

MIT Open Access Articles

On frame synchronization for multiple access channels

The MIT Faculty has made this article openly available. **Please share** how this access benefits you. Your story matters.

Citation: Shen, D. et al. "On Frame Synchronization for Multiple Access Channels." Communications Workshops, 2009. ICC Workshops 2009. IEEE International Conference on. 2009. 1-5. © 2009 IEEE.

As Published: <http://dx.doi.org/10.1109/ICCW.2009.5208107>

Publisher: Institute of Electrical and Electronics Engineers

Persistent URL: <http://hdl.handle.net/1721.1/61421>

Version: Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

Terms of Use: Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.



On Frame Synchronization for Multiple Access Channels

Dawei Shen

Media Lab, Viral Comm. Group
Massachusetts Institute of Technology
Cambridge, MA, 02139
Email: dawei@media.mit.edu

Wenyi Zhang

Ming Hsieh Dept. of Electrical Engineering
University of Southern California
Los Angeles, CA, 90089
Email: wenyizha@usc.edu

David P. Reed, Andrew B. Lippman

Media Lab, Viral Comm. Group
Massachusetts Institute of Technology
Cambridge, MA, 02139
Email: {dpreed, lip}@media.mit.edu

Abstract—The problem of frame synchronization is formulated and investigated for multiple access channels (MAC). Several decision rules for locating the starting positions in continuously transmitted frames are proposed and compared, for both user-synchronous and user-asynchronous cases. It is shown that the common decision rule based on the correlation statistic is suboptimal, and that correction terms need be added in order to achieve an improved detection performance. For the user-asynchronous case, the optimal joint decision rule is derived in analytical form for two-user MAC and is highly complex, and is shown to suffer from a high computational complexity. To reduce the complexity, suboptimal separate decision algorithms with and without Gaussian approximation are derived, and it is illustrated using Monte Carlo simulation that those low-complexity algorithms only incur a slight degradation in optimality.

I. INTRODUCTION

Frame synchronization is an integral functionality in many modern communication systems, in which a “pattern” or “sync word” consisting of a fixed symbol sequence is inserted periodically into the transmitted data stream. Assuming that symbol-level synchronization has already been accomplished, the receiver, before demodulating data symbols, needs to locate the positions of the sync words in its received stream. In a pioneer work [1], Massey derived the optimal decision rule for frame synchronizing a single-user channel with antipodal inputs. The key observation therein is that the optimal decision rule is not simply maximizing the correlation statistic between the sync word and the received signal stream, but further including a correction term which is a deterministic nonlinear function of the received signal. Intuitively, the correlation statistic only indicates how a segment of the received signal “looks similar” to the sync word, but loses useful information captured by the correction term which further indicates how the remaining parts of the received signal statistically “look different” than the sync word.

Motivated by the discovery in [1], researchers have made a series of extensions and further studies; see, *e.g.* [2], [3], [4], [5] and references therein. To date, most of the development has been confined for various scenarios in single-user channels; meanwhile, the corresponding results for frame synchronization beyond simple correlation in multiple access channels (MAC) still largely remain unexplored. Besides its obvious relevance in data communication systems with multiuser detection or multi-packet reception capabilities, our study of frame synchronization for MAC is also motivated

by the specific problem of RFID signal separation [6]. When multiple RFID tags are activated, symbol-synchronous signals from these tags are simultaneously transmitted in a continuous and repeated fashion, with fixed sync-words inserted into the data streams. Such a system model exactly fits into the research domain of frame synchronization.

In this paper, we investigate both the user-synchronous and user-asynchronous cases. In the user-synchronous case, transmitters are scheduled such that their transmitted frames align synchronously. We derive the optimal maximum a posteriori (MAP) decision rule in this case, which akin to single-user channels includes a correction term in addition to the correlation statistic. In the more general and complicated user-asynchronous case, multiple transmitters start transmitting frames at different time instants and thus there are multiple frame positions to locate. The computational complexity of the MAP joint decision rule grows exponentially with the number of transmitters thus rendering it infeasible for practical implementation. Consequently, an alternative approach which performs frame synchronization for each transmitter separately is proposed, with and without the further complexity reduction via Gaussian approximation of multiple access interference. The performance of the various approaches is evaluated through Monte Carlo simulation, in which different design parameters like the choice of sync words, the relative signal strength between transmitters, and the number of transmitters are examined.

The remainder of this paper is organized as follows. We introduce the channel model in Section II. In section III, we derive the optimal decision rule for the user-synchronous case. In Section IV, we focus on the various approaches to the user-asynchronous case. Finally, we present the simulation results in Section V.

