IV. PHYSICAL ACOUSTICS*

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A. STUDIES OF SUPERSONIC JETS

Our study of supersonic jets continues using a modified version of the corona probe described in a previous report.[1] The following modifications have been incorporated: A current collector has been placed around the central discharge wire. This makes the discharge more stable and less dependent on the flow velocity and the geometry of the surrounding system. A 10-MΩ resistor has been placed in series with the central discharge wire, thereby decreasing the discharge current and prolonging probe life. The probe current is now measured as the voltage drop across a 1000-Ω resistor between the current collector and ground. In Fig. IV-1 a longitudinal trace of the probe output as a function of distance along the center line of the jet is shown. This curve represents the density variation along the jet, and the shock cell structure of the jet is clearly noticeable.

As well as this steady-state response, we have also investigated the spectrum of the fluctuations in the probe current. Under certain operating conditions of the jets this spectrum contains lines which undoubtedly correspond to shock cell oscillations. Of particular interest is the amplitude variation of these oscillations with distance along the jet. A typical example of such a variation is shown in Fig. IV-2, which corresponds to an oscillation frequency of 8.8 Hz. It is interesting to note that the maximum amplitude is in the third cell, and no detectable oscillations occur in the first and second cells.

While engaged in this work we have also studied some of the characteristics of the probe itself. For example, the variation of the probe output voltage with density for some different probe bias voltages is shown in Fig. IV-3. Self-oscillation of the probe will occur under certain conditions. As can be seen from Fig. IV-4, the frequency of these oscillations depends on both operating

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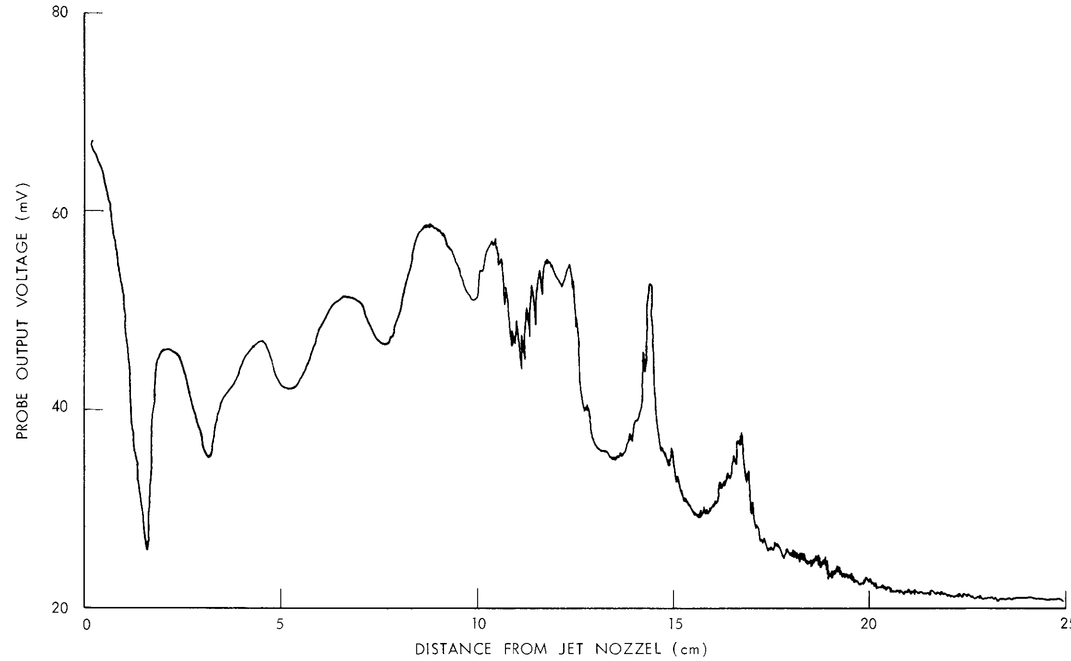

Fig. IV-1. Probe output voltage measured along the axis of the jet as a function of distance from the jet nozzle for a pressure ratio of 18.6 and a probe bias of 900 V.

