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Economic Aspects of Optimal  
Disability Benefits

P. Diamond and E. Sheshinski

No. 601

Rev. Jan. 1992

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## Economic Aspects of Optimal Disability Benefits

P. Diamond and E. Sheshinski\*

### 1. Introduction

The US social security system began with retirement benefits but no disability benefits. When disability insurance was added to the system (1956), the formula for retirement benefits was adapted to determine disability benefits. With the introduction of early retirement benefits at age 62 (1956 for women, 1961 for men), there has been a three year overlap when workers could apply for either retirement or disability benefits. Economists have been studying the relationship between labor supply of older workers and the availability of retirement benefits.<sup>1</sup> Economists have also studied the optimal design of retirement benefits in recognition of the effect of benefits on labor supply.<sup>2</sup> While it has been recognized that disability benefits also affect labor supply,<sup>3</sup> we are unaware of any studies of the optimal design of disability and retirement benefits in recognition of the effect of benefits on labor supply.<sup>4</sup> This paper begins such a study, using a simple static model to analyze optimal levels of disability and welfare (or retirement) benefits

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<sup>1</sup>Diamond and Hausman (1984), Burtless and Moffitt (1984), and Gustman and Steinmeier (1985).

<sup>2</sup>Diamond and Mirrlees (1978), (1982), (1986).

<sup>3</sup>Halpern and Hausman (1986), Leonard (1986).

<sup>4</sup>The paper by N. Stern (1982) can be interpreted as contrasting welfare benefit and disability programs. For another analysis of disability insurance that recognizes imperfect measurement of disability, see D. Parsons (1990). See also G. Akerlof (1978) for optimal benefits with one type of classification error, but not both types. For a general discussion of disability programs in eight countries, see Haveman, Halberstadt, and Burkhauser (1984).

with recognition of the imperfect nature of disability evaluation.

We begin with a discussion of the problems of modeling disability and disability screening mechanisms. We next analyze the optimal structure of benefits for a given disability screening mechanism in a model with a continuous distribution of disability levels. We then specialize the model to the case of only two disability levels to simplify the analysis of comparative statics and to examine optimal disability standards and benefits levels. We return to the continuous model for additional analysis of optimal standards and benefits. We conclude with a logarithmic example and suggestions for further research.

## 2. Modelling Disability

At any time there is a vast array of actual and potential marginal products among the employed and not-employed individuals in the economy. Similarly, there is a vast spread in the difficulties or disutilities associated with working. For some, the alternative source of income should they not work is disability income rather than retirement income or welfare benefits. In principle, social security disability benefits are available for those unable to find any remunerative employment as a consequence of their disabilities. Yet, some people with considerable handicaps are employed, while others with seemingly similar handicaps are receiving benefits. While eligibility for benefits is straightforward for some handicaps, with others it is difficult to determine. For some handicaps it is hard for any outsider to judge the ability to work or its difficulty.

Our problem is to model this complex picture in a way that is helpful for the analysis of economic variables such as benefit levels. Since we will analyze a static model, we will not explicitly model individual histories

(including, e.g., industrial accidents) which may shed light on abilities to work. We have identified two separate aspects of working – the ability to carry on economically useful activities and the difficulty (including physical pain) in carrying them out. For convenience, we will work with a single variable by assuming that all individuals have the same marginal products, thus ignoring correlations between productivity and disability incidence. In contrast, we will assume that each individual has an additive disutility of work, with disutilities distributed over all nonnegative values. For the modeling purposes here, it makes little difference whether someone is totally unable to work or has an infinite disutility of work. We further simplify the model by assuming that all workers have the same (concave, increasing) utility of consumption,  $u(c)$ , while all nonworkers have the same (concave, increasing) utility<sup>5</sup> of consumption  $v(c)$ . Thus the distribution of disutilities plays a central role in determining willingness to work. We take work to be a 0-1 variable and do not model varying hours or work intensity.

Recognizing the problem of determining disability, we assume that disutility is not perfectly observable. For the start, we assume that there is a disability evaluation mechanism in place.<sup>6</sup> When someone with disutility  $\theta$  is evaluated for disability, there is a probability,  $p(\theta)$  that the individual will be judged disabled.<sup>7</sup> Naturally, we assume that  $p(\theta)$  is increasing in  $\theta$  and, for notational convenience, that  $p(0) = 0$ . In Section 6 we look

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<sup>5</sup>We also assume that both  $u'$  and  $v'$  go from  $\infty$  to 0 as  $c$  goes from 0 to  $+\infty$ .

<sup>6</sup>We assume that the evaluation process is costless for both the applicant and the evaluator. It would be interesting to introduce costs.

<sup>7</sup>We assume that  $p(\theta)$  is known to the government decision makers. Obviously, it is not precisely known, but one can make reasonable hypotheses about its basic form. This paper can be interpreted as a mapping from beliefs about the accuracy of disability evaluation into desired benefit levels. For some evidence on disability evaluation, see S. Nagi (1969). For evidence that some people denied disability benefits work while others do not, see J. Bound (1989).

into the public decisions which affect the shape of  $p(\theta)$ . These decisions relate the sizes of type I and type II errors to the strictness of the disability standard.

### 3. Continuous Model

We consider three different consumption levels,  $c_a$  for active workers,  $c_b$  for welfare (or early retirement) beneficiaries, and  $c_d$  for disabled beneficiaries. The utility function for a worker with labor disutility level  $\theta$  is written  $u(c_a) - \theta$ . We assume that  $\theta$  is nonnegative and distributed in the population with distribution  $F(\theta)$  (and density  $f(\theta)$ ). For convenience, we assume that  $f(\theta)$  is continuous and positive for all nonnegative values of  $\theta$ . The utility function for nonworkers is  $v(c)$ , where  $c$  will take on the values  $c_b$  or  $c_d$  depending on whether the nonworker is receiving welfare or disability benefits. We assume that disability benefits are at least as large as welfare benefits (otherwise, there is no reason to have a separate disability program with a selective screening mechanism).

$$c_d \geq c_b \quad (1)$$

Given the utility functions and the consumption levels for workers and nonworkers, we can analyze which individuals would prefer working to receiving retirement benefits and which would prefer working to receiving disability benefits (if eligible). Those with disutilities of labor below the threshold values which equate utilities will choose to work. Thus the threshold values satisfy

$$\begin{aligned} \theta_b &= \text{Max}[0, u(c_a) - v(c_b)] \\ \theta_d &= \text{Max}[0, u(c_a) - v(c_d)] \end{aligned} \quad (2)$$



Since disability benefits are greater than retirement benefits (1), the threshold labor disutility for persons who apply for disability benefits is lower than the threshold for those who apply for retirement benefits. Thus we have:

$$\theta_b \geq \theta_d \quad (3)$$

With the fraction of the population with disutility  $\theta$  who are eligible for disability benefits written as  $p(\theta)$ , individuals' labor supply choices and consumption levels are as shown in Figure 1.

We normalize by setting the marginal product of a worker equal to one. We can now state the problem of the selection of consumption levels to maximize social welfare as

$$\begin{aligned} \text{Maximize } & \int_0^{\theta_d} [u(c_a) - \theta] dF(\theta) \\ & + \int_{\theta_d}^{\theta_b} [p(\theta)v(c_d) + (1-p(\theta))(u(c_a) - \theta)] dF(\theta) \\ & + \int_{\theta_b}^{\infty} [p(\theta)v(c_d) + (1-p(\theta))v(c_b)] dF(\theta) \end{aligned} \quad (4a)$$

Subject to:

$$\begin{aligned} & \int_0^{\theta_d} (c_a - 1) dF + \int_{\theta_d}^{\theta_b} [p(\theta)c_d + (1-p(\theta))(c_a - 1)] dF \\ & + \int_{\theta_b}^{\infty} [p(\theta)c_d + (1-p(\theta))c_b] dF = R \end{aligned} \quad (4b)$$

where  $R$  are the resources available to the economy. We first discuss the first order conditions under the assumptions that  $\theta_d > 0$  and  $\theta_b > \theta_d$ . We then state sufficient conditions for these strict inequalities to hold at the



optimum.

Using Lagrange multiplier  $\lambda$  and assuming also that  $\theta_d > 0$ , we can state the first order conditions as the resource constraint (4b) and the three equations

$$\begin{aligned}
 & [u'(c_a) - \lambda] \int_0^{\theta_d} dF + [u'(c_a) - \lambda] \int_{\theta_d}^{\theta_b} (1-p(\theta)) dF \\
 & = \lambda (c_a - 1 - c_d) p(\theta_d) f(\theta_d) \frac{\partial \theta_d}{\partial c_a} \\
 & + \lambda (c_a - 1 - c_b) (1-p(\theta_b)) f(\theta_b) \frac{\partial \theta_b}{\partial c_a} \\
 & = \lambda [(c_a - 1 - c_d) p(\theta_d) f(\theta_d) + (c_a - 1 - c_b) (1-p(\theta_b)) f(\theta_b)] u'(c_a) \quad (5a)
 \end{aligned}$$

$$[v'(c_d) - \lambda] \int_{\theta_d}^{\infty} p(\theta) dF = -\lambda (c_a - 1 - c_d) p(\theta_d) f(\theta_d) v'(c_d) \quad (5b)$$

$$[v'(c_b) - \lambda] \int_{\theta_b}^{\infty} (1-p(\theta)) dF = -\lambda (c_a - 1 - c_b) (1-p(\theta_b)) f(\theta_b) v'(c_b) \quad (5c)$$

where we have used (2) to obtain the derivatives of  $\theta_b$  and  $\theta_d$ . If, at the optimum,  $\theta_d$  is zero, we have the same conditions since  $p(0) = 0$ . The left sides of all three expressions are the social values of the differences between giving consumption to the agents in the three different positions rather than holding the resources. The right sides are the social values of the resource savings from the induced changes in labor supply as a consequence of altered benefits. The private return to working for someone ineligible for disability benefits is  $c_a - c_b$ . Comparing this to the marginal product, we see that there is an implicit tax on work when  $c_a - c_b < 1$ . From (5c), we

will have an implicit tax on work if, at the optimum  $v'(c_b) > \lambda$ . Below we will note plausible sufficient conditions for this conclusion.

