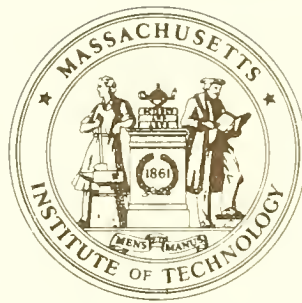


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RESOURCE EXTRACTION WHEN A FUTURE  
SUBSTITUTE HAS AN UNCERTAIN COST\*

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Working Paper Number 196

April 1977

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## 1. Introduction

Economic theory treating the extraction of exhaustible natural resources has usually neglected most types of uncertainty, at least until recently. However, in the last couple of years several economists have treated various aspects of uncertainty. In particular, Dasgupta and Heal [2] and Dasgupta and Stiglitz [3] have studied cases where a perfect substitute for the natural resource will become available at some future date. Both these studies assume that this future date is uncertain, but that the characteristics of the substitute product, represented by the costs of producing it, are known with certainty. This is, of course, highly unrealistic. In the real world both the date of availability and the costs of producing a substitute, for instance fusion energy, will be uncertain. However, the analysis quickly gets very complex when too many things are taken into consideration at once. Dasgupta, Heal and Stiglitz have studied cases where the date of availability of a substitute is uncertain, the present paper will therefore consider this date as known, but assume that the costs of producing the substitute are uncertain.

In Section 2 a simple model for studying the type of uncertainty mentioned above is presented. Section 3 shows how uncertainty affects the socially optimal resource

extraction under various assumptions about the society's risk aversion. In Section 4 the competitive solution is derived and compared with the social optimum. Finally, some conclusions are drawn in Section 5.

## 2. A Simple Model of Resource Extraction with Uncertain Costs of Substitute Production

Let  $x(t)$  and  $y(t)$  be the rate of resource extraction and of production of a perfect substitute, respectively. Assume that the gross benefits to society from using  $x(t) + y(t)$  of the resource plus substitute at time  $t$  are given by a strictly increasing, strictly concave function  $U^*(x(t)+y(t))$ , and that the unit costs of extraction of  $x$  and production of  $y$  are given by the non-negative constants  $c_1$  and  $c_2$ , respectively. With an exogenously given constant positive discount factor  $r$ , the discounted net benefits to society are

$$W = \int_0^{\infty} e^{-rt} [U^*(x(t)+y(t)) - c_1 x(t) - c_2 y(t)] dt. \quad (1)$$

Defining the function

$$U(x+y) = U^*(x+y) - c_1(x+y),$$

we can rewrite (1) as

$$W = \int_0^{\infty} e^{-rt} U(x+y) - cy, \quad (2)$$

where  $c = c_2 - c_1$  is the cost difference between producing the substitute and extracting the resource. We shall assume that  $c_1$  is known with certainty, but that  $c_2$  is uncertain

before a given date  $T$ . However, we shall assume that the whole probability distribution of  $c_2$  lies above  $c_1$ , so that there are only positive values in the probability distribution of  $c$ . Before  $T$  it is impossible to produce the substitute, and at  $T$  the true value of  $c_2$ , and therefore  $c$ , is revealed.

The constraints which the resource extraction and the substitute production must satisfy are

$$\begin{aligned}\dot{R}(t) &= -x(t), \quad R(0) \text{ given,} \\ R(t) &\geq 0, \\ x(t) &\geq 0, \\ y(t) &\geq 0, \\ y(t) &= 0 \text{ for } t < T,\end{aligned}\tag{3}$$

where  $R(t)$  is the resource stock at time  $t$  and  $R(0)$  is the given initial resource stock.

Before stating the society's optimization problem, let us define two functions which will play an important role in the following analysis. The first of these functions is

$$F(S) = \text{Max}_{x(t)} \int_0^T e^{-rt} U(x(t)) dt \tag{4}$$

subject to

$$\begin{aligned}\dot{R}(t) &= -x(t), \\ R(0) &\text{ given, } R(T) \geq S, \\ x(t) &\geq 0.\end{aligned}$$

In other words,  $F(S)$  is the maximum of the discounted net benefits to society from 0 till  $T$  given that the resource stock at  $T$  is not less than  $S$ . Obviously, for any given value of  $S$ , an optimal program of resource extraction (until  $T$ ) exists. We shall denote this path by  $\tilde{x}(t, S)$ , showing that the path depends on  $S$ .

The second function we shall define is

$$G(S, c) = \text{Max}_{\substack{x(t) \\ y(t)}} \int_0^{\infty} e^{-rt} [U(x(t) + y(t)) - cy(t)] dt \quad (5)$$

subject to

$$\begin{aligned} \dot{R}(t) &= -x(t), \\ R(T) &= S, \quad R(t) \geq 0, \\ x(t) &\geq 0, \\ y(t) &\geq 0. \end{aligned}$$

In other words,  $G(S, c)$  is the maximum of the discounted net benefits to society after  $T$  given that the resource stock at  $T$  is  $S$ . Note that  $G(S, c)$  depends on  $c$ , which is known at  $T$ , but uncertain before  $T$ . For any given value of  $S$  and  $c$ , an optimal program of resource extraction and substitute production after  $T$  exists. We shall denote these paths by  $\hat{x}(t, S, c)$  and  $\hat{y}(t, S, c)$ .