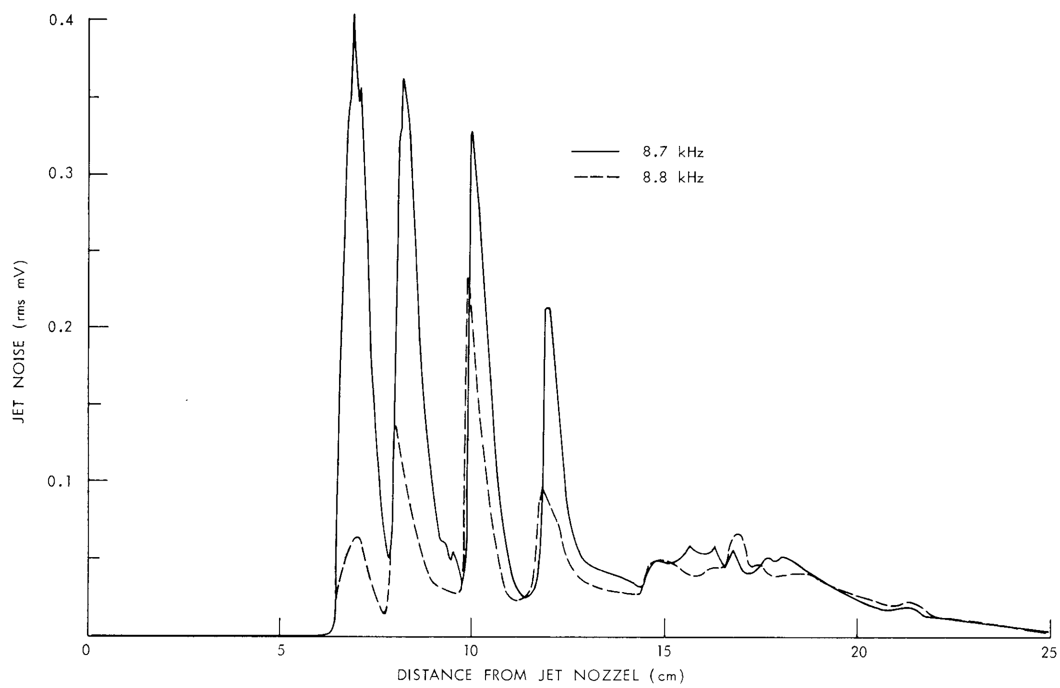

Fig. IV-2. The rms jet noise along the jet axis as measured by the fluctuations of the probe output voltage. The bandwidth of the spectrum analyzer is ±6 Hz and the probe bias is 900 V. The curves have not been corrected for changes in the small-signal gain of the probe, which are due to changes of the average gas density. Note that the higher frequencies appear closer to the jet nozzle in each shock cell.

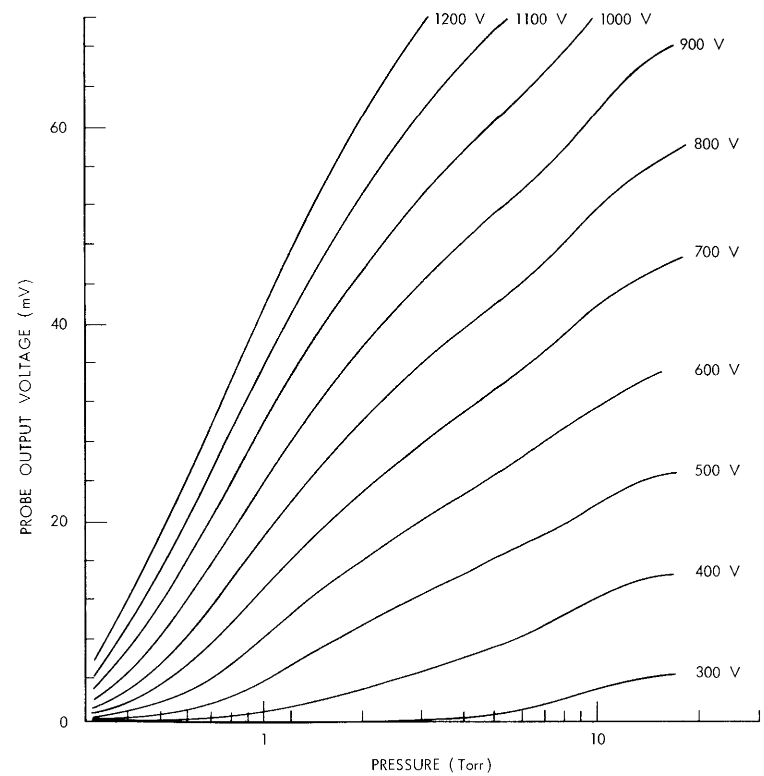

Fig. IV-3. Probe output voltage as a function of ambient pressure under low flow-rate conditions, for various probe bias voltages. These curves are used to calibrate the probe for measuring the gas density in the supersonic argon jet.