II. CHANNEL MODEL

We consider the MAC model with n scalar transmitters and a single scalar receiver,

$$\mathbf{y} = \underline{h}^T \mathbf{x} + z = \sum_{l=1}^n h_l x_l + z, \quad (1)$$

where \underline{h} is a length- n complex vector whose l -th element h_l denotes the channel gain of the link between the l -th transmitter (sending x_l) and the receiver, and z denotes the

memoryless additive noise modeled as a zero-mean circularly-symmetric complex Gaussian random variable with variance σ^2 . Throughout the paper, we assume that the channel gain vector \underline{h} is perfectly known at the receiver. On the other hand, there is no specific need for the transmitter to have any knowledge of the value of \underline{h} .

We consider the case where all the transmitted data frames from the n transmitters have the same frame length N . Each frame has at its beginning a length- L input sequence called the sync word, whose value is fixed for each specific transmitter, but is generally different for different transmitters. Each input symbol x_l corresponds to a point in a two-dimensional M -constellation, for example, PSK or QAM. We further assume that all the transmitters use the same M -constellation, and that all the M possible points in the constellation, denoted $\{w_1, w_2, \dots, w_M\}$, are transmitted equally probably with a probability of $1/M$.

III. USER-SYNCHRONOUS CASE

In this case, transmitters are scheduled such that their transmitted frames align synchronously. All transmitted symbols within a frame are denoted using the elements in an $n \times N$ matrix $\mathbf{T} =$

$$\begin{pmatrix} s_{1,0} & \cdots & s_{1,L-1} & \mathbf{d}_{1,L} & \cdots & \mathbf{d}_{1,N-1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ s_{n,0} & \cdots & s_{n,L-1} & \mathbf{d}_{n,L} & \cdots & \mathbf{d}_{n,N-1} \end{pmatrix}. \quad (2)$$

The l^{th} row of \mathbf{T} is the frame transmitted by the l -th transmitter. The $n \times (N - L)$ sub-matrix \mathbf{D} , with elements $\mathbf{d}_{i,j}$ ($1 \leq i \leq n, L \leq j \leq N - 1$), denotes the random data symbols where $\mathbf{d}_{i,j}$ are statistically independent. The $n \times L$ sub-matrix \mathbf{S} , with elements $s_{i,j}$ ($1 \leq i \leq n, 0 \leq j \leq L - 1$), denotes the deterministic sync words matrix. At the stage of frame synchronization, \mathbf{D} is unknown at the receiver but \mathbf{S} is. Note that since each $\mathbf{d}_{i,j}$ takes on values from M equally probable points $\{w_1, w_2, \dots, w_M\}$ of the constellation, each column of \mathbf{D} is uniformly distributed over M^n vectors $\{\underline{v}_1, \underline{v}_2, \dots, \underline{v}_{M^n}\} := \{w_1, w_2, \dots, w_M\}^n$.

Under frame synchronism, the received frame without noise is a length- N vector

$$\underline{q} = \underline{h}^T \mathbf{T} = (t_0, \dots, t_{L-1}, \mathbf{c}_L, \dots, \mathbf{c}_{N-1}), \quad (3)$$

where

$$t_i = \sum_{l=1}^n h_l s_{l,i}, \quad 0 \leq i \leq L - 1 \quad (4)$$

$$\mathbf{c}_j = \sum_{l=1}^n h_l \mathbf{d}_{l,j}, \quad L \leq j \leq N - 1. \quad (5)$$

Modeling the uncertainty in the frame timing as a cyclic right shift operator, the actual received frame with noise can be expressed as

$$\underline{r} = \mathcal{T}^\mu(\underline{q}) + \underline{z}, \quad (6)$$

where \mathcal{T}^μ is the cyclic right shift operator, shifting its length- N operand by μ symbol positions in a cyclic fashion. We

use \underline{r} , the observation of \underline{r} , to decide the sync word position μ , which is assumed to be uniformly distributed over $\{0, 1, \dots, N - 1\}$. The MAP decision rule minimizes the decision error probability, and seeks for the value of μ that maximizes $\Pr[\mu = \mu | \underline{r} = \underline{r}] = p_{\underline{r}}(\underline{r} | \mu = \mu) \Pr[\mu = \mu] / p_{\underline{r}}(\underline{r})$. Under the assumption that $\Pr[\mu = \mu] = 1/N$ for all μ , we may equivalently maximize $p_{\underline{r}}(\underline{r} | \mu = \mu)$, which is further equivalent to

$$\max_{0 \leq \mu \leq N-1} \sum_{\text{all } \underline{D}} p_{\underline{z}}(\underline{r} - \mathcal{T}^\mu(\underline{q})). \quad (7)$$