Obviously, whatever value of resource stock the society decides to leave to extract after  $T$ , i.e. whatever value  $S$



has, a necessary condition for a social optimum is that  $x(t) = \tilde{x}(t, S)$  for  $t < T$ , and  $x(t) = \hat{x}(t, S, c)$  and  $y(t) = \hat{y}(t, S, c)$  to  $t > T$ . This means that we can write

$$W = F(X) + G(S, c). \quad (6)$$

At the initial time point ( $t = 0$ ),  $c$  is an uncertain variable. It therefore has no meaning to choose  $S$  so that  $W$  is maximized. The appropriate procedure in such cases, under reasonable assumptions (cf. Arrow [1]), is to maximize the expected value of some strictly increasing function of  $W$ . Disregarding risk aversion for the time being, the social objective will simply be to maximize the expected value of  $W$ , i.e.

$$EW = F(S) + EG(S, c), \quad (7)$$

with respect to  $S$ . Once the optimal value of  $S$  is found, the optimal resource extraction and substitute production follow from  $\tilde{x}(t, S)$ ,  $\hat{x}(t, S, c)$  and  $\hat{y}(t, S, c)$ .

To be able to study the properties of the optimal solution, we must first study the properties of the functions  $F(S)$  and  $G(S, c)$ . Let us first look at the function  $F$ . Disregarding the possibility of  $S$  not being an effective constraint, it is well known that the function  $\tilde{x}(t, S)$  must imply that  $e^{-rt} U'(\tilde{x}(t, S))$  is independent of  $t$ , i.e. that the discounted net benefits to society of a marginal increase in the resource use is the same throughout the period.

Furthermore, it is intuitively obvious, and easy to prove formally, that a marginal increase in  $R(0) - S$  increases  $F(S)$  by precisely  $e^{-rt} U'(\tilde{x}(t,S))$ , i.e.

$$F'(S) = -e^{-rT} U'(\tilde{x}(T,S)) < 0. \quad (8)$$

Differentiating  $F'_S(S)$  gives us

$$F''(S) = -e^{-rT} U''(\tilde{x}(T,S)) \frac{\partial \tilde{x}(T,S)}{\partial S}.$$

Since  $e^{-rt} U'(\tilde{x}(t,S))$  is independent of  $t$  and  $U'' < 0$ , a change in  $S$  must change  $\tilde{x}(t,S)$  in the same direction for all  $t$ . Together with the resource constraint

$$\int_0^T \tilde{x}(t,S) dt = R(0) - S$$

this means that  $\partial \tilde{x}(T,S)/\partial S < 0$ , so that we must have

$$F''(S) < 0. \quad (9)$$

Let us now turn to the function  $G(S,c)$ . The optimal resource extraction and substitute production corresponding to a given  $S$  and  $c$  have the following properties, assuming that the distribution of  $c$  is such that  $U'(y) = c$  gives a positive solution for  $y$  whatever value of  $c$  turns out to be the true one (for details, see for instance [3]):

To begin with the substitute is not produced, and  $U'(x)$  increases with the rate  $r$ . This development continues until the resource is completely exhausted, which occurs exactly when  $U'(x)$  reaches  $c$ . We shall denote this date by  $N(S,c)$ , indicating that the date of exhaustion depends on  $S$  and  $c$ . After  $N(S,c)$  we of course have no resource extraction, and substitute production takes place on the level that makes  $U'(y) = c$ . To summarize, we have

$$\left. \begin{aligned} U'(\hat{x}(t,S,c)) &= e^{-r(N(S,c)-t)}c \\ \hat{y}(t,S,c) &= 0 \end{aligned} \right\} \text{ for } t < N(S,c)$$

$$\left. \begin{aligned} \hat{x}(t,S,c) &= 0 \\ U'(\hat{y}(t,s,c)) &= c \end{aligned} \right\} \text{ for } t > N(S,c)$$
(10)

where  $N(S,c)$  is determined by

$$\int_T^{N(S,c)} \hat{x}(t,S,c) dt = S. \quad (11)$$

From (10) it is easy to see that

$$\hat{x}(N(S,c),S,c) = \hat{y}(N(S,c),S,c) = k(c), \quad (12)$$

where  $k(c)$  is a constant which only depends on  $c$  and is determined by

$$U'(k(c)) = c,$$

so that

$$k'(c) = \frac{1}{U''(k(c))} < 0. \quad (13)$$

Like for  $F'$ , it is intuitively obvious, and easy to prove, that

$$G_S(S, c) = e^{-rt} U'(\hat{x}(t, S, c)) = e^{-rN(S, c)} c > 0. \quad (14)$$

It is also obvious that  $G(S, c)$  must be higher the lower  $c$  is, i.e.  $G_c(S, c) < 0$ . Differentiating (14) with respect to  $S$  gives us

$$G_{SS}(S, c) = -e^{-rN(S, c)} c r N_S(S, c).$$

Furthermore, by using Equations (10)-(11) we can show that (see the Appendix for details)

$$N_S(S, c) = \frac{1}{k(c) - rcZ}, \quad (15)$$

where

$$Z = \int_T^{N(S, c)} \frac{1}{U''(\hat{x}(t, S, c))} e^{-r(N(S, c) - t)} dt < 0 \quad (16)$$