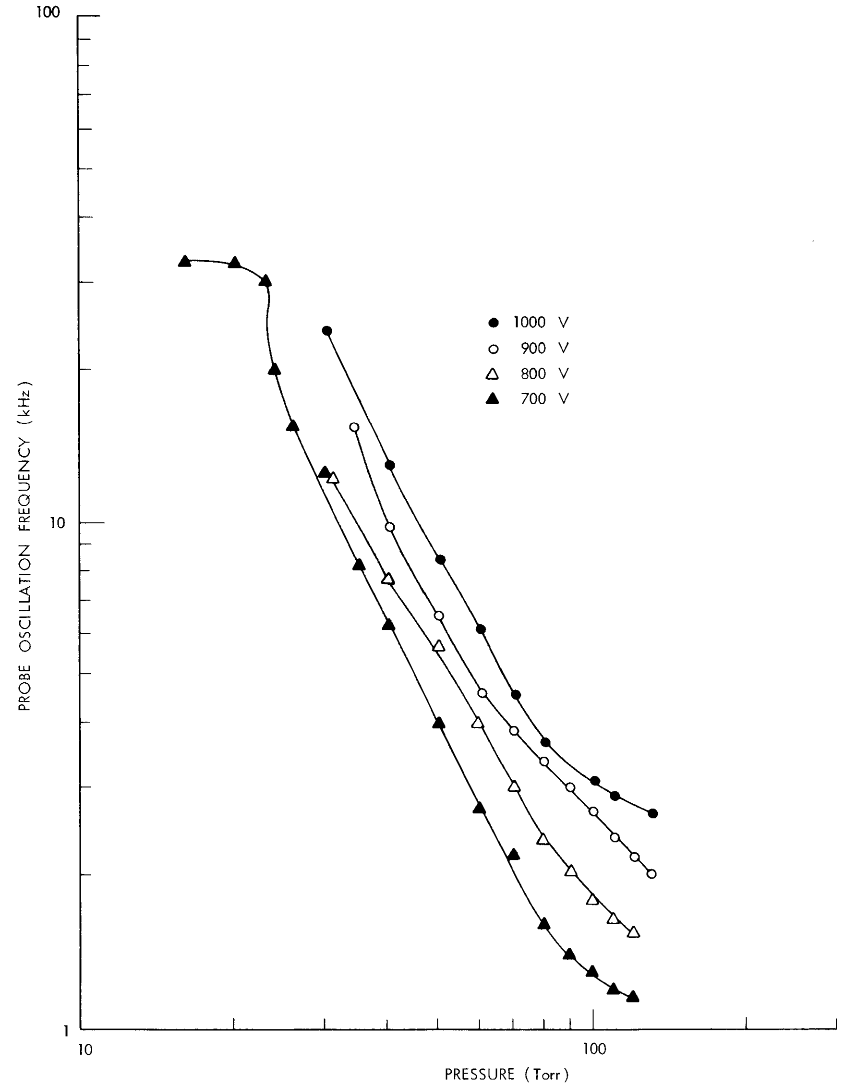

Fig. IV-4. Probe self-oscillation frequency as a function of pressure for various probe bias voltages.

voltage and gas density. These oscillations do not appear to be present when the probe operates in the jet. A more detailed analysis of this matter will be presented in a future report.

R. H. Price

References

1. R. H. Price and U. Ingard, "Techniques for Measuring Density Fluctuations in Jets," Quarterly Progress Report No. 100, January 15, 1971, pp. 70-72.

B. NONLINEAR TRANSMISSION OF SOUND THROUGH AN ORIFICE

Experimental data have already been reported on nonlinear transmission of sound through a sharp-edged orifice.[1] At high enough levels of sound (i.e., >130 dB) separation of flow occurs[2] and the average velocity u at the orifice becomes a nonlinear function of Δp (the pressure drop across the orifice)[3]

$$\Delta p = \frac{\rho}{2C_2^2} |u|u, \tag{1}$$

where ρ is the density of the air, and C_1 is the orifice coefficient. If A_D and A_o are the duct and orifice cross section, respectively, then (1) can be written in terms of the average duct velocity U.

