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Passive Reduced Order Modeling of Multiport Interconnects via Semidefinite Programming

Zohaib Mahmood, Brad Bond, Tarek Moselhy, Alexandre Megretski, and Luca Daniel

Abstract—In this paper we present a passive reduced order modeling algorithm for linear multiport interconnect structures. The proposed technique uses rational fitting via semidefinite programming to identify a passive transfer matrix from given frequency domain data samples. Numerical results are presented for a power distribution grid and an array of inductors, and the proposed approach is compared to two existing rational fitting techniques.

I. INTRODUCTION

Automatic generation of accurate, compact, and passive models for multiport interconnect structures (e.g. power grid) is a crucial part of the design process for complex analog and packaging systems. Conventionally, interconnect structures are laid out in a field solver and then simulated for frequency response in the desired frequency band. Based on the frequency samples or system matrices extracted by the solver, a reduced model is developed which can be incorporated into a circuit simulator (e.g. Spice or Spectre). Inside of the circuit simulator, these reduced models may be used in lengthy time domain simulations of a larger system containing also transistors and other nonlinear devices. A small violation of any basic property of the structure, such as passivity, can cause huge errors in the response of the overall system, and the results become completely nonphysical. Passivity is a particularly important property, compared to stability for instance, since arbitrary connections of passive systems are guaranteed to be passive, while the same is not true for stable systems.

Traditional projection based model reduction approaches, such as PRIMA [1], are capable of preserving passivity only when the system matrices possess a very specific structure (i.e. positive definiteness of the system matrices). This is highly restrictive as the field-solver is often not capable of guaranteeing such structured system matrices. Additionally, when capturing full-wave or substrate effects in the field solver, the resulting system matrices possess a non-rational frequency dependence. To capture such effects in a finite-order state-space model, one must approximate this frequency dependence, using for instance polynomial fitting of matrices [2], making preservation of passivity through projection even more challenging. Furthermore, often times only transfer function samples are available, obtained possibly from measurement of a fabricated device or from a commercial field solver, eliminating projection as a possible approach. Several passivity-preserving rational fitting approaches exist for fitting reduced models directly from transfer function samples [3], [4], however these methods have only been tested for single port case. For multiport structures, the only available approach so far is to identify a stable, but non-passive, multiport model, and

then perturb the model to make it passive [5]–[7]. However such approaches suffer from limitations if the initial non-passive model has significant passivity violations, resulting into an ill-posed problem. Also these perturbations may cost significant amount in terms of accuracy. Hence it is worth investigating the implications of enforcing passivity as part of the optimization process, rather than as a post-processing step.

In this paper we propose a new technique for passive modeling of *multiport* interconnect structures. Given transfer function samples, we identify a rational transfer function reduced model that minimizes the transfer function mismatch at the given frequencies subject to a global passivity constraint. After a convex relaxation this problem can be formulated as a semidefinite optimization problem, which can be solved efficiently using existing techniques. The remainder of this paper is organized as follows: in Section II we summarize related background; in Section III we present our approach to passive modeling of linear multiport systems; finally in Section IV we show the effectiveness of the proposed approach in modeling practical multiport interconnect structures.

II. BACKGROUND - RATIONAL FITTING OF TRANSFER FUNCTIONS

Rational approximation of single port systems consists of finding scalar polynomials $p(s), q(s)$ such that the reduced model defined by the transfer function $\hat{H}(s) = p(s)/q(s)$ minimizes the error between the reduced model and the original system in either the L_2 or L_∞ sense, i.e. p, q satisfy one of the following

$$L_2 : \min_{p,q} \sum_i \left| H_i - \frac{p(j\omega_i)}{q(j\omega_i)} \right|^2 \quad (1)$$

$$L_\infty : \min_{p,q} \max_i \left| H_i - \frac{p(j\omega_i)}{q(j\omega_i)} \right| \quad (2)$$

where $H_i = H(j\omega_i)$ are given transfer function samples. To enforce passivity for an impedance or admittance system the transfer function must be ‘positive real’, i.e. it must satisfy the following constraints

$$\overline{H(\bar{s})} = H(s) \quad (3a)$$

$$H(s) \text{ is analytic in } \Re\{s\} > 0 \quad (3b)$$

$$H(j\omega) + H(j\omega)^* \geq 0 \quad \forall \omega \quad (3c)$$

Where $\Re\{ \}$ denotes the real part. Several convex and quasi-convex relaxations have been proposed to solve the L_2 and L_∞ matching problem for the single port case [3], [4].