In other words, we have

$$G_{SS}(S, c) < 0. \quad (17)$$

Differentiating  $G_S(S, c)$  with respect to  $c$  gives us

$$G_{SC}(S,c) = e^{-rN(S,c)} [1 - rcN_c(S,c)]. \quad (18)$$

By using Equations (10)-(11), we find (cf. the Appendix) that

$$N_c(S,c) = \frac{1}{rc - k(c)/Z}, \quad (19)$$

i.e.  $N_c(S,c) > 0$  and  $1 - rcN_c(S,c) > 0$ . This means that we have

$$G_{SC}(S,c) > 0. \quad (20)$$

In particular, it is clear from (11) that for  $S = 0$  we must have  $N(S,c) = T$ , which of course implies  $N_c(0,c) = 0$ . From (18) we therefore have

$$G_{SC}(0,c) = e^{-rT}. \quad (20')$$

Finally, we will need the sign of  $G_{SCC}(S,c)$ . From (20') it follows that  $G_{SCC}(0,c) = 0$ . When  $S > 0$  it is proved in the Appendix that

$$cU'''(x) \leq 2(U''(x))^2 \Rightarrow G_{SCC}(S,c) < 0. \quad (21)$$

The sign of  $U'''(x)$  depends on the curvature of the function  $U'(x)$ , which can be interpreted as the demand function for the resource. The condition (21) therefore states that a sufficient condition for  $G_{SCC}(S,c)$  to be negative is that



the demand function is either concave or convex with a sufficiently weak curvature. In the Appendix it is also indicated that even if the demand function is convex with a curvature so strong that  $cU'''(x) > 2(U''(x))^2$ , we may well have  $G_{scc}(S,c) < 0$ . However, we cannot exclude the possibility of  $G_{scc}(S,c)$  being positive, at least for some relevant  $S$  and  $c$ .

Let us finish this section by summarizing what properties the functions  $F(S)$  and  $G(S,c)$  have. The function  $F(S)$  is strictly decreasing and strictly concave in  $S$  (cf. (8) and (9)). The function  $G(S,c)$  is strictly increasing and strictly concave in  $S$  (cf. (14) and (17)), and is strictly decreasing in  $c$ . Finally, the function  $G_s(S,c)$  is strictly increasing in  $c$  (cf. (20)). For most reasonable demand functions we will also expect to find  $G_s(S,c)$  strictly concave in  $c$  (cf. (21) and the discussion after (21)). In the next sections we shall utilize this information to study the effects of uncertain costs of producing the substitute.

### 3. The Effects of Uncertainty

Let us first assume that society as a whole is risk neutral, so that the socially optimal value of  $S$  is the value that maximizes the expected value of  $W$ , i.e. maximizes the expression given by (7). Denoting the optimal  $S$  by  $S^*$ , we therefore have

$$\frac{\partial F(S^*)}{\partial S} + \frac{EG(S^*, c)}{\partial S} = 0,$$

or

$$F'(S^*) + EG_S(S^*, c) \leq 0, \quad (22)$$

where a strict inequality implies  $S^* = 0$ . Let us compare  $S^*$  with the optimal value of  $S$  in the case of certainty, which we shall denote by  $S^\circ$ . If  $c$  is known with certainty to be equal to its expected value  $\bar{c}$ ,  $S^\circ$  is given by

$$F'(S^\circ) + G_S(S^\circ, \bar{c}) \leq 0, \quad (23)$$

where a strict inequality implies  $S^\circ = 0$ .

From (20') we have  $G_{SCC}(0, c) = 0$ , so that

$$EG_S(0, c) = G_S(0, \bar{c}).$$

Together with (22) and (23) this means that  $S^* = 0$  if and only if  $S^\circ = 0$ . We therefore have the following proposition,

remembering how the resource extraction before  $T$  depends on  $S$  (cf. p. 7):

Proposition 1. When society is risk neutral and the cost of producing a substitute for the resource is uncertain, it will be optimal to use up all of the resource stock before the time  $T$  when substitute production becomes technically feasible if and only if this is optimal also when this cost is known with certainty. If the entire resource stock is exhausted at  $T$ , the whole path of extraction will be independent of whether or not the cost of producing the substitute is uncertain.

Let us now turn to the case where  $S^*$  and  $S^\circ$  are positive. In the end of Section 2 we argued that  $G_S(S, c)$  usually would be strictly concave in  $c$ . If this is the case, we must have

$$EG_S(S^*, c) < G_S(S^*, \bar{c}),$$

which together with (22) implies that

$$F'(S^*) + G_S(S^*, \bar{c}) > 0. \quad (24)$$

But  $F'(S) + G_S(S, \bar{c})$  is a strictly decreasing function of  $S$ , so (24) and (23) therefore imply that

$$S^* < S^\circ. \quad (25)$$

In other words, more of the natural resource is used up before  $T$  in the case of uncertainty than in the case of certainty.