Dividing the expressions in (5) by the marginal utilities on the right hand side and adding, we see that the inverse of the Lagrangian equals the average of the inverses of the marginal utilities of consumption

$$\begin{aligned} \int_0^{\theta_d} (u'(c_a))^{-1} dF + \int_{\theta_d}^{\theta_b} [(1-p(\theta)) (u'(c_a))^{-1} + p(\theta) (v'(c_d))^{-1}] dF \\ + \int_{\theta_b}^{\infty} [(1-p(\theta)) (v'(c_b))^{-1} + p(\theta) (v'(c_d))^{-1}] dF = \lambda^{-1} \end{aligned} \quad (6)$$

We turn next to sufficient conditions for the solution to be internal. The question of when it is desirable to have some work in the economy ( $\theta_b > 0$ ) is a question of the wealth of the economy; i.e. the size of  $R$ . If no one works, it is optimal for everyone to have the same consumption ( $c_b = c_d = R$ ). This allocation will not be optimal if those with the least disutility of labor would choose to work for the additional consumption equal to their marginal products.

$$u(R+1) > v(R) \quad (7)$$

We assume that (7) is satisfied (implying  $\theta_b > 0$ ) and refer to it as the poverty condition.

We turn next to sufficient conditions for disability benefits to exceed welfare benefits. Consider the economy with a single benefit program. The level of benefits is chosen relative to the consumption of workers to balance the effect on labor supply with the desire to allocate consumption efficiently over workers and nonworkers. Introducing a small disability program permits

higher benefits for a fraction of nonworkers,  $\int_{\theta_b}^{\infty} p(\theta) dF$ , at a cost of the

decrease in labor supply of size  $p(\theta_b) f(\theta_b)$ . Without a separate disability program, the attempt to pay more to all nonworkers raises benefits for the  $1-F(\theta_b)$  nonworkers and decreases the labor supply by  $f(\theta_b)$ . Thus, having  $p(\theta)$  increasing in  $\theta$  is sufficient to make a disability program desirable provided the marginal utility of consumption of nonworkers exceeds that of workers at the optimum without a disability program.<sup>8</sup>

A sufficient condition for this latter relation can be expressed in terms of the Diamond and Mirrlees (1978) moral hazard condition that equating utilities between nonworkers and workers who like work the most leaves marginal utility higher for nonworkers:

$$u(x) = v(y) \text{ implies } u'(x) < v'(y) \quad (8)$$

By (7) there is some work at the optimal allocation. Thus at least the most able worker ( $\theta = 0$ ) must work, implying  $u(c_a) > v(c_b)$ . And so, by (8),  $u'(c_a) < v'(c_b)$ . While it is not so plausible that (8) holds for workers with  $\theta = 0$ , it is plausible that the same condition does hold for a sufficient range of values of  $\theta$  to yield the condition needed, that, at the optimum  $u'(c_a) < v'(c_b)$ . We assume that (8) is satisfied and conclude that  $c_d > c_b$ .

From the equation for  $\lambda$ , (6), we then have  $\lambda < v'(c_b)$ . From the first order condition, (5c), we then have an implicit tax on work,  $1 > c_a - c_b$ . It follows that  $1 > c_a - c_d$  and so, from (5b)  $\lambda \leq v'(c_d)$ , with equality if  $\theta_d = 0$ .

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<sup>8</sup>To see this formally, add (5b) and (5c) under the condition  $c_b = c_d$  and so  $\theta_b = \theta_d$ . The fact that  $p(\theta)$  is increasing will then imply that the left hand side of (5b) exceeds the right hand side provided that  $v'(c_d) > \lambda$ .

We can then conclude that  $\lambda > u'(c_a)$ . Thus both welfare and disability programs have benefit levels below the level which would equate the marginal utilities of beneficiaries with that of workers.

#### 4. Labor Disutility Observable

We have used the realistic assumption that labor disutility (or disability) is not perfectly observable. It may assist in the interpretation of the first order conditions to analyze the full optimum where disability is observable. The maximization problem in (4) is changed by making  $\theta_b$  and  $\theta_d$  control variables rather than endogenously determined by benefit levels in (2). In this case, there is no reason for a distinction between welfare and disability benefits ( $c_b^* = c_d^*$ ). Consumption is optimally allocated to equate marginal utilities of consumption

$$u'(c_a^*) = v'(c_d^*) \quad (9)$$

All individuals with disutility levels below some cutoff  $\theta^*$  should work. The cutoff is determined by comparing the utility gain from extra work

$u(c_a^*) - \theta^* - v(c_d^*)$  with the value of extra net consumption as a consequence of work, which is the sum of marginal product and the change in consumption which results from the change in status

$$u(c_a^*) - \theta^* - v(c_d^*) = -u'(c_a^*)(1 - c_a^* + c_d^*) \quad (10)$$

The allocation determined by (9), (10), and the resource constraint (4b) differs from that analyzed in Section 3 in that  $\theta^*$  is determined by social needs while  $\theta_b$  and  $\theta_d$  are determined by workers to equate utilities with and without work. From the moral hazard condition we know that an attempt to implement an allocation satisfying (9) results in no one choosing to work when

disutility is not observable. Thus, without observability it is optimal to adjust consumption levels away from the equal marginal utility condition to induce a more appropriate labor supply.

### 5. Two Class Model

To get some additional feeling for the nature of the first order conditions, we consider a case where there are only two types in the economy,  $\theta_1$  and  $\theta_2$  ( $\theta_1 < \theta_2$ ) with population weights  $f_1$  and  $f_2 = 1 - f_1$ . The probability of a type- $i$  person being judged disabled is  $p_i$ , with  $p_2 > p_1$ . We assume that the optimum has the form that the  $\theta_1$  types accept disability benefits if judged disabled but otherwise work; while the  $\theta_2$  types don't work whatever their eligibility for disability benefits. For a logarithmic example, sufficient conditions for the optimum to have this form will come out of analysis of that special case.

The social welfare optimization now takes the form

$$\begin{aligned} \text{Max } & [(u(c_a) - \theta_1)(1-p_1) + v(c_d)p_1]f_1 \\ & + [v(c_b)(1-p_2) + v(c_d)p_2]f_2 \end{aligned} \quad (11a)$$

subject to

$$(c_a - 1)(1-p_1)f_1 + c_dp_1f_1 + c_b(1-p_2)f_2 + c_dp_2f_2 = R \quad (11b)$$

$$v(c_d) \geq u(c_a) - \theta_1 \geq v(c_b) \geq u(c_a) - \theta_2 \quad (11c)$$

$$c_d \geq c_b \quad (11d)$$

The constraints in (11c) ensure that individual behavior coincides with that described in the objective function and resource constraint. One can check that the condition  $u(c_a) - \theta_1 \geq v(c_b)$  and the moral hazard condition (8) imply that at the optimum



$$u(c_a) - \theta_1 = v(c_b) \quad (12)$$

Thus the optimum is described by the resource constraint (11b), the equal utility condition (12) and a single first order condition which can be obtained by differentiating social welfare in (11a) with respect to  $c_d$ , with  $c_a$  and  $c_b$  functions of  $c_d$  given by (11b) and (12). Upon differentiation we have

$$\begin{aligned} N \equiv & \{ (1-p_1) f_1 v'(c_b) [v'(c_d) - u'(c_a)] \\ & + (1-p_2) f_2 u'(c_a) [v'(c_d) - v'(c_b)] \} \end{aligned} \quad (13)$$

$$\frac{dW}{dc_d} = N[p_1 f_1 + p_2 f_2] [v'(c_b) (1-p_1) f_1 + u'(c_a) (1-p_2) f_2]^{-1}$$

The moral hazard condition implies that  $\frac{dW}{dc_d}$  is positive at  $c_d = c_b$ . Thus the optimum has a separate disability program. Since  $c_d$  does not affect behavior at the margin,  $v'(c_d)$  is equal to the Lagrangian on the resource constraint.

Of the various comparative static exercises one could do, we restrict ourselves to one simple one. Consider a set of different economies which happen to have chosen the same levels of  $c_a$  and  $c_b$ . Assume that the different economies have different levels of resources  $R$  and different levels of one of the variables  $p_1$ ,  $p_2$ , or  $f_1$ . Then, comparative statics is done by examining the equation,  $N = 0$ . Differentiating  $N$ , and evaluating where  $N = 0$ , we have

$$\frac{\partial N}{\partial c_d} < 0, \frac{\partial N}{\partial p_1} < 0, \frac{\partial N}{\partial p_2} > 0, \frac{\partial N}{\partial f_1} > 0 \quad (14)$$

Thus, across these different economies, those with a more discriminating disability system ( $p_1$  smaller or  $p_2$  larger) or a smaller population with high disability have larger benefit levels. While the former proposition is

intuitive, the latter rests on the fact that for a program increase there must be decreases in both wages and welfare benefits to finance the change and this is more attractive the smaller the welfare population.

## 6. Disability Standard

Relative to the set of people one would ideally like to see receiving disability benefits, any attempt to evaluate abilities to work will be subject to two types of error – admission of people ideally omitted and exclusion of people ideally included.<sup>9</sup> In this section, we generalize the model of Section 5 to relate the probabilities of a disability finding,  $p_1$  and  $p_2$ , to a disability standard,  $\theta^*$ . We also consider the first order condition for optimization of  $\theta^*$ . In the next section we will extend the analysis to the continuous case.

Individuals are now characterized by two variables,  $\theta$  and  $\theta_e$ , being true and estimated disutilities of labor. We assume that the distributions<sup>10</sup> of  $\theta_e$  for the two types are  $G_1(\theta_e)$  and  $G_2(\theta_e)$ . Since observation of  $\theta_e$  is merely a device for making inferences about  $\theta$ , it is possible to define measured disability so that two conditions are met (see Milgrom (1979)). First (which is implied by the second), the assumption that  $p_1 < p_2$  for any disability standard  $\theta^*$

$$p_1(\theta^*) = 1 - G_1(\theta^*) < 1 - G_2(\theta^*) = p_2(\theta^*) \quad (15)$$

Second, the appropriate definition of measured disability gives us the monotone likelihood ratio condition for all  $\theta^*$

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<sup>9</sup>We draw on the analysis of auto insurance underwriting by Smallwood (1975).