Noting that \underline{z} is a length- N circularly-symmetric complex Gaussian random vector with zero mean and covariance matrix $\sigma^2 \cdot \mathbf{I}_{N \times N}$, we can further express the statistic in (7) as

$$\left[\prod_{i=0}^{L-1} \prod_{l=1}^n e^{(r_{i+\mu} h_l^* s_{l,i}^* + r_{i+\mu}^* h_l s_{l,i}) / \sigma^2} \right] \cdot \prod_{j=L}^{N-1} \left(\sum_{\underline{v}=\underline{v}_1}^{\underline{v}_{M^n}} e^{(\sum_{l=1}^n (r_{j+\mu} h_l^* v_l^* + r_{j+\mu}^* h_l v_l) - |\sum_{l=1}^n h_l v_l|^2) / \sigma^2} \right).$$

Taking logarithm and further removing redundant terms independent of μ , we obtain the final expression of the statistic for MAP frame synchronization,

$$S = \frac{1}{\sigma^2} \sum_{i=0}^{L-1} \sum_{l=1}^n (2\text{Re}(r_{i+\mu} h_l^* s_{l,i}^*)) - \sum_{i=0}^{L-1} \ln \left(\sum_{\underline{v}=\underline{v}_1}^{\underline{v}_{M^n}} e^{(\sum_{l=1}^n (2\text{Re}(r_{i+\mu} h_l^* v_l^*)) - |\sum_{l=1}^n h_l v_l|^2) / \sigma^2} \right). \quad (8)$$

In the correlation decision rule, only the first term in S is computed and maximized over μ . In view of (8), we notice that the optimal MAP decision rule should be adjusted by adding a correction term to the correlation. When there is only one transmitter ($n = 1$) and the constellation is antipodal $\{1, -1\}$, the MAP decision rule (8) reduces to that in [1] for single-user channels with antipodal inputs.

IV. USER-ASYNCHRONOUS CASE

Now we consider the more general case where transmitters start transmitting frames at different time instants. We assume that all the transmitters are synchronous at the symbol level so the discrete-time channel model in Section II still applies. Instead of finding one single sync word position μ in the user-synchronous case in Section III, here we need to find n sync word positions each for one transmitter. We hence represent the sync word positions by a random vector $\underline{\mu} = (\mu_1, \mu_2, \dots, \mu_n)^T$, in which each component is uniformly distributed over $\{0, 1, \dots, N - 1\}$, and is independent of the others.

There are two approaches for deciding $\underline{\mu}$. In the first approach, we can make decision on μ_l ($1 \leq l \leq n$) separately for each transmitter. In this approach, we further assume that all the other $n - 1$ transmitters constantly transmit random data symbols spanning the whole length- N frame, and thus ignore the knowledge of the deterministic sync words contained in the other transmitters' frames. In the second approach, we can jointly make decision on $\underline{\mu}$ as a single entity using the MAP decision rule. This approach is optimal, but requires a high-complexity algorithm to compute statistics. In the remainder of this section, we derive the two approaches in details.

A. Separate Decision Approach

Without loss of generality, we only consider deciding the sync word position μ_1 for the first transmitter. Treating the sync words for the other transmitters as random data symbols, we can denote the transmitted symbols in a frame of the n transmitters as a matrix $\mathbf{T}^s =$

$$\begin{pmatrix} s_0 & \cdots & s_{L-1} & \mathbf{d}_{1,L} & \cdots & \mathbf{d}_{1,N-1} \\ \mathbf{d}_{2,0} & \cdots & \mathbf{d}_{2,L-1} & \mathbf{d}_{2,L} & \cdots & \mathbf{d}_{2,N-1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{d}_{n,0} & \cdots & \mathbf{d}_{n,L-1} & \mathbf{d}_{n,L} & \cdots & \mathbf{d}_{n,N-1} \end{pmatrix}. \quad (9)$$