If the function  $G_s(S, c)$  is strictly convex in  $c$ , we of course get the opposite result from (25), i.e. we find that  $S^* > S^0$ . If the sign of  $G_{scc}(S, c)$  depends on  $S$  and/or  $c$ , we cannot state how uncertainty will affect the optimal  $S$  without more knowledge about the demand function, the probability distribution of the cost of producing the substitute, and the initial resource stock.

From the reasoning above and what we have said about how the resource extraction before  $T$  depends on  $S$ , we therefore have the following proposition:

Proposition 2. The optimal path of resource extraction, up till the time  $T$  when substitute production becomes technically feasible, will be higher, equal or lower when the cost of producing a substitute is uncertain than when this cost is certain. One will usually expect to find  $G_{scc}(S, c) < 0$ , in this case the resource extraction before  $T$  will be higher when the cost of producing the substitute is uncertain, provided that society is risk neutral and that some of the resource stock remains unextracted at  $T$ .

It may seem somewhat strange that uncertainty in the most usual cases will tend to increase the optimal resource extraction before substitute production is possible. As we soon shall see, however, this result is only valid when society is risk neutral or at least not too strongly risk averse.

Assume that society is risk averse, so that instead of maximizing the expected value of  $W$ , it wishes to maximize the expected value of  $\phi(W)$ , where  $\phi(W)$  is a strictly increasing, strictly concave function. The optimal resource stock at time  $T$  is now given by the value of  $S$  which maximizes

$$E\phi(W) = E\phi(F(S)+G(S,c)). \quad (26)$$

Calling the optimal value of  $S$  in this case  $S^{**}$ , we therefore must have

$$E\{\phi'(F(S^{**})+G(S^{**},c))(F'(S^{**})+G_S(S^{**},c))\} \leq 0, \quad (27)$$

where a strict inequality implies  $S^{**} = 0$ . Equation (27) states that a weighted average of  $(F'(S^{**})+G_S(S^{**},c))$  over all outcomes of  $c$  is equal to zero. Since  $U''(W) < 0$ ,  $G_c(S^{**},c) < 0$  and  $G_{sc}(S^{**},c) > 0$ , the weights for higher values of  $F'(S^{**}) + G_S(S^{**},c)$  compared with the weights for lower values of  $F'(S^{**}) + G_S(S^{**},c)$  in (27) are higher than they would have been with  $U''(W) = 0$ . This implies that

$$E\{F'(S^{**})+G_S(S^{**},c)\} < 0,$$

i.e.

$$F'(S^{**}) + EG_S(S^{**},c) < 0. \quad (28)$$



Comparing this with (22) and remembering that  $F'_S(S) + EG'_S(S,c)$  is a strictly decreasing function of  $S$  gives us

$$S^{**} > S^*,$$

unless  $S^{**} = S^* = 0$ . In other words, less of the natural resource is used up before  $T$  in the case with risk aversion than in the case with risk neutrality. From what we have said before we therefore have the following proposition:

Proposition 3. When the cost of producing a substitute is uncertain, the optimal resource extraction at any point of time before such production is technically feasible is lower when society is risk averse than when society is risk neutral, unless the entire resource stock is exhausted at  $T$  in both cases.

It is not possible to say anything in general about the relationship between  $S^{**}$  and  $S^0$ . Obviously, for a sufficiently weak degree of risk aversion we can get  $S^{**} < S^0$ , since  $S^* - S^{**}$  approaches zero as  $\phi''(W)$  approaches zero. To see if  $S^{**} > S^0$  is possible, look at in what way risk aversion affects society's choice of  $S$ . From (27), it is clear that risk aversion, i.e.  $\phi''(W) < 0$ , simply has the effect that society gives relatively more weight to the outcomes of  $c$  which give a low value of  $G(S,c)$ , i.e. to the high values of  $c$ . The stronger the degree of risk aversion, the stronger is this effect. The limiting case of risk aversion is to

only consider the highest possible outcomes of  $c$ , i.e. to choose  $S$  so that the lowest possible outcome of  $W$  is maximized. The optimal value  $S^{**}$  in this case simply follows from (23), with  $S^0$  replaced by  $S^{**}$  and  $\bar{c}$  replaced by the highest possible outcome of  $c$ , which obviously is higher than  $\bar{c}$ . From the properties of the functions  $F(S)$  and  $G(S, c)$  it is clear that  $S^{**}$  in this case must be higher than  $S^0$ . From what we have said before we therefore have the following proposition:

Proposition 4. When the cost of producing a substitute is uncertain and society has a sufficiently strong degree of risk aversion, the optimal resource extraction at any point of time before substitute production becomes technically feasible is lower than it would have been if the cost of producing the substitute was known with certainty, unless the entire resource stock is exhausted at  $T$  in both cases.

#### 4. The Competitive Solution

The demand for the resource in a competitive economy will depend on its price, such that the following equality must hold:

$$p(t) = U'(x(t)), \quad (30)$$

where  $p(t)$  is the resource price (net of extraction costs).

