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PRODUCTION-THEORETIC INPUT PRICE INDICES AND THE
MEASUREMENT OF REAL AGGREGATE INPUT USE

Franklin M. Fisher

Number 384

July 1985

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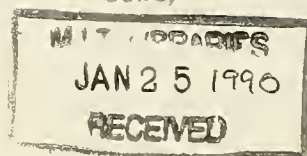
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AND THE MEASUREMENT OF REAL AGGREGATE INPUT USE

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* Paper prepared for the Fourth Karlsruhe Seminar on Measurement in Economics (Theory and Application of Indices), July 1985. I am indebted to Karl Shell for helpful conversations but retain responsibility for error.

1. Introduction

Index number theory, and the theory of price indices in particular, tends to run in terms of arithmetic properties. Such questions as whether a particular index satisfies a chain property or a reversal test are well discussed and relatively well understood. Important as such properties and such discussions are, however, they are only one side of the index number story. The other side lies in what I shall term the economic theory of index numbers: What are we trying to measure? What would we do if we had all the information possible? What, if anything, are we attempting to approximate with the index numbers computed in practice?

In the case of the theory of the individual consumer, to be sure, the economic theory of index numbers is well established.

We are accustomed to regarding computed cost-of-living indices as approximations to the ratio of expenditures required to attain a given unchanging indifference curve at base and current prices. Where the analytic questions are properly asked, the theory can be extended to cast light on what to do in the presence of taste and quality change. (See, for example, Fisher and Shell 1972, Essay I.) While the aggregation problems accompanying an extension to many consumers are formidable, they present no problem of principle.

The situation is somewhat different when we turn to the production side of the economy and the measurement of real output and input. Here there is relatively little agreement on the analytic basis for index numbers. Indeed, the notion that what one means by aggregate real output is a Laspeyres output index is ~~fairly~~ widespread. Yet while anyone is free to choose his or her own meaning for words, it is still appropriate to inquire what light production theory can cast on the questions that index numbers are supposed to answer. Putting aside the question of established usage, what index numbers would one compute if one knew the technology perfectly but had to summarize prices and outputs on the one hand or factor prices and inputs on the other in a pair of aggregate measures. Only by thinking about the problem in this way can one investigate the strengths and weaknesses of actually computable indices as approximations to analytically satisfactory constructs.

More than a decade ago, Karl Shell and I began thinking along these lines (Fisher and Shell 1972, Essay II). We consi-

dered the production-theoretic foundation for output deflation and presented a number of results. The position that we took (and to which we still adhere) is that real output must be defined relative to a given technology, with points on the same production possibility frontier (PPF) considered as involving the same real output. This leads to a theory of output deflation that is isomorphic to the theory of the cost-of-living index. We developed that theory on the assumption that the money output to be deflated is that of an entire closed economy but have since gone on to consider the more complex case in which inputs can be purchased by the productive unit involved (Fisher and Shell 1981).

I shall not discuss that work in detail but shall follow up a closely related subject. Upon reading a preliminary version of the output-deflation section of our book, John Muellbauer (1972) pointed out that an isomorphic theory can be built for the case of input deflation and proceeded to do so. That topic -- the deflation of inputs and the measurement of real aggregate input use -- is the subject of the present paper. Dealing with the conceptual problems involved casts light on the parallel problems of output deflation; further, there are some interesting new results to present.

2. Measuring Real Aggregate Input Use

I begin with the simplest case, that of a single firm producing one output from several inputs with a well-behaved neo-classical production function.

Any solution to the problem of measuring real aggregate

input use must begin with an answer to the following question: Since we shall be reducing input vectors to index-number scalars, and since we wish to obtain a complete ordering for the resulting scalars and the input vectors to which they correspond, we must choose a set of equivalence classes for input vectors such that all vectors in the same equivalent class will be said to involve the same aggregate input. How is this to be done?

In the case of the single-output firm, the answer to this question immediately suggests itself -- the use of the isoquants of the production function to define equivalence classes. With a given technology, we will say that two input vectors corresponding to the same output involve the same aggregate input use. If factor prices change and the firm remains on the same isoquant, then any change in money input costs will be considered as a pure price phenomenon.

In Figure 1, the base-period isoquant is drawn with base-period factor prices, \hat{w} , represented by the solid line. Optimal input use is \hat{v} , and total input costs $\hat{C} = \hat{w}\hat{v}$.¹ In the current

1. For ease of notation, I omit transposition signs when writing inner products, relying on the context to make clear what is intended.

period, factor prices, w , are denoted by the dashed lines. Actual factor usage is v^* and actual money costs are $C^* = wv^*$. Had factor prices been w instead of \hat{w} in the base period, the output corresponding to the base-period isoquant would have been efficiently produced with inputs v rather than \hat{v} and money costs would have been C rather than \hat{C} . The view of input-deflation