Compared with \mathbf{T} in (2), the symbols below the sync word (s_0, s_1, \dots, s_{L-1}) are no longer deterministic sync words, but unknown random data symbols. We thus denote the sub-matrix formed by elements $\mathbf{d}_{l,i}$ ($2 \leq l \leq n, 0 \leq i \leq L-1$) as \mathbf{D}' . Let $\underline{\mathbf{d}}_j$ ($L \leq j \leq N-1$) denote $(d_{1,j}, d_{2,j}, \dots, d_{n,j})^T$, the j th column of the sub-matrix \mathbf{D} , and $\underline{\mathbf{d}}'_i$ ($0 \leq i \leq L-1$) denote $(d_{2,i}, d_{3,i}, \dots, d_{n,i})^T$, the i th column of the sub-matrix \mathbf{D}' . As analyzed in Section III, $\underline{\mathbf{d}}_j$ is uniformly distributed over M^n possible vectors $\{\underline{v}_1, \underline{v}_2, \dots, \underline{v}_{M^n}\}$. Similarly, $\underline{\mathbf{d}}'_i$ is uniformly distributed over M^{n-1} possible vectors $\{\underline{u}_1, \underline{u}_2, \dots, \underline{u}_{M^{n-1}}\} := \{w_1, w_2, \dots, w_M\}^{n-1}$. The received synchronous frame without noise is

$$\underline{\mathbf{q}}^s = \underline{\mathbf{h}}^T \mathbf{T}^s = (\mathbf{t}_0, \mathbf{t}_1, \dots, \mathbf{t}_{L-1}, \mathbf{c}_L, \mathbf{c}_{L+1}, \dots, \mathbf{c}_{N-1}), \quad (10)$$

where

$$\mathbf{t}_i = h_1 s_i + \sum_{l=2}^n h_l \mathbf{d}_{l,i}, \quad 0 \leq i \leq L-1 \quad (11)$$

$$\mathbf{c}_j = \sum_{l=1}^n h_l \mathbf{d}_{l,j}, \quad L \leq j \leq N-1. \quad (12)$$

Taking the random shift μ_1 and noise into consideration, the actual received frame at the receiver is

$$\underline{\mathbf{r}}^s = \mathcal{T}^{\mu_1}(\underline{\mathbf{q}}^s) + \underline{\mathbf{z}}, \quad (13)$$

Analogous to the user-synchronous case, the MAP decision rule for μ_1 leads us to maximize

$$\begin{aligned} S_1^s &= \sum_{\text{all } D} \sum_{\text{all } D'} p_{\underline{\mathbf{r}}^s}(\underline{\mathbf{r}}^s | D = D, D' = D', \mu = \mu) \\ &\propto \sum_{\text{all } D} \sum_{\text{all } D'} p_{\underline{\mathbf{z}}}(\underline{\mathbf{r}}^s - \mathcal{T}^{\mu}(\underline{\mathbf{q}}^s)). \end{aligned}$$

We can hence obtain the decision rule for separate decision in the user-asynchronous case, which is to select μ ($0 \leq \mu \leq N-1$) that maximizes

$$\begin{aligned} S^s &= \frac{1}{\sigma^2} \sum_{i=0}^{L-1} (2\text{Re}(r_{i+\mu} h_1^* s_i^*)) + \sum_{i=0}^{L-1} \left[\ln \left(\sum_{\underline{u}=\underline{u}_1}^{\underline{u}_{M^{n-1}}} e^{(\sum_{l=2}^n (2\text{Re}(r_{i+\mu} h_l^* u_l^*)) - |h_1 s_i + \sum_{l=2}^n h_l u_l|^2) / \sigma^2} \right) \right. \\ &\quad \left. - \ln \left(\sum_{\underline{v}=\underline{v}_1}^{\underline{v}_{M^n}} e^{(\sum_{l=1}^n (2\text{Re}(r_{i+\mu} h_l^* v_l^*)) - |\sum_{l=1}^n h_l v_l|^2) / \sigma^2} \right) \right], \end{aligned} \quad (14)$$

The detailed derivation steps are omitted. Note that the first term in S^s is again the correlation between the sync word and the received signal, and that the remaining terms are

corrections to the correlation term. The computational complexity of the separate decision algorithm can be shown to be dominated by $\Theta(n^2 N L M^n)$, for frame synchronizing all the n transmitters.

B. Separate Decision with Gaussian Approximation

In the channel model, $\mathbf{d}_{i,j}$ is a random data symbol drawn from a two-dimensional M -constellation on the complex plane, with zero mean $E[\mathbf{d}_{i,j}] = 0$. The variance of $\mathbf{d}_{i,j}$ is the average energy of the constellation, i.e., $\text{Var}[\mathbf{d}_{i,j}] = E_{\text{avg}}$. A simplification of the separate decision approach can thus be obtained by approximating the randomness in \mathbf{t}_i and \mathbf{c}_j as complex Gaussian random variables

$$\mathbf{t}_i \sim \mathcal{CN}(h_1 s_i; \sigma_t^2), \quad 0 \leq i \leq L-1, \quad \sigma_t^2 = E_{\text{avg}} \sum_{l=2}^n |h_l|^2 \quad (15)$$

$$\mathbf{c}_j \sim \mathcal{CN}(0; \sigma_c^2), \quad L \leq j \leq N-1, \quad \sigma_c^2 = E_{\text{avg}} \sum_{l=1}^n |h_l|^2. \quad (16)$$

