

## Chapter 2. Theory of Coulomb Blockade in Semiconductor Devices

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## 2.1 Project Description

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The conductance through a small grain weakly coupled to metallic leads shows periodic dependence on the voltage applied to a gate electrode. This phenomenon is observed in both metallic<sup>1</sup> and semiconductor<sup>2</sup> devices, and is commonly referred to as the Coulomb blockade. This behavior of conductance is caused by electrostatic energy arising due to a change in the charge of the grain (quantum dot) by an electron tunneling through it.

The Coulomb blockade is typically observed in structures with a well defined quantum dot, which is separated from the leads by tunneling barriers. Recently, the experimental observation of Coulomb blockade has become possible in more sophisticated devices. An experimental group at Harvard has studied the Coulomb blockade in double quantum dots.<sup>3</sup> In a number of other experiments,<sup>4</sup> the phenomenon of Coulomb blockade was observed at unusually high temperatures  $T \sim 100$  K, where the transport may be due to the activation

of electrons over the barriers. In the past year, our research group worked on the theoretical problems raised by these experiments.

### 2.1.1 Transport through Double Quantum Dots

In these experiments,<sup>3</sup> the Coulomb blockade oscillations of conductance through a system of two quantum dots (figure 1) were studied. The contacts between the quantum dots and the leads were in the weak tunneling regime, with conductances  $G_{l,r} \ll e^2/h$ . On the other hand, the contact between the dots was tuned by adjusting the gate voltage  $V_0$  in such a way that the corresponding transmission coefficient  $T_0$  scanned the whole region from 0 to 1. As a result, a series of conductance dependences on the gate voltage, at different values of  $T_0$ , were measured.<sup>5</sup> The most basic features of the experimental data, including the splitting of the Coulomb blockade peaks when  $T_0$  increases from 0 to 1, were explained in our previous work.<sup>6</sup> We have now extended the theoretical description to include the effects of temperature and the small asymmetry of the double dot system on the heights and shapes of the Coulomb blockade peaks observed in the experiments.<sup>3</sup>

The experiments have demonstrated that small tunneling between the dots gives rise to a small splitting of the Coulomb blockade peaks. At non-zero

<sup>1</sup> H. Grabert and M.H. Devoret, *Single Charge Tunneling*, (New York: Plenum Press, 1992).

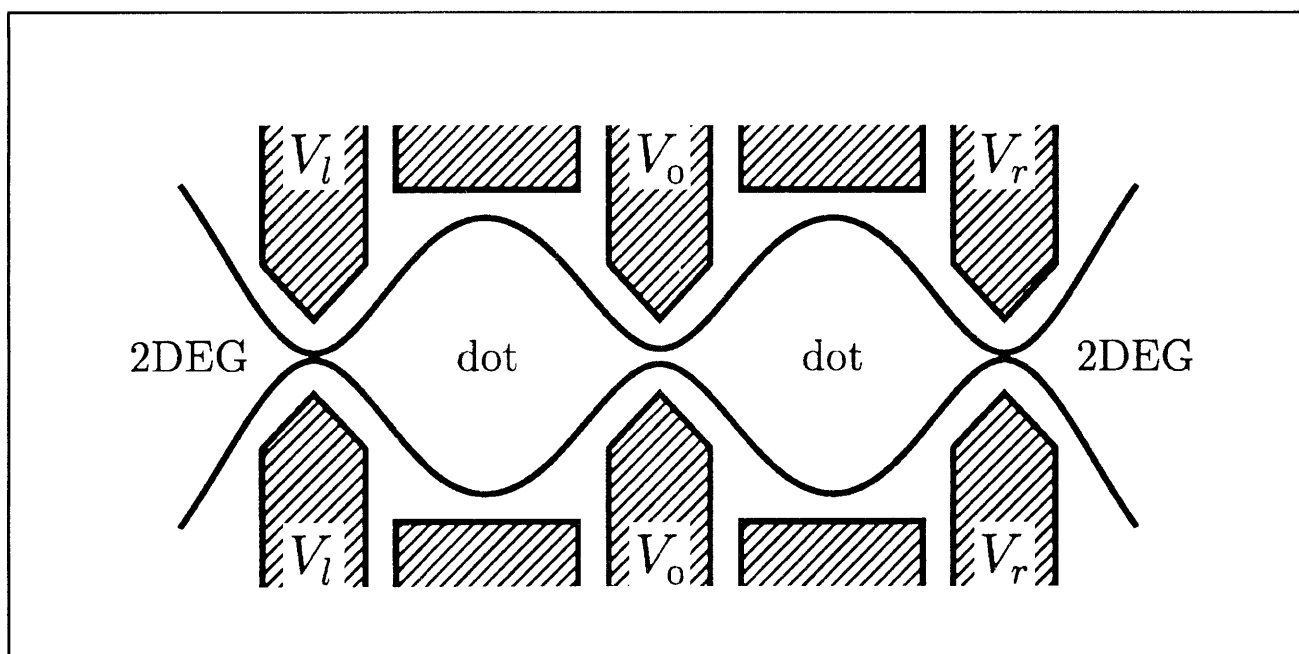
<sup>2</sup> M.A. Kastner, *Rev. Mod. Phys.* 64: 849 (1992).