<sup>10</sup>We assume that  $G_1$  and  $G_2$  have positive support on the entire positive halfline.

$$\frac{g_1'(\theta^*)}{g_1(\theta^*)} < \frac{g_2'(\theta^*)}{g_2(\theta^*)} \quad (16)$$

In keeping with the analysis of the previous section, we would like to exclude all type ones from disability benefits while including all type twos. However some type ones are included for any finite standard and some type twos are excluded for any positive standard. Thus it is inevitable that some errors of classification are made. The problem is to strike the appropriate balance between the two types of error.

Continuing to assume that the optimum occurs with type ones accepting disability but not welfare benefits while type twos do not work, we have the same maximization problem as in (11) with  $p_i$  now functions of the control variable  $\theta^*$ . Recalling that the Lagrangian is equal to  $v'(c_d)$ , the first order condition for  $\theta^*$  is

$$\begin{aligned} & [v(c_d) - v(c_b)] (f_1 g_1 + f_2 g_2(\theta^*)) \\ & = v'(c_d) [(c_d + 1 - c_a) f_1 g_1(\theta^*) + (c_d - c_b) f_2 g_2(\theta^*)] \end{aligned} \quad (17)$$

Rearranging terms it is instructive to write the first order condition as

$$\begin{aligned} & [v(c_d) - v(c_b) - v'(c_d) (c_d - c_b)] f_2 g_2(\theta^*) \\ & = [v'(c_d) (c_d + 1 - c_a) - (v(c_a) - v(c_b))] f_1 g_1(\theta^*) \end{aligned} \quad (18)$$

By the concavity of  $v$ , the left side is positive at the optimum. Thus the right side is positive, implying that  $1 - c_a + c_d$  is positive; this also follows from the presence of an implicit tax on work as part of the optimal structure which was argued in Section 3.

Using the monotone likelihood ration condition, (16), one can draw comparative statics from (18). Let us consider a  $\theta^*$  and a set of economies

that have the same levels of benefits and wages, the same  $g_1$  functions, but different levels of  $R$  and  $f$ . Considering (13) and (18), this can only happen where  $\frac{g_2 G_1}{g_1 G_2}$  is independent of  $\theta$  for some range. Thus, we can differentiate

(18) alone, knowing that (13) continues to be satisfied. By implicit differentiation of (18), we see that  $\theta^*$  is larger the greater the size of the type one disability eligible population.

### 7. Disability Standard: Continuous Case

In a similar fashion, we can introduce a choice of disability standard into the continuous case. We write the distribution of observed disutility,  $\theta_e$ , conditional on true disutility being  $\theta$  as  $G(\theta_e; \theta)$ , with density<sup>11</sup>  $g(\theta_e; \theta)$ . Then, for disability standard  $\theta^*$ , we have

$$p(\theta) = 1 - G(\theta^*; \theta) \quad (19)$$

Returning to the welfare maximization problem, (3.4), we can now add a further first order condition by differentiating with respect to  $\theta^*$  (assuming  $\theta^* > 0$ )

$$\begin{aligned} & \int_{\theta_d}^{\theta_b} [v(c_d) - u(c_a) + \theta] g(\theta^*; \theta) dF(\theta) \\ & + \int_{\theta_b}^{\infty} [v(c_d) - v(c_b)] g(\theta^*; \theta) dF(\theta) \\ & = \lambda \left[ \int_{\theta_d}^{\theta_b} [c_d - c_a + 1] g(\theta^*; \theta) dF(\theta) \right. \\ & \left. + \int_{\theta_b}^{\infty} [c_d - c_b] g(\theta^*; \theta) dF(\theta) \right] \end{aligned} \quad (20)$$

---

<sup>11</sup>For convenience, we assume that  $g$  is positive for all positive values of  $\theta_e$ .

Thus a loosening of the standard attracts some people to the disability program from the labor force while shifting others from retirement. The former shift has a net social value  $v(c_d) - \lambda c_d - (u(c_a) - \theta - \lambda(c_a - 1))$  while the latter has a social value  $v(c_d) - \lambda c_d - (v(c_b) - \lambda c_b)$ . We write these differences as  $\Delta_a + \theta$  and  $\Delta_b$  respectively. Across the set of people shifted, these net gains must balance out for an optimum. That is, the first order condition (20) can be written as

$$\Delta_b \int_{\theta_b}^{\infty} g(\theta^*; \theta) dF(\theta) + \Delta_a \int_{\theta_d}^{\theta_b} g(\theta^*; \theta) dF(\theta) + \int_{\theta_d}^{\theta_b} \theta g(\theta^*; \theta) dF(\theta) = 0 \quad (21)$$

From the concavity of  $v$  and the fact that  $v'(c_d) \geq \lambda$  we have

$$\begin{aligned} v(c_d) - v(c_b) &> v'(c_d) (c_d - c_b) \\ &\geq \lambda (c_d - c_b) \end{aligned} \quad (22)$$

Thus, at the optimum, with a decrease in disability standards, social welfare goes up from additional nonworkers added to the disability rolls ( $\Delta_b > 0$ ) while it goes down from at least some of the workers switching to disability ( $\Delta_a < 0$ ).

## 8. Multiple Disability Standards

If individuals with different health problems (e.g. physical vs. mental, or sight, lower back, etc.) have different distributions of actual and observed disutilities, we can think of the population as made up of the sum of distributions  $F_i(\theta)$  with conditional distributions of observed disutilities  $G_i(\theta_o; \theta)$ . With the same benefits for all recipients (as with the US disability program) and the same Lagrangian for the single government budget con-



straint, the first order conditions (20) tell us how the standard  $\theta_i^*$  should vary with distribution  $G_i(\theta_e; \theta)$ , while the other first order conditions are modified by adding over different health problems. In (21),  $\theta_i^*$ ,  $F_i$ , and  $g_i$  are the only elements to vary with respect to  $i$ .

To proceed, we now assume the analog of (18), which Milgrom (1979) calls the local monotone ratio property

$$\frac{\partial}{\partial \theta} \log \frac{\partial}{\partial \theta_e} g_i(\theta_e; \theta) > 0 \quad (23)$$

Unlike the two-class case, this is an assumption on the joint distribution of  $\theta_e$  and  $\theta$  rather than following from a suitable definition of  $\theta_e$ . Given (23), the left-hand side of (21) is increasing in  $\theta^*$ . It is natural to ask how the standard should vary with the distribution. That is, across different health problems, how does  $\theta_i^*$  vary with  $F_i$  and  $g_i$  in equation (21). We shall examine the special case where the distributions of disutilities are the same,  $F_1(\theta) = F_2(\theta)$ , but the conditional distributions  $G_i$  are different. A simple rightward shift for the difference between distributions is obviously offset by a comparable shift in  $\theta^*$ . The interesting case to analyze would be for a difference in the shape of the distributions.

### 9. Logarithmic Example

This section explores in more detail the two class model of Section 5 with a specification of logarithmic utility for workers and non-workers. There are six possible configurations of the economy. Workers of each type might either work (case I), apply for disability and work if it is not granted (case II), or apply for disability and retire if it is not granted (case III). Since disutility of work is greater for type 2 workers than for type 1 workers

$(\theta_1 < \theta_2)$ , there are six possible configurations as follows.

Configuration 1-(I,I). Both types work and do not apply for disability. The resource constraint implies  $c_a = R + 1$ , social welfare;

$$W_1 = \log(R+1) - \theta_1 f_1 - \theta_2 f_2$$

Configuration 2-(I,II). Type 1 works, type 2 applies for disability and works if it is not granted. The incentive compatibility constraints are

$$\log(c_b) \leq \log(c_a) - \theta_2 \leq \log(c_d) \leq \log(c_a) - \theta_1 \quad (24)$$

From (24), the marginal utility for persons on disability is higher than for those working. The social planner thus wants to make  $c_d$  as high as possible, consistent with (24). Hence

$$\log(c_d) = \log(c_a) - \theta_1 \quad \text{or} \quad c_d = c_a e^{-\theta_1} \quad (25)$$

The resource constraint is

$$(c_a - 1)f_1 + c_d p_2 f_2 + (c_a - 1)(1 - p_2)f_2 = R \quad (26)$$

Solving (9.2) and (9.3) gives

$$c_a = \frac{R + 1 - p_2 f_2}{e^{-\theta_1} p_2 f_2 + 1 - p_2 f_2} \quad (27)$$

Social welfare is

$$W_2 = (\log(c_a) - \theta_1)f_1 + \log(c_d)p_2 f_2 + (\log(c_a) - \theta_2)(1 - p_2)f_2$$

Using (25) and (27) gives

$$W_2 = \log\left(\frac{R + 1 - p_2 f_2}{e^{-\theta_1} p_2 f_2 + 1 - p_2 f_2}\right) - \theta_1(f_1 + p_2 f_2) - \theta_2(1 - p_2)f_2$$

Configuration 3-(I,III). Type 1 works, type 2 applies for disability and

retires if it is not granted. The incentive compatibility constraints are

$$\log(c_a) - \theta_2 \leq \log(c_b) \leq \log(c_d) \leq \log(c_a) - \theta_1 \quad (28)$$

From (28) the marginal utility of persons on retirement or disability is greater than that of workers. Thus the social planner sets  $c_d$  and  $c_b$  as high as possible consistent with (28):

$$\log(c_b) = \log(c_d) = \log(c_a) - \theta_1 \quad \text{or} \quad c_b = c_d = c_a e^{-\theta_1} \quad (29)$$

The resource constraint is

$$(c_a - 1)f_1 + c_d p_2 f_2 + c_b (1 - p_2) f_2 = R \quad (30)$$

Solving (9.6) and (9.7) gives

$$c_a = \frac{R + f_1}{f_2 e^{-\theta_1} + f_1} \quad (31)$$

Social welfare is

$$W_3 = (\log(c_a) - \theta_1) f_1 + \log(c_d) p_2 f_2 + \log(c_b) (1 - p_2) f_2$$

Using (9.6) and (9.8) gives

$$W_3 = \log\left(\frac{R + f_1}{f_2 e^{-\theta_1} + f_1}\right) - \theta_1$$

Configuration 4-(II,II). Both types apply for disability and work if it is not granted. The incentive compatibility constraints are

$$\log(c_b) \leq \log(c_a) - \theta_2 \leq \log(c_a) - \theta_1 \leq \log(c_d)$$

The social planner equalizes marginal utility for workers and disabled persons by setting

$$c_a = c_d \quad (32)$$

The resource constraint is

$$(c_a - 1)((1 - p_1)f_1 + (1 - p_2)f_2) + c_d(p_1f_1 + p_2f_2) = R \quad (33)$$

Solving (32) and (33) gives

$$c_a = R + 1 - p_1f_1 - p_2f_2 \quad (34)$$

Social welfare is

$$W_4 = (\log(c_a) - \theta_1)(1 - p_1)f_1 + \log(c_d)p_1f_1 + (\log(c_a) - \theta_2)(1 - p_2)f_2 + \log(c_d)p_2f_2 \quad (35)$$

