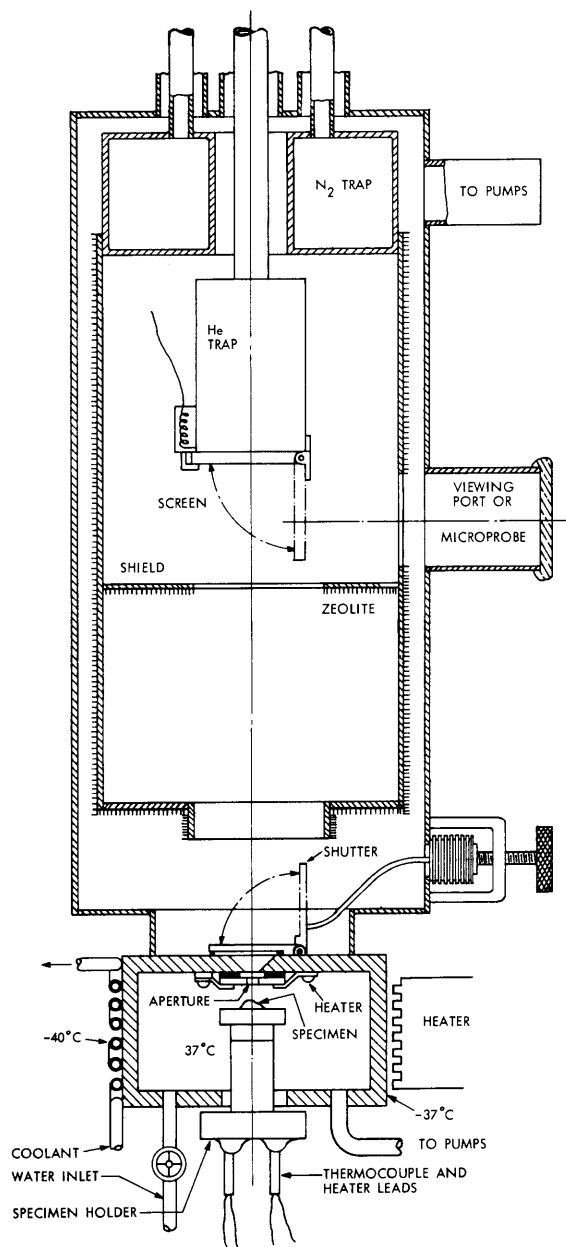

Fig. I-4. H_2O molecular microscope.

specimen will be investigated. A low vapor pressure of water will be maintained in the specimen chamber so that there will be a negligible number of molecule-molecule collisions. Some of those molecules that bounce off the specimen, which is maintained at a higher temperature than the chamber walls, will travel in a straight line through the aperture and will freeze out on the screen as shown in Fig. I-5. The distance between
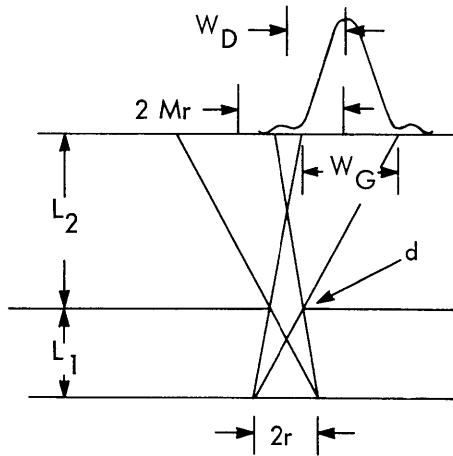

Fig. I-5. Geometry of molecular rays.

the specimen and the aperture is L_1, and that between the aperture and the screen is L_2. Therefore the magnification to be expected is $M = L_2/L_1$. If we let the diameter of the minimum resolvable spot on the specimen be 2r, then the magnified spot on the screen will have a diameter of 2Mr with additional broadening attributable partly to the geometrical spread of the molecular rays and partly to their diffraction at the aperture. If we make these two sources of spreading equal to each other and to one-half the width of the minimum resolvable spot, namely, Mr, we have

$$W_D = L_2\lambda/d = W_G = d(L_2+L_1)/L_1 = Mr,$$

where W_D and W_G are the widths contributed by diffraction and geometry, respectively, d is the diameter of the aperture, and λ is the de Broglie wave length, $h/m\bar{v}$.

We can now inquire how many molecules emitted from the minimum resolvable spot will strike the screen per second. Since $\rho\bar{v}\pi r^2/4$ molecules per second strike the minimum resolvable area per second, this will also be the number evaporating (ρ is the molecule number density in the specimen chamber). The solid angle subtended by the aperture is d^2/L_1^2, and hence the number of molecules striking the screen per second is simply $d^2\rho\bar{v}\pi r^2/4L_1^2$.