For multiport systems, it is common practice to identify elements of the transfer matrix individually, and then concatenate the entries into a single matrix. However, since passivity is a property of the entire transfer matrix, such approaches cannot guarantee passivity for the overall system.

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III. PASSIVE FITTING FOR MULTIPOINT LTI SYSTEMS

Given transfer matrix samples H_i corresponding to frequencies $\omega_i \in \mathbb{R}$, we wish to identify reduced model $\hat{H}(s)$ such that the mismatch between $\hat{H}(j\omega_i)$ and H_i is minimized, and $\hat{H}(s)$ is passive. Specially we search for reduced models in the following form:

$$\begin{aligned}\hat{H}(s) &= \hat{H}_+(s) + \hat{H}_0(s), \\ \hat{H}_+(s) &= P(s)/q(s), \text{ and } \hat{H}_0(s) = P_0(s)/q_0(s), \\ P, P_0 : \mathbb{C} &\mapsto \mathbb{C}^{n \times n}, \text{ are symmetric matrix-valued polynomials} \\ q, q_0 : \mathbb{C} &\mapsto \mathbb{C}, \text{ are scalar polynomials} \\ q : \text{all roots of } q &\text{ are in the open left half plane} \\ q_0 : \text{all roots of } q_0 &\text{ are on the imaginary axis}\end{aligned}$$

Here $\hat{H}_+(s)$ is a strictly passive transfer matrix, and $\hat{H}_0(s)$ is a marginally stable system that may be needed to capture effects in the data resulting from non-physical behavior outside the frequency range of interest. Such effects are often numerical artifacts introduced by the field solvers. Since this term is purely imaginary, it does not affect passivity and can therefore be fit using a simple least squares fit.

The transfer matrix of a stable system is completely defined by its real part on the $j\omega$ axis, hence for $\hat{H}_+(s)$ we shall instead identify matrix polynomial $B = B(\lambda)$ and a scalar polynomial $a = a(\lambda)$ such that the real part,

$$\Re\{\hat{H}_+(s)\} = \frac{B(\omega^2)}{a(\omega^2)} \quad (4)$$

for all purely imaginary $s = j\omega$. Once $B(\lambda)$ and $a(\lambda)$ are known, $P(s)$ and $q(s)$ can be uniquely (modulo normalization) constructed as in [4], which requires $B()$ and $a()$ to be functions of ω^2 . To enforce passivity, we require $B(\omega^2) = B(\omega^2)^T$ to be positive definite, and $a(\omega^2) > 0$. The resulting optimization problem,

$$\begin{aligned}\min_{B,a} \quad & \sum_i \left| \Re\{H_i\} - \frac{B(\omega_i^2)}{a(\omega_i^2)} \right|^2 \\ \text{subject to} \quad & B(\omega^2) = B(\omega^2)^T > 0 \\ & a(\omega^2) > 0\end{aligned} \quad (5)$$

is non-convex and therefore difficult to solve.

A useful relaxation to the non-convex problem (5) is the following

$$\begin{aligned}\min_{B,a} \quad & \sum_i \frac{\left| a(\omega_i^2) \Re\{H_i\} - B(\omega_i^2) \right|^2}{a(\omega_i^2)} \\ \text{subject to} \quad & B(\omega^2) = B(\omega^2)^T > 0 \\ & a(\omega^2) > 0 \\ & \sum_i a(\omega_i^2) = 1\end{aligned} \quad (6)$$

This Second Order Cone Program (SOCP) can easily be solved by transforming it into a semidefinite program (SDP), which can be solved using public domain solvers such as SeDuMi [8].

