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**Citation:** Sander, Michelle Y., and Franz X. Kartner. "Carrier-envelope phase dynamics of octavespanning titanium:sapphire lasers." Lasers and Electro-Optics, 2009 and 2009 Conference on Quantum electronics and Laser Science Conference. CLEO/QELS 2009. Conference on. 2009. 1-2. © 2009 IEEE/Optical Society of America.

**As Published:** http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5224417

**Publisher:** Institute of Electrical and Electronics Engineers

**Persistent URL:** <http://hdl.handle.net/1721.1/61631>

**Version:** Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

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### **Carrier-Envelope Phase Dynamics of Octave-Spanning Titanium:Sapphire Lasers**

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**Abstract**: The carrier-envelope phase dynamics of octave-spanning Ti:sapphire lasers are analyzed based on a one-dimensional laser. It is found that self-steepening is the major contributor to the energy dependent carrier-envelope phase and that center-frequency-shifts are negligible. ©2009 Optical Society of America

**OCIS codes**: (320.7090) Ultrafast lasers; (320.5550) Pulses

### **1. Introduction**

A pulse train generated by a mode-locked laser is characterized by a carrier-envelope phase (CEP) shift per roundtrip, due to the difference in phase and group velocity of pulses in the cavity. For frequency metrology, such as optical clock applications and control of extreme nonlinear optical processes, it is crucial to control and stabilize the CEP. To achieve this locking, the carrier-envelope phase shift per round-trip is most often changed via the pump power, i.e. via the intracavity pulse energy. Therefore, the variation of the carrier-envelope phase slip with intracavity energy is of major importance and the subject of this study.

In previous work, we presented a one-dimensional laser model for octave-spanning Ti:sapphire lasers based on dispersion managed mode-locking for sub-two-cycle pulses [1, 2]. The model included gain saturation, self-phase modulation, saturable absorber action, higher order dispersive terms and measured transfer functions for the dispersion compensating cavity mirrors. This allowed us to quantitatively predict the octave-spanning spectra and pulse shape of sub-two-cycle pulses in excellent agreement with experimental results [1, 2].

In this paper, this model is extended to investigate the impact of self-steepening on the pulse shaping and carrierenvelope phase dynamics of these lasers. This nonlinear term alters the group delay of the pulse and has to be taken into account when determining the carrier-envelope phase shift, as predicted by Haus and Ippen [3].

#### **2. Results**

Fig. 1a) illustrates the experimental setup of the modeled Ti:sapphire laser [4]. Fig. 1 b) and c) show the simulated output spectra with and without self-steepening (ss) in the Ti:sapphire crystal and both spectra agree very well with the measured data. The self-steepening process slightly enhances the shorter wavelengths while the longer wavelengths are attenuated in turn. However, the impact of this nonlinear term is not a dominant factor for the pulse shaping as the pulse duration within the Ti:sapphire crystal usually does not fall significantly below three optical cycles.



Fig.1. a) Schematic of the studied 500 MHz ring laser [4]; comparison of simulated output spectra and measured results, with self-steepening (ss) or without: b) main output spectrum, c) 1f-2f output spectrum.

In the temporal domain, self-steepening causes nonlinear timing shifts for the pulse envelope. With *u(t)* denoting the pulse envelope and  $\omega_c = 2.356 \cdot 10^{-15}$  Hz the carrier frequency in Eq. (1), the pulse can be described in a timeframe *t'* that follows the peak of the carrier-envelope in contrast to an absolute temporal scale *t*. During each round-trip, the pulse accumulates a nonlinear phase *Φ* and a timing shift *τ*. We therefore obtain the following description for the nonlinear contribution to the CEP *ΔΦCE, NL*:

$$
u(t-\tau)e^{j\Phi}e^{j\omega_c t} = u(t')e^{j\Phi}e^{j\omega_c \tau}e^{j\omega_c t'} \implies \Delta \Phi_{CE, NL} = \Phi + \omega_c \tau
$$
 (1)