Combining (32), (34), and (35) gives

$$W_4 = \log(R + 1 - p_1f_1 - p_2f_2) - \theta_1(1 - p_1)f_1 - \theta_2(1 - p_2)f_2$$

Configuration 5-(II,III). Both types apply for disability. If disability is not granted, type 1's work while type 2's retire. This is the configuration analyzed in Section 5. With  $u(x)$  and  $v(x)$  logarithmic, solving (11b), (12), and (13) gives

$$c_a = (R + (1 - p_1)f_1) \frac{(1 - p_1)f_1 + (1 - p_2)f_2}{(1 - p_1)f_1 + e^{-\theta_1}(1 - p_2)f_2} \quad (36)$$

$$c_b = c_a e^{-\theta_1} \quad (37)$$

$$c_d = R + (1 - p_1)f_1 \quad (38)$$

$$W_5 = \log(R + (1 - p_1)f_1) + \left[ \log \frac{(1 - p_1)f_1 + (1 - p_2)f_2}{(1 - p_1)f_1 + e^{-\theta_1}(1 - p_2)f_2} - \theta_1 \right] \cdot [(1 - p_1)f_1 + (1 - p_2)f_2]$$

Configuration 6-(III,III). Both types apply for disability and retire if it is not granted. The incentive compatibility constraints are

$$\log(c_a) - \theta_2 \leq \log(c_a) - \theta_1 \leq \log(c_b) \leq \log(c_d)$$

Setting  $c_b = c_d$  equalizes marginal utility for persons on retirement and

persons on disability. From the resource constraint  $C_d = C_b = R$ ; social welfare is  $W_6 = \log(R)$ .

Figures 2-6 present social welfare and benefit levels under the optimal program for parameter variations around a base setting of

$$\theta_1 = 1, \theta_2 = 4, f_1 = f_2 = .5, p_1 = .25, p_2 = .75, \text{ and } R = .45.^{12}$$

Figure 2 shows how different configurations are optimal for different levels of  $R$ . Note that configuration 4, in which both types apply for disability and work if it is not granted, is not optimal for any value of  $R$ , given the above base settings for the other parameters. The difference between  $p_1$  and  $p_2$ , i.e. the effectiveness of disability screening, needs to be increased significantly<sup>13</sup> to generate configuration 4. That is not surprising, since in configuration 4 both types make the same labor supply choice at the optimal consumption levels. Disability screening has to compensate for differences in disutility of work in order to have this pattern of benefits optimal.<sup>14</sup> Social welfare is continuous in resources, although consumption levels are not, with discontinuous jumps when the configuration

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<sup>12</sup>If workers were free to choose their hours  $h$  given wage  $\frac{1}{8}$  per hour and disutility of work  $\frac{\theta_i}{8}$  per hour, they would choose  $h$  so as to maximize  $\log\left(\frac{h}{8}\right) - h\frac{\theta_i}{8}$ . The  $\theta_i$ 's were chosen so that type 1 would want to work 8 hours while type 2 would want to work 2 hours. The base setting for  $R$  was chosen so as to be in the middle of the range of  $R$  for which configuration 5 is optimal (see Figure 2).

<sup>13</sup>For example, there is configuration 4 with  $p_1 < .08$  for  $p_2 = .75$  at the base setting of the other parameters.

<sup>14</sup>Configurations 1 and 6, which also involve symmetric labor supply choices, can always be generated by sufficiently extreme values of  $R$ .



changes. As resources increase, avoiding the disutility of work becomes more attractive. Each configuration change is an increase in the number of people not working financed in part from a fall in wages. Presumably the discontinuity would not be present with a continuous distribution of types.

Figure 3 shows the effects of varying  $f_1$  given the base settings for the other parameters. If  $f_1$  is less than .05, configuration 3 is optimal; otherwise, configuration 5 is optimal, as is the case for the range shown. From (38) one can verify that  $c_d$  is linear in  $f_1$  in configuration 5. From (37) one sees that  $c_b$  is a constant fraction of  $c_a$ . It is interesting that  $c_a$  and  $c_b$  are not monotone in  $f_1$ , although social welfare rises as  $f_1$  increases, since a rise in  $f_1$  means that the share of workers with low disutility has increased.

Figures 4-6 show the effects of variations in the disability screening parameters  $p_1$  and  $p_2$ . By definition  $p_2 > p_1$ . Increases in  $p_1$ , holding  $p_2$  constant at the base setting .75, cause a fall in social welfare and benefit levels (see Figure 4). This happens because a larger fraction of type 1's are granted disability, hence a smaller fraction work, and there is less income generated in the economy. If  $p_1$  is pushed into the range .62 - .75, configuration 5 is no longer optimal; instead, configuration 3 is.

An increase in  $p_2$ , holding  $p_1$  constant at the base setting .25, causes social welfare to rise (see Figure 5). An increase in the probability of type 2 being awarded disability means that a larger share of income is directed toward type 2's. Since the marginal utility of income is greater for type 2's, social welfare rises. Note from (38) that  $c_d$  is independent of  $p_2$ . Thus  $c_a$  and  $c_b$  fall to finance the increased size of the disability program.

Figure 6 shows the effect of varying the ratio  $p_2/p_1$ , holding the

fraction awarded disability,  $p_1 f_1 + p_2 f_2$ , constant at the base level of .5. This amounts to varying the efficiency of disability screening while holding awards constant. As the screening technology improves ( $p_2/p_1$  increases), the incentive compatibility constraint weakens and hence social welfare rises. In the limit as  $p_2/p_1 \rightarrow \infty$ , the same benefits can be given to workers of type 1 and to non-workers of type 2, i.e.  $c_d = c_a$ . Note that  $c_b$  is fixed at a constant fraction of  $c_a$  by (37), but  $c_b$  becomes irrelevant as the fraction of type 2's awarded disability approaches 1.

#### 10. Concluding Remarks

We have examined how benefits and disability standards would be set by a very knowledgeable social welfare maximizing government. We have assumed that it is costless to evaluate disability applicants. That is obviously wrong. If there were a social cost but no private cost to applying, the social cost would be paid by all the  $1-F(\theta_d)$  individuals who would apply for benefits. By introducing a fee for applying, the government could cut down the number of applicants. The value of such a fee depends on the distribution of disabilities among individuals dissuaded from applying. Particularly attractive would be a fee negatively related to the disutility of work. The absence of work for a prior period would be such a fee, although it represents a social cost for those who are rejected and for whom  $\theta$  lies between  $\theta_d$  and  $\theta_b$ . Purely financial application fees avoid some of the social cost, but possibly not all since they may involve undesirable income redistributions (as would be apparent with wealth as well as disutility differences in the population). More important is the distribution of disincentives to apply. The optimal application fee structure would be an interesting extension of the model.