The objective function in (6) can be interpreted as a weighted version of the original objective function in (5) with normalized

weights. As a result, the optimal solution to (6) provides a lower bound for the uniformly optimal solution

$$\begin{aligned}\sum_i \frac{|a(\omega_i^2) \Re\{H_i\} - B(\omega_i^2)|^2}{a^2(\omega_i^2)} a(\omega_i^2) &= \sum_i \left| \Re\{H_i\} - \frac{B(\omega_i^2)}{a(\omega_i^2)} \right|^2 a(\omega_i^2) \\ &\leq \max_i \left| \Re\{H_i\} - \frac{B(\omega_i^2)}{a(\omega_i^2)} \right|^2\end{aligned} \quad (7)$$

Once the transfer matrix is identified, it can be transformed into a state-space model, thereby allowing the application of existing passivity-preserving techniques for state-space systems, such as balanced truncation, to potentially further reduce the size of the reduced model.

IV. RESULTS

In this section we shall present two examples illustrating the usefulness of our proposed methodology on modeling interconnect structures. Comparisons with component-wise single port rational fit [9] and stable rational fit [4] are also provided, where transfer functions are identified individually, and then stacked together in a larger matrix to represent a multipoint system.

A. Power & Ground Distribution Grid

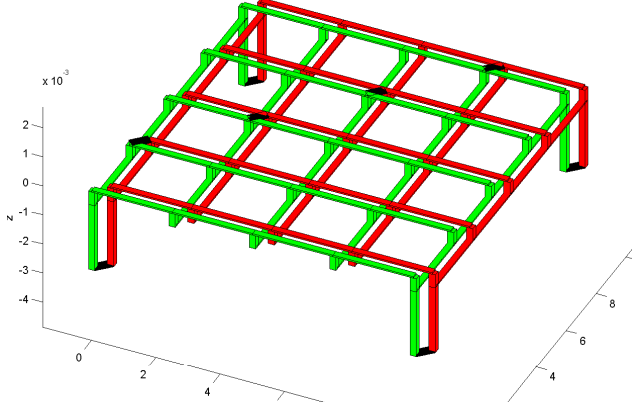
The first example we present is a power & ground distribution grid used in systems on chip or on package. The 3D layout for this power grid is shown in Figure 1(a), and is composed of five *Vdd* (red or dark grey) and *Gnd* (green or light grey) segments placed along both x and y axes. External connections given by solder balls in a flip chip technology, are modeled with bond wires running vertically. Important parameters of this power grid are as follows: die size = $10\text{mm} \times 10\text{mm}$, wire width = $20\mu\text{m}$, wire height = $5\mu\text{m}$, vertical separation = $4\mu\text{m}$, gnd-vdd separation = $20\mu\text{m}$, bond-wire lengths = $500\mu\text{m}$ and solder ball radius = $20\mu\text{m}$. This structure was simulated using 52390 unknowns in the full wave mixed potential integral equation (MPIE) solver, FastMaxwell [10], to obtain frequency response samples up to 12 GHz. The multipoint simulation was arranged by placing eight ports: four at the grid corners and four inside the grid. Ports are illustrated in Figure 1(a) as black strips.

For this example our proposed algorithm identified an 8×8 passive transfer matrix of order $m = 400$. Note that 400 is a small order for such a system with 8 ports, since it means each transfer function is roughly captured by 6 poles. Figures 2(a) and 2(b) compare the real and imaginary impedance respectively of our reduced model with the field solver data. Figure 1(b)[top] plots the error $e_{i,k}(\omega)$ for each entry of the transfer matrix of the identified model, defined as

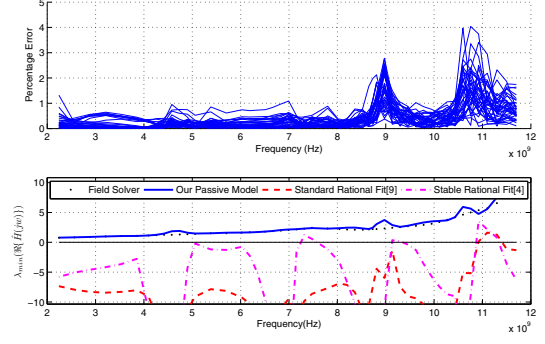
$$e_{i,k}(\omega) = \frac{|H_{i,k}(j\omega) - \hat{H}_{i,k}(j\omega)|}{\max_{i,k,\omega} |H_{i,k}(j\omega)|} \quad (8)$$

where $H(j\omega)$ is original transfer function, $\hat{H}(j\omega)$ is the identified transfer function, and the maximum is taken over frequencies between 2GHz and 12GHz.