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The difference between the phase velocity  $v_p$  and group velocity  $v_g$  given in Eq. (2) determines a linear contribution to the CEP. It can be expressed in terms of the sum over the dispersive material phases  $\varphi_{Mat} = kL$  of the different intracavity elements (described by a wavenumber  $k_i$  and the material length  $L_i$ ):

$$
\Delta \Phi_{CE,Lin}(\omega_c) = \omega_c L \left( \frac{1}{\nu_p} - \frac{1}{\nu_g} \right) = \omega_c \sum_i L_i \left( \frac{k_i}{\omega_c} - \frac{\partial k_i}{\partial \omega} \right)
$$
(2)

To determine the change in the CEP  $\Delta \Phi_{CE} = \Delta \Phi_{CE, NL} + \Delta \Phi_{CE, Lin}$  depending on the intracavity energy *W*, the shift of the carrier frequency with energy has to be considered. Since the material phases do not depend on energy and the length of the cavity elements remain constant, *ΔΦCE, Lin* contributes the last term to Eq. (3). For the inserted materials this second order derivative is equal to the dispersion parameter  $D = \partial^2 \varphi_{Mat}/\partial \omega^2 = L \partial^2 k/\partial \omega^2$  at  $\omega = \omega_c$ . In total, the obtained relation consists of the change in round-trip phase and nonlinear timing shift with energy and of a contribution from the energy-dependent shift of the carrier frequency, as presented in Eq. (3).

$$
\frac{\partial \Delta \Phi_{CE}}{\partial W} = \frac{\partial \Phi}{\partial W} + \frac{\partial (\omega_c \tau)}{\partial W} + \frac{\partial}{\partial W} \left( \omega_c \sum_i L_i \left( \frac{k_i}{\omega_c} - \frac{\partial k_i}{\partial \omega} \right) \right) = \frac{\partial \Phi}{\partial W} + \omega_c \frac{\partial \tau}{\partial W} + \tau \frac{\partial \omega_c}{\partial W} - \omega_c \sum_i L_i \frac{\partial^2 k_i}{\partial \omega^2} \bigg|_{\omega_c} \frac{\partial \omega_c}{\partial W} \tag{3}
$$

By varying the intracavity energy for the laser system, we found a linear relationship for the nonlinear time shift and the round-trip phase as shown in Fig. 2a) and b). The timing shift  $\tau$  varies from 3 fs to 8 fs for the considered range of energies, while the phase *Φ* decreases from a value close to -π/2 to -3π/2. The CEP change due to the carrier frequency shift is relatively small in comparison. Since octave-spanning laser systems operate close to zero net dispersion, around  $D = \pm 10$  fs<sup>2</sup>, and they fill up the whole gain bandwidth, the shift of the carrier frequency is less significant. In Eq. (4) the dominant contribution arises from the timing shift of the pulse center: This term is about four times as large as the contribution from the nonlinear phase shift. They are both of opposite sign, as was shown for exact solutions of the Nonlinear Schrödinger Equation for dispersion managed solitons perturbed by selfsteepening [3], where the timing shift change was established to be twice as large as the round-trip phase change. Theoretical and experimental evaluations of the CEP depending on intensity were also conducted in [5, 6], however, to the authors' knowledge octave-spanning, dispersion managed lasers have not been studied numerically so far.



Fig.2. a) Intensity dependent timing shift due to self-steepening in the Ti:sapphire crystal, b) decrease of the round-trip phase shift with higher energies, c) shift of the carrier frequency  $\omega_c = 2.356$  PHz approximated by a linear fit.

#### **3. Conclusion**

The control of the carrier-envelope phase slip in octave-spanning Ti:sapphire laser via the intracavity pulse energy was studied. While the output spectral shape is barely altered by the impact of self-steepening, it is found that the nonlinear temporal shift of the pulse center due to self-steepening dominates the carrier-envelope phase shift. Based on this analysis, we found a predominantly linear dependence of the carrier-envelope frequency on pulse energy.

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