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THE PRODUCTION-THEORETIC MEASUREMENT
OF INPUT PRICE AND QUANTITY INDICES

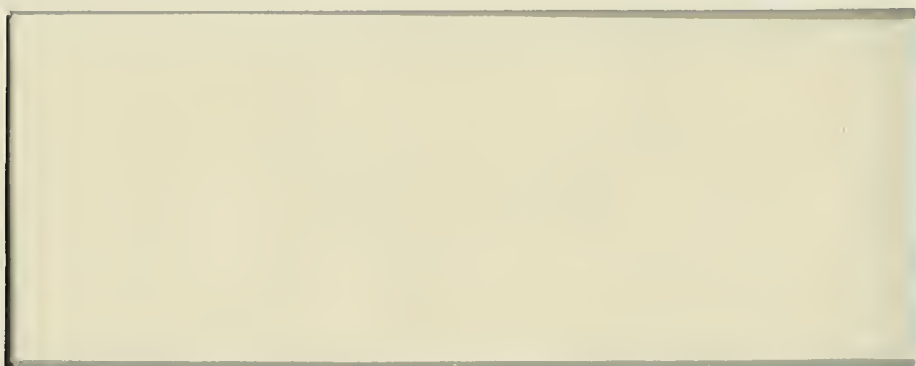
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No. 575

April 1991

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THE PRODUCTION-THEORETIC MEASUREMENT OF INPUT PRICE AND QUANTITY INDICES

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* Paper prepared for Conference in Honor of Zvi Griliches, Jerusalem, May 1991. This paper adapts material from my (eventually) forthcoming book with Karl Shell on the production-theoretic approach to price and quantity measurement.

1. Introduction

It has long been understood that conventional measures of the cost of living such as the consumer price index are but approximations to a "true" index based on consumer theory. It is less well accepted that measurement of prices and quantities in production are or ought to be focussed on approximating "true" indices based on production theory. Indeed, in calculating aggregates for production, concentration has been on arithmetical properties. In a real sense, practice has come before theory, and practice has dominated. Yet the question of what it is that we are trying to measure is of obvious importance.

In the form considered here, that question was first raised in Fisher and Shell (1972). Snell and I considered the case of the measurement of output and, especially, output prices when the production system is described by the production possibility frontier (PPF) of a closed economy. Following that, John Muellbauer (1972) pointed out that an isomorphic theory can be built for the case of input deflation and proceeded to do so, deriving

many interesting results.

Unfortunately, however, Muellbauer's isomorphic theory does not seem of great direct interest. That is because the closed-economy characterization of a PPF by fixed factor supplies has as its parallel the characterization of output as a fixed vector. This means that the index number question being asked has to do with the relative input usage required to produce a fixed output vector under two different regimes. Since firms and production systems usually do not face fixed demand vectors, this seems too narrow a treatment. (The exception is the case of the single-output competitive firm.)

Fortunately, widening that treatment does enable us to follow up on Muellbauer's insight that input deflation and measurement requires a production-theoretic treatment. In the simplest case, the demand conditions that a competitive firm takes as given are not represented by a vector of fixed output quantities but by a vector of fixed output prices at which the firm can sell. The input deflation and usage problem ought thus to be analyzed using this assumption.

Such a treatment does not end the matter, however, for input deflation and measurement is seldom done at the level of the individual competitive firm. While the same treatment of outputs sold at fixed prices will serve for the case of a small fully open economy trading outputs on world markets at fixed prices, it will not cover the case of a large economy or of a firm or industry facing downward sloping demand curves. As it turns out, such cases can involve substantial complexities and cast doubt on the use of Paasche and Laspeyres indices as appropriate bounds.

2. Actions or Reactions? Quantity vs. Price Effects

A monetary magnitude changes. It is desired to separate that change into a price effect and a real change. Thus, for example, the money value of output is observed to increase, and we wish to speak of a real output change versus an inflationary (or deflationary) effect. Similarly, when money expenditures on inputs rise, we may wish to speak of the extent to which this is simply due to changes in factor prices and the extent to which it represents a real increase (or decrease) in the use of resources. In the case of the consumer, a change in the value of money income is to be separated into a price effect and a change in real income.

One important thing that all such examples have in common is that it is natural to think of the price effect as coming from changes on the side of the markets being studied faced by the unit under analysis -- supply prices for factors in the case of input deflation; demand prices for goods in the case of output deflation; supply prices for goods in the case of the consumer price index. Then any change in real magnitudes must involve an action on the part of the unit that is different from that which would have been observed had the unit, with other circumstances unchanging, been faced with the price changes.

Consider a competitive firm producing a single output. A change in costs is to be divided into a factor-price effect and a real change in input usage. If we consider factor price changes as coming from outside the firm, then it is natural to ask what would have happened to costs had the other circumstances of the

firm remained constant. Clearly, this amounted to asking what effect the changes in factor prices would have had if the firm had remained on the same isoquant as in the base period. That change is the pure price effect; any further change in costs represents a real change.

There is another way to think about this. Any solution to the problem of measuring real aggregate input use must begin with an answer to the following question: Since we shall be reducing input vectors to index-number scalars, and since we wish to obtain a complete ordering for the resulting scalars and the input vectors to which they correspond, we must choose a set of equivalence classes for input vectors such that all vectors in the same equivalent class will be said to involve the same aggregate input. How is this to be done?