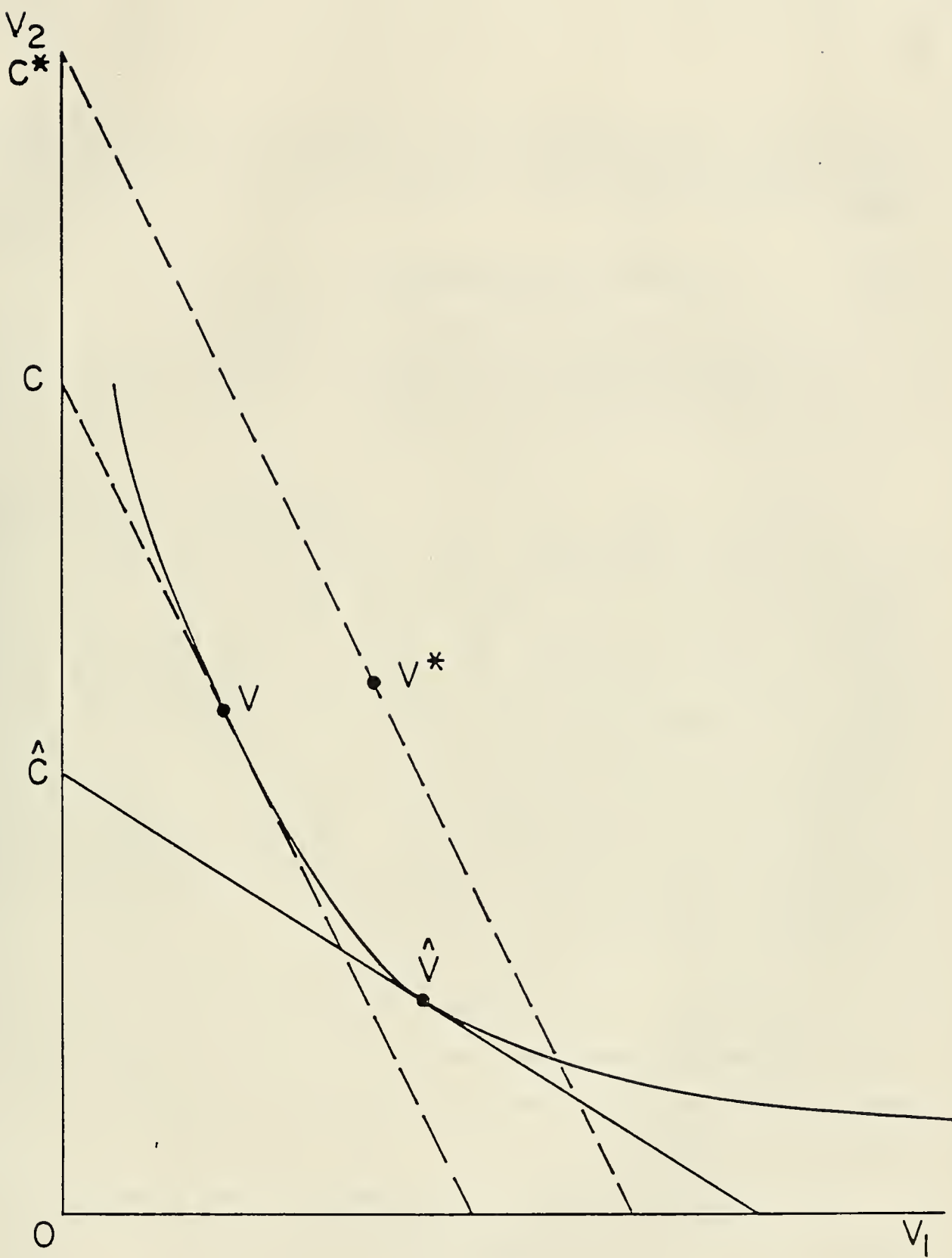


Figure 1

taken here is that the change in costs from \hat{C} to C^* should be thought of as:

$$(2.1) \quad C^*/\hat{C} = (C^*/C)(C/\hat{C}) \quad ,$$

with the first factor the increase in real aggregate input usage and the second reflecting price changes.

[Figure 1 here]

In more general terms, let the firm's cost function be $C(w, x)$, where x is output. The construction just given uses base-period output, \hat{x} , and calculates the deflator for money input costs as $C(w, \hat{x})/C(\hat{w}, \hat{x})$. This deflator is divided into money input costs to give the measure of real aggregate input usage relative to the base period.

Obviously, this approach will lead to a theory largely isomorphic to that of the cost-of-living index (as well as to the Fisher-Shell theory of output deflation). In considering possible objections to the theory being advanced, therefore, it is well to consider what those same objections imply about the relatively well-established theory of the cost-of-living index. Consideration of such objections leads to insight as to what is involved in the present analysis.

The first such objection is conceptual. The procedure just described treats input vectors as identical if they can produce the same output and treats a movement to a higher-numbered isoquant as an input increase. But we are trying to build a theory of input aggregation and measurement. Is it not odd that levels of outputs become central to the theory? Moreover,

different firms with different technologies facing the same set of input prices will have different factor price deflators constructed for them.

The answer lies in consideration of the object of the enterprise. We are treating the firm as the object of interest with factor prices given from outside.² Any production-theoretic

2. Cases of monopsony can also be treated but are irrelevant to the present discussion.

view of input deflation must involve the production function of the firm. Just as the cost-of-living index describes price changes from the point of view of the individual consumer, so the production-theoretic input price index describes factor price changes from the point of view of the individual firm. The fact that different firms have different points of view, so to speak, is not a valid objection. The aggregation problem to which it points cannot be solved by choosing a firm-independent measure of input prices. To do that is merely to impose on all firms a measure not relevant to any one of them.³

3. To say this is not, of course, to say that there is no interest in the variation of the production-theoretic index as the production function varies over firms (or changes over time). Some results on such variation are presented below.

3. Homogeneity and Related Properties

The second objection to the approach taken here is somewhat more troublesome, at least at first glance. Suppose that input

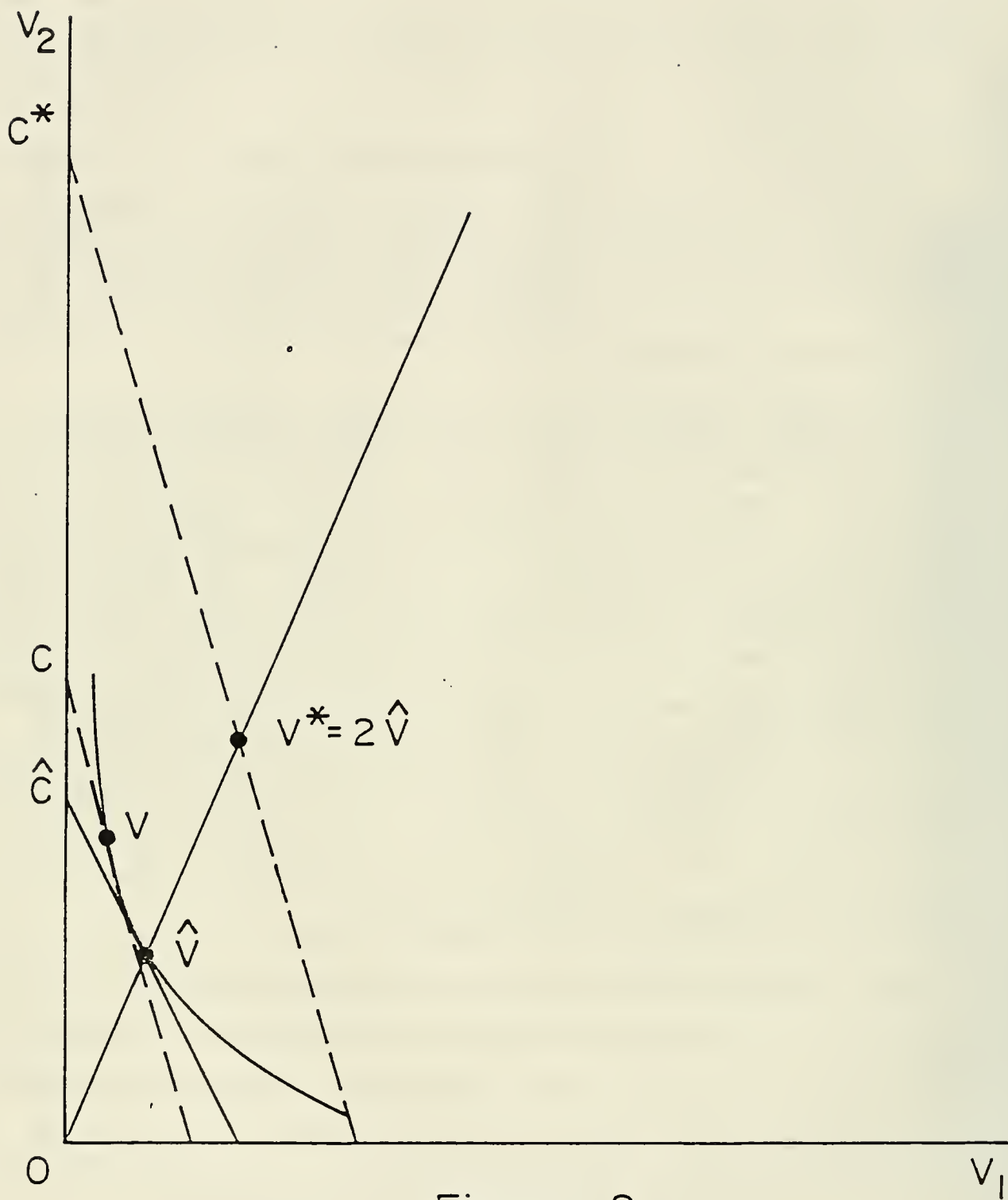


Figure 2

prices change but that it just so happens that the usage of every factor changes in the same proportion, so that the actual input point in the current period is on the ray through \hat{v} in Figure 2. To fix ideas, suppose that the usage of every input exactly doubles. It is evident that the production-theoretic view taken here does not lead to a measure of increase in aggregate real input usage which doubles. In terms of Figure 2, real input usage will be said to increase by a factor of $C^*/C > 2$, and it is easy to see that the sense of the inequality is no accident.⁴

4. Because a Laspeyres price index must bound the production-theoretic input-price deflator from above, the corresponding Paasche quantity index bounds the production-theoretic aggregate input index from below. The Paasche quantity index, $C^*/(C^*/2)$, is obviously 2, however.