For negligible scattering in the specimen chamber the mfp should be approximately equal to $L_1 = 1/\rho\sigma$, where σ is the cross section for scattering. Combining these various expressions, we obtain for n_s, the number of molecules striking the screen

per second, the following expression;

$$n_s = \pi/4 (h^2/\sqrt{2k}) \sqrt{mT} (1/m^2\sigma)(1/L_1),$$

where we have used the facts that $\bar{v} = \sqrt{2kT/M}$ and the mass of one atom m is the molecular weight divided by Avogadro's number M/N.

Specializing for the case of $L_1 = 0.3$ cm and $M = 18$ (water), $T = 300\,°K$ and taking the cross section for scattering of the atoms to be 1.5×10^{-14} cm^2, we obtain $n_s = 150$ counts per second, which indicates that a 10-sec "exposure" provides a sufficient number of counts so that shot noise will not be a problem. That is, successive 10-sec exposures from such a spot should in principle agree within approximately 3 per cent. Notice that both L_2 and M are arbitrary as long as there is negligible scattering in the region beyond the aperture. It seems reasonable to make $L_2 = 30$ cm, which would yield a magnification of ~100. The magnification to be chosen will depend, however, on the methods used for detecting the molecules that have frozen out on the screen. Note that the cross section assumed is that derived from viscosity approximately corrected for the small angles appropriate to this experiment.

The diffraction condition required $Mr = L_2\lambda/d$, whence $r = L_1/\lambda d$. Equating the diffraction width to the geometrical width, we have $r^2 = L_1\lambda$, and again for $L_1 = 0.3$ cm and typical water molecule wavelengths of 0.35 Å we get $r = d = 3000$ Å. Thus the minimum resolvable spot diameter would be 6000 Å, with an aperture of 3000 Å. The mean-free path condition requires that the density ρ should be equal to $1/L_1\sigma$, whence the vapor pressure should be 10^{-1} Torr approximately, which for water implies a tempera-. ture of $-40\,°C$. The walls of the specimen chamber could be maintained slightly warmer, except for one region where ice would collect. The specimen would be on its own support which could be maintained at any temperature desired, including $37\,°C$. The aperture 0.3 cm above it should be kept quite a bit warmer to prevent the condensation of water molecules in it.

The kind of specimen that one would use would be the kind that is customarily observed under a light microscope. One might inquire, Do water molecules evaporate uniformly from red blood corpuscles, or sections of bones, frozen cells, and so forth?

When the water molecules impinge on the screen, which is maintained at an adequately low temperature somewhere between $77\,°K$ and $1.6\,°K$, there are several important questions to be answered. First, What is the probability that a molecule will stick as a function of its velocity, the temperature of the surface, and the nature of that surface? Second, having stuck, How likely is it to stay in place as a function of temperature and nature of the surface? Third, being stuck, What sort of methods can be used to make it visible? Although there is some information about the first two questions, we cannot say that these experimental tests have been made at very low temperatures

or under suitably clean vacuum conditions. Certainly, in the early work designed to investigate molecular beam condensation targets, and even in some of the more recent work of this nature where radioactive detection in atomic-beam resonance experiments has been used, conditions were not usually very clean, such methods as heating platinum buttons in a flame before placing them in vacuum being typical.

It is also essential to know what background to expect from the screen in the absence of molecules from the beam. The contribution from the residual gas, at a pressure of 10^{-15} Torr in the 10 seconds during which the beam deposits 150 molecules on an area 3×10^{-5} cm^2 on the screen, is 300 molecules, most of which would not be water. Much lower pressures than these conservative estimates should be attainable in a cryosorptively pumped enclosure.

As to the question of making the small number of molecules deposited on the surface visible, it is possible that some chemical method of development may be discovered in which even a single molecule can serve as a condensation nucleus so that large deposits of some other vapor can be formed which will reveal the arrangement of molecules on the surface. One can think, of course, about what the necessary conditions are for this to happen, but there will be a certain element of magic and luck in finding the right procedure.

A brute force technique fortunately also exists which would probably be necessary in any case when higher over-all magnification is sought in the future. One need only scan the surface with an electron beam from an electron microprobe system. If the beam has adequate power density, as it does, it could raise the temperature of the screen rapidly enough that the molecules in that region would evaporate without having a chance to move sideways. The electron current densities are such that the probability of ionizing the evaporated water would be high. Once the water molecules are ionized the resultant bundle of ions can be turned into a visible image in one of a number of familiar ways.

The following numbers seem encouraging. Apparently one can obtain commercially electron microprobes that produce 10^{-7} A in a 10^{-4} cm spot. Suppose that on this spot there are 10^4 atoms each with a cross section of 10^{-15} cm^2. Then 10^{-3} of the area is covered with atoms and in each second 10^9 out of the 10^{12} incident electrons will strike atoms; that is, 10^5 electrons will strike each atom, which should give adequate ionization probability even at the rather high energies required for good focusing of the electron beam. A 10 kV 10^{-7} A beam gives 1 mW, which, in 1 sec, should certainly heat the surface up adequately. In fact, it may heat too much, but by keeping an adequate amount of superfluid helium behind a very thin foil it is probable that the foil can be kept within the correct temperature range.

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