Again, following the MAP decision rule, we obtain the decision statistic which is to be maximized over μ , as

$$S^{sa} = \sum_{i=0}^{L-1} (2\text{Re}(r_{i+\mu} h_1^* s_i^*)) - \left(\frac{\sigma_c^2 - \sigma_t^2}{\sigma_c^2 + \sigma_t^2} \right) \cdot \sum_{i=0}^{L-1} |r_{i+\mu}|^2. \quad (17)$$

By the Gaussian approximation, we find that the correction term is considerably simplified compared with S^s in (14), while the correlation term still remaining the same. Note that here $(\sigma_c^2 - \sigma_t^2) = E_{\text{avg}} |h_1|^2$ since we consider the decision of μ_1 . The computational complexity of the separate decision algorithm with Gaussian approximation is $\Theta(nNL)$. Furthermore, the execution of the algorithm does not involve logarithm or exponential functions, which in practice may only be approximately implemented by quantization or table-lookup.

C. Joint Decision Approach

Joint decision of frame positions exponentially enlarges the search space to N^n possibilities, and leads to complicated analytic expression for the MAP decision rule. In this subsection, we demonstrate its derivation through the two-user case, and the general n -user case can be derived in a similar but substantially more tedious way.

The MAP decision rule is to choose (μ_1, μ_2) ($0 \leq \mu_1, \mu_2 \leq N-1$) that maximizes $\Pr[\mu_1 = \mu_1, \mu_2 = \mu_2 | \underline{\mathbf{r}}^j = \underline{\mathbf{r}}^j] = p_{\underline{\mathbf{r}}^j}(\underline{\mathbf{r}}^j | \mu_1 = \mu_1, \mu_2 = \mu_2) \Pr[\mu_1 = \mu_1, \mu_2 = \mu_2] / p_{\underline{\mathbf{r}}^j}(\underline{\mathbf{r}}^j)$. By the independence assumption, $\Pr[\mu_1 = \mu_1, \mu_2 = \mu_2] = 1/N^2$ for all possible pairs of (μ_1, μ_2) . Thus we may equivalently search over (μ_1, μ_2) to maximize $p_{\underline{\mathbf{r}}^j}(\underline{\mathbf{r}}^j | \mu_1 = \mu_1, \mu_2 = \mu_2)$.

Three possible cases need to be considered separately in order to evaluate $p_{\underline{\mathbf{r}}^j}(\underline{\mathbf{r}}^j | \mu_1 = \mu_1, \mu_2 = \mu_2)$. First, if $\mu_1 = \mu_2$, we return back to the user-synchronous case, and the corresponding decision statistic is

$$\begin{aligned} S^j &= \frac{1}{\sigma^2} \sum_{i=0}^{L-1} (2\text{Re}(r_{i+\mu} h_1^* s_{1,i}^*) + 2\text{Re}(r_{i+\mu} h_2^* s_{2,i}^*)) \\ &\quad - \frac{1}{\sigma^2} \sum_{i=0}^{L-1} |h_1 s_{1,i} + h_2 s_{2,i}|^2 + \sum_{i=L}^{N-1} \ln \left(\sum_{\underline{v}=\underline{v}_1}^{\underline{v}_{M^2}} e^{(\sum_{l=1}^2 (2\text{Re}(r_{i+\mu} h_l^* v_l^*)) - |\sum_{l=1}^2 h_l v_l|^2) / \sigma^2} \right). \end{aligned} \quad (18)$$

Here we have an additional term $-\frac{1}{\sigma^2} \sum_{i=0}^{L-1} |h_1 s_{1,i} + h_2 s_{2,i}|^2$, because this term would not be independent of (μ_1, μ_2) if μ_1 and μ_2 are different, as considered in the following.