Hence the production-theoretic index of aggregate input usage is not homogeneous of degree one in the inputs being aggregated.

[Figure 2 here]

This objection was made by Diewert (1983, pp. 16-26) to the parallel theory of the measurement of real output. At first glance, it seems a telling one. Careful consideration, however, reveals that its force is far less than first appears.

Suppose first that the technology is one of constant returns and that there has been no technological change between the base and the current periods. Then the isoquant through the point $2\hat{v}$ must be parallel along rays to that through \hat{v} . In that case, however, the situation pictured in Figure 2 cannot occur. In

such circumstances, the only way that $2\hat{v}$ can be the input point for the current period is if relative input prices do not change. But if relative input prices do not change, then it is easy to see that the production-theoretic approach (like every other sensible approach in such circumstances) will lead to an input-usage index of 2.

Now suppose (still with no technical change) that the technology does not exhibit constant returns. If the isoquant map is homothetic, the situation pictured in Figure 2 still cannot occur; in more general circumstances, however, it can. Once one leaves constant returns, however, it ceases to be obvious that it is desirable for an index of aggregate input use to be homogeneous of degree one in the individual inputs. Inputs are important because they are employed in production. The measurement of aggregate input usage properly ought to be from the point of view of that employment. Without homotheticity in the underlying technology it is not at all clear why movements along a ray should play any special role in aggregate input measurement.

If this view that doubling does not mean doubling seems hard to swallow, consider the parallel issue that arises in the theory of the cost-of-living index. There is nothing about Figure 2 that could not apply to that theory with isoquants and costs being replaced by indifference curves and expenditures. Indeed, in the case of the cost-of-living index, there is relatively little interest in homothetic maps and none at all in constant returns, so situations such as Figure 2 are the rule rather than the exception. In such a context, one surely hesitates to insist

that a doubling of the consumption of every commodity must mean a doubling of real income. Yet the apparent appeal of the homogeneity property is every bit as strong in the consumer context as in the production one -- precisely because that appeal rests on no consideration of the context involved or the question being asked.

This is, of course, not to say that the homogeneity property may not be an interesting one. One can perfectly well construct an input index by asking by what factor the current-period input vector must be multiplied to place it on the base period isoquant.⁵ Such a construction guarantees the homogeneity property,

5. In the case of real output measurement this is what Diewert (1983, p. 18) terms the "Malmquist-Bergson-Moorsteen" approach. He gives an extensive bibliography and discussion of these matters in that context.

but concentrates on movements along a ray. Once one leaves homothetic isoquant maps, it is not clear why such movements should be of special interest.

One can think of such a ray-centered construction in a different way. Instead of asking by what factor the current period input point must be multiplied to place it on the base period isoquant, ask the equivalent question: By what factor must the base period isoquant be expanded (or contracted) parallel along rays to pass through the current-period input point. Obviously, this amounts to the same thing.

Thinking about matters in this way, however, points up the difference between such an approach and the one taken here. The

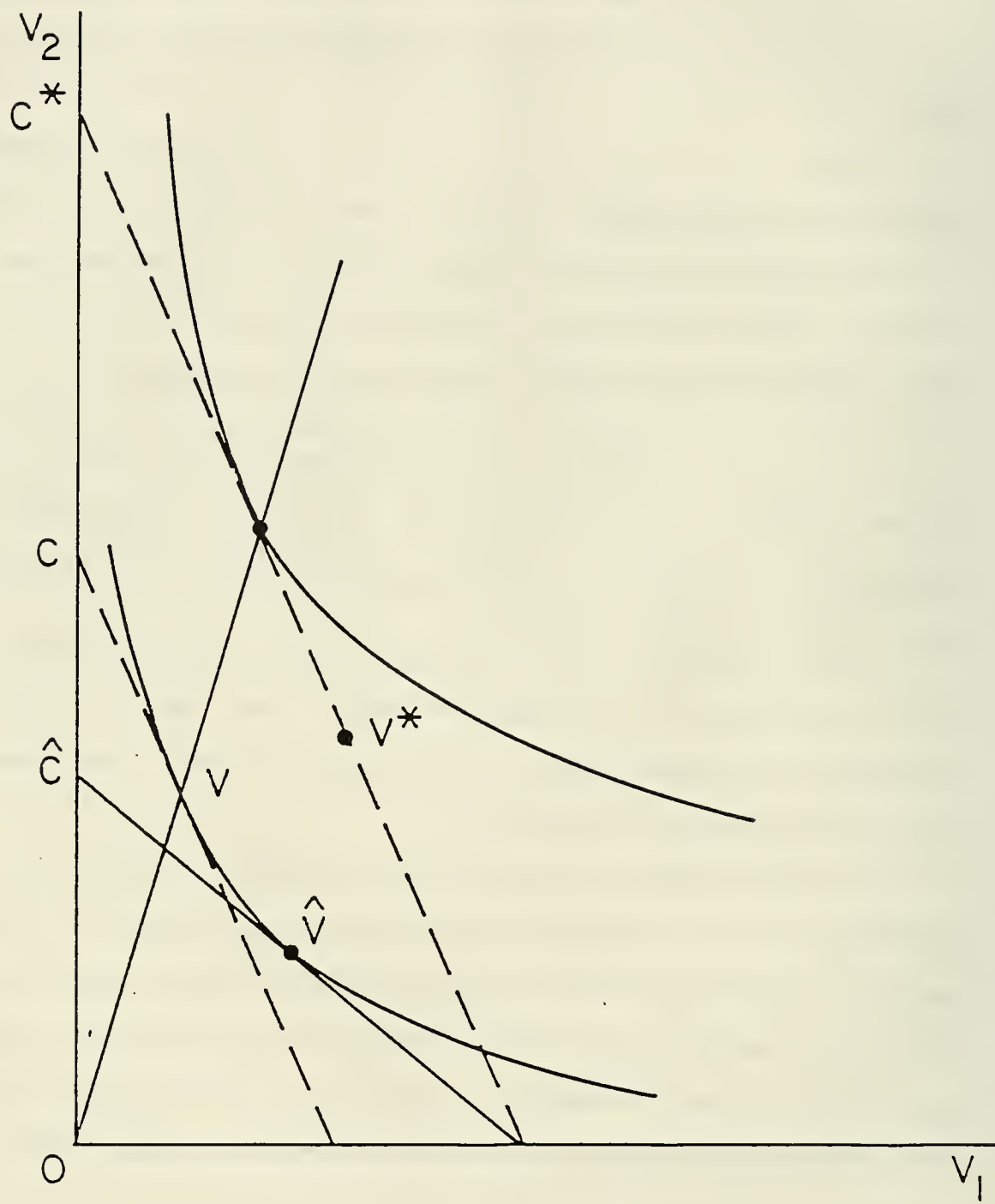


Figure 3

aggregate real input index constructed in the theory here being espoused answers the question: By what factor must the base period isoquant be expanded (or contracted) parallel along rays to become tangent to the isocost line at the new prices through the new input point? (See Figure 3.) Where the isoquant map is homothetic (and unchanging), this construction gives the same answer as does the ray-centered one. Where homotheticity is lacking (or the isoquant map changes), the answers are different.