We have compared our algorithm with standard rational fitting [9] and stable rational fitting algorithms [4] on individual transfer functions. While both alternative methods produce accurate fits to all elements of the transfer function matrix with order $m = 640$, the resulting models are not passive.

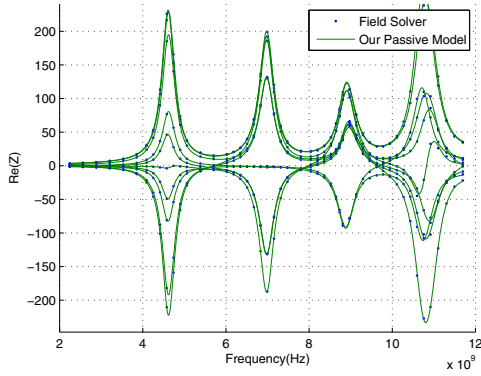


(a) 3D layout of the distribution grid (not to scale) showing Vdd (red or dark grey) and Gnd (green or light grey) lines. Black strips represent location of ports

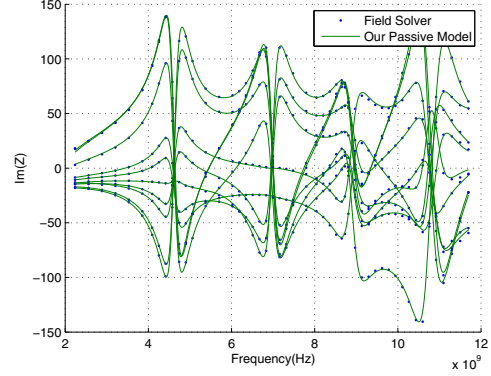


(b) Top: Percentage error defined by (8) Bottom: $\lambda_{\min}(\Re\{\hat{H}(j\omega_i)\})$

Fig. 1. Power Grid: Layout, Error and Eigen values



(a) Comparing real part of impedance from our passive model (solid line) and from field solver (dots)



(b) Comparing imaginary part of impedance from our passive model (solid line) and from field solver (dots)

Fig. 2. Power Grid: Impedance Parameters

Since passivity requires condition (3) to hold for all ω , a reduced model is non-passive if $\lambda_{\min}(\Re\{\hat{H}(j\omega_i)\}) < 0$ for some ω_i . Figure 1(b)[bottom] plots $\lambda_{\min}(\Re\{\hat{H}(j\omega_i)\})$ for the three reduced models. It is clear from Figure 1(b)[bottom] that both alternative methods generate models which are non-passive.

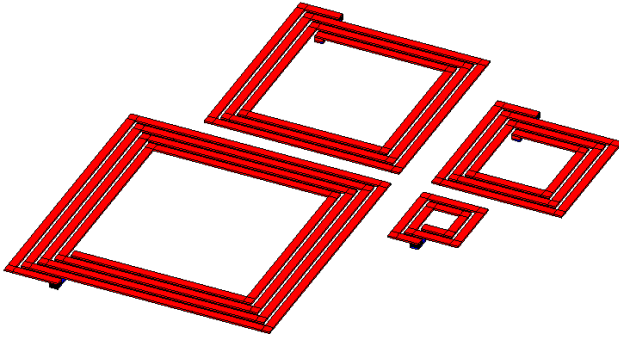
B. Nonuniform Inductor Array for Multichannel Transceiver Environment