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Figure 1

Labor Supply and Consumption Levels

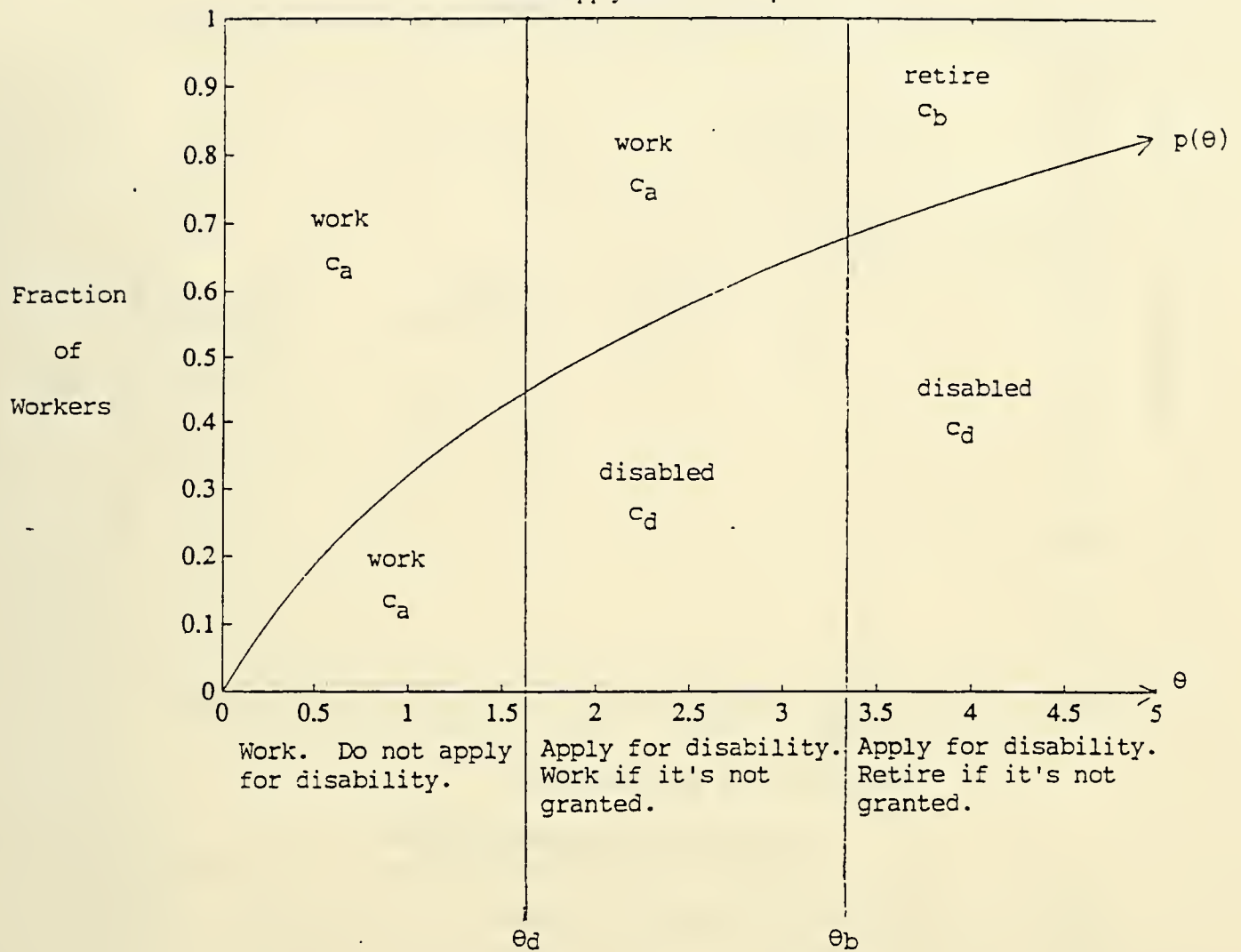




Figure 2

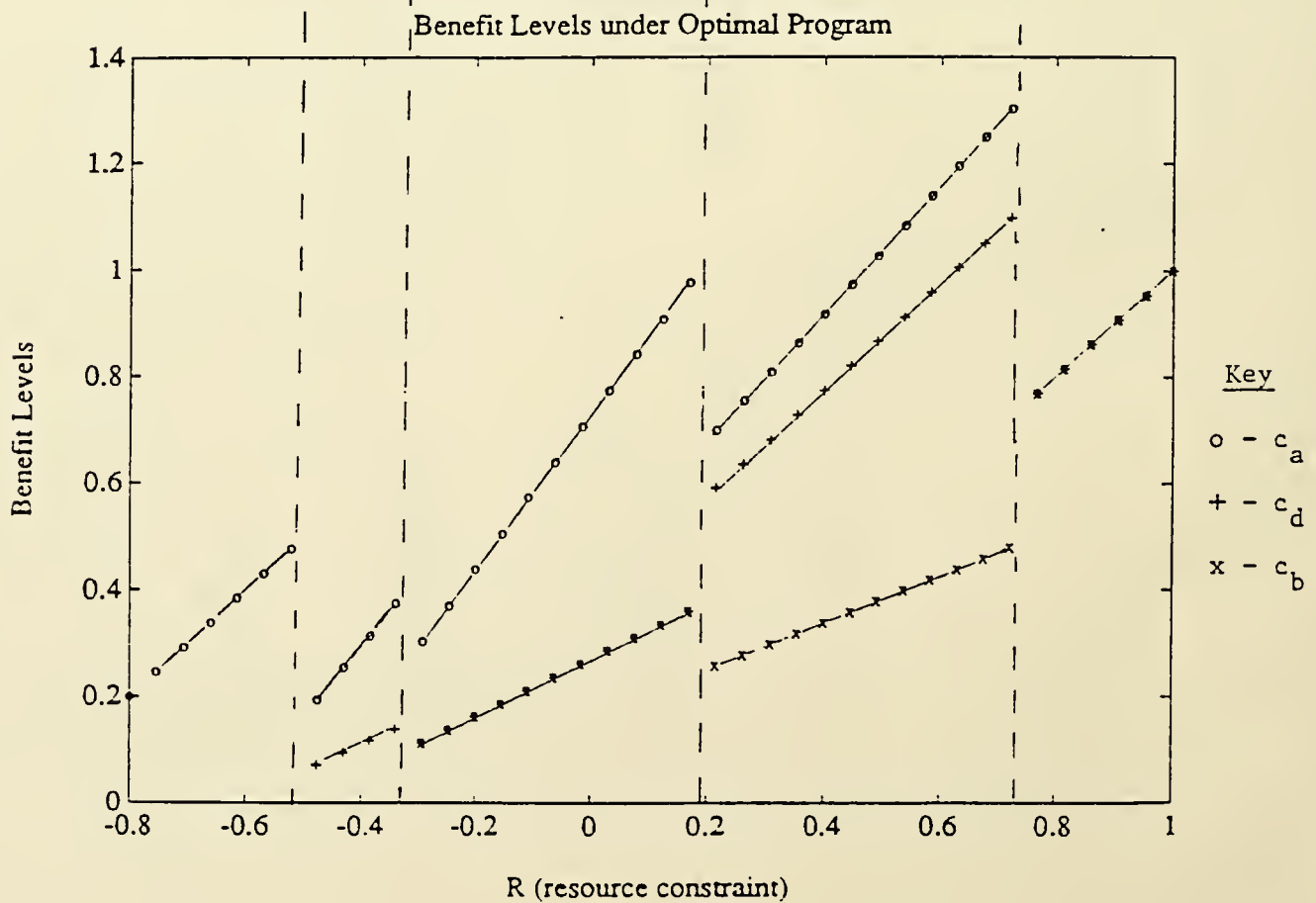
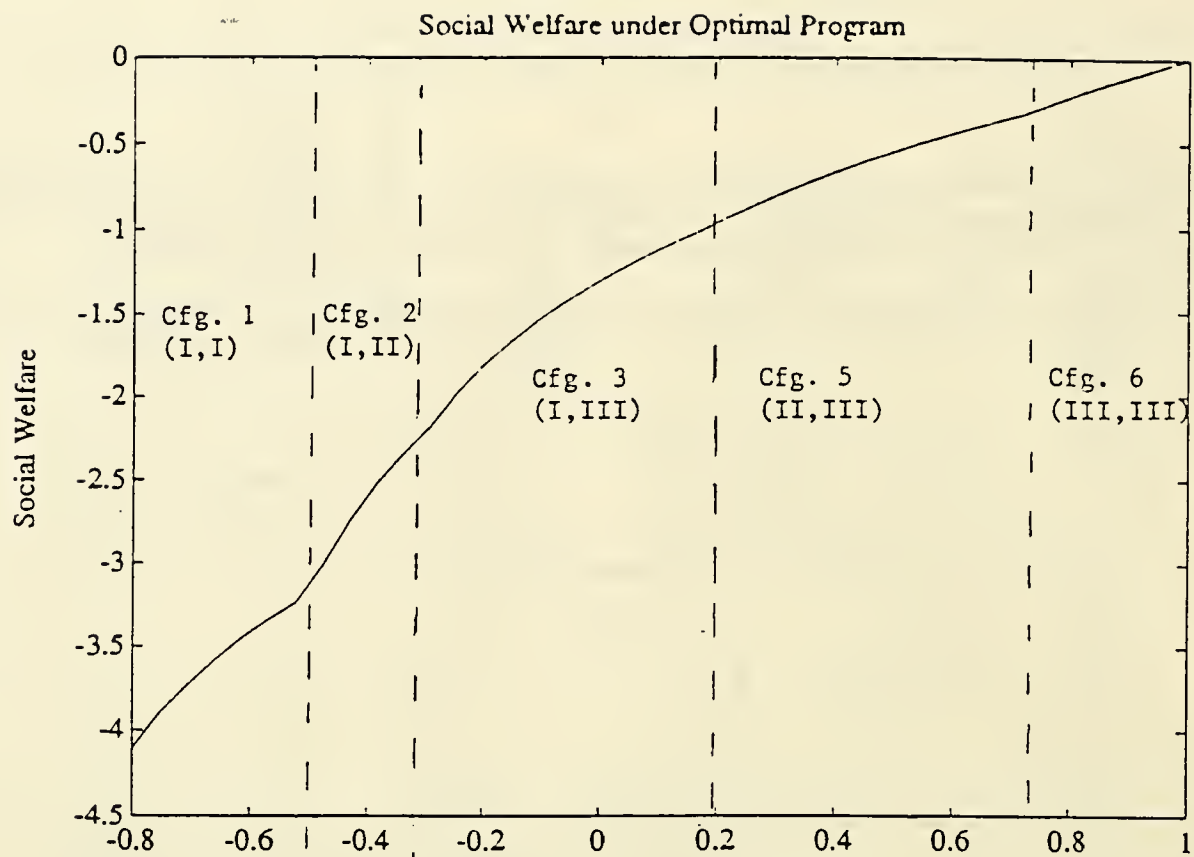