For an equilibrium with positive resource extraction for all  $t < T$  to exist, the resource price  $p(t)$  must rise with a rate equal to  $r$  (see for instance [6]). Together with (30) this implies that  $e^{-rt}U'(x(t))$  must be constant for all  $t < T$ . But it is precisely the path  $\tilde{x}(t,S)$ , defined on pages 4-5, which has this property. Before  $T$  the price is therefore given by

$$p(t) = U'(\tilde{x}(t,S)) \text{ for } t < T. \quad (31)$$

The level of the price path  $p(t)$  will be determined once  $S$  is determined. We shall return to how  $S$  is determined in the competitive economy shortly, after seeing how  $p(t)$  develops after  $T$ .

After  $T$  substitute production is technically feasible with a known cost. The equilibrium development of  $p(t)$  in a competitive economy, assuming  $S > 0$ , has been discussed by Dasgupta and Stiglitz [3], Hoel [5] and Nordhaus [7]: The price  $p(t)$  will start at some value lower than  $c$ , rise with the rate  $r$  and reach  $c$  at the same moment as the resource stock is completely exhausted. Together with (30) this means that  $U'(x(t))$  must rise with the rate  $r$  until it reaches  $c$  at the same time as  $R(t)$  reaches zero. But from (10)-(11) we see that it is precisely the path  $\tilde{x}(t,S,c)$  which has this property, i.e.

$$p(t) = U'(\hat{x}(t,S,c)) \text{ for } T < t < N(S,c). \quad (32)$$

The level of this price path will depend both on  $S$  and on what the cost of producing the substitute turns out to be.

Whatever value of  $S$  the competitive economy gives, it is clear that there is nothing to prevent a jump in  $p(t)$  at  $T$ . The reason why such a jump can occur (and will occur except by chance) is that  $c$  is not known just before  $T$ , while it becomes known at  $T$ . From (31) and (32) we therefore see that whatever value  $S$  has,  $p(t)$  will make a jump at  $T$  for all outcomes of  $c$  except one. To distinguish  $p(t)$  just before and just after  $T$ , we shall denote these values by  $p(T^-)$  and  $p(T^+)$ . Note that whatever value  $S$  has,  $p(T^+)$  will be a random variable before  $T$ .

If all resource extracting firms are risk neutral, a necessary condition for an equilibrium with  $S > 0$  is that the expected value of  $p(T^+)$  is equal to  $p(T^-)$ .  $p(T^-)$  follows from (31), and  $p(T^+)$  follows from (32); using  $\bar{S}^*$  to denote the market determined value of  $S$  we therefore have

$$EU'(\hat{x}(T, \bar{S}^*, c)) = U'(\tilde{x}(T, \bar{S}^*)).$$

But from (8) and (14) this is seen to be equivalent to

$$F'(\bar{S}^*) + EG_S(\bar{S}^*, c) = 0. \quad (33)$$

Comparing this equation with (22), it is clear that  $\bar{S}^* = S^*$ . As the extension to the case where  $\bar{S}^* = 0$  is straightforward, we therefore have the following proposition:

Proposition 5. If society as a whole and all resource extracting firms in a competitive economy are risk neutral, the resource extraction and substitute production in the competitive economy is socially optimal.

Let us now turn to the case where all firms are risk averse. In this case the expected value of  $p(T^+)$  must exceed  $p(T^-)$  for any resource extraction to be postponed till after  $T$ . Using the same reasoning as we used to derive (33), we therefore get

$$F'(\bar{S}^{**}) + EG_S(\bar{S}^{**}, c) > 0, \quad (34)$$

where  $\bar{S}^{**}$  is the competitive solution of  $S$  when all firms are risk averse. Since  $F'(S) + EG_S(S, c)$  is strictly decreasing in  $S$ , (33) and (34) imply that  $\bar{S}^{**} < \bar{S}^*$  when  $\bar{S}^* > 0$ . It is straightforward to verify that  $\bar{S}^{**} = \bar{S}^*$  if and only if  $\bar{S}^* = 0$ . From what we know about the function  $\tilde{x}(t, S)$ , we therefore have the following proposition:

Proposition 6. When the cost of producing a substitute is uncertain, the resource extraction in a competitive economy at any point of time before substitute production becomes technically feasible is higher when all resource extracting firms are risk averse than when all resource extracting firms are risk neutral, unless the entire resource stock is exhausted at  $T$  in the latter case.



From Propositions 3 and 6 we immediately see that the following proposition must hold:

Proposition 7: When the cost of producing a substitute is uncertain, and society as a whole and/or all resource extracting firms have risk aversion, the resource extraction in a competitive economy at any point of time before substitute production becomes technically feasible is higher than the socially optimal level of resource extraction, unless it is socially optimal to have no extraction after  $T$ .

It should be noted that we purposely have distinguished between the cases where all resource extracting firms are risk neutral and the case where all resource extracting firms are risk averse. Let us now briefly consider the case where some firms are risk neutral and some firms are risk averse. In this case we may still get  $E p(T^+) = p(T^-)$ , so that Proposition 5 (with "all resource extracting firms" replaced by "some resource extracting firms") will still hold. In such a case all risk averse firms will extract all of their resource stock before  $T$ , while the risk neutral firms will have a total stock  $\bar{S}^*$  left at  $T$ . Obviously,  $E p(T^+) = p(T^-)$  can only occur if the total initial resource stock of the risk neutral firms is not less than  $\bar{S}^*$ . If this requirement does not hold, the equilibrium solution must imply  $E p(T^+) > p(T^-)$ , so that Propositions 5 and 6 (with "all resource extracting firms" replaced by "some

resource extracting firms") will hold in this case (see [4] for similar reasoning in the case of an uncertain resource stock).