<sup>3</sup> F.R. Waugh et al., *Phys. Rev. Lett.* 75: 705 (1995); F.R. Waugh et al., *Phys. Rev. B* 53: 1413 (1996).

<sup>4</sup> K. Murase et al., *Microelectr. Eng.* 28: 399 (1995); E. Leobandung et al., *Appl. Phys. Lett.* 67: 2338 (1995); E. Leobandung et al., *Appl. Phys. Lett.* 67: 938 (1995).

<sup>5</sup> K.A. Matveev, L.I. Glazman, and H.U. Baranger, *Phys. Rev. B* 54: 5637 (1996).

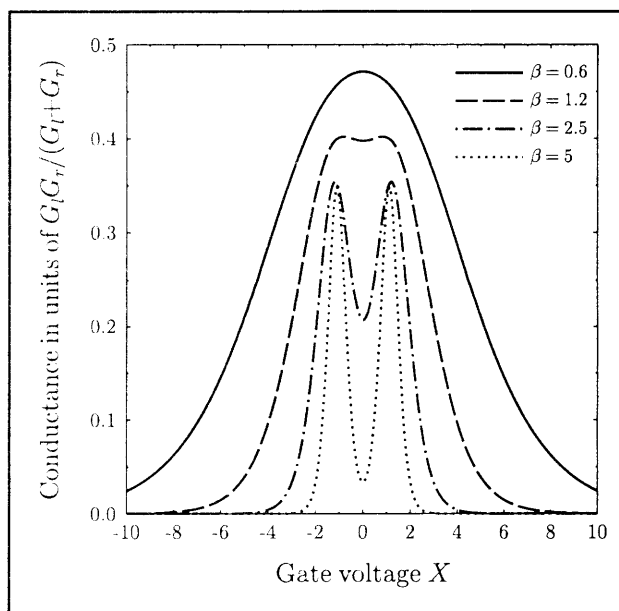
<sup>6</sup> K.A. Matveev, L.I. Glazman, and H.U. Baranger, *Phys. Rev. B* 53: 1034 (1996).



**Figure 1.** Schematic view of the double quantum dot system. The dots are formed by applying negative voltage to the gates (shaded); the solid line shows the boundary of the 2D electron gas (2DEG).  $V_l$  and  $V_r$  create tunnel barriers between the dots and the leads while  $V_0$  controls the transmission coefficient through the constriction connecting the dots.