Second, if (μ_1, μ_2) satisfies $0 < |\mu_1 - \mu_2| < L$, we encounter a scenario in which the two sync words have partial overlap. Without loss of generality, we assume $0 \leq \mu_1 < \mu_2 \leq N - 1$, and introduce $\delta := \mu_2 - \mu_1, 1 \leq \delta \leq L - 1$. The transmitted symbols within a frame are represented using a matrix $\mathbf{T}^j =$

$$\begin{pmatrix} s_{1,0} & \cdots & s_{1,\delta-1} & s_{1,\delta} & \cdots & s_{1,L-1} \\ \mathbf{d}_{2,0} & \cdots & \mathbf{d}_{2,\delta-1} & s_{2,0} & \cdots & s_{2,L-\delta-1} \\ \mathbf{d}_{1,L} & \cdots & \mathbf{d}_{1,L+\delta-1} & \mathbf{d}_{1,L+\delta} & \cdots & \mathbf{d}_{1,N-1} \\ s_{2,L-\delta} & \cdots & s_{2,L-1} & \mathbf{d}_{2,L+\delta} & \cdots & \mathbf{d}_{2,N-1} \end{pmatrix}$$

The resulting decision statistic is

$$\begin{aligned} S^j = & \frac{1}{\sigma^2} \sum_{i=0}^{L-1} (2\text{Re}(r_{i+\mu_1} h_1^* s_{1,i}^*) + 2\text{Re}(r_{i+\mu_1+\delta} h_2^* s_{2,i}^*)) \\ & - \frac{1}{\sigma^2} \sum_{i=\delta}^{L-1} |h_1 s_{1,i} + h_2 s_{2,i-\delta}|^2 + \sum_{i=L+\delta}^{N-1} \left[\ln \right. \\ & \left. \left(\sum_{v=v_1}^{v_M} e^{(2\text{Re}(r_{i+\mu_1} (h_1^* v_1^* + h_2^* v_2^*)) - |h_1 v_1 + h_2 v_2|^2)/\sigma^2)} \right) \right] + \sum_{i=0}^{\delta-1} \\ & \left[\ln \left(\sum_{w=w_1}^{w_M} e^{(2\text{Re}(r_{i+\mu_1} h_2^* w^*) - |h_1 s_{1,i} + h_2 w|^2)/\sigma^2)} \right) \right] + \sum_{i=L}^{L+\delta-1} \\ & \left[\ln \left(\sum_{w=w_1}^{w_M} e^{(2\text{Re}(r_{i+\mu_1} h_1^* w^*) - |h_1 w + h_2 s_{2,i-\delta}|^2)/\sigma^2)} \right) \right]. \end{aligned} \quad (19)$$

The last case occurs if $|\mu_2 - \mu_1| \geq L$, so that the two sync words have no overlap. Without loss of generality, we assume $0 \leq \mu_1 < \mu_2 \leq N - 1$ and still use δ to denote $\delta = \mu_2 - \mu_1, L \leq \delta \leq N - L$. The decision statistic can then be obtained as

$$\begin{aligned} S^j = & \frac{1}{\sigma^2} \sum_{i=0}^{L-1} ((2\text{Re}(r_{i+\mu_1} h_1^* s_{1,i}^*) + 2\text{Re}(r_{i+\mu_1+\delta} h_2^* s_{2,i}^*))) \\ & + \sum_{i=0}^{L-1} \left[\ln \left(\sum_{w=w_1}^{w_M} e^{(2\text{Re}(r_{i+\mu_1} h_2^* w^*) - |h_1 s_{1,i} + h_2 w|^2)/\sigma^2)} \right) \right] + \\ & \sum_{i=\delta}^{L+\delta-1} \left[\ln \left(\sum_{w=w_1}^{w_M} e^{(2\text{Re}(r_{i+\mu_1} h_1^* w^*) - |h_1 w + h_2 s_{2,i-\delta}|^2)/\sigma^2)} \right) \right] \\ & + \sum_{L \leq i \leq \delta-1, L+\delta \leq i \leq N-1} \left[\ln \left(\sum_{v=v_1}^{v_M} e^{(2\text{Re}(r_{i+\mu_1} (h_1^* v_1^* + h_2^* v_2^*)) - |h_1 v_1 + h_2 v_2|^2)/\sigma^2)} \right) \right]. \end{aligned} \quad (20)$$

As anticipated, the joint decision rule for frame synchronization appears involved even for the two-user case. In general, the computational complexity of the joint decision rule scales like $\Theta(nN^{n+1}M^n)$, because the search space for $\underline{\mu}$ has a size of N^n , and for each $\underline{\mu}$, $\Theta(nNM^n)$ operations need to be executed in computing the decision statistic.

V. SIMULATION RESULTS

Obtaining an analytical expression for the probability of erroneous frame synchronization appears elusive given the complexity of the decision rules. In this paper, we utilize Monte Carlo simulation to compare the performance of different frame synchronization methods.