## 5. Concluding Comments

Our conclusions so far are given in Propositions 1-7 in Sections 3 and 4, and will not be repeated here. However, we shall make some further comments on some of the propositions.

Proposition 2 implies that without risk aversion it is optimal in most, but not all, cases to have a higher initial resource extraction when there is cost uncertainty than when the cost of producing the substitute is known. Dasgupta and Stiglitz [3] only study how an uncertain date of availability of the substitute affects the initial resource extraction in the special case where the demand function for the resource has a constant price elasticity. In this case they show that the optimal initial resource extraction can be both higher (for a "small" initial resource stock) and lower (for a "large" initial resource stock) than the initial resource extraction when the date of substitute availability is known. As we mentioned in the Introduction, the real world is characterized by an uncertain date of availability of the substitute and uncertain costs of producing this substitute. From our result in Proposition 2 and the result of Dasgupta and Stiglitz it does not seem possible to give any general conclusion about how the optimal initial resource extraction will be affected by this kind of simultaneous

uncertainty. It seems that whether the initial extraction will be higher or lower than in the case of full certainty will depend on the initial resource stock, the demand function for the resource, the cost of producing the resource, and on the properties of the joint probability distribution of the date of substitute availability and the cost of producing the substitute.

Proposition 5 states that without risk aversion the competitive resource extraction will be socially optimal. A similar result holds when the cost of producing the substitute is certain, but the date of availability of the substitute is uncertain (cf. [3]). This suggests that when both types of uncertainty occur simultaneously, the competitive resource extraction will be socially optimal in the absence of risk aversion.

Propositions 3, 6 and 7 show that while risk aversion tends to reduce the socially optimal resource extraction before  $T$ , risk aversion tends to increase the resource extraction in a competitive economy before  $T$ . The reason for this is the following: Risk aversion always makes whatever decision-maker one is studying give more weight to the outcomes of the uncertain variable which this decision-maker regards as least desirable. For society as a whole the least desirable outcomes are obviously those with high values of  $c$ . For resource extracting firms, on the other

hand, the least desirable outcomes are those giving a low future resource price, i.e. those with low values of  $c$ . The effects of risk aversion in the case of an uncertain date of availability do not seem to have been studied by anyone. However, reasoning similar to the above applies also in this case: For society as a whole the least desirable outcomes are obviously those with a long period before substitute production becomes possible, while resource extracting firms consider these outcomes as the most desirable, as they in these cases can get a resource price higher than the cost of producing the substitute for a long period. It therefore seems reasonable to guess that Propositions 3, 6 and 7 can be generalized to the case where we have an uncertain date of availability of the substitute in addition to an uncertain cost of producing this substitute.

Proposition 7 states that risk aversion will imply that the resource extraction in a competitive economy is higher than optimal before  $T$ . A natural question to rise is what kind of policy could make the competitive firms extract their resources at an optimal rate. The most obvious policy would be to impose a tax on extraction before  $T$  (and/or subsidize extraction after  $T$ ). Such a policy would reduce the resource extraction before  $T$ , provided that the resource extracting firms really believed that the tax would be lower after  $T$  than before  $T$ . However, it may be difficult to

convince the resource extracting firms that this will be the case. The firms may well believe that once the date  $T$  is reached, the tax will stay unchanged. After all, reducing the tax at  $T$  will mean that the state will lose revenue which must be raised elsewhere in the economy, at the same time that the after tax profits of the resource extracting firms, which already are above the normal return on capital, will increase. Reducing the tax on resource extracting firms at  $T$  may therefore be regarded as a very undesirable policy by whatever government is in power at time  $T$ . It is not much help for a government to make a binding tax agreement with resource extracting firms at the initial time point ( $t = 0$ ) either: If the government at  $t = T$  wishes to keep the revenues from resource extracting firms unchanged after  $T$ , there will often be several ways to achieve this without breaking a legal tax agreement, for instance through price regulation. It may therefore be impossible to reduce the competitive resource extracting before  $T$  to the socially optimal level only by imposing a tax on the extraction before  $T$ . In this case the most obvious policy which suggests itself in a competitive economy with privately owned natural resources is direct output regulation of the resource extracting firms.

## Appendix

From (10) we obtain (A.1)

$$U''(\hat{x}(t, S, c)) \bullet \frac{\partial \hat{x}(t, S, c)}{\partial S} = e^{-r(N(S, c) - t)} (-crN_S(S, c)) \quad (A.1)$$

and from (11) we get

$$\hat{x}(N(S, c), S, c) \bullet N_S(S, c) + \int_T^{N(S, c)} \frac{\partial \hat{x}(t, S, c)}{\partial S} dt = 1. \quad (A.2)$$

Inserting the expression for  $\partial \hat{x}(t, S, c) / \partial S$  from (A.1) into (A.2) gives us

$$\left[ \hat{x}(N(S, c), S, c) - cr \int_T^{N(S, c)} \frac{1}{U''(\hat{x}(t, S, c))} e^{-r(N(S, c) - t)} dt \right] N_S(S, c) = 1$$

or, by using (12),

$$N_S(S, c) = \frac{1}{k(c) - crZ}$$

which is identical to (15), with  $Z$  given by (16).

