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GMTI MIMO Radar

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Abstract— Multiple-input multiple-output (MIMO) extensions to radar systems enable a number of advantages compared to traditional approaches. These advantages include improved angle estimation and target detection. In this paper, MIMO ground moving target indication (GMTI) radar is addressed. The concept of coherent MIMO radar is introduced. Comparisons are presented comparing MIMO GMTI and traditional radar performance. Simulations and theoretical bounds for MIMO GMTI angle estimation and minimum detectable velocity are presented. The simulations are evaluated in the time domain, enabling waveform design studies. For some applications, these results indicate significant potential improvements in clutter-mitigation SINR loss and reduction in angle-estimation error for slow-moving targets.

I. INTRODUCTION

MIMO radar is an emerging active sensing technology [1], [2], [3], [4], [5]. The notion of MIMO radar is simply that there are multiple radiating and receiving sites [1], as shown in Figure 1. The collected information is then processed together. In some sense, MIMO radars are a generalization of multistatic radar concepts.

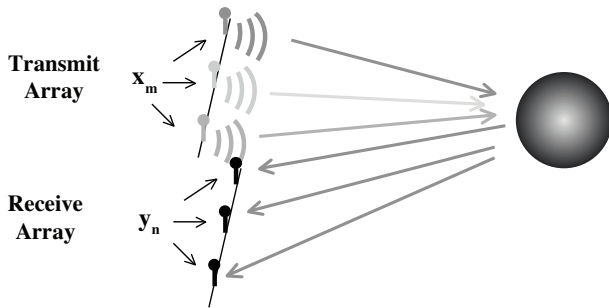


Fig. 1. Illustration of the basic MIMO radar. The location of the m^{th} transmitter is given by \mathbf{x}_m , and the location of the n^{th} receiver is given by \mathbf{y}_n .

There is a continuum of MIMO radar systems concepts; however, there are two basic regimes of operation considered in the current literature. In the first regime, the transmit array elements (and receive array elements) are broadly spaced, providing independent scattering responses for each antenna pairing, sometimes referred to as statistical MIMO radar. In the second regime, the transmit array elements (and receive array elements) are closely spaced so that the target is in the far field of the transmit and receive array. This is sometimes referred to as coherent MIMO radar, and the target response

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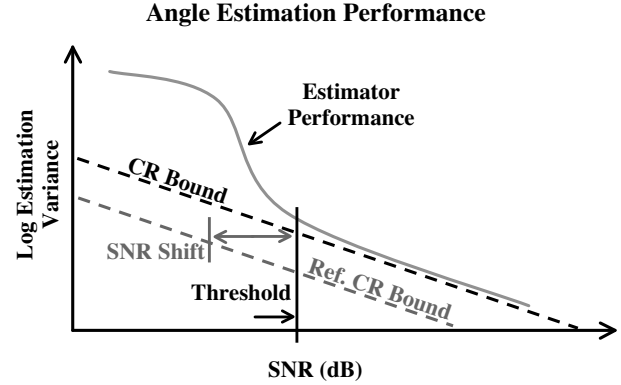


Fig. 2. Notional performance angle estimation is depicted. Asymptotically, the estimator converges to the Cramer-Rao performance bound. SNR shift is the difference between a given Cramer-Rao bound and a bound for a different array geometry.

is essentially the same for each pairing up to the relative time delay. For the rest of the paper it is assumed that the MIMO radar is operating as a coherent MIMO radar.

The fundamental advantage of coherent MIMO radars is that they enable the use of sparse arrays without the adverse effects of sidelobes. For ground moving target indicator (GMTI) radars there are two implications of the availability of larger, sparse arrays: improved angle estimation and minimum detectable velocity.

The performance of many estimators, including a MIMO radar angle estimator, often exhibit similar characteristics. As seen in Figure 2, at low signal-to-noise ratio (SNR) the variance of the estimator is poor and far from the Cramer-Rao bound (CRB) [6]. This is because the noise is of sufficient strength to allow the estimator to be confused by any near ambiguities of the angle-estimation statistic. As the SNR is increased to some threshold, the estimator's performance approaches the CRB. While the CRB is a useful tool for characterizing system performance, the threshold SNR is also important. The threshold point is dependent upon sidelobes of the array. A large, sparse array will have better CRB performance, but it will have this at the expense of a larger threshold point. MIMO radars circumvent this effect by using a virtual array that is filled, constructed from sparse real arrays.