[Figure 3 here]

That difference can be seen in another way. The ray-centered construction takes as its reference point the point where the ray through the current-period input point crosses the base period isoquant. In effect, it begins by asking the question: What would inputs have been in the base period had the firm been restricted to the current period's input proportions? By contrast, the production-theoretic index takes as its base point the point on the base-period isoquant where that isoquant is tangent to an isocost line corresponding to the current period's input prices. In effect, it begins by asking the question: What would inputs have been in the base period had the firm faced current period input prices? Both ways of looking at the problem are interesting, but I believe the approach taken here is much more of an economics-oriented one. Certainly it lends itself readily (as the ray-centered approach does not) to an accompanying theory of input-price deflation.

Having said all this, I should point out that much of the

comparative-static analysis given below is of interest primarily in the case of homothetic technologies. As already observed, under homotheticity, if the technology does not change, both the ray-centered approach and the one used here give the same answers and the real aggregate input use index here constructed does in fact have the homogeneity property.

But what if technology does change between the base period and the current period? Here the ray-centered and the production-theoretic approach will give different answers. Perhaps surprisingly, despite the failure of homogeneity of degree one, there is a powerful case to be made that the answers of the production-theoretic approach are superior.

The reason for this lies in the fact that every way of constructing a real input-usage index implies the construction of a corresponding input-price deflator and conversely. Hence, in considering the properties of any approach to the problem, one must consider the properties of both price and quantity indices, not merely the properties of only one of them.

Implicit in the idea that homogeneity of degree one in the input vector is a desirable property for a real input index is the view that a weaker property is even more naturally desirable: The measure of aggregate input should not change if the input vector itself does not change. Parallel to this is a similar statement about the input-price deflator: The input price deflator should not change if input prices do not change (and, we may add, should be homogeneous of degree one in those prices).

In addition, one wants the two indices to have certain natural consistency properties. In particular, it is natural to

require the "circle property". In terms of prices, the price index should have the property that the change in relative input prices from situation A to situation B multiplied by the change in relative prices from situation B to situation C should equal the change in relative prices from situation A to situation C. A similar property should hold for the index of real input usage.

Unfortunately, one cannot have all these desirable properties at the same time. In particular:

Theorem 3.1: A. The production-theoretic input-price deflator and its associated index of real input use are the only indices having the circle property and also the following properties:

1. The relative change in the index of factor prices multiplied by the relative change in the index of real input usage equals the relative change in expenditures on inputs.

2. With an unchanging technology, a movement along the base period isoquant leaves the index of real input usage unchanged.

3. The index of factor prices does not change if factor prices remain constant.

In addition, the production-theoretic input-price deflator is homogeneous of degree one in the input prices.

B. There exists no way of constructing a pair of input-price and real input-usage indices that have properties 1-3 just given, the circle property, and also the following property:

4. The index of real input usage does not change if inputs remain constant. which lead to those amounts being chosen.

Proof. A. It is obvious that the indices resulting from the production-theoretic approach have the circle property and properties 1-3. Further, the resulting input-price deflator is obviously homogeneous of degree one in the input prices. Consider Figure 1 once again. Since v and \hat{v} both lie on the base-period isoquant, property 2 implies that the movement in costs from \hat{C} to C^* is entirely a price phenomenon. Since input prices (as shown by the dashed lines) are the same at v as at v^* , property 3 implies that there is no price change between those two points. Hence, by the circle property, the movement of money costs from C to C^* is entirely a change in real input usage. This division of the movement from \hat{C} to C^* into monetary and real changes, however, is just that of the production-theoretic approach. (See equation (2.1).)

B. In view of part A of the theorem, it suffices to show that the production-theoretic approach does not have property 4. Consider Figure 4. Here, the technology changes between the base and the current periods, and it just so happens that input usage remains at \hat{v} despite the fact that relative input prices change. (Obviously, this requires a change in the isoquant map.) The production-theoretic input-price deflator, however, will still call the change in money costs from \hat{C} to C a pure price phenomenon and the change from C to C^* an increase in real input usage. It follows that real input usage will be said to have changed from \hat{v} to v^* , violating property 4.

[Figure 4 here]