Figure 3

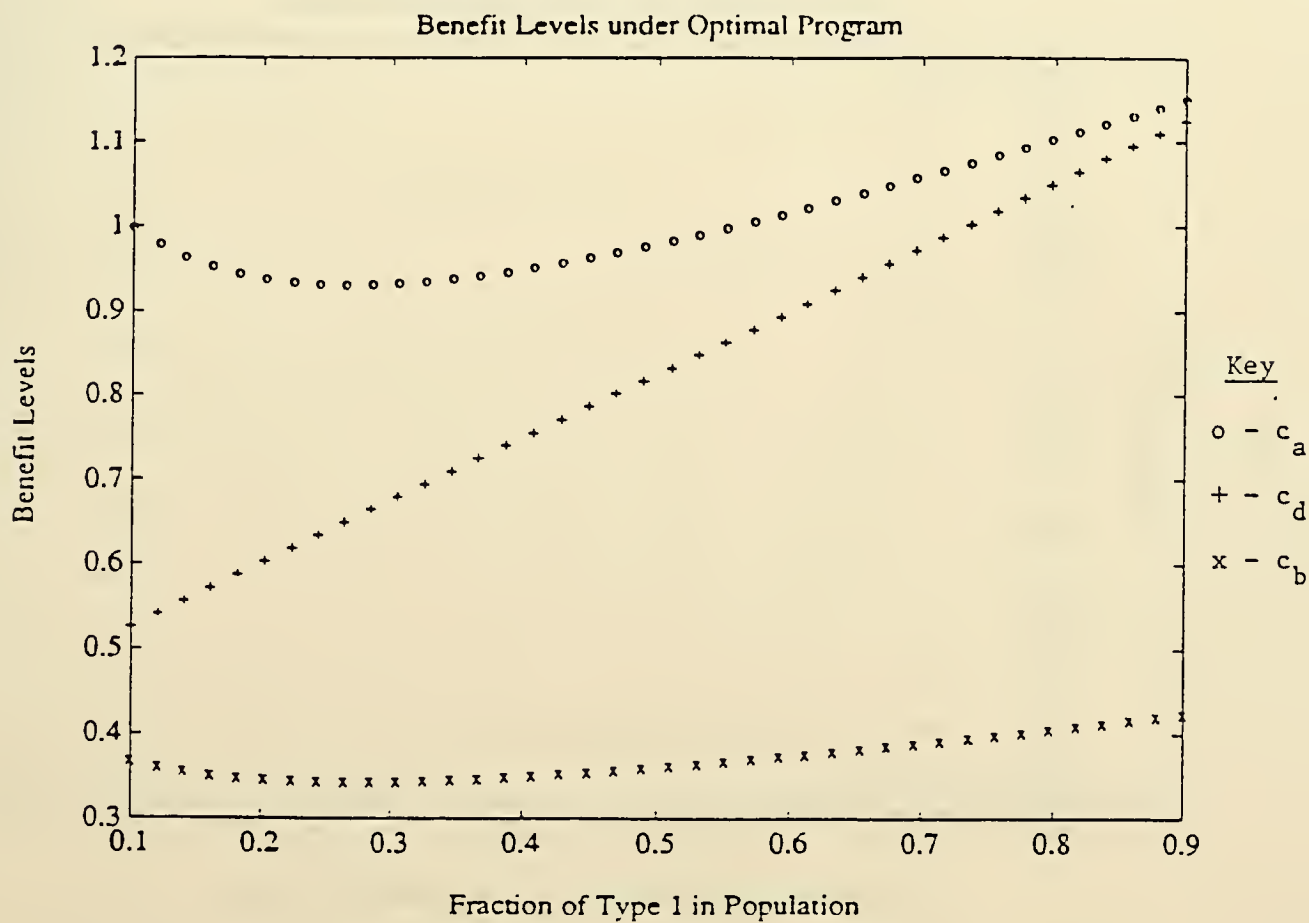
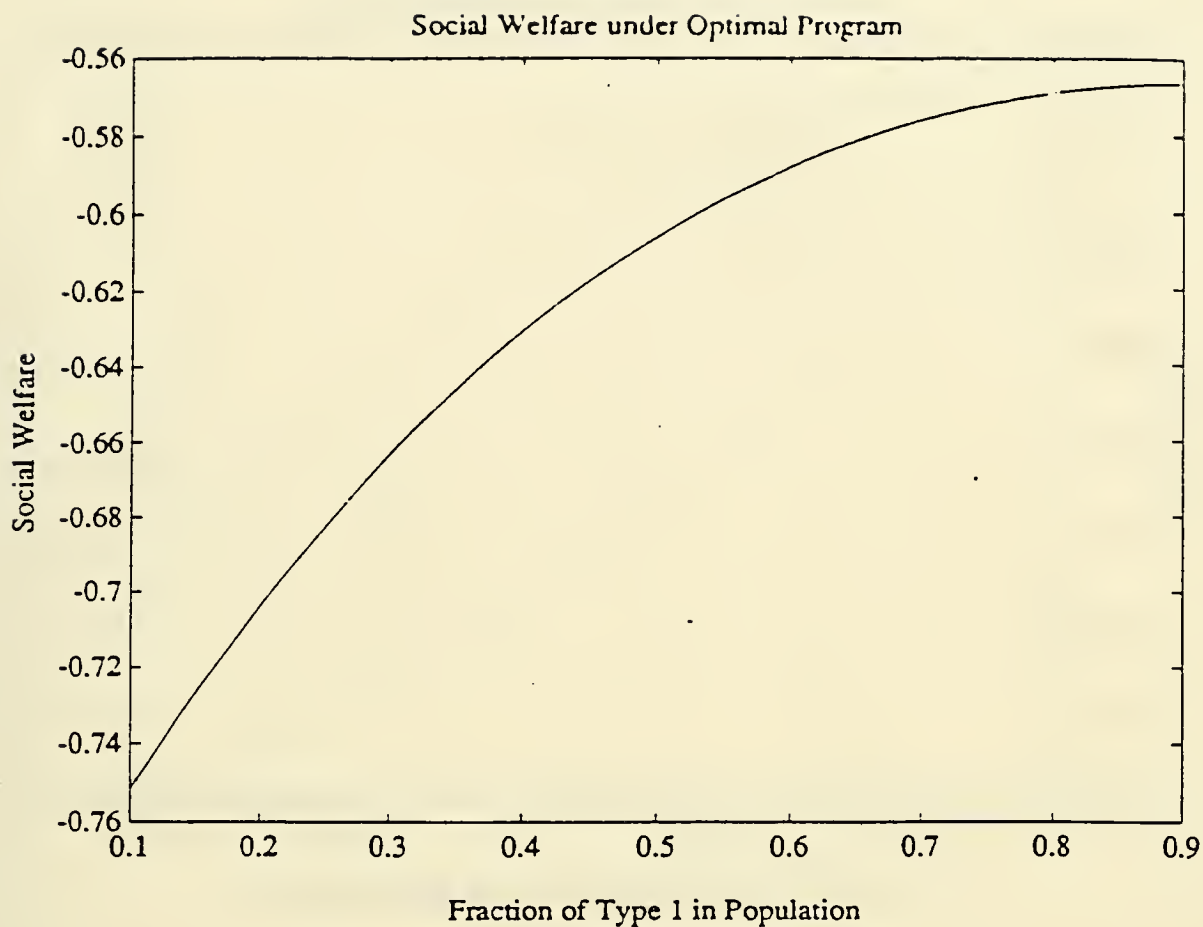


Figure 4

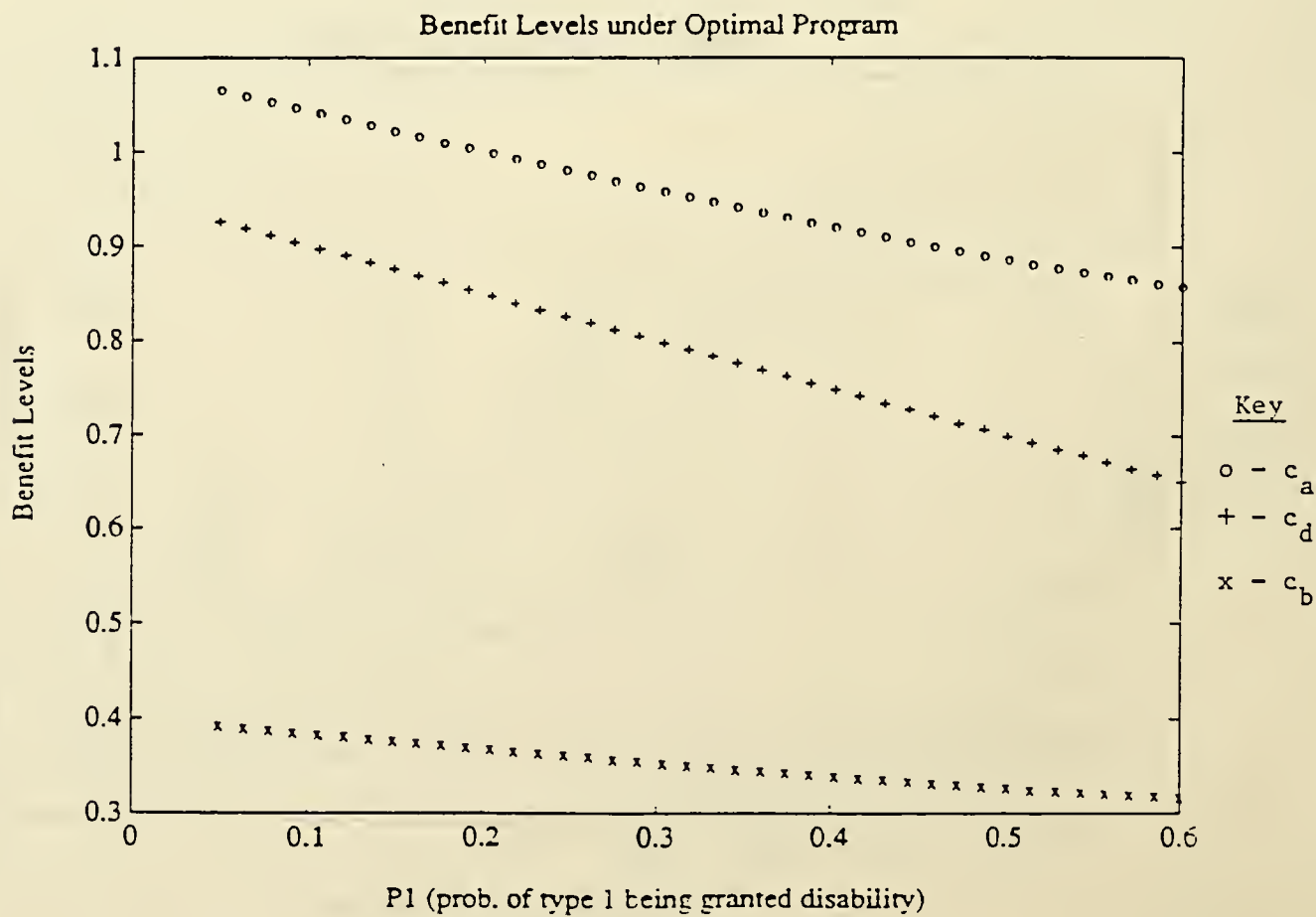
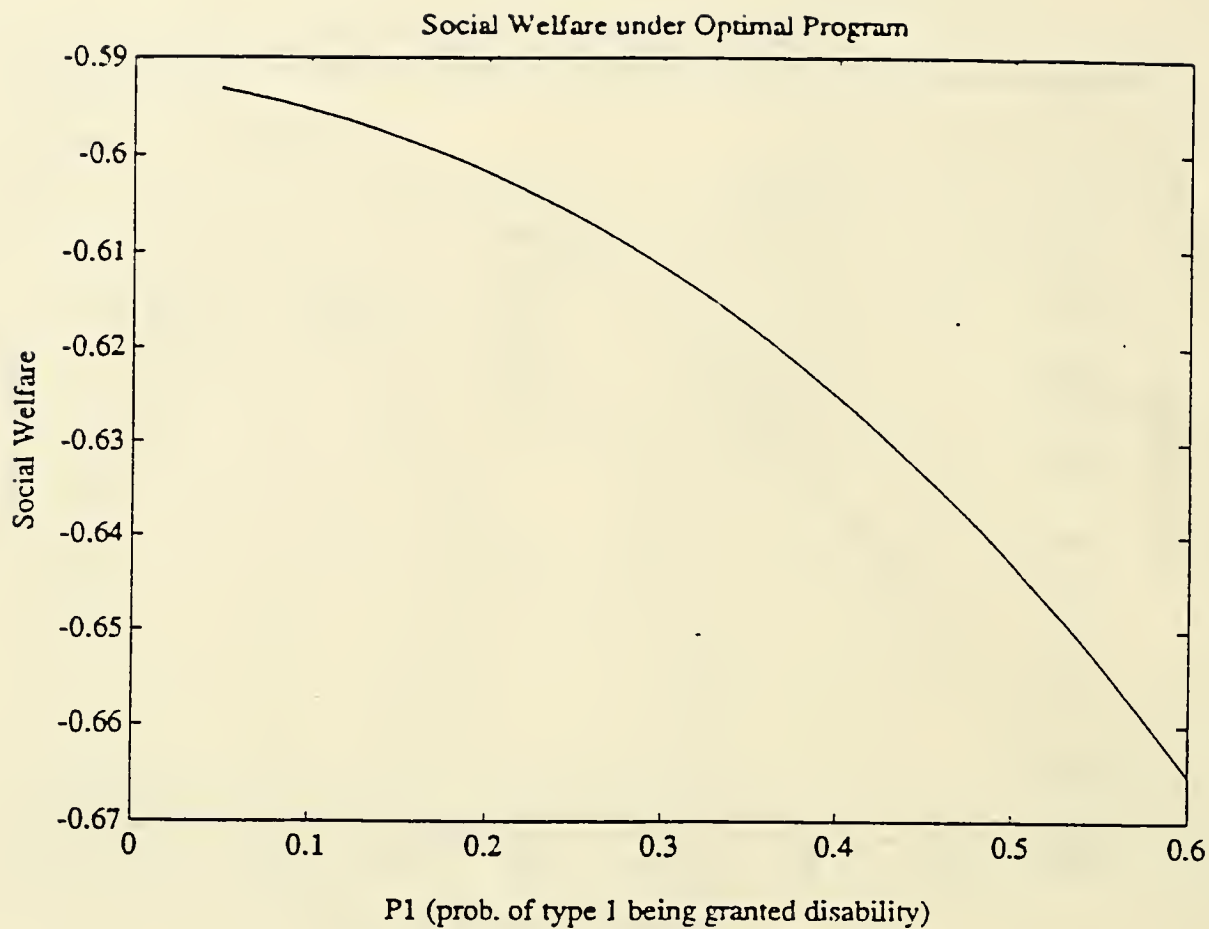