Compared to traditional single-input multiple-output (SIMO) systems, MIMO GMTI radars can be employed to improve minimum detectable velocities [2]. Minimum detectable velocity is sensitive to both the aperture size and integration interval. Both of these characteristics can be improved by using MIMO radars. There is a question of how to make a fair comparison. Performance criteria

are different depending upon whether the radar is doing wide-area surveillance or tracking a particular target. For the moment, wide-area surveillance is considered. In this mode, typical GMTI systems either transmit from a single element (or subarray) covering a larger area or scan a beam from the transmit array over the area of interest. For a comparison with a traditional GMTI transmitting from a single element, the MIMO system may have n_T transmitters illuminating the same area. Assuming that the MIMO system is transmitting independent sequences simultaneously, so that the radiated power combines incoherently, the MIMO system may illuminate the ground with n_T times as much power. If the traditional GMTI system uses the entire transmit array coherently, sweeping a beam to perform its surveillance, as seen in Figure 3, then the traditional system illuminates the ground with n_T^2 as much power as that of a single antenna. However, the beam must be swept over the region of interest. The integration interval is about $1/n_{beams} \approx 1/n_T$. As a consequence, the integrated power is proportional to n_T^2/n_T . The MIMO system would illuminate this same total region continuously, so that the average power on the ground for the MIMO system and the swept beam is approximately the same. The combination of the longer illumination and the larger aperture of the MIMO radar provides for the possibility of improved minimum detectable velocity for GMTI systems.

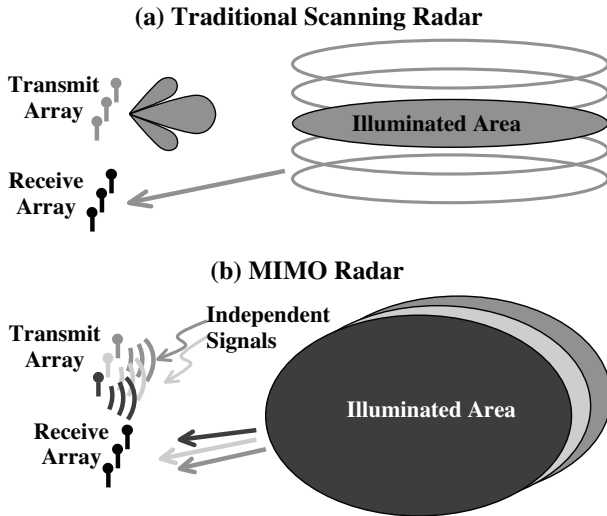


Fig. 3. Illumination patterns of (a) scanning SIMO radar and (b) MIMO radar assuming independent transmit signals.

II. MIMO RADAR VIRTUAL APERTURE

In this section, the MIMO channel is introduced. This is followed by a discussion of the MIMO virtual array. For the analysis discussed in this section, it is assumed that the waveforms transmitted from each transmit antenna (contained within a row of the matrix \mathbf{S}_δ below) are independent.

Between the transmitter and receiver is the channel. In some sense, the role of radar processing is to estimate and interpret this channel, which includes moving targets as well as clutter.

Without loss of generality, a baseband sampled signal can be considered. The $n_R \times n_S$ received data matrix \mathbf{Z} is given by

$$\mathbf{Z} = \sum_{\delta} \mathbf{H}_{\delta} \mathbf{S}_{\delta} + \mathbf{N}, \quad (1)$$

where \mathbf{N} contains the sum of noise and external interference. The summation in Equation 1 is over delays, δ , which correspond to different range cells.

If the illuminated region contained a single simple scatterer (as is displayed in Figure 1) in the far field at delay δ , then the channel matrices at all delays would be zero with the exception of \mathbf{H}_{δ} , which would have the structure

$$(\mathbf{H}_{\delta})_{n,m} \propto e^{ik\mathbf{u} \cdot (\mathbf{y}_n + \mathbf{x}_m)}, \quad (2)$$

where $k\mathbf{u}$ is the wave vector ($k \equiv 2\pi/\lambda$) and \mathbf{x}_m and \mathbf{y}_n are 3-vectors of physical locations for the transmitter and receiver phase centers, respectively. The argument of the exponential reflects differential path lengths between transmitter and receiver phase centers, given a far-field target in direction \mathbf{u} . As an example, if both the transmitter and receiver arrays have three antennas with antenna separation d , located along a line at $\{-d, 0, d\}$, in direction \mathbf{d} , then the channel matrix is given by

$$\mathbf{H}_{\delta} \propto \begin{pmatrix} e^{i\eta 2d} & e^{i\eta d} & e^{i\eta 0} \\ e^{i\eta d} & e^{i\eta 0d} & e^{-i\eta d} \\ e^{i\eta 0} & e^{-i\eta d} & e^{-i\eta 2d} \end{pmatrix}, \quad (3)$$