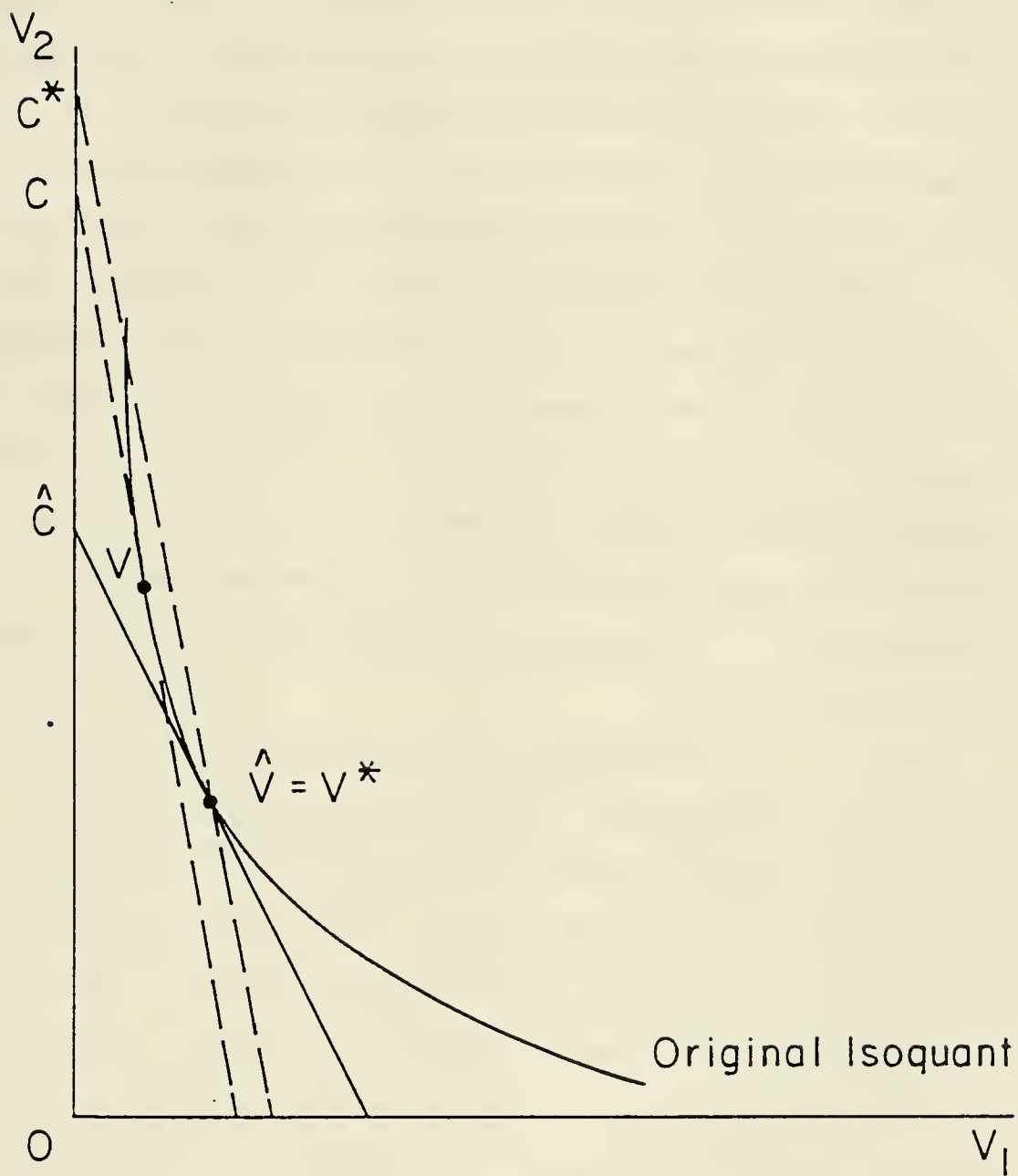


Figure 4

Lest the violation of property 4 just exemplified in the proof be considered particularly damaging, it is instructive to examine the situation in Figure 4 again to see the contradiction between property 4 and property 3. In Figure 4, the prices corresponding to costs C and C^* are the same. By property 2, real input usage at v is the same as at \hat{v} . It follows that any approach (the ray-centered one, for example) that satisfies property 4 and has real input usage the same at v^* as at \hat{v} must describe the change in costs from C to C^* as purely monetary to satisfy property 1 and the circle property. Hence any such approach will show a change in input prices between the situation corresponding to C and that corresponding to C^* even though no input price changes.

Thus, in the presence of properties 1 and 2, properties 3 and 4 are contradictory. One cannot have both an input price index that depends only on input prices and an input-usage index that depends only on input usage, have them multiply in the obvious way, retain the natural circle property, and still have movements along an isoquant represent no change in input usage. Something has to give. In the case of Paasche or Laspeyres indices, the missing property is property 2 -- equivalence along an isoquant. In the case of more sophisticated approaches, either property 3 or property 4 must be abandoned once we leave homotheticity and an unchanging technology.

Which property should be retained? That depends on what it is that one is trying to do. I take the view that input price deflation means looking at input prices from the point of view of

the input-using unit -- the firm in the simplest case. Those prices are among the givens of that unit's problem, whereas the quantities of input used are functions of those prices. If we wish to form aggregates in this context, then, it is property 3 that should be retained. The resulting input-price index should depend only on the input prices; the fact that the corresponding input usage index will depend both on prices and on quantities merely reflects the fact that quantities themselves depend on prices.

I add two points in this connection that may serve to make the argument more convincing. First, we have already seen that

the tension between properties 3 and 4 does not arise until we leave the case of an unchanging homothetic isoquant map. When we leave homotheticity behind, the case in favor of an input-usage index homogeneous of degree one in the input vector stops being a convincing one. With or without a homogeneous isoquant map, however, the case in favor of an input-price index homogeneous of degree one in the input prices remains convincing. The cost function continues to have that property even when the underlying isoquant map is not homothetic.

Second, consider again the case of the cost-of-living index and interpret Figure 4 as an indifference map. Insistence on property 4 in that context leads to a case in which unchanging consumer prices (the dashed lines) imply a change in the cost-of-living index. Retention of property 3, on the other hand, does not do this but does lead to the proposition that the change from C to C^* involves an increase in real income despite the fact that v and v^* appear to be on the same base-period indifference curve. Once one realizes that v^* must be on a higher-numbered indifference curve than v according to the current period's indifference map, that property does not seem so odd. Plainly, the same argument must apply to input (or output) deflation and measurement.

Thus, while it might be nice to have both property 3 and property 4, this is impossible in the presence of properties 1 and 2. Given that, it seems sensible to me to retain property 3 and abandon property 4. Certainly, the implications of an approach that does so are well worth investigating; it is far from a fatal objection that such an approach fails to make the mea-

surement of real input usage depend only on the input vector and fails to be homogeneous of degree one in the elements of that vector.

4. Generalization to Many Outputs

I now continue with the development of the production-theoretic input-price deflator. To do so, it is necessary to generalize the case of a single firm producing a single output. While the generalization to many outputs is an easy one, as we shall see, the generalization to many firms is not always so straightforward. This is because of the possibility that the aggregation of firms involved is large enough to have an effect on either output or input prices. I shall avoid this issue in the present paper, however, and shall assume that the productive unit involved is small enough so that it takes prices as given. In view of this, we may as well keep on thinking of the productive unit as a competitive firm.⁶

6. See Fisher and Shell, 1981, for a discussion of the problems involved in more general situations in the case of output deflation.

Even the case of the competitive firm, however, requires generalization to allow several outputs, and that generalization, while not hard, has some interesting features. In particular, we must ask how the underlying isoquant map is to be constructed.

In the isomorphic case of output deflation, this question is

that of how to construct the production possibility frontier. In that case, one obvious possibility is to take the vector of inputs as fixed -- at least up to scalar multiplication. (This is the approach taken in Fisher and Shell, 1972, Essay II.) Such a choice is natural when dealing with output price deflation in a closed economy, but it is not inevitable, however. If the productive unit being studied purchases inputs at fixed prices, for example, then it becomes natural to draw the production possibility frontier as the locus of outputs that can be produced at constant input cost. (This is studied in Fisher and Shell, 1981.)

In the present case of input deflation and measurement, the isomorphic choice to that of the closed economy with fixed inputs is to take the output vector of the firm as fixed -- at least up to scalar multiplication. This is the choice made in Muellbauer (1972), but it is not an appealing one. Even apart from the assumption that the productive unit being analyzed takes prices as given, there is no interesting problem in which output proportions are fixed. Rather, given that we are dealing with a competitive firm, it makes sense to analyze matters from the same point of view as does the firm. This means fixing output prices and drawing an isoquant as the locus of all input combinations that can produce a given value of output.

Naturally, this makes the isoquant map depend on the particular output prices used. Since we shall study the effect of changes in the isoquant map on the constructed indices, this makes it important to study the effect of changing output prices in particular.⁷

7. Note that the case of unchanged output prices is no different from that of a single output or of fixed output proportions, so that the only difference in terms of exposition (but not in terms of actual index construction) lies in whether comparative static exercises are done with changing prices or with changing output proportions.

5. The Input-Price Deflator: Formal Statement

I now give a formal description of the production-theoretic input-price deflator. The givens of the problem are: the vector of base-period factor prices, \hat{w} ; the vector of current period factor prices, w ; total base-period costs, \hat{C} ; and the vector of output prices, p , the same in both periods. Production takes place according to the production function:

$$(5.1) \quad F(x, v) = 0 \quad ,$$

where x is the vector of outputs produced by the firm and v the vector of inputs it uses. Hats will be used to describe base-period values.

Consider the solution to the following problem:

$$(5.2) \quad \begin{aligned} &\text{Maximize } \hat{y} \equiv p\hat{x} \\ &\text{subject to } F(\hat{x}, \hat{v}) = 0 \\ &\text{and } \hat{w}\hat{v} = \hat{C} \end{aligned} .$$

Call the resulting value of y , \hat{y} .

Next, solve the following problem:

$$\begin{aligned}
 (5.3) \quad & \text{Minimize } C \equiv wv \\
 & \text{subject to } F(x, v) = 0 \\
 & \text{and } px = \hat{y} \quad .
 \end{aligned}$$

The production-theoretic input-price deflator is then the ratio, $J \equiv C/\hat{C}$.