For the user-synchronous case, a simulation is set up with three transmitters, each of which transmits antipodal signals. Channel coefficients are assumed to be known as $h_1 = 1, h_2 = 0.75$, and $h_3 = 0.5$. The frame length is $N = 91$, and a Barker sequence or a Neuman-Hofman sequence (both of length $L = 13$) is used as the common sync word for the three transmitters. Figure 1 displays the probability of error versus SNR with the simple correlation rule, the optimal decision rule

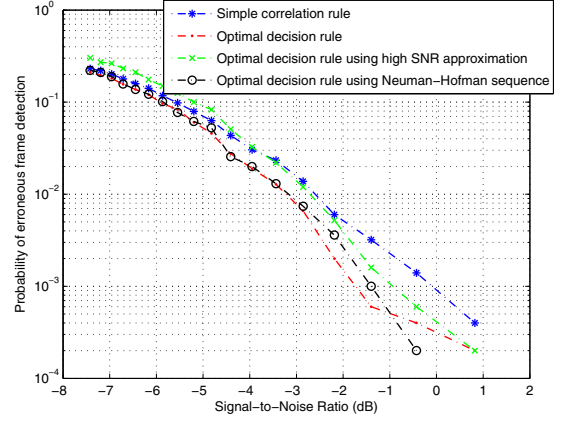


Fig. 1. Probability of error in the user-synchronous case.

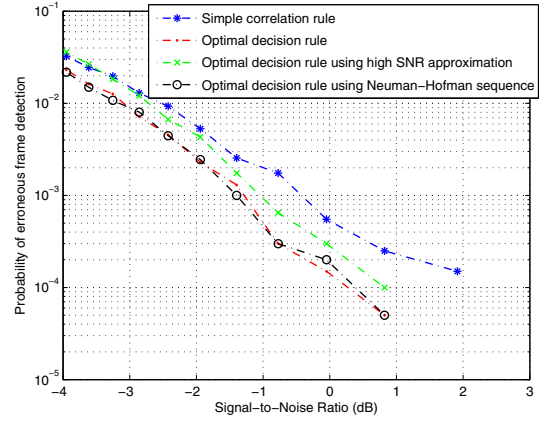


Fig. 2. High-SNR zoom-in display of Figure 1.

and its high-SNR approximation obtained in an analogous way as that in [1]. Each operating point is simulated from 5000 trials. The optimal decision rules outperform the correlation rule significantly. We note that due to the power combining of the common sync word among transmitters, the decisions rules achieve low error rate for rather low SNR, say, below 0 dB.

Figure 2 zooms into the (relatively) high-SNR regime, where the high-SNR approximation decision rule performs approximately optimally and outperforms the correlation rule. On the other hand, as the noise variance gets larger, the high-SNR approximation becomes loose, and hence its performance degrades substantially.

Figure 3 displays the probability of error versus SNR for the user-asynchronous case. The simulation is set up with two transmitters, with $h_1 = 1, h_2 = 0.5$. The SNR of x-axis is defined as $\text{SNR} = |h_1|^2/\sigma^2$, and the y-axis displays the probability of error for transmitter 1. In the presence of the secondary transmitter, which mainly acts as interference due to the lack of synchronism, a much larger SNR is required to achieve low error rate. It is clear that the performance of

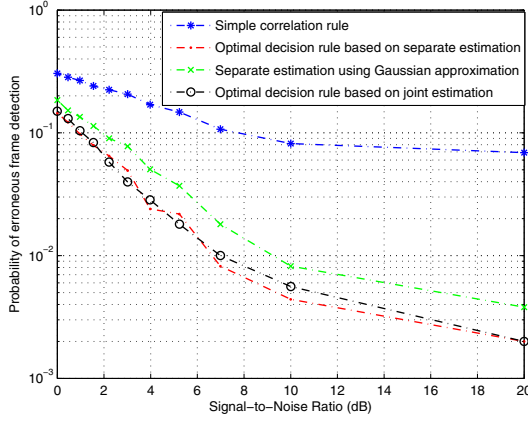


Fig. 3. Probability of error in the user-asynchronous case.

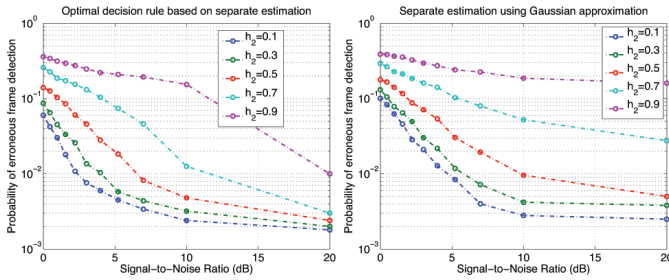


Fig. 4. Probability of error under different power levels of the secondary transmitter.

the simple correlation rule may not be tolerable. The separate decision rule offers a significant performance enhancement. Its Gaussian approximation greatly reduces the complexity, while losing only 3 ~ 4dB compared with the rule without approximation. The joint decision approach does not provide notable benefits compared with the separate approach. These observations imply that modeling other users' symbols as random or even Gaussian without considering the existence of known sync words may not degrade the performance much.