$$\Delta p = \frac{\rho}{2C_1^2}\left(\frac{A_D}{A_o}\right)^2 |U|U. \tag{2}$$

To be thorough, we include in the right-hand side of (2) a resistive, as well as a reactive, term to account for viscous and inertial effects in the vicinity of the orifice. The equation of motion then becomes

$$\Delta p = Q|U|U - \rho c\theta U + \mu \frac{d}{dt} U \qquad \text{for } x = 0, \tag{3}$$

where

$$Q = \left(\rho/2C_1^2\right)(A_D/A_o)^2, \qquad \mu = \rho t_m(A_D/A_o),$$

with C the speed of sound, t_m the effective thickness of the orifice,[4] and θ a resistive coefficient that can be expressed in terms of the geometry of the orifice and the coefficient of viscosity.

As a result of the nonlinearity the transmitted wave will contain a line spectrum representing the fundamental and overtones of the driving frequency. In a previous attempt to explain the experimentally obtained spectrum[5] the tube was assumed to be infinite in both directions from the plate. The theoretically obtained spectrum then contained only odd harmonics of the frequency of the incident wave. Although these harmonics were indeed predominant in the experimental data, even harmonics did occur. In the following analysis we shall re-examine this matter, using a more realistic model of the experimental arrangement.

In the experiment[1] the orifice is at $x = 0$, while a highimpedance piston located at $x = -L$ moves according to

$$u_p = u_o \cos \omega t, \tag{4}$$

thereby generating a wave that is partially transmitted through the orifice. In the steady state the incident, reflected, and transmitted waves have the form

$$\begin{pmatrix} P_I(x,t) \\ P_R(x,t) \\ P_T(x,t) \end{pmatrix} = \sum_{n=1}^{\infty} \begin{pmatrix} I_n \cos n\phi + A_n \sin n\phi \\ R_n \cos n\psi + U_n \sin n\psi \\ T_n \cos n\phi + V_n \sin n\phi \end{pmatrix} \tag{5}$$

and the corresponding velocities are

$$\begin{pmatrix} U_I(x,t) \\ U_R(x,t) \\ U_T(x,t) \end{pmatrix} = \sum_{n=1}^{\infty} \frac{1}{\rho c} \begin{pmatrix} I_n \cos n\phi + A_n \sin n\phi \\ -R_n \cos n\psi - U_n \sin n\psi \\ T_n \cos n\phi + V_n \sin n\phi \end{pmatrix}, \tag{6}$$

where $\phi = kx - \omega t$, $\psi = kx + \omega t$. The coefficients of (5) will be obtained by substituting (5) and (6) in (3), as well as in the two boundary conditions that express continuity of velocity:

$$U_I(-L,t) + U_R(-L,t) = u_p \tag{7}$$

$$U_I(0,t) + U_R(0,t) = U_T(0,t) \equiv U. \tag{8}$$

The calculations are straightforward and yield

$$\begin{aligned} A_n + U_n &= V_n \\ I_n - R_n &= T_n \end{aligned} \tag{9}$$

$$\begin{aligned} (I_n-R_n)c_n + (U_n-A_n)s_n &= \rho c u_o \delta_{1n} \\ (I_n+R_n)s_n + (U_n+A_n)c_n &= 0 \end{aligned} \tag{10}$$

$$\begin{aligned} I_n + R_n + (\theta-1)T_n + q_n V_n &= B \sum_{j=1}^{n-1} [1-\delta_{1n}][T_{n-j}T_j - V_{n-j}V_j] + 2B \sum_{j=1}^{\infty} [T_{n+j}T_j + V_{n+j}V_j] \\ A_n - U_n + (\theta-1)V_n - q_n T_n &= B \sum_{j=1}^{n-1} [1-\delta_{1n}][T_{n-j}V_j + T_j V_{n-j}] + 2B \sum_{j=1}^{\infty} [T_{n+j}V_j - T_j V_{n+j}], \end{aligned} \tag{11}$$

where

$$c_n \equiv \cos nKL, \qquad s_n \equiv \sin nKL$$

$$q_n \equiv nKt_m(A_D/A_o), \qquad B \equiv \frac{Q}{2p^2c^2}.$$

This system of equations has been simplified further, and computer work is being carried out to obtain approximate values of the coefficients for a given set of parameters. The results will be forthcoming.

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3. S. Whitaker, Introduction to Fluid Mechanics (Prentice-Hall, Inc., Englewood Cliffs, N.J., 1968).
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