Figure 5

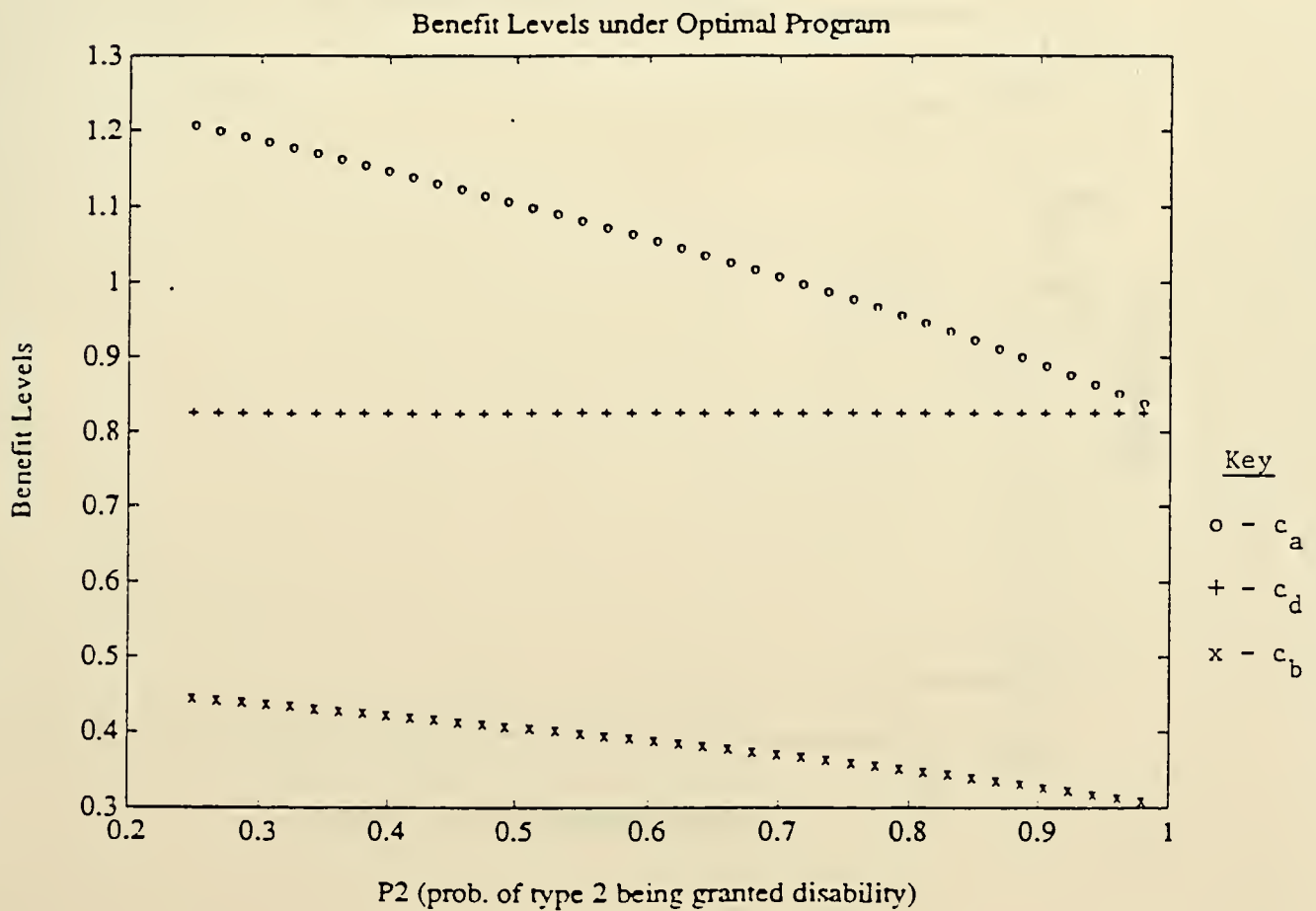
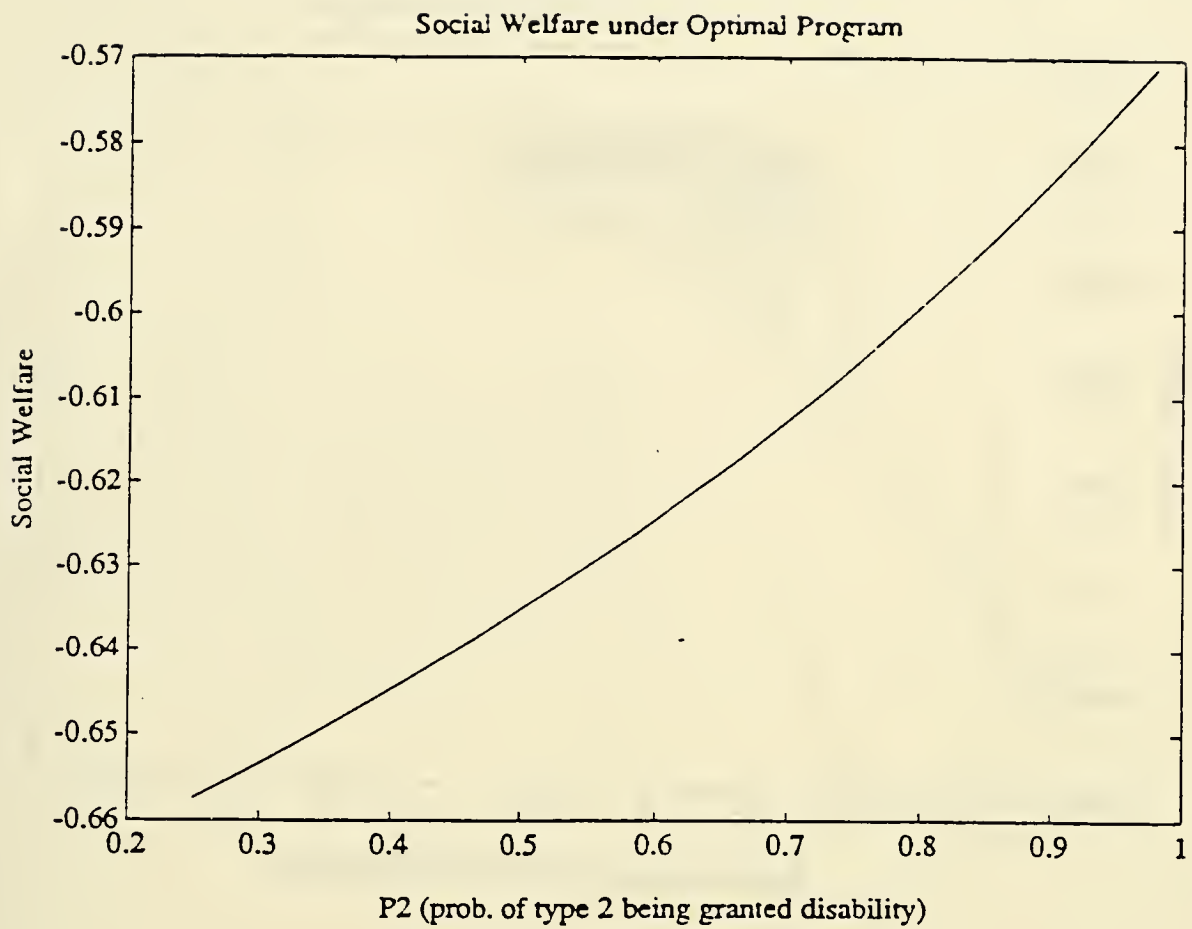
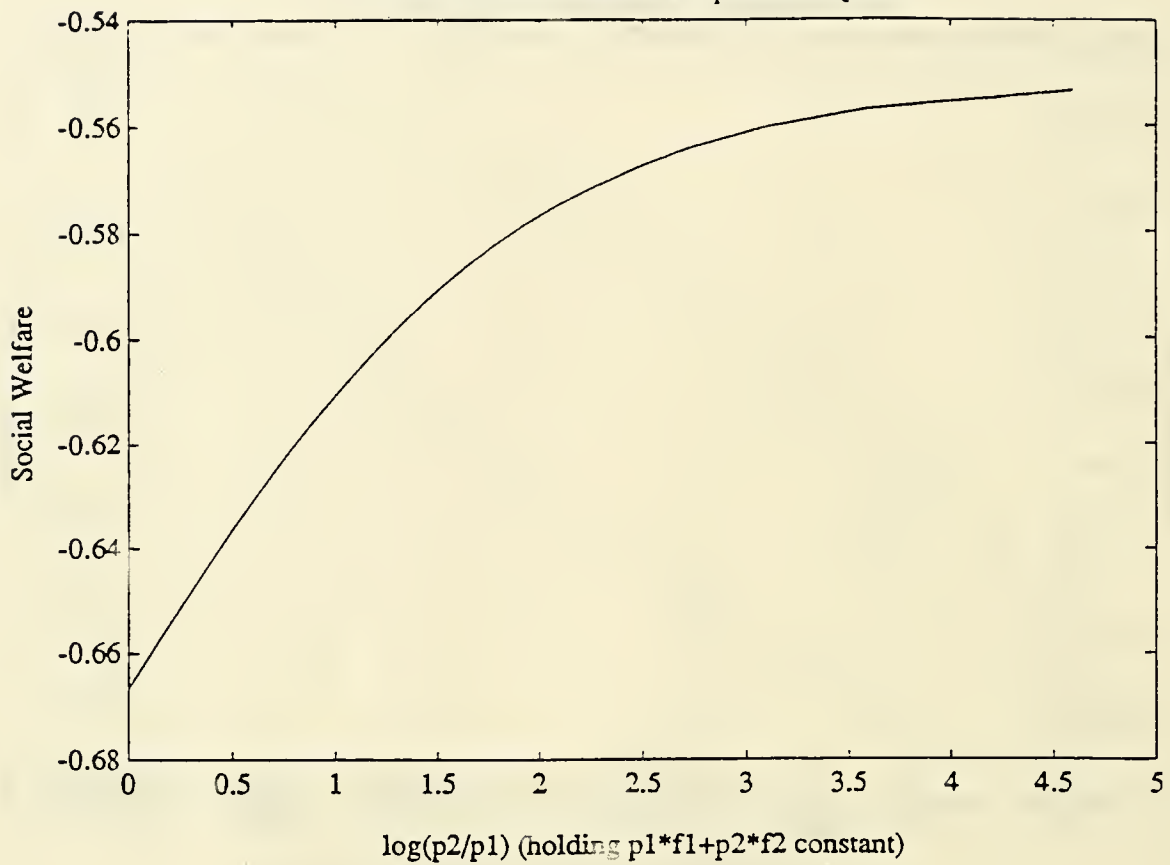
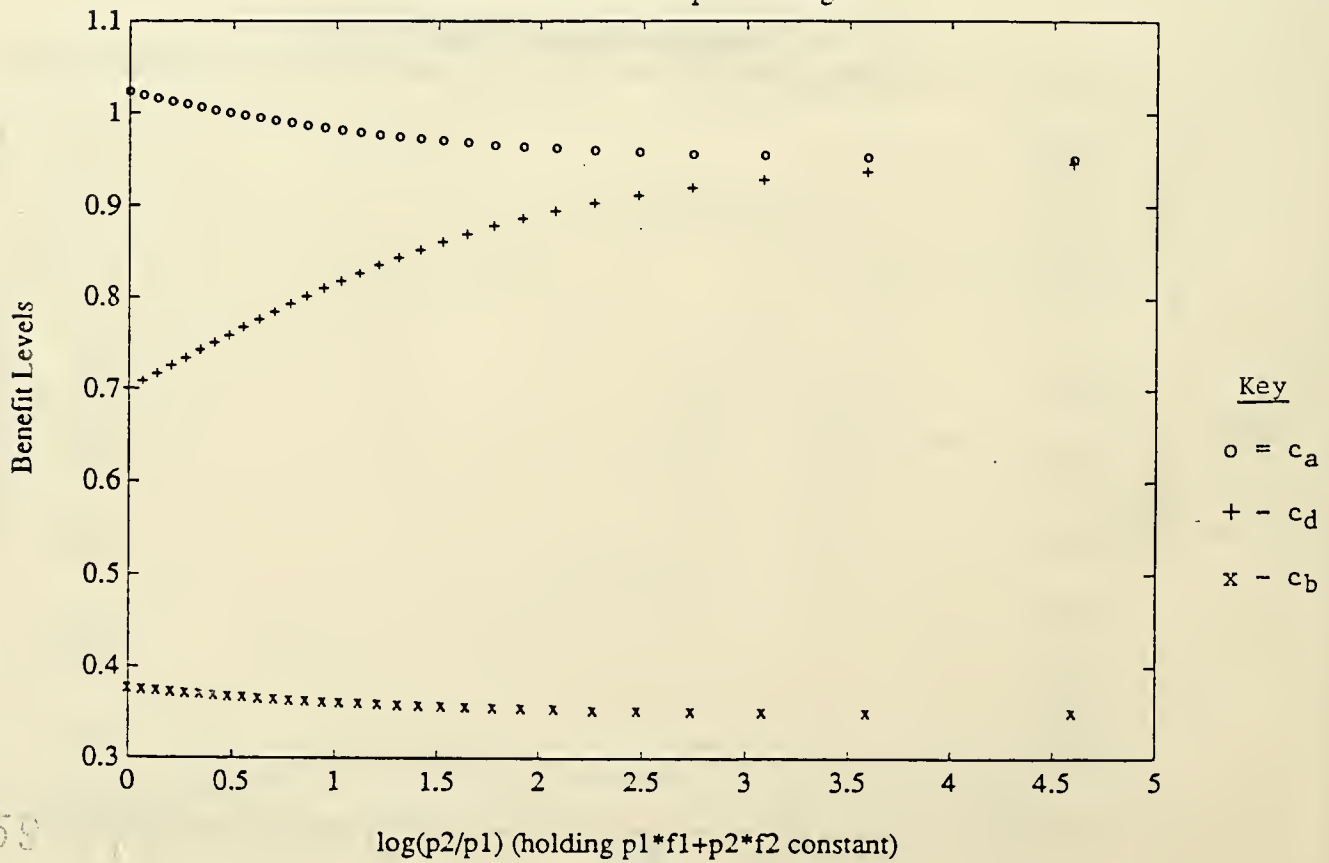