From (10) we obtain

$$U''(\hat{x}(t, S, c)) \bullet \frac{\partial \hat{x}(t, S, c)}{\partial c} = e^{-r(N(S, c) - t)} (1 - crN_c(S, c)) \quad (A.3)$$

and from (11) we get



$$\hat{x}(N(S,c), S, c) N_c(S, c) + \int_T^{N(S,c)} \frac{\partial \hat{x}(t, S, c)}{\partial c} dt = 0 \quad (A.4)$$

Inserting the expression for  $\partial \hat{x}(t, S, c) / \partial S$  from (A.3) into (A.4) gives us

$$\begin{aligned} & \left( \hat{x}(N(S,c), S, c) - cr \int_T^{N(S,c)} \frac{1}{U''(\hat{x}(t, S, c))} e^{-r(N(S,c)-t)} dt \right) N_c(S, c) \\ & = - \int_T^{N(S,c)} \frac{1}{U''(\hat{x}(t, S, c))} e^{-r(N(S,c)-t)} dt, \end{aligned}$$

which, using (12), can be rewritten as

$$N_c(S, c) = \frac{-Z}{k(c) - crZ} = \frac{1}{rc - k(c)/Z}, \quad (A.5)$$

which is identical to (19), with  $Z$  given by (16).

Differentiating  $G_{sc}(S, c)$ , given by (18), with respect to  $c$  gives us

$$\begin{aligned} G_{sc}(S, c) &= e^{-rN(S,c)} [-rN_c(S, c) - rcN_{cc}(S, c) \\ &\quad + (1 - rcN_c(S, c)) (-rN_c(S, c))] \\ &= e^{-rN(S,c)} [r^2c(N_c(S, c))^2 - 2rN_c(S, c) \\ &\quad - rcN_{cc}(S, c)]. \end{aligned} \quad (A.6)$$

Differentiating (A.5 with respect to  $c$  gives us

$$N_{cc}(S, c) = (N_c(S, c))^2 \left[ \frac{\partial k(c)/Z}{\partial c} - r \right],$$

which inserted in (A.6) gives us

$$\begin{aligned}
 G_{\text{SCC}}(S, c) &= e^{-rN(S, c)} rN_c(S, c) \left[ 2(rN_c(S, c) - 1) \right. \\
 &\quad \left. - cN_c(S, c) \cdot \frac{k(c)/Z}{c} \right] \\
 &= e^{-rN(S, c)} r(N_c(S, c))^2 \left[ \frac{2(rN_c(S, c) - 1)}{N_c(S, c)} \right. \\
 &\quad \left. - \frac{\partial k(c)/Z}{\partial c} \right].
 \end{aligned} \tag{A.7}$$

Furthermore, from (A.5) it is easy to see that

$$\frac{2(rN_c(S, c) - 1)}{cN_c(S, c)} = \frac{2}{c} \frac{k(c)}{Z},$$

which inserted into (A.7) gives us

$$G_{\text{SCC}}(S, c) = e^{-rN(S, c)} r(N_c(S, c))^2 \left[ \frac{2}{c} \frac{k(c)}{Z} - \frac{\partial k(c)/Z}{\partial c} \right]. \tag{A.8}$$

From (A.8) we therefore see that

$$G_{\text{SCC}}(S, c) < 0 \Leftrightarrow \frac{\partial \left( \frac{k(c)}{Z} \right)}{\partial c} > \frac{2k(c)}{cZ},$$

or, after some manipulation

$$G_{\text{SCC}}(S, c) < 0 \Leftrightarrow \frac{\partial Z}{\partial c} \frac{c}{Z} > -2 + k'(c) \frac{c}{k}. \tag{A.9}$$

From (12) and (13) we know that  $k'(c)c/k < 0$ , a sufficient condition for  $G_{\text{SCC}}(S, c) < 0$  is therefore that

$$\frac{\partial Z}{\partial c} \frac{c}{Z} > -2. \quad (\text{A.10})$$

Let us see when this inequality holds. By differentiating  $Z$ , given by (16), and using (12), we obtain

$$\begin{aligned} \frac{\partial Z}{\partial c} &= \frac{N_c(S, c)}{U''(k(c))} - rN_c(S, c)Z \\ &\quad - \int_T^{N(S, c)} \frac{U'''(\hat{x}(t, S, c))}{(U''(\hat{x}(t, S, c)))^2} \frac{\partial \hat{x}(t, S, c)}{\partial c} e^{-r(N(S, c)-t)} dt \end{aligned} \quad (\text{A.11})$$