Some comments are in order. First, the solution to the problem in (5.2) amounts to finding the isoquant (for the production of value, $y \equiv px$) tangent to the plane corresponding to base-period input prices and cost. The solution to the problem in (5.3) then takes that isoquant and minimizes cost along it at the current period's factor prices, w . This construction is readily seen to be equivalent to that in Figure 1 above.

The reason for beginning with the maximization problem in (5.2) instead of directly with the base period's isoquant has to do with the analysis of comparative statics given below. Since we shall wish to ask how the deflator (and its associated index of real input usage) would differ if the isoquant map were different, it is necessary to have a method which unambiguously picks out the isoquant with which the deflator is to be constructed. If the isoquant map is the actual base-period one, then this will be the actual base-period isoquant. Otherwise, it will be the isoquant tangent to the plane representing base-period factor prices and costs. This approach is consonant with the general view that the givens of the problem are factor prices and base-period costs (current-period costs are to be deflated), not actual base-period inputs.

Next, the deflator can be defined in terms of cost functions. Remembering that "output" here is really y , we can think

of the firms cost function as $C(w, y)$. Letting \hat{y} be actual base period revenue, the deflator is readily seen to be $C(w, \hat{y})/C(\hat{w}, \hat{y})$. This form is not so helpful for comparative static analysis, however, as is the more extensive description given above because it presupposes that the "output", \hat{y} , will remain the same when the isoquant map changes.

It is obvious that, as in the case of the cost-of-living index, the input-price deflator here defined (J) is bounded above by the Laspeyres price index, $L \equiv \hat{w}\hat{v}/\hat{\hat{w}}\hat{\hat{v}}$. The denominators of both J and L are the same (\hat{C}). The numerator of J , however, is the minimum cost of producing "output" \hat{y} , while the numerator of L is the cost of doing so by using base-period inputs, \hat{v} . (A glance at Figure 1 will confirm that the cost of \hat{v} at the dashed-line prices is greater than C .) It follows that the production-theoretic index of real input usage is bounded below by the Paasche quantity index, $wv/\hat{w}\hat{v}$.

Now, the whole exposition so far has used the base-period isoquant to make comparisons. Equally valid comparisons can be made using the current-period isoquant. An analysis isomorphic to that just given shows that the production-theoretic input price deflator formed using the current-period isoquant is bounded below by the Paasche input-price index, $P \equiv wv/\hat{w}\hat{v}$, while the corresponding production-theoretic index of input usage is bounded above by the Laspeyres quantity index, $\hat{w}\hat{v}/\hat{\hat{w}}\hat{\hat{v}}$.

If the isoquant map is homothetic and unchanging, then the production-theoretic indices that use the base-period isoquant and those that use the current-period isoquant will be the same.

In that case, the production-theoretic input-price deflator will be bounded below by L and above by P, with a similar statement holding for the index of input usage.

While the assumption of homotheticity is of considerable interest, however, the assumption of an unchanging isoquant map is not. Merely a change in relative output prices will alter the isoquant map and may destroy the relation between the Paasche price index and the deflator constructed using the base-period isoquant (and similarly for other inequalities). One important way of looking at the comparative static analysis given below is thus as an analysis of the ways in which the production-theoretic indices using the current-period isoquant differ from those using the base-period isoquant and of the changes that must therefore be made in the Paasche input-price index to restore the bounding inequality.⁸

8. Of course, the same analysis gives the changes that should be made in a Laspeyres input-price index to restore its relation to the production-theoretic deflator constructed using the current-period isoquant. It is tedious to keep repeating things like this, and I shall not do so henceforth.

Notice, however, that such an interpretation of comparative statics requires that the inequalities in question apply if there is no change in the isoquant map. This means that either the base period or the current-period isoquant map must be homothetic. Since such homotheticity is the interesting leading case, I shall assume it for the rest of this paper, pointing out where it is needed explicitly for purposes other than the interpretation

of results just discussed.⁹

9. The assumption involved is weaker than that of homogeneity (of any degree) of the production function (5.1). There seems little point in going into details here, however. See Fisher and Shell (1981, pp. 80-81) for a discussion of the parallel issues in the case of output price deflation.

Of course, the comparative static analysis given below can be viewed as interesting in itself. It shows the ways in which the deflator (and the input-usage index) constructed from the point of view of one firm differs from that constructed from the point of view of another. For this purpose, homotheticity is not required except where stated explicitly below.

6. Comparative Statics: Changing Output Prices

I now turn to a leading case of comparative static analysis to exemplify what is involved therein. It will be convenient to exhibit the Lagrangians for (5.2) and (5.3), respectively. They are:

$$(6.1) \quad \hat{L} = p\hat{x} + \hat{\gamma} F(\hat{x}, \hat{v}) - (1/\hat{\mu})(\hat{w}\hat{v} - \hat{C})$$

and

$$(6.2) \quad L = wv + \gamma F(x, v) - \mu(px - \hat{y}) \quad .$$

Here, γ , $\hat{\gamma}$, μ , and $(1/\hat{\mu})$ are Lagrange multipliers, the last one being written as a reciprocal for reasons of symmetry of interpretation.

As already remarked, the obvious first case to examine is that of a change in one of the output prices, p_k . We prove the following theorem (isomorphic to Fisher and Shell 1981, Theorem 10.c.1, pp. 82-83) .

Theorem 6.1: A. $\partial C / \partial p_k = \mu(\hat{x}_k - x_k)$.

B. If a rise in the price of the i th factor, w_i (with output prices unchanged), would increase (reduce) output of the k th good, given the base-period isoquant, then a rise in the price of the k th good will reduce (increase) the importance of the i th factor price in the production-theoretic input-price deflator.

Proof: A. Apply the Envelope Theorem first to (6.2) and then to (6.1), obtaining:

$$\begin{aligned} (6.3) \quad \partial C / \partial p_k &= \partial L / \partial p_k = -\mu(x_k - \partial \hat{y} / \partial p_k) \\ &= -\mu(x_k - \partial \hat{L} / \partial p_k) = \mu(\hat{x}_k - x_k) . \end{aligned}$$

B. Suppose a rise in w_i would increase production of the k th good, given the base-period isoquant. Consider such a rise from the base to the current period, with all other factor prices held constant. Then the input-price deflator must be greater than one whether the comparison is made using the isoquant map before the output price change or using the isoquant map after that change. Since production of the k th good goes up, given the base-period isoquant, $x_k > \hat{x}_k$. Since the Lagrange multiplier, μ , is positive (It is the marginal cost of "output", \hat{y} .), part A of the theorem implies that the input-price deflator must be lower if the isoquant map after the output price change is used than if the isoquant map before the output price change is. Since the

only factor price that changes is w_i , it follows that the effect of an increase in p_k is to reduce the importance of w_i in the input-price deflator. (A similar analysis applies to the case in which a rise in w_i decreases production of the k th good, given the base-period isoquant.)

It is illuminating to relate these results directly to the way in which the weights in a Paasche input-price index would shift with changes in output prices. To do this, we first prove two lemmas (isomorphic, respectively, to Lemmas 10.c.1 and 10.c.2, pp. 84-85 of Fisher and Shell, 1981).

Lemma 6.1: Under homotheticity, $\partial \mu / \partial w_i = \mu (v_i / C)$.