where

$$\eta = k\mathbf{u} \cdot \frac{\mathbf{d}}{\|\mathbf{d}\|}. \quad (4)$$

There are two important concepts illustrated by the structure of the channel matrix in Equation (3). First, the largest phase offsets $e^{i\eta 2d}$ and $e^{-i\eta 2d}$ are larger than those created by the real arrays. This is the motivation for the MIMO virtual array concept. In this case, the virtual array has five virtual array locations: $-2d, -d, 0, d$, and $2d$. Second, some entries are overrepresented. For the particular real arrays used in this example, the entries in the Hankel channel matrix are repeated. This motivates the exploration of sparse real arrays to minimize the number of repeated phase measurements.

In particular, when a dense set of receive antennas is paired with the appropriate sparse set of transmit antennas, the virtual array, which is constructed by convolving the locations of the transmit and the receive antennas, can be constructed to be filled. For example, a 4×4 MIMO radar is considered. In the following notation, a 1 indicates the existence of an antenna and a 0 indicates no antenna at that point in a regular array. If the receive array is filled, given by

$$\{1 \ 1 \ 1 \ 1\},$$

and the transmit array is sparse with antennas spaced by the length of the receive array, given by

$$\{1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1\},$$

then the MIMO virtual array is given by the 16 virtual-element array,

$$\{1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1\}.$$

This is denoted a Nyquist virtual array. Similar studies can be performed assuming transmit/receive modules; however, potential performance improvements are smaller although still significant.

III. THEORETICAL PERFORMANCE IMPROVEMENTS

Theoretical performance improvements for GMTI are discussed in [2]. As an example, a toy problem involving a slowly moving target with moderate SNR is discussed. The MIMO system has a 5-element sparse transmit array and a 5-element filled receive array. For the reference SIMO system, both the transmit and receive elements form filled arrays. It is assumed that the SIMO system operates by beamforming and sweeping this beam over the area of interest.

In Figure 4, the loss in target SINR as a function of target radial velocity is displayed for a fixed-target cross section and total transmit power. The improvement of the MIMO system compared to the SIMO system is dramatic, particularly at lower velocities. This improvement is the result of two effects. First, the MIMO system is integrating 5 times longer than the SIMO system, which has to periodically switch the transmit beam position to cover the same search area. Longer integration time also results in smaller resolution cells and thus less integrated clutter per cell. Second, the MIMO system has adaptive control of a much larger virtual array.

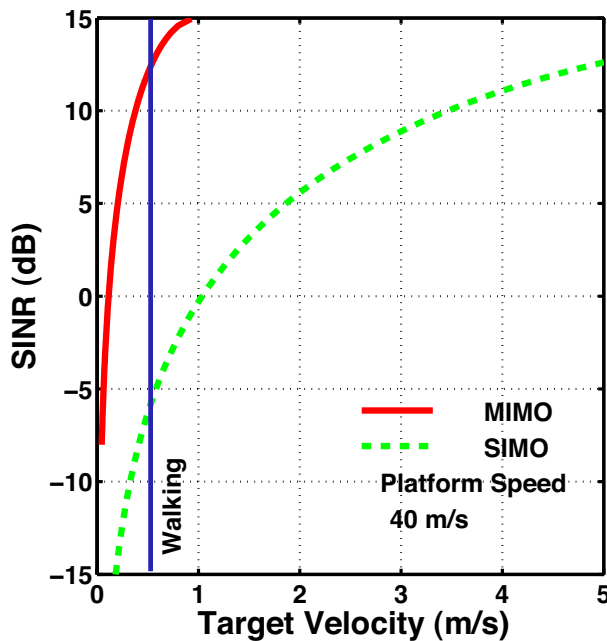


Fig. 4. Comparison of MIMO and SIMO target SINR as a function of target velocity.

Assuming a target velocity of 0.5 m/s, the Cramer-Rao bounds for the SIMO and MIMO systems are displayed in

Figure 5. The difference in angle-estimation performance is displayed as a function of the integrated SIMO array SNR (ASNR). The significant difference in angle-estimation performance arises from two effects. First, the MIMO system has better target SINR at the given velocity, as seen in Figure 4. Second, the MIMO system has access to the much larger MIMO virtual array, which improves angle-estimation performance even in the absence of clutter.