Figure 6

## Social Welfare under the Optimal Program



## Benefit Levels under Optimal Program

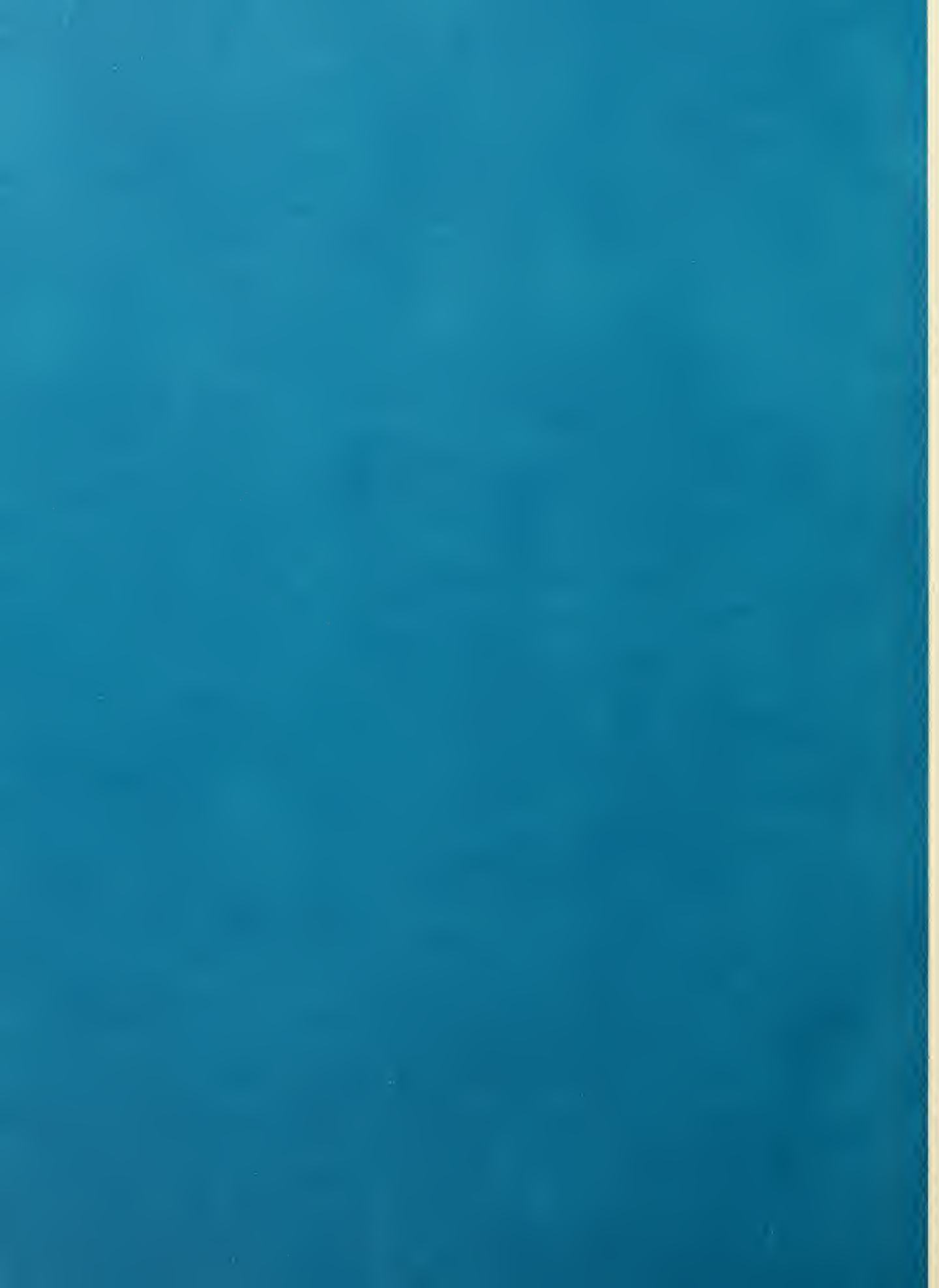




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