Proof: Differentiating (6.2) yields

$$(6.4) \quad \partial L / \partial w_i = v_i \quad ; \quad \partial L / \partial \hat{y} = \mu .$$

Hence

$$(6.5) \quad \partial \mu / \partial w_i = \partial^2 L / \partial \hat{y} \partial w_i = \partial v_i / \partial \hat{y} .$$

Now, we can evaluate $\partial v_i / \partial \hat{y}$ in two steps. First, consider $\partial v_i / \partial C$; that is, consider what happens to employment of the i th factor as costs increase with factor prices constant. By homotheticity, this is just v_i / C . To evaluate $\partial v_i / \partial \hat{y}$, therefore, it suffices to evaluate $\partial C / \partial \hat{y}$. From the Envelope Theorem applied to (6.2), however, this is just μ . Hence

$$(6.6) \quad \partial v_i / \partial \hat{y} = (\partial v_i / \partial C) (\partial C / \partial \hat{y}) = \mu (w_i / C) ,$$

and the lemma now follows from (6.4) and (6.5).

Lemma 6.2: Under homotheticity,

$$\frac{\partial v_i}{\partial p_k} = -\mu \frac{\partial x_k}{\partial w_i} - \frac{\mu^{x_k} v_i}{c}.$$

Proof: Differentiate (6.2) with respect to w_i and p_k (this time treating \hat{C} as a constant) to obtain:

$$(6.7) \quad \partial L / \partial w_i = v_i \quad ; \quad \partial L / \partial p_k = -\mu^{x_k}.$$

Hence

$$(6.8) \quad \begin{aligned} \frac{\partial v_i}{\partial p_k} &= \frac{\partial^2 L}{\partial p_k \partial w_i} = \frac{\partial (-\mu^{x_k})}{\partial w_i} \\ &= -\mu \frac{\partial x_k}{\partial w_i} - \frac{\mu^{x_k} v_i}{c}, \end{aligned}$$

where the last step follows from Lemma 6.1.

We can now prove (isomorphic to Theorem 10.c.2 of Fisher and Shell 1981, p. 86):

Theorem 6.2: Under homotheticity,

$$\frac{\partial (v_i/c)}{\partial p_k} = -(\mu/c) (\partial x_k / \partial w_i).$$

Proof:

$$(6.9) \quad \begin{aligned} \frac{\partial (v_i/c)}{\partial p_k} &= \frac{c(\partial v_i / \partial p_k) - v_i(\partial c / \partial p_k)}{c^2} \\ &= \frac{-\mu(\partial x_k / \partial w_i) - (\mu^{x_k} v_i/c)}{c} + \frac{\mu^{v_i} x_k}{c^2} \end{aligned}$$

$$= - (\mu/C) (\partial x_k / \partial w_i) ,$$

using Lemma 6.2 and applying the Envelope Theorem to (6.2).

Combining this with Theorem 6.1, we see that an increase in p_k will increase the importance of w_i in the input-price deflator if and only if the "weight", v_i/C , naturally associated with w_i in an index of total costs is also increased. This sort of duality is typical of comparative static results in this area.¹⁰

10. Although it seems unlikely that such relationships are restricted to the case of homothetic technologies, I have been unable to find a proof of Theorem 6.2 that does not make explicit use of homotheticity.

A great many other comparative results can be proved. In particular, the results just given can be adapted to cover the case of a Hicks-neutral technical change in the production of a given output. The presentation of such particular results will have to await a different occasion, however, for I want to use the remaining space to return to more general problems -- those of aggregation.

7. Aggregation over Input Prices

There are two types of aggregation problem that can be considered. These are aggregation over input prices and aggregation over firms (or industries or sectors). When one is dealing with the question of output-price deflation, these two types of aggregation tend to coincide because it is natural to associate

particular kinds of output with particular firms. When dealing with input-price deflation, on the other hand, the two types of aggregation problem are less naturally associated, and it is best to take them up separately. I begin by considering aggregation over input prices.

The production-theoretic input-price index can be written as a function of current-period factor prices, holding base-period factor prices fixed:

$$(7.1) \quad J = J(w_1, \dots, w_r) \quad .$$

Suppose that we wish to form an aggregate of the first t factor prices, $1 < t < r$, so that J can be written as

$$(7.2) \quad J = H(A(w_1, \dots, w_t), w_{t+1}, \dots, w_r) \quad ,$$

where $A(\cdot)$ is a scalar-valued function. By Leontief's well-known aggregation theorem (Leontief, 1947), this can be done if and only if

$$(7.3) \quad \frac{\partial (J_i/J_j)}{\partial w_k} \equiv 0 \quad \begin{array}{l} (i, j = 1, \dots, t; \\ k = t+1, \dots, r) \end{array} \quad ,$$

where $J_i \equiv \partial J / \partial w_i$. This means that the J -constant marginal rate of substitution between any pair of prices in the aggregate must be independent of any price not in the aggregate.

Now, application of the Envelope Theorem to (6.2) above shows that

$$(7.4) \quad J_i/J_j = v_i/v_j \quad (i, j = 1, \dots, t).$$

Hence (7.3) requires that changes in w_k leave unchanged the ratio in which v_i and v_j are employed. It follows that such ratios can

depend only on the first t factor prices, the ones to be included in the aggregate. Under homotheticity, that dependence will be on the $t-1$ ratios of the factor prices to be aggregated.

Now such price ratios, of course, will also be the marginal rates of substitution in production among the factors to be included in the aggregate. Since the employment of any factor not so included certainly depends on that factor's own price, the aggregation condition just described is equivalent to the condition that the marginal rates of substitution among factors included in the aggregate be independent of the employment of factors not so included. A second application of Leontief's Theorem now shows that aggregation over the first t input prices is possible if and only if the efficient production surface can be written as:

$$(7.5) \quad 0 = F(x, v) = G(x, B(v_1, \dots, v_t), v_{t+1}, \dots, v_r),$$

where $B(\cdot)$ is scalar-valued and homothetic (the latter property being guaranteed if the underlying technology is homothetic).

In other words, factor-price aggregation in the production-theoretic input-price deflator is possible if and only if the corresponding factor aggregation is possible in the production function itself.

If the productive unit under study is a firm and $F(\cdot, \cdot)$ simply given as its efficient technology, there is nothing more to say. If, however, the productive unit is an aggregate and its efficient technology built up from the technologies of underlying firms by allocating a total stock of factors and assigning out-

firms by allocating a total stock of factors and assigning outputs to achieve efficient production, then there is a great deal more that can be said. That case has been extensively studied in the literature, and the conditions that permit such aggregation shown to be extremely restrictive.¹² It follows that aggregation

11. See, for example, Fisher (1969, 1982) and Blackorby and Schworm (1984).

of factor prices in the production-theoretic input-price deflator is unlikely to be possible.

8. Vertical Aggregation over Productive Units

The other type of aggregation problem is that of aggregation over productive units. Here there are two cases: aggregation over units that do not buy or sell from each other ("horizontal" or "conglomerate" aggregation) and aggregation over units that trade directly with each other ("vertical" aggregation).

Horizontal or conglomerate aggregation presents problems that are beyond the scope of the present paper. That is because as one includes more and more productive units it becomes less easy to maintain the assumption that output prices are independent of the activities of the aggregate productive unit. General equilibrium considerations come to the fore, and I shall not discuss such considerations here.¹²

12. See Fisher and Shell, 1981, for a discussion of the isomorphic problem of output deflation when input prices are not independent of the activities of the productive unit.

Vertical aggregation, on the other hand, raises some interesting questions that can be discussed here.