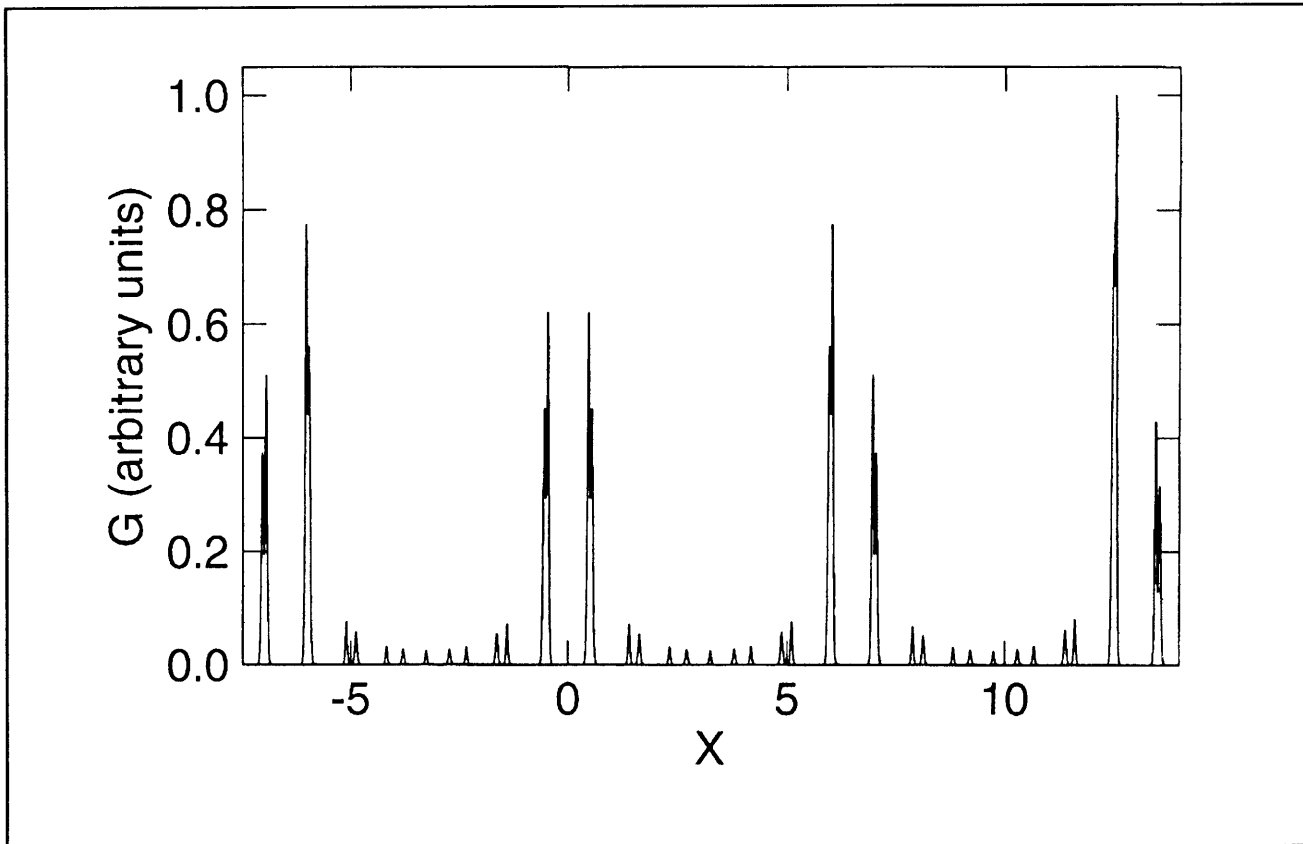
temperatures, the splitting may or may not be resolved depending on the relation between the temperature  $T$  and the charging energy  $E_C$ . We have studied the whole temperature dependence of the split peaks analytically; the results are presented graphically in figure 2.

In the experiments with double dots, one usually tries to make the two quantum dots symmetric. However, a small asymmetry is unavoidable, and is reflected in the actual experimental data. For example, when the gate voltage is chosen in such a way that an electron can tunnel to the first dot without changing its energy, tunneling to the other dot can still cost some electrostatic energy. This energy itself depends on the gate voltage  $X$ , and leads to a periodic in  $X$  suppression of the peak heights. Our results are summarized in figure 3, which strongly resembles the actual experimental data.<sup>7</sup>



**Figure 2.** The evolution of the split peaks with temperature. The gate voltage  $X$  is plotted in units of the distance between splitted peaks. The peak splitting is observable at a sufficiently low temperature,  $\beta \equiv E_C/T > 2$ .