In the case of the single-output firm, the answer to this question immediately suggests itself -- the use of the isoquants of the production function to define equivalence classes. With a given technology, we will say that two input vectors corresponding to the same output involve the same aggregate input use. If factor prices change and the firm remains on the same isoquant, then any change in money input costs will be considered as a pure price phenomenon.

Obviously, this is isomorphic to the theory of the consumer price index. There the consumer faces prices as given from outside and the equivalence classes are provided by the indifference curves.

In the case of production, however, matters need not always

be quite so simple. For one thing, the production unit involved may have monopoly or monopsony power, so that prices cannot be taken as given from outside. I shall treat such cases in detail later. Suffice it to say now that they involve a generalization of the principles just described. Thus, in the case of input deflation and a firm with monopsony power, input prices cannot be taken as given, but supply conditions can. A change in the money value of input can then be separated into the change that would have taken place had the production unit merely responded to altered supply conditions and the remaining -- real -- change.

In all cases, we separate the world into price effects -- caused by the other side of the markets being studied -- and real effects -- the actions of the productive unit being studied that are not merely reactions to the price effects. But the case of monopoly or monopsony points up the somewhat artificial nature of such a division. After all, when we study a unit as large as the entire productive system, the prices at which outputs can be sold will not be independent of the actions of the production system itself, even if no monopoly power is involved.

The answer here is simple. The division of a change in monetary value into a single measurement of price change and a single measurement of quantity change is necessarily arbitrary. In the true world of general equilibrium effects, no such change is truly possible. Instead, all we can do is to ask "what if" questions. In particular, if we wish to examine the production sector, we can ask what would have happened if the original production sector had faced different input prices (or supply conditions). Answering such questions provides insight into

whether real input usage has increased or decreased, even though any actual change has consequences beyond the production sector itself.

3..Simple_Input_Deflation_in_More_Detail

I now concentrate on the easy case of simple input deflation -- the deflation of inputs in the case of a single competitive firm. This case is quite instructive, and the propositions discussed will be seen to be of quite general applicability, so that a good deal more than an example is involved.

[FIGURE 3.1 HERE]

In Figure 3.1, the base-period isoquant is drawn with base-period factor prices, \hat{w} , represented by the solid line. Optimal input use is \hat{v} , and total input costs $\hat{C} = \hat{w}\hat{v}$. (For ease of notation, I omit transposition signs when writing inner products, relying on the context to make clear what is intended.) In the current period, factor prices, w , are denoted by the dashed lines. Actual factor usage is v^* and actual money costs are $C^* = wv^*$. Had factor prices been w instead of \hat{w} in the base period, the output corresponding to the base-period isoquant would have been efficiently produced with inputs v rather than \hat{v} and money costs would have been C rather than \hat{C} . The view of input-deflation taken here is that the change in costs from \hat{C} to C^* should be thought of as:

$$(3.1) \quad C^*/\hat{C} = (C^*/C)(C/\hat{C}) \quad ,$$

with the first factor the increase in real aggregate input usage and the second reflecting price changes.

In more general terms, let the firm's cost function be $C(w, x)$, where x is output. The construction just given uses base-period output, \hat{x} , and calculates the deflator for money input costs as $C(w, \hat{x})/C(\hat{w}, \hat{x})$. This deflator is divided into the relative change in money input costs to give the measure of real aggregate input usage relative to the base period.

It is not hard to see that the deflator just produced is bounded above by a Laspeyres index of input prices. Similarly, the deflator based on the current period's isoquant will be bounded below by a Paasche index. In the case of unchanging technology and a homothetic isoquant map, both deflators will be the same and both bounds will apply.

Evidently, this approach leads to a theory largely isomorphic to that of the cost-of-living index (as well as to the Fisher-Shell theory of output deflation). In considering possible objections to the theory being advanced, therefore, it is well to consider what those same objections imply about the relatively well-established theory of the cost-of-living index.¹

1. Space does not permit consideration of all possible objections here. In particular, Diewert (1983) points out (using the parallel case of output deflation) that the method here given will often not lead to a measure of real input usage that doubles if the firm exactly doubles its usage of every input. Such matters are taken up in Fisher (1988).

One such objection is as follows. The procedure just described treats input vectors as identical if they can produce the

same output and treats a movement to a higher-numbered isoquant as an input increase. But we are trying to build a theory of input aggregation and measurement. Is it not odd that levels of outputs become central to the theory? Moreover, different firms with different technologies facing the same set of input prices will have different factor price deflators constructed for them.

The answer lies in consideration of the object of the enterprise. We are treating the firm as the object of interest with factor prices given from outside. Any production-theoretic view of input deflation must involve the production function of the firm. Just as the cost-of-living index describes price changes from the point of view of the individual consumer, so the production-theoretic input price index describes factor price changes from the point of view of the individual firm. The fact that different firms have different points of view, so to speak, is not a valid objection. The aggregation problem to which it points cannot be solved by choosing a firm-independent measure of input prices. To do that is merely to impose on all firms a measure not relevant to any one of them.