To fix ideas, suppose first that there are only two firms involved. Suppose further that the first of these firms, the "Seller", buys only primary factors and sells only to the second firm, the "Buyer". Finally, suppose that the Buyer buys only from the Seller, using no primary factors, and sells only to consumers.

Obviously, the prices faced by consumers -- the output prices of the Buyer -- are influenced by the Buyer's input prices -- the output prices of the Seller -- which in turn are influenced by the prices of primary factors -- the input prices of the Seller. A sensible set of questions to ask is how to measure the relative contributions of the Seller and of primary factors to inflation as seen by the Buyer and how to measure the relative contributions of primary factors, the Seller, and the Buyer to inflation as seen by consumers.

To study such questions, it is convenient to begin with an even simpler case, that in which a single firm buys only primary factors and sells only to consumers, so that there is only one stage of production. Here, an obvious thought is to calculate the single firm's production-theoretic output-price index and its production-theoretic input-price index and to take the ratio of these two indices as the contribution of the firm to inflation. It is important to understand that this "obvious thought" is totally mistaken.

There is more than one way to see this. I begin with a relatively formal way. The production-theoretic output-price

index is constructed by assuming output prices to be given outside the firm by demand conditions and having the firm optimize in various ways given those prices and the conditions of factor supply. The construction of the production-theoretic input-price index, on the other hand, assumes input prices to be given from outside the firm and performs certain optimization problems taking those prices and the conditions of demand as given. To compare the production-theoretic output-price and input-price indices is to take as simultaneously valid two sets of conditions that can hold simultaneously only if all prices are determined outside the firm, in which case the question of the firm's own contribution to inflation is vacuous.

There is more to this than the fact that we have been assuming the productive unit analyzed too small to influence prices. The extension of the theory to relax that assumption (whether or not monopoly elements are involved) still constructs the input-price deflator by assuming input prices fixed outside the productive unit and the output-price deflator by assuming output prices so fixed. Comparison of the two assumes both sets of prices fixed outside the unit, and this is not a useful assumption in the present context.

The underlying reason for this problem is not hard to find. The method suggested by the "obvious thought" cannot provide an appropriate answer to the question being asked. That question has to do (in this example) with contributions to inflation as seen by consumers. Inflation as seen by consumers, however, is measured by the cost-of-living index and not by the production-

theoretic output-price index. The latter index takes output prices as demand determined, as reflective of the prices at which firms can sell. It is the cost-of-living index that takes output prices as production determined, as reflective of the prices at which consumers can buy. Each measure has its uses, but if we seek to evaluate contributions to inflation as seen by consumers, it is the cost-of-living index that must be used.

Once this is realized, it is possible to see how to proceed. As observed in Fisher and Shell (1972, pp. 7-8), in discussing the difference between taste and quality changes, there is a choice as to where we take the interface between the household and its opportunity set. Consumers buy goods and use them in activities inside the household to maximize utility. Some of those activities can be considered as production activities inside the household rather than directly as consumption (food preparation, for example). Now suppose that the productive system were organized differently with the activities now carried on in the productive unit under analysis being carried on inside the household.¹³ Then the household would buy primary factors and

13. As usual, I ignore difficulties of aggregation over households in the cost-of-living index.

use them in its productive activities to maximize utility. The prices it would face would be those of primary factors. Its cost-of-living index would be computed using those prices. Comparison of such a cost-of-living index with the actual cost-of-living index computed using the output prices of the productive unit being studied thus measures the extent to which that produc-

tive unit contributes to inflation as seen by the household.

If such an approach seems somewhat forced, consider its generalization to the case of two firms described earlier -- the Buyer and Seller. Consider inflation from the point of view of the Buyer; this is measured by the Buyer's production-theoretic input-price index. The relative contributions of the Seller and of primary factor prices can be analyzed quite naturally by asking how different the production-theoretic input-price index would be if the Seller did not exist and the Buyer were vertically integrated from purchase of primary factors to sale to consumers. (Note that the Seller's output-price index plays no role in this.) This is the same procedure as that involving households but in a more familiar context.

It is easy to see that the same procedure applies to less restrictive cases. For example, there is nothing in it that requires the Buyer to use no primary inputs. Whether or not such inputs are used, the comparison to be made is that of the production-theoretic input-price index with and without vertical integration.

Note that a similar procedure can be used to measure the Buyer's and consumers' relative contributions to output-price inflation as seen by the Seller. This might be of some interest when considering demand-pull rather than cost-push inflation. I leave the details to the reader.

Now, it is interesting to ask how such vertically aggregated indices relate to the corresponding unaggregated indices. In particular, can the same comparison of Seller-contribution and

primary-factor-contribution to inflation as seen by the Buyer be made using the input-price index of the Buyer and the input-price index of the Seller? One would hope that it could. Returning to the case in which the Buyer buys only from the Seller, the Buyer's input-price index reflects inflation as the Buyer sees it, while the Seller's input-price index reflects inflation as the Seller sees it -- inflation as reflected in changes in the prices of primary factors. It seems natural to compare the two to evaluate the Seller's contribution to inflation as seen by the Buyer. Perhaps construction of an input-price index for a vertically integrated firm is not required.

Unfortunately, this will not work, for reasons similar to those given earlier for the failure of the "obvious thought." The Seller's input-price index is calculated taking as given the Seller's output prices -- the prices at which the Seller sells to the Buyer. But it is the change in those prices that lead to changes in the Buyer's input-price index. The contribution of the Seller to inflation as seen by the Buyer cannot be assessed using a construct that assumes there is no such inflation.

So long as we remain in the theoretical world in which there is sufficient information to construct all these production-theoretic indices, this presents no great problem. The difficulty of constructing a vertically aggregated production-theoretic input price index is not analytically greater than that of constructing such an index without vertical integration. The difference merely lies in what production processes are considered to be under control of the productive unit being analyzed.

When we come to the question of approximations used in

practice, on the other hand, the difficulties are considerably greater. Consider in particular the use of Laspeyres input-price indices.

We have already seen that (in the simple case considered above¹⁴) a Laspeyres input-price index for the Seller will bound

14. Where the productive unit is large enough to affect input prices (even if it is made up of competitive firms) Laspeyres and Paasche bounds do not apply. See Fisher and Shell (1981, pp. 59-68) for a discussion of the parallel case of output-price deflation.

from above the production-theoretic input-price index for the Seller. Further, it is not hard to see that such a Laspeyres index will also bound from above the production-theoretic input-price index that would apply under vertical integration. So far so good.

Unfortunately, the reason for constructing such a vertically aggregated input-price index was to compare it with the production-theoretic input-price index of the Buyer and thus to assess relative contributions to inflation. In practice, however, all we are likely to have for the Buyer is another Laspeyres input-price index, and this merely provides an upper bound on the production-theoretic index we need. Comparison of the Laspeyres input-price indices for the Seller and the Buyer, therefore, merely provides a comparison of two upper bounds. That comparison may or may not provide a close approximation to the comparison of the production-theoretic input-price indices themselves,

and it is obviously not possible to say anything general as to the sign of the approximation error.

This is, of course, a problem, but I regard it as a problem for the use of Laspeyres indices and not for the theory of the production-theoretic input-price index. As is true in the area of index numbers and aggregation generally, I regard it as important to ask the right questions and examine the defects of the answers that can be given in practice. That seems to me preferable to tailoring the questions to suit the currently available answers.

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