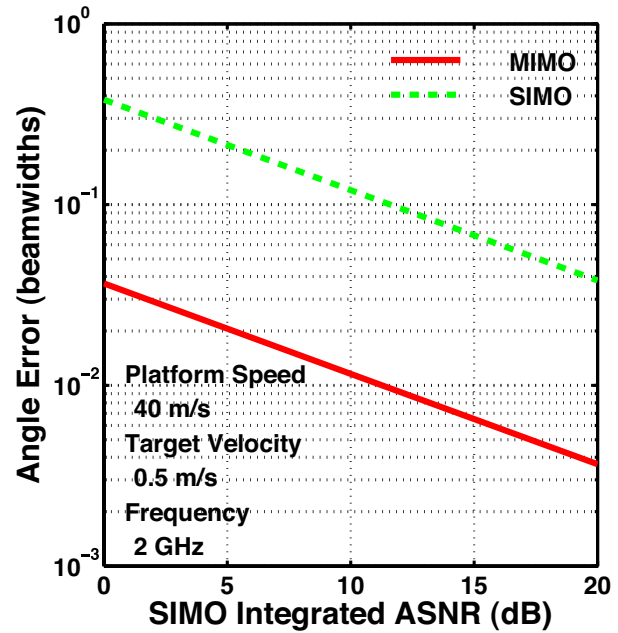


Fig. 5. Comparison of MIMO and SIMO angle-estimation Cramer-Rao bounds as a function ASNR.

IV. WAVEFORMS FOR GMTI

Implicit in the waveform (and receiver) design is the notion that the channel must be estimated accurately. The scattering response of the clutter and targets associated with each transmitter must be disentangled by the receiver. If the length of the scattering field divided by the speed of light is relatively small compared to pulse duration, this is a relatively easy problem. However, given the typical geometries of GMTI systems, the length of the observed scattering field increases as the pulse length increases. In the literature on MIMO radar, it is often assumed that the transmitted waveforms associated with each transmit antenna or subarray have perfect waveform cross-correlation properties. In practice, this is impossible to achieve. Consequently, the problem deserves investigation.

Motivated by an analog to the communications literature, it is tempting to consider approaches such as frequency division multiple access (FDMA), code division multiple access (CDMA), or time division multiple access (TDMA). The FDMA approach can be discarded quickly because, under the assumption of uniformly distributed clutter, if the frequency offsets are sufficiently different to separate the transmitters, then they are also sufficiently different to cause independent

scattering responses by the various transmitters in each given range cell; thus coherence is lost.

There are a wide range of possible CDMA waveforms. An example in this context would be random waveforms. If random signals, transmitted from subarray 1 and 2, during a given pulse are denoted $s_1(t)$ and $s_2(t)$, then the typical cross correlation for a given delay offset is given by

$$\left| \int dt s_2^*(t + \tau) s_1(t) \right|^2 \sim \frac{1}{BT}, \quad (5)$$

where BT is the time-bandwidth product of the waveforms. This result indicates that the errant clutter contribution in a given range bin due to another range bin associated with another transmitter is reduced by the time-bandwidth product of the pulse. Unfortunately, under the assumption of uniform distributed clutter, the integrated cross-transmitter errant clutter contribution from all ranges is proportional to

$$B \int d\tau \left| \int dt s_2^*(t + \tau) s_1(t) \right|^2 \sim 1. \quad (6)$$

Because a matched filter receiver will observe contributions of contamination from each of the transmitters and each transmitter pairing is incoherent with respect to another pairing, the clutter fills n_t degrees of freedom of the $(n_t \cdot n_r) \times (n_t \cdot n_r)$ MIMO virtual spatial covariance matrix associated with a given Doppler bin. While less disastrous than it might sound, the increased clutter subspace size is undesirable.

A natural question to ask is, can this effect be overcome by careful design of the waveforms? It can be proven that waveforms with perfect characteristics do not exist. In fact, for clutter occupying an extended domain of range-Doppler, if any two of the waveforms are not identical, then the rank of the MIMO virtual spatial covariance associated with the target Doppler bin must be greater than 1, for all but isolated values of the target Doppler. Further, for any set of n_t uncorrelated waveforms, the rank is n_t for all target Dopplers. These facts hold for both single-pulse and multi-pulse waveforms [7]. However, when the clutter is sufficiently limited in range and Doppler (for example, by the antenna footprints on the ground and $(\text{range})^4$ losses), there are multiple-pulse waveforms that circumvent this issue.

The first example of this is TDMA. TDMA can work in principle. However, it has the unfortunate characteristic that only one transmitter is radiating during a pulse. While some low-cost system concepts might take advantage of this characteristic, for many systems, this is undesirable because it would significantly reduce the average transmitted power. In addition, to use the TDMA approach the GMTI system must have sufficient freedom in pulse repetition frequency (PRF) such that the interleaved pulses can be employed.