By differentiating (10) we see that

$$\frac{\partial \hat{x}(t, S, c)}{\partial c} = \frac{1 - rcN_c(S, c)}{U''(\hat{x}(t, S, c))} e^{-r(N(S, c)-t)} > \frac{1 - rcN_c(S, c)}{U''(\hat{x}(t, S, c))}$$

for  $t < N(S, c)$  (for  $t = N(S, c)$  we get an equality instead of an inequality). Using this inequality, we therefore must have

$$\begin{aligned} & - \int_T^{N(S, c)} \frac{U'''(\hat{x}(t, S, c))}{(U''(\hat{x}(t, S, c)))^2} \frac{\partial \hat{x}(t, S, c)}{\partial c} e^{-r(N(S, c)-t)} dt \\ & \leq -\xi(S, c)(1 - rcN_c(S, c))Z, \end{aligned} \quad (\text{A.12})$$

where we have used the definition of  $Z$  and defined  $\xi(S, c)$  by

$$\xi(S, c) = \text{Max} \left[ 0, \text{Max}_t \frac{U'''(\hat{x}(t, S, c))}{(U''(\hat{x}(t, S, c)))^2} \right].$$

Using (A.12) together with (A.11) gives us

$$\frac{\partial Z}{\partial c} \leq \frac{N_c(S, c)}{U''(k(c))} - rN_c(S, c)Z - \xi(S, c)(1 - rN_c(S, c)Z)$$

or

$$\frac{\partial Z}{\partial c} \frac{c}{Z} \geq \frac{cN_c(S, c)}{U''(k(c))Z} - [rN'_c(S, c) + c\xi(S, c)(1 - rN_c(S, c))] . \quad (A.14)$$

The first term in (A.14) is positive, a sufficient condition for (A.10) to hold is therefore that the terms in brackets do not exceed 2. But  $0 < rN_c(S, c) < 1$  (cf. (19)), a sufficient condition for this to hold is therefore that

$$c\xi(S, c) < 2. \quad (A.15)$$

From the definition of  $\xi(S, c)$  it is clear that if the demand curve  $U'(x)$  is linear or concave,  $\xi(S, c)$  will be equal to zero, so that (A.15) holds. It is also clear that even if the demand function is convex, which is often assumed, (A.15) will hold as long as the curvature of  $U'(x)$  is not too strong or  $c$  is not too high. And even if  $U'''(x)$  and/or  $c$  is so large that (A.15) does not hold, the condition (A.10) may still hold. Finally, even if (A.10) doesn't hold, we may have  $G_{SCC}(S, c) < 0$ .

The reasoning above indicates that one will usually expect  $G_{SCC}(S, c)$  to be negative. To get an idea about what condition (A.15) states, let us look at the example  $U(x) =$

$(1-\beta)^{-1}x^{1-\beta}$ , where  $\beta > 0$  and  $\beta \neq 1$ . This function implies that the demand function  $U'(x)$  has a constant price elasticity. With this function we get

$$c\xi(S,c) = c \frac{1+\beta}{\beta} \left( \max_t [\hat{x}(t,S,c)] \right)^\beta$$

The condition (A.15) can therefore be rewritten

$$\left( \max_t [\hat{x}(t,S,c)] \right)^{-\beta} > \frac{1+\beta}{\beta} \frac{c}{2}$$

or

$$\min_t \{U'(\hat{x}(t,S,c))\} > \frac{1+\beta}{\beta} \frac{c}{2}.$$

In this inequality the left-hand side is simply the price of the resource at  $T$ , and we see that for (A.15) to hold, the resource price at  $T$  must exceed  $(1+\beta)c/2\beta$ . This means that if  $\beta < 1$ , (A.15) can never hold. If  $\beta > 1$ , whether or not (A.15) will be satisfied will depend on  $S$  and  $c$ .

## References

- [1] Arrow, K. J. Aspects of the Theory of Risk-Bearing (Helsinki: Yrjö Jahnessonin Säätiö, 1965).
- [2] Dasgupta, P. and Heal, G. M. "The Optimal Depletion of Exhaustible Resources," Review of Economic Studies, Symposium on the Economics of Exhaustible Resources (1974), pp. 3-28.
- [3] Dasgupta, P. and Stiglitz, J. E. "Uncertainty and the Rate of Extraction under Alternative Institutional Arrangements," Technical Report No. 179, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1976.
- [4] Heal, G. M. "Economic Aspects of Natural Resource Depletion," in D. W. Pearce and J. Rose (eds.): The Economics of Natural Resource Depletion (New York: John Wiley and Sons, 1975), pp. 118-139.
- [5] Hoel, M. "Resource Extraction under Some Alternative Market Structures," Memorandum from the Institute of Economics, University of Oslo, May 20, 1976.
- [6] Hotelling, M. "The Economics of Exhaustible Resources," Journal of Political Economy, Vol. 39, No. 2 (April), pp. 137-175.
- [7] Nordhaus, W. D. "The Allocation of Energy Resources," Brookings Papers on Economic Activity, No. 3 (1973), pp. 529-576.

- [8] Weinstein, M. C. and Zeckhauser, R. J. "The Optimal Consumption of Depletable Natural Resources," Quarterly Journal of Economics, Vol. 89, No. 3 (August, 1975), pp. 371-392.





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