The effect of the secondary asynchronous transmitter is examined in Figure 4, where h_2 increases from 0.1 to 0.9. As anticipated, the error rate increases with the interference power. In the high SNR regime, when h_2 is comparable with h_1 , separate decision with Gaussian approximation exhibits a performance slump compared with the optimal decision rule.

To examine the effect of the frame size N , we fix the length of the sync word as 13 and compare the performance with different frame sizes in Figure 5. Again we fix $h_1 = 1$ and $h_2 = 0.5$ as in Figure 3. We notice that, increasing the frame size degrades the frame synchronization performance. Empirically an increase in the frame size by 40 causes about 3dB loss in terms of SNR.

Finally, we examine the effect of the number of transmitters. In the simulation, h_1 is fixed as 1; while a fixed amount of power, 0.25, is distributed evenly among all the other transmitters. As we observe from Figure 6, the curves for different

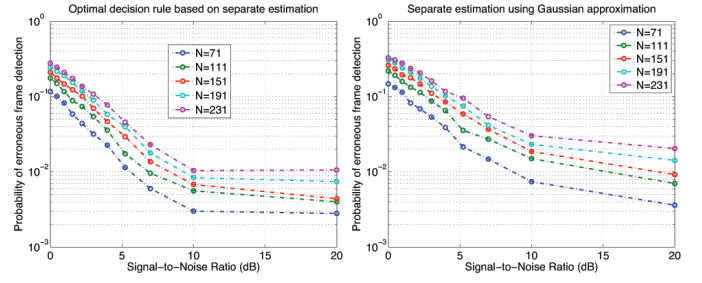


Fig. 5. Probability of error for different frame sizes.

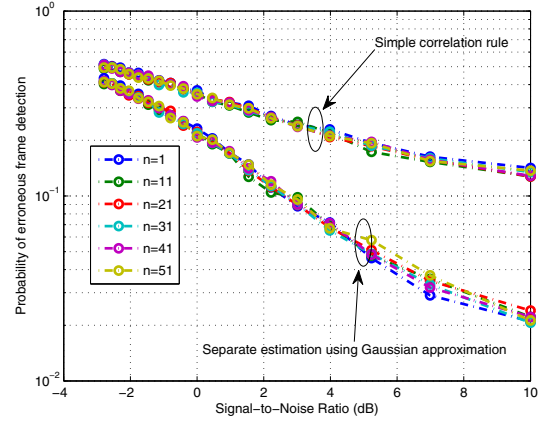


Fig. 6. Probability of error for different transmitter numbers

numbers of transmitters almost overlap with each other. This observation reveals that the performance is mostly determined by the signal-to-interference-plus-noise ratio (SINR), rather than the number of transmitters. It is also seen that the Gaussian approximation works well even when the number of transmitters is small.

ACKNOWLEDGMENT

The work of D. Shen has been supported in part by Communications Futures Program (CFP) at MIT. The work of W. Zhang has been supported in part by NSF OCE0520324.

REFERENCES

- [1] J. L. Massey, "Optimum Frame Synchronization," *IEEE Trans. Commun.*, vol. COM-20, pp. 115–119, Apr. 1972.
- [2] G. L. Lui and H. H. Tan, "Frame Synchronization for Gaussian Channels," *IEEE Trans. Commun.*, vol. COM-35, pp. 818–829, Aug. 1987.
- [3] R. Mehlman and H. Meyr, "Optimum Frame Synchronization for Asynchronous Packet Transmission," in *Proc. ICC*, vol. 2, 1993, pp. 826–830.
- [4] J. A. Gansman, M. P. Fitz, and J. V. Krogmeier, "Optimum and Sub-optimum Frame Synchronization for Pilot-Symbol-Assisted Modulation," *IEEE Trans. Commun.*, vol. COM-45, pp. 1327–1337, Oct. 1997.
- [5] Y. Wang, K. Shi, and E. Serpedin, "Continuous-Mode Frame Synchronization for Frequency-Selective Channels," *IEEE Trans. Veh. Technol.*, vol. 53, no. 3, pp. 865–871, May 2004.
- [6] D. Shen, G. R. Woo, A. Lippman, and D. P. Reed, "Separation of Multiple Passive RFID Signals Using Software Defined Radio," in *IEEE International Conference on RFID*, Apr. 2009.