A simple alternative approach is to use the same waveform on each transmitter, but to apply a modulation in slow time (pulse-to-pulse). This approach has been recreated independently in a variety of contexts (an example is given in [8]). One slow-time modulation approach is to shift each transmitter in Doppler frequency as seen in Figure 6, in which the range

Doppler image associated receiver 1 is displayed. Here, this is denoted Doppler division multiple access (DDMA). Similar to TDMA, DDMA requires sufficient PRF design freedom. In addition, fast-moving targets suffer from angle ambiguities. However, these ambiguities can be mitigated by employing different PRFs over successive coherent processing intervals (CPIs). The transmitters are distinguished by their locations in Doppler. This image is produced using a time-domain GMTI radar simulation. The 5×5 MIMO array assumes a filled receive array and a sparse transmit array such that the MIMO virtual array is Nyquist sampled. The SIMO result is produced with 5 transmit/receive elements, assuming a sweeping transmit beamformer. The platform speed is 40 m/s and the PRF is 7.2 KHz. The clutter is assumed to be Gaussian.

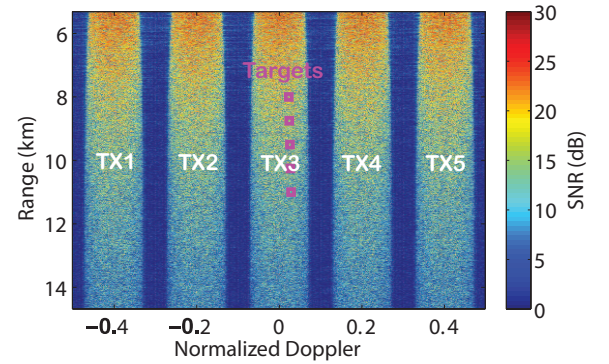


Fig. 6. Range-Doppler image before clutter mitigation for a MIMO system employing DDMA.

Applying post-Doppler space-time adaptive processing (STAP), the resulting estimated SINR loss is display in Figure 7. The angle estimations of the targets for the SIMO and MIMO systems are displayed in Figure 8. The null for the MIMO system is significantly narrower.

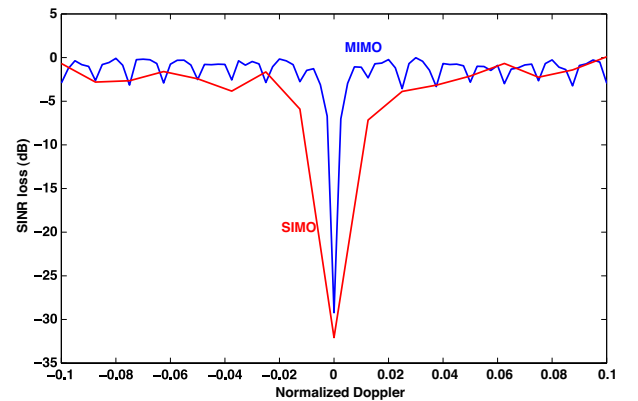


Fig. 7. Comparison of SIMO and MIMO target SINR loss as a function of normalized Doppler shift. Sidelooking array with platform speed = 40 m/s, PRF = 7.2 kHz, and $\lambda = 15$ cm.

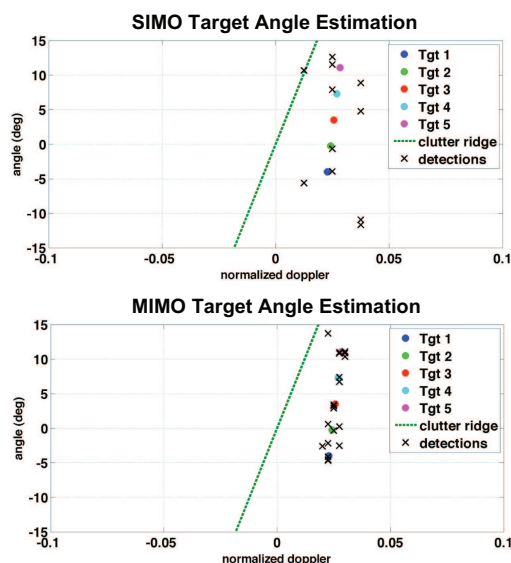


Fig. 8. Comparison of SIMO and MIMO angle-estimation performance.

V. CONCLUSIONS

In this paper, coherent MIMO processing was introduced for GMTI. The MIMO virtual array was described and theoretical performance bounds for SIMO and MIMO systems were compared. In these comparisons, significant performance improvements were demonstrated for MIMO radars. The issues for waveforms specific to GMTI were discussed, and TDMA and DDMA approaches were identified as viable approaches for a subset of system concepts.

VI. ACKNOWLEDGMENTS

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