Two more comments on this before proceeding. First, from the point of view of the input-producing sector (which could also be made up of firms if inputs are intermediate products), the construction of an index of prices for its outputs ought not to depend on the production functions of its customers. But output deflation is a different enterprise with price changes taken as originating in the purchasing sector. Here, in the case of input deflation, price changes are taken as originating in the selling sector. This is consonant with the general view of price and

quantity indexation taken above.

Second, the fact that the production-theoretic input price index will be different for firms with different production functions does not mean that nothing can be said about the nature of such dependence. On the contrary, interest certainly attaches to the way in which such indices change as the production function varies over firms or over time. Such matters can be treated with comparative static analysis (although space does not permit doing so in the present paper).

When we examine more general (and more interesting) cases than that of the single-output competitive firm, the same principles will apply. In each case, we shall examine what would have happened had the productive unit in question faced the new set of input prices (or, more generally, input supply conditions). We will restrict the productive unit to an isoquant or to a generalization thereof.

Before proceeding to the details of that analysis, however, two subjects can be discussed at the present, general level. They are the treatment of corner solutions and the treatment of quality changes in inputs.

3. Corner Solutions

The issues involved in corner solutions here are readily resolved. The isomorphic case to that of new or disappearing goods in the analysis of the cost-of-living index or of the production-theoretic output price index is that of a new or disappearing factor of production. This does not seem to be a particularly interesting case unless we are considering as fac-

tors not primary factors but inputs of goods or materials which one sector or country buys from another. If we do consider such cases, then it is not hard to imagine a technical discovery which, in effect, produces a good that serves as a previously unknown factor of production from the point of view of the sector or economy that purchases it.

Despite such cases, there is no need to give a lengthy analysis -- in part because of the isomorphisms already mentioned. It is plain from consideration of Fisher and Shell (1972, pp. 99-105) that the problem of what factor price to use for a new factor is only a problem for a Paasche index and not for the production-theoretic input price index. The bounding property of a Paasche index will be preserved if the factor price so chosen is any price at or above its base period's demand reservation price. (This is the price at which the productive sector being considered would be just indifferent between employing the factor and not doing so; it is the intercept on the base-period factor demand curve.) Of all prices in that range, the demand reservation price produces the most efficient Paasche lower bound. Further, that price can also be interpreted as the shadow price of the constraint involved in not being able to employ the factor in question in the base period.

It is important to realize, however, that corner solutions which arise through the appearance of new goods or the disappearance of old ones do not raise similar problems. This is because the isoquant relative to which the production-theoretic input price deflator is calculated is defined for a given set of output

conditions. Just how those conditions are properly defined is a matter we consider below, but however they are defined they will be the same for both periods. New goods do therefore raise a question; the appearance of a new good, like any other technical change, will generally alter the isoquant which determines the production-theoretic input price index. This is quite a different class of effects from those involved in corner solutions in factor space.

To take the simplest example, suppose that the relevant isoquant describes the efficient factor combinations for producing a given set of goods in specified quantities. That set of goods either includes the new one or it does not. What the set is certainly affects the shape of the isoquant and the value of the resulting input price deflator but the fact that a different set of goods would produce a different isoquant and a different deflator raises no problem for the construction of the deflator using the given isoquant corresponding to a given set of goods.

4. Quality Change: The Repackaging Theorem

The analysis of quality change presents somewhat deeper problems.

In the theory of the cost-of-living index, a fairly natural question is that of the treatment of quality change in one or more of the goods consumed. While such changes can be handled in principle as the disappearance of one good (with the old quality) and the appearance of a new good (with the new quality), this is not done in practice and, indeed, is rather awkward. In the theory of the cost-of-living index, quality improvement in a

consumed good is equivalent to an improvement in the opportunity set facing consumers and therefore equivalent to a fall in the cost of living if prices remain constant. Fisher and Shell (1972, pp. 26-37) analyzed such effects and considered whether one can treat a quality improvement as equivalent to a virtual price decrease in the good whose quality had changed. The principal result was the demonstration that, while this could generally be done, it could only be done with the virtual price change involved independent of all other prices and amounts consumed (e.g. dependent only on the physical nature of the quality change) under very special circumstances. The necessary and sufficient conditions for such treatment are that the consumer view the new quality of good as exactly equivalent to a fixed number of units of the old quality, i.e., that the quality change enter the utility function as a shift in a good-augmenting parameter (as b in $U(bx_1, x_2, \dots, x_n)$, where the x_i are the quantities consumed and $U(\cdot)$ is the utility function) We called this case "repackaging" since the consumer regards the new quality good as a "repackaged" version of the old one, so to speak.

Obviously, this is a very special case, and the result calls into question a whole class of methods used to adjust for quality change, a class of which econometrics is the most sophisticated representative.

The case of output price deflation has an isomorphic problem (considered in Fisher and Shell 1972, pp. 105-7)). There it turns out that one would still wish to treat a quality change as equivalent to a price decrease. This is because the resources used to produce the new quality of good could have been used to

produce different amounts of the original quality had consumers desired it. In effect, quality change is interpreted as simply a shift in demand. The isomorphic statement to the repackaging theorem is that quality improvement can be treated as a virtual decrease in the price of the good whose quality has changed independent of prices and quantities of all goods produced only under special circumstances. The necessary and sufficient conditions for such treatment are that the production possibility frontier (PPF) with the new quality good differ from the PPF with the old one by a shift in a good-augmenting parameter. This is equivalent to requiring that the production functions for the new and old quality good differ by a Hicks-neutral technical change so that, in terms of resource use (rather than utility), one unit of the new quality of good is equivalent to a fixed number of units of the old.

Leaving the repackaging theorem aside for the moment, it is important to recognize the difference between the treatment of quality change and that of technological change in the theory of output deflation. The production-theoretic output price deflator is constructed from the point of view of a specific PPF. A technical change will shift that PPF and (usually) change the deflator. However, the PPF used to construct the deflator will be constant when comparing base and current period prices even if the technical change occurred between the two periods. This is because the question to be answered concerns the responses which an economy with a given PPF would have made when faced with two sets of prices. A shift in the PPF may increase real output --

if it leads to a greater value of production than would have been achieved with the base period's PPF and the current period's prices -- but this is not because the change is equivalent to a price decrease. With base-period and current-period prices the same, the deflator will be unity ~~whichever~~ PPF is used to construct it.

Quality change, on the other hand, is treated as unambiguous. It is treated as though the choice between new and old quality of goods reflected not a change in supply but a change in demand conditions. Hence, a quality change requiring more resources to produce the new quality than to produce the old leads to an unambiguous increase in real output if the number of units of the new quality of good produced in the current period is the same as the number of units of the old quality of good produced in the base period, with the production of all other goods held constant. The quality change is thus equivalent to a price decrease. Indeed, prices will be treated as declining solely because of the quality change, even if money prices remain the same in the two periods.

Obviously, it is a matter of some importance to decide whether a given change should be treated as technological and due to supply or as a quality change and due to demand. (Similarly, in the theory of the cost-of-living index, it is important to decide whether a given change is a taste change, due to demand, or a quality change due to supply.²) The basic question to be

2. This is not always as simple as it looks. See Fisher and Shell (1972, pp. 7-8).

asked is always whether, if prices do not change, one wishes to consider the value of the deflator as necessarily altered by the change under consideration. If so, then it is a quality change.

Now, the question of how a given change should be treated arises again when we consider input deflation. Here the question to ask is whether, with money input prices the same, we wish to say that input prices have gone down as a result of the change. If so, then the change will be treated as a quality change; if not, then it will be treated as a technical change.

It is important to realize here that a change that occurs inside the unit whose input prices are to be deflated will be treated as a technical change (just as such a change in the case of households is treated as a shift in taste). In general, the discovery of a more efficient process within the production unit itself will not be considered as a decrease in input prices. An input price index is to measure the cost of inputs, not the usefulness of inputs once they are used. A quality change on the other hand, means a decrease in input costs for any technology used by the production sector. Thus, the kind of quality change with which we are here concerned is change in input quality. Changes in output quality will simply be treated as technical changes for purposes of input deflation. It is not reasonable to treat an output quality improvement as a virtual decline in input prices.³

3. Note the reversal of roles from the case of output price deflation. There a quality change in an input was treated as a technical change -- a change in the PPF. Here it is treated as a

quality change. On the other hand, a quality change in an output -- there treated as such -- is here treated as a technical change -- a change in the isoquant.

To fix ideas, suppose that the change in question concerns the quality of labor, so that labor (other factors constant) becomes more productive. If that increase in productivity is due to a discovery on the part of firms as to how to use labor more efficiently, then this will be treated as a technical change and not as a decrease in the effective cost of inputs. If, on the other hand, the increase is due to better education of labor, then one may very well wish to treat it as a virtual decrease in input costs and a reduction in the input price deflator.

The issue is not a simple one, however. Better educated workers may represent an effective decrease in input prices to the firm that hires them; this is not so (or at least not obviously so) if the education comes through on-the-job training. Now suppose that the production sector whose input costs are being considered is that of the entire economy. Do better educated workers represent lower virtual input prices? Or do they simply reflect a more efficient way of using a given set of inputs -- "raw" labor. Either answer is possible.

For those cases in which a given change is treated as a quality change in an input, a result isomorphic to that of the repackaging theorem applies and is very natural. A quality improvement in an input can always be treated as equivalent to a decrease in the price of that input with input quality constant.

That price decrease will be independent of input prices and purchases only under special conditions, however. Necessary and sufficient conditions for such independence are that the quality change can be represented within the productive technology as a shift in a parameter augmenting the factor whose quality has changed. In other words, the quality change must be such that one unit of the new quality of factor is exactly equivalent in production to a fixed number of units of the old quality of factor.

It may be thought that this version of the repackaging condition seems more likely to be satisfied in the present case of quality changes in factors than in either the case of good-augmenting changes in the utility function or Hicks-neutral changes in production functions -- the conditions that apply to the cost-of-living index and to the output price deflator, respectively. Once one has decided that a given change is to be considered as a quality change in a factor rather than as a change in technology affecting how that factor is used, it may not seem too stringent to suppose that the change is factor-augmenting. But is this really so? Aside from the fact that the decision to treat the change as a quality change already supposes a fairly restrictive set of circumstances -- perhaps more restrictive than in the case of output price deflation or the cost-of-living index -- it is not obvious that factor augmentation is then a totally natural assumption. Can an educated worker, for example, really do everything better than an uneducated one and better in the same proportion no matter what the

task? If not, then the quality change involved in education is not merely factor-augmenting in the way required by the repackaging theorem. No simple adjustment in wages will suffice to account for the effect of the change on the production-theoretic input price deflator or on the corresponding production-theoretic index of real input use.

5. The Fully Open Case

As already indicated, the easiest case for the analysis of economy-wide input price deflation is that of a fully open economy. In the present circumstances this amounts to assuming that the productive sector (like a multiproduct competitive firm) can sell all the outputs it wants at fixed output prices. The isoquant relative to which the input price deflator is defined then becomes the locus of all efficient input combinations which will produce output bundles of equal value.

The economic unit's technologically efficient production plans can be summarized in terms of the production relation

$$(5.1) \quad F(x, v) = 0,$$

where x is the output vector and v is the input vector. We are also given a vector of output prices $p = (p_1, \dots, p_r)$ and a money value of output, y .

The isoquant to be used in constructing the production-theoretic input price deflator is defined in the following way: Choose any output vector, x , whose value at prices p is y . Next consider the set of v corresponding to that x and the production function $F(x, v) = 0$. This is an isoquant for the production of

the given vector x . Consider the family of isoquants generated in this way by varying x over all vectors satisfying $px = y$. The isoquant that will be used for input price deflation is the lower envelope of this family. Each point, v , on that envelope also lies on some isoquant defined for x fixed for some x with total value of output at the given output prices is equal to y .

Formally, we have:

$$(5.2) \quad I_{FO} \equiv \{v \mid v \text{ is minimal subject to } F(x,v) = 0 \text{ and } px = y\}.$$

Given the isoquant I_{FO} , the construction of the corresponding production-theoretic input price index is a straightforward application of the general approach discussed above. We are given two input price vectors, w^A and w^B . First we define

$$(5.3) \quad C^A \equiv \min_v w^A v \text{ subject to } v \text{ lying on } I_{FO}.$$

Let v^A be the minimizing value of v in Problem (5.3); thus, we have $C^A = w^A v^A$.

Similarly, we define

$$(5.4) \quad C^B \equiv \min_v w^B v \text{ subject to } v \text{ lying on } I_{FO}.$$

Let v^B be the minimizing value of v in Problem (5.4); thus, we have $C^B = w^B v^B$.

The production-theoretic input price index for comparing money costs at input prices w^A to money costs at input prices w^B when outputs can be freely sold at fixed prices, p , and output value is held constant at y is then (C^A/C^B) .

This index is, of course, defined relative to a given isoquant, I_{FO} , which is determined by technology (4.1), output

prices, p , and output value, y . If these are the actual technology, output prices, and output value of the base period (the period indexed as B), then a Laspeyres input price index will bound the corresponding production-theoretic index from above. If they are the technology, output prices, and output value of the current period (indexed as A), then a Paasche input price index will bound the corresponding production-theoretic index from below. If the two isoquants are parallel along rays, then the two production-theoretic indices will be equal and both bounds will apply.

When will such isoquants be parallel in this way? Sufficient conditions are: (i) The production technology $F(.,.)$ is common to both periods; (ii) $F(.,.)$ is constant returns to scale; and (iii) the output prices, p , are proportional in both periods. These conditions -- particularly that of constant returns -- are far stronger than necessary.

When such parallelism along rays is not present, the two production-theoretic input price indices will differ and the Paasche and Laspeyres bounds need not both hold simultaneously.

6. General Demand Conditions: The Monopolistic Case

The fully open case just analyzed in which output demands are perfectly elastic is the appropriate one for input deflation where the productive sector involved is small -- a firm or a small group of firms in competition, or a small country in international trade. Larger units or aggregates, however, cannot be so simply treated. We must therefore analyze the case of declining demand curves.

It turns out to matter a good deal whether or not the agents in the production sector realize that they face declining demand curves and take that fact into account in their decision making. If they do -- the case of monopoly -- the analysis is somewhat simpler than if they do not -- the competitive case where each firm behaves as if it were in the fully open situation. The present section takes up the monopoly case; the more difficult (and more interesting) competitive case is treated later.

As before, efficient production plans are given by the relation

$$(6.1) \quad F(x, v) = 0 ,$$

where x is an r -vector of outputs and v is an m -vector of inputs. Let p be the corresponding r -vector of output prices; outputs are sold according to the demand schedule

$$(6.2) \quad x = x^D(p) .$$

If demand for some good is perfectly elastic at a constant price, then the corresponding component of x^D is infinite below that price, zero above it, and any value in $[0, +\infty]$ at the exogenously given price.

Suppose that we observe the economic unit in question (for convenience, the "economy") producing an output vector x^* at prices p^* so that its total revenue is $y^* = p^*x^*$. Fixing total revenue, y^* , what are the output combinations which the economy could have sold? The answer depends on what is assumed about the output demand conditions which the economy faces. In Figure 6.1,

the point x^* is indicated. For the fully open economy, the dashed line shows the output combinations consistent with total revenue, y^* , i.e. $\{x \mid p^*x^* = y^*\}$. Points to the northeast of the dashed line produce more revenue for the fully open economy than it receives at x^* . The solid curve represents the output combinations consistent with y^* for the economy facing declining demand schedules, i.e. $\{x \mid x = x^D(p) \text{ and } px = y^*\}$, where $x^* = x^D(p^*)$. The solid curve lies to the northeast of the dashed line reflecting the fact that increased outputs can only be sold at lower prices.

From Figure 6.1, we see that the fully open model provides a more "optimistic" isoquant than does the declining demand schedule model. This is reflected in the fact that the isoquant showing the efficient factor combinations required to generate the given revenue y^* shows greater required inputs with declining demand than in the fully open case.

It is not hard to see, therefore, that the production-theoretic input price deflator constructed in the declining-demand-curve case will be less than that for the corresponding fully open case because the fully open economic unit with a given isoquant responds more fully to factor price changes than does the unit facing declining demand curves. Hence a given change in the money value of costs will be considered more of a real change and less of a monetary one when declining demand curves are present than when they are not.

The isoquant for the general case of a monopoly economic unit facing declining demand curves, I_{II} , is defined as follows:

$$(6.3) \quad I_M \equiv \{v \mid v \text{ is minimal subject to } F(x,v) = 0, \\ x = x^D(p), \text{ and } px = y\}.$$

It is important to realize that this isoquant is based on the assumption that the economic unit "sees" and acts upon its entire demand schedule. It does not take output prices as given but rather minimizes its cost subject to the constraint $px^D(p) = y$. In other words, in deriving I_M , we have assumed the economic unit to have monopoly power in its product markets. The case of competition with declining demand is studied later.

Now, assume that I_M is derived from technological and output market conditions actually prevailing in the base period. Denote these conditions by the superscript B, so that the isoquant becomes

$$(6.4) \quad I_M^B \equiv \{v \mid v \text{ is minimal subject to } F^B(x,v) = 0, \\ x = x^{DB}(p), \text{ and } px = y\}.$$

Assume that v^B , the actual vector of base period inputs minimizes costs at factor prices w^B subject to the technological constraint, $F^B(x,v) = 0$, the demand constraint $x = x^{DB}(p)$ and the revenue constraint $px = y^B$. Then the two isoquants, I_M^B , defined in (6.4) and I_{FO}^B , the corresponding fully open isoquant (see (5.2)) share a common point, but I_M^B lies above and to the right of I_{FO}^B .⁴ Let J_M^B denote the production-theoretic input price

4. I_M^B would lie below and to the left of a "fully closed" isoquant defined with perfectly inelastic demands -- a fixed market basket of outputs -- but the latter is of very little interest.

deflator defined relative to I_M^B and J_{FO}^B the production-theoretic input price deflator defined relative to I_{FO}^B . Then we have:

$$(6.5) \quad L \geq J_M^B \geq J_{FO}^B ,$$

where L denotes the Laspeyres input price index. Similarly, for the indices derived from current-period conditions, (superscripted A) we have:

$$(6.6) \quad P \leq J_M^A \leq J_{FO}^A ,$$

where P denotes the Paasche input price index.

Note that because the production-theoretic input price deflator for the monopoly case is always greater than or equal to the corresponding deflator for the fully open case, a given increase in money costs will be attributed more to real input usage in the fully open case than in the case of monopoly. This corresponds to the isoquants drawn in Figure 6.1. The fully open case has a flatter isoquant, so that factor price changes will induce a greater movement in factor usage than in the monopoly case. Cost changes resulting from movements along an isoquant are counted as monetary only.

7. The General Competitive Case

I now turn to the important case of a competitive economic unit (an industry, say) facing declining output demand schedules. There is no monopoly power. The economic unit faces the same general demand conditions as in the preceding section -- but it does not know it. The firms which make up the unit optimize taking output prices as given as in the fully open case of

Section 5 above. In fact, however, taken all together, they are not in a fully open environment, and output prices do depend on the sum of their decisions.

This fact raises a new problem. The production-theoretic view of input deflation asks what the economic unit would have spent on inputs at the new input prices holding the value of output constant. But now holding output value constant is not a simple matter. It cannot be done, as in the fully open case, by restricting the output vector to an isovalue line at fixed output prices, because output prices are not fixed. On the other hand, it cannot be done, as in the monopoly case, by assuming that the economic unit minimizes cost subject to outputs lying on an isorevenue curve, because the economic unit does not in fact solve such a problem. Indeed, the construction of the production-theoretic input price deflator in this case is not merely a problem in constrained optimization; that construction also involves a fixed-point argument ensuring equality of supply and demand in all output markets. This makes comparative statics (not treated in this paper) far more difficult than in the other cases so far considered. Further, Paasche and Laspeyres bounds need no longer apply.

As in the preceding section, technology is summarized by

$$(7.1) \quad F(x, v) = 0 \quad 5$$

5. Note that (7.1) gives the production technology for the industry. In the absence of constant returns, this will general-

ly not also be the technology for the individual firm. This is a matter of no consequence here, however.

and output demand by

$$(7.2) \quad x = x^D(p) \quad .$$

Firms in our competitive industry face a given output price vector, p , and act as if they do not affect it. Thus, the firms perceive themselves as operating in a fully open economy as in Section 5. If that perception were correct, the isoquant for the industry facing output prices, p , and earning total revenue, y , would be

$$(7.3) \quad I_{FO}(p) \equiv \{v \mid v \text{ is minimal subject to } F(x,v) = 0 \\ \text{and } px = y\} \quad .$$

By fixing total revenue, y , and varying output prices, p , we can derive from (7.3) the implied industry supply schedule for outputs (parametric on y , of course):

$$(7.4) \quad x = x^S(p) \quad .$$

Now, we cannot base the analysis of our competitive industry on the isoquant, $I_{FO}(p)$, defined in (7.3). First, $I_{FO}(p)$ is derived for fixed output prices, and output prices are not fixed at the industry level. Second, $I_{FO}(p)$ does not take into account the industry-wide constraint that supply and demand for outputs must be equal, i.e., that $x^D(p) = x^S(p)$.

The fact that this constraint is not recognized by the competitive industry makes the analysis complex. This is because

the equilibrium output prices which equate output supplies and demands themselves depend on factor prices since output supplies so depend. Thus, while the competitive industry, given y , minimizes costs while remaining on $I_{FO}(p)$ for some p , which p that is depends on factor prices, w . Were w different, p would also be different, and the competitive economy would solve a different problem.

We must therefore take this into account and (in principle) use the p that corresponds to equilibrium in output markets given the factor prices involved.

In defining the production-theoretic input price index for the competitive general case, J_{CG} , we are given the production relation (7.1), total revenue, y , the output demand schedules (7.2), and two input price vectors, w^A and w^B .

First, take the output price vector, p , as a parameter and let

$$(7.5) \quad C^A(p) \equiv \min_{x,v} w^A v$$

subject to $F(x,v) = 0$ and $px = y$.

($C^A(p)$ is thus money cost at factor prices, w^A , given that input is on $I_{FO}(p)$.) Let the minimizing input vector be $v^A(p)$ and the resulting vector of optimal output supplies be $x^{SA}(p)$. Now find that value of p (for convenience assumed to be unique) such that $x^{SA}(p) = x^D(p)$; call it p^A .

Similarly, let

$$(7.6) \quad C^B(p) \equiv \min_{x,v} w^B v$$

subject to $F(x,v) = 0$ and $px = y$.

Let the minimizing input vector be $v^B(p)$ and the resulting vector of optimal output supplies be $x^{SB}(p)$. Now find the value of p (assumed unique) such that $x^{SB}(p) = x^D(p)$; call it p^B .

The production-theoretic input price index appropriate to the competitive general case is then defined by

$$(7.7) \quad J_{CG} = C^A(p^A)/C^B(p^B) \quad .$$

The fact that the firms making up the competitive economic unit behave as though they face flat demand curves whereas the unit as a whole does not creates practical difficulties for further analysis. First, comparative static analysis (not here treated) becomes difficult. The effect of a shift in a given parameter now does not only involve changes in the solution to the unit's optimizing problem given that shift (which are readily handled by the Envelope Theorem). Such a shift also involves shifts in the other parameters of the unit's optimization problem through changes in the position of equilibrium in all output markets. Without more information as to demand schedules, such shifts cannot be studied.

Second, and more important for the practice of price index construction, Laspeyres and Paasche bounds are no longer guaranteed to hold. To see this, Let B denote actual base-period and A actual current-period conditions, respectively. Then $C^B(p^B)$, the denominator of J_{CG} and the denominator of the Laspeyres input

price index will be the same. However, the usual method of establishing the bound would be to show that the numerator of J_{CG} , $C^A(p^A)$ is the value of the solution to a minimum problem in which v^B (the actual input combination used in the base period) was feasible. This is no longer guaranteed. Figure 7.1 shows a situation in which the value of v^B at factor prices w^A (represented by the slope of the dashed lines) is lower than $C^A(p^A)$. Further (just for good measure), the value of v^A at prices $w^{6,B}$ (represented by the slopes of the solid lines) is lower than $C^B(p^B)$. Plainly, Laspeyres and Paasche bounds are inapplicable here.

The moral is clear. Except where demand curves can in fact be taken as approximately flat because the unit is very small, Paasche and Laspeyres input price indices will not bound the production-theoretic input price indices. The case is even worse than for output price deflation. There, if the unit is large enough, it is appropriate to treat factor supplies as fixed, generating another case -- that of the fully colsed economy -- in which Paasche and Laspeyres bounds apply. (See Essay II of Fisher and Shell 1972.) Here, no matter how big the unit, it is inappropriate to assume that the demand curves it faces are perfectly inelastic.

Paasche and Laspeyres input price indices will thus give a misleading picture except for very small economic units. For other cases, detailed knowledge of demand schedules (and, in general, of technology as well) will be required. One cannot assume that changes in input prices leave output prices unaffected, and this creates a serious problem.

Of course, this difficulty with Paasche and Laspeyres indices is a form of aggregation problem. It comes about because the situation facing an entire industry is not that perceived by the firms that make it up. If (parallel to the case of the cost of living index and an individual household) we were content to look at input prices from the point of view of an individual firm, such problems would not arise (although even then Paasche and Laspeyres quantity weights would have to be those corresponding to an individual firm's purchases). Input deflation and input quantity measurement, however, is seldom done from so narrow a point of view. When Paasche or Laspeyres indices are used on an industry or economy-wide basis, their grounding in production theory is not a strong one.

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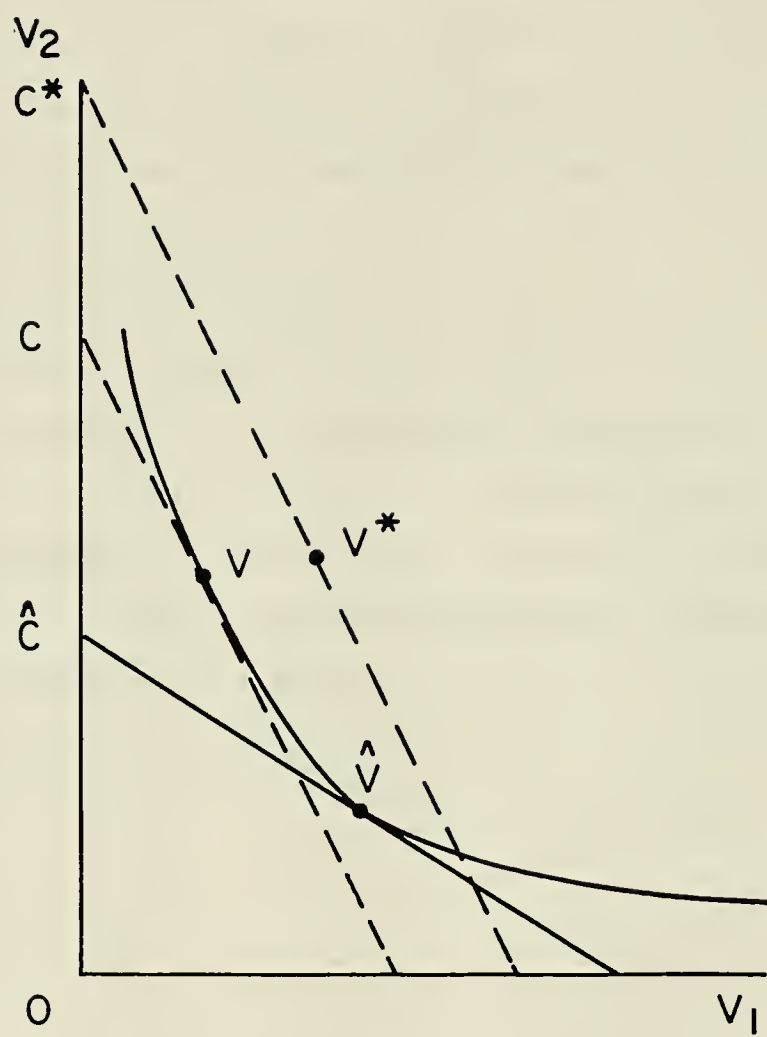


Figure 3.1

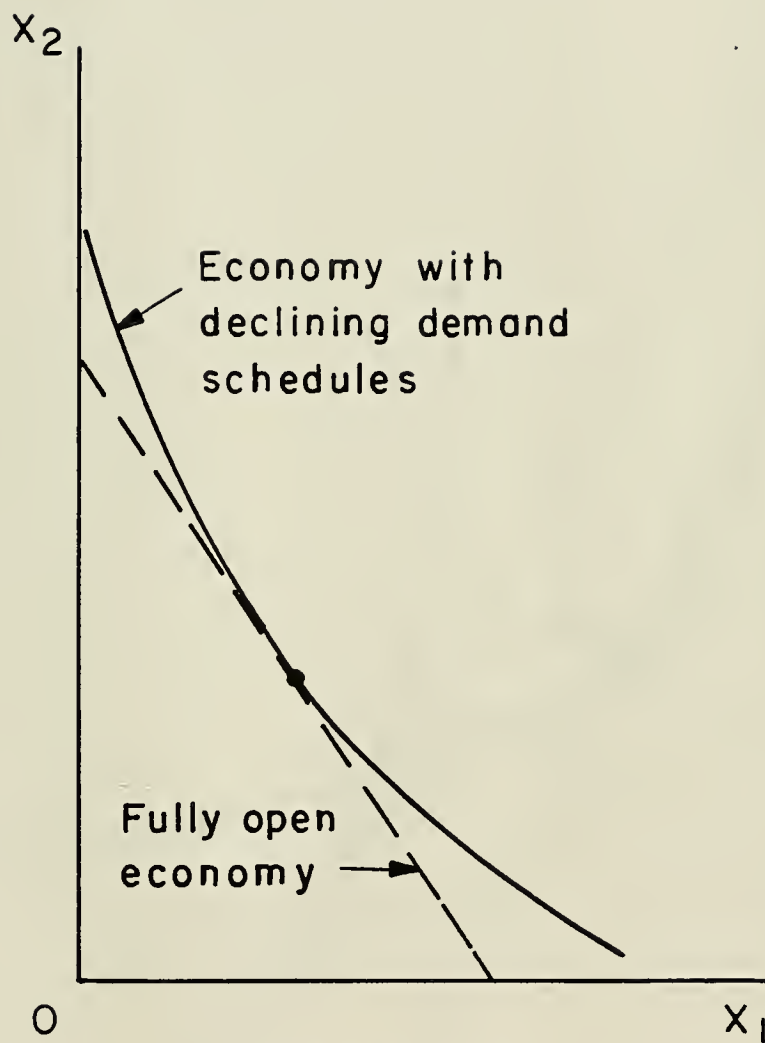


Figure 6.1