The second example we shall discuss is an inductor array which is normally used in the design of multichannel receivers or transmitters on chip or on a package. The layout for this array is shown in Figure 3(a). The array is comprised of four inductors laid out in the form of a 2x2 matrix. Important dimensions of this array are as follows: wire width= $10\mu m$, wire height= $4\mu m$, height of inductors above substrate= $20\mu m$, horizontal separation between sides of two adjacent inductors= $400\mu m$, length of sides of each inductor= $800\mu m, 600\mu m, 400\mu m, 200\mu m$, and having 4,3,3,2 turns respectively. The array has four ports in total, configured at the input of each inductor. This structure was

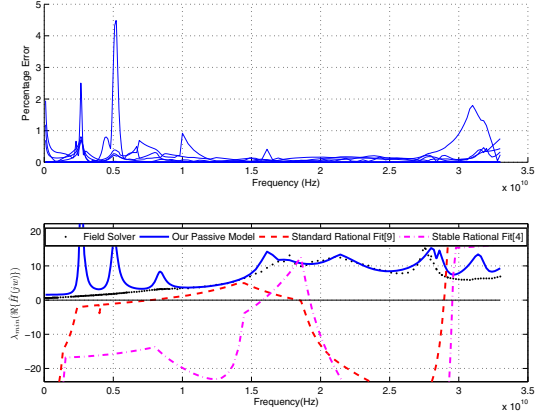
simulated using 10356 unknowns in the full wave field solver, FastMaxwell [10] which captures substrate using a Green function complex image method.

For this example a 4×4 passive transfer matrix of order $m = 96$ was identified. Note that on such a system with 16 transfer functions an order of 96 is actually small (i.e. roughly 6 poles per transfer function). Figure 4 shows impedance parameters both from the field solver and from our identified model. Figure 3(b)[top] plots error $e_{i,k}(\omega)$ of the identified model as defined in (8), which attains a maximum of 4.5% error.

To emphasize the importance of preserving passivity, two additional models were identified from this example using the standard rational fit [9] and stable rational fit [4] approaches. Although for the same model-complexity ($m = 96$) the rational fits identified quite accurate models, passivity was still not preserved, as is evident from the negative eigenvalues plotted in Figure 3(b)[bottom] corresponding to the two alternative models.

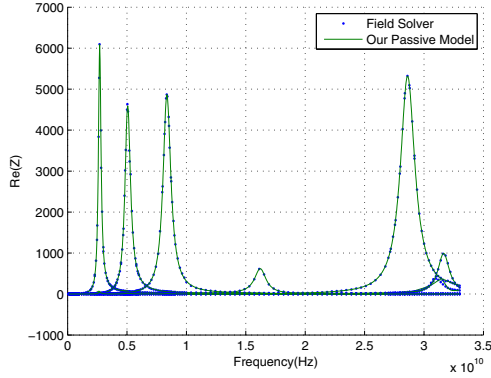


(a) 3D layout of inductor array (wire widths not to scale)

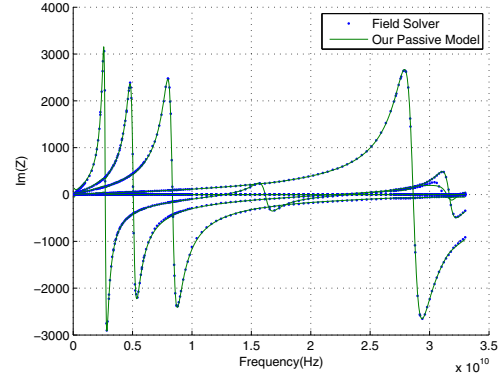


(b) Top: Percentage error defined by (8) Bottom: $\lambda_{\min}(\Re\{\hat{H}(j\omega_i)\})$

Fig. 3. Inductor Array: Layout, Error and Eigen values



(a) Comparing real part of impedance from our passive model (solid line) and from field solver (dots)



(b) Comparing imaginary part of impedance from our passive model (solid line) and from field solver (dots)

Fig. 4. Inductor Array: Impedance Parameters

V. CONCLUSION

In this paper we have presented a new technique for identifying passive models of multiport interconnect structures from frequency-domain transfer matrix data samples. The technique uses semidefinite programming to optimally fit the given data samples while simultaneously globally enforcing passivity of the identified model through a semidefinite constraint. The proposed approach has been tested on different multiport interconnect structures and compared to standard rational fitting algorithms.

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