<sup>7</sup> F.R. Waugh et al., *Phys. Rev. Lett.* 75: 705 (1995); F.R. Waugh et al., *Phys. Rev. B* 53: 1413 (1996).



**Figure 3.** The conductance as a function of dimensionless gate voltage  $X$  in the case of 15 percent asymmetry between the dots. Note the correlation between the modulation of the peak height and the separation of adjacent peaks: when the peaks are high the splitting is small, while when the peaks are small they are well separated.

## 2.2 Coulomb Blockade in the Regime of Activated Conduction

In recent experiments,<sup>8</sup> the periodic oscillations of conductance through a quantum dot were observed at temperatures exceeding 100 K. In this regime, it is not obvious that the transport through the barriers is due to tunneling rather than activation of electrons over the barriers. A simple theoretical investigation shows that for any barrier there is a characteristic temperature  $T^*$  determined by the shape of the barrier, such that at temperatures exceeding  $T^*$  the transport is activated. Though it does not seem possible to determine  $T^*$  for the barriers used in the experiments,<sup>8</sup> the unusually high temperatures at which the conductance oscillations were observed raised the fundamental problem of the Coulomb blockade of activated conduction.

Our studies<sup>9</sup> show that the Coulomb blockade in this regime is still possible, but there are several features which distinguish it from the standard tunneling regime. In particular, we have found that one has to consider the dependence of the heights of the barriers on the electric field created by the charge in the dot. Unlike in the tunneling case, without the effect of the charge of the dot on barrier heights, there is no Coulomb blockade.

When the effects of the charge on the barriers are included, the Coulomb blockade is restored. The shape of conductance peaks is given by

$$G(N) \propto \frac{e^{E_c \Lambda X / T}}{e^{E_c X / T} + e^{-E_c X / T}} \quad (1)$$

where  $X$  is a dimensionless gate voltage, and  $\Lambda$  is a geometrical parameter depending on the sym-

<sup>8</sup> K. Murase et al., *Microelectr. Eng.* 28: 399 (1995); E. Leobandung et al., *Appl. Phys. Lett.* 67: 2338 (1995); E. Leobandung et al., *Appl. Phys. Lett.* 67: 938 (1995).

<sup>9</sup> K.A. Mateev and L.I. Glazman, *Phys. Rev. B* 54: 10339 (1996).

metry of the barrier,  $-1 < \Lambda < 1$ . For symmetric barriers  $\Lambda = 0$ , and the peaks are similar to those<sup>10</sup> of the blockade of tunneling conductance. It is interesting, however, that the activation energy  $E_c X$  for the conductance (1) at off-peak values of the gate voltage is smaller than in the tunneling case<sup>10</sup> by a factor of two.

For an asymmetric barrier  $\Lambda \neq 0$ , and the peaks (1) have an unusual asymmetric shapes. This effect is specific to the activated regime. Thus, by measuring the Coulomb blockade peaks, one may be able to identify the mechanism of transport through the quantum dot.

## 2.3 Publications

Matveev, K.A., L.I. Glazman, and H.U. Baranger. "Coulomb Blockade of Tunneling Through a Double Quantum Dot." *Phys. Rev. B* 54: 5637 (1996).

Matveev, K.A., and L.I. Glazman, "Coulomb Blockade of Activated Conduction." *Phys. Rev. B* 54: 10339 (1996).

Matveev, K.A., L.I. Glazman, and H.U. Baranger. "Tunneling Spectroscopy of Quantum Charge Fluctuations in the Coulomb Blockade." *Phys. Rev. B* 53: 1034 (1996).

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<sup>10</sup> L.I. Glazman and R.I. Shekhter, *J. Phys. CM* 1: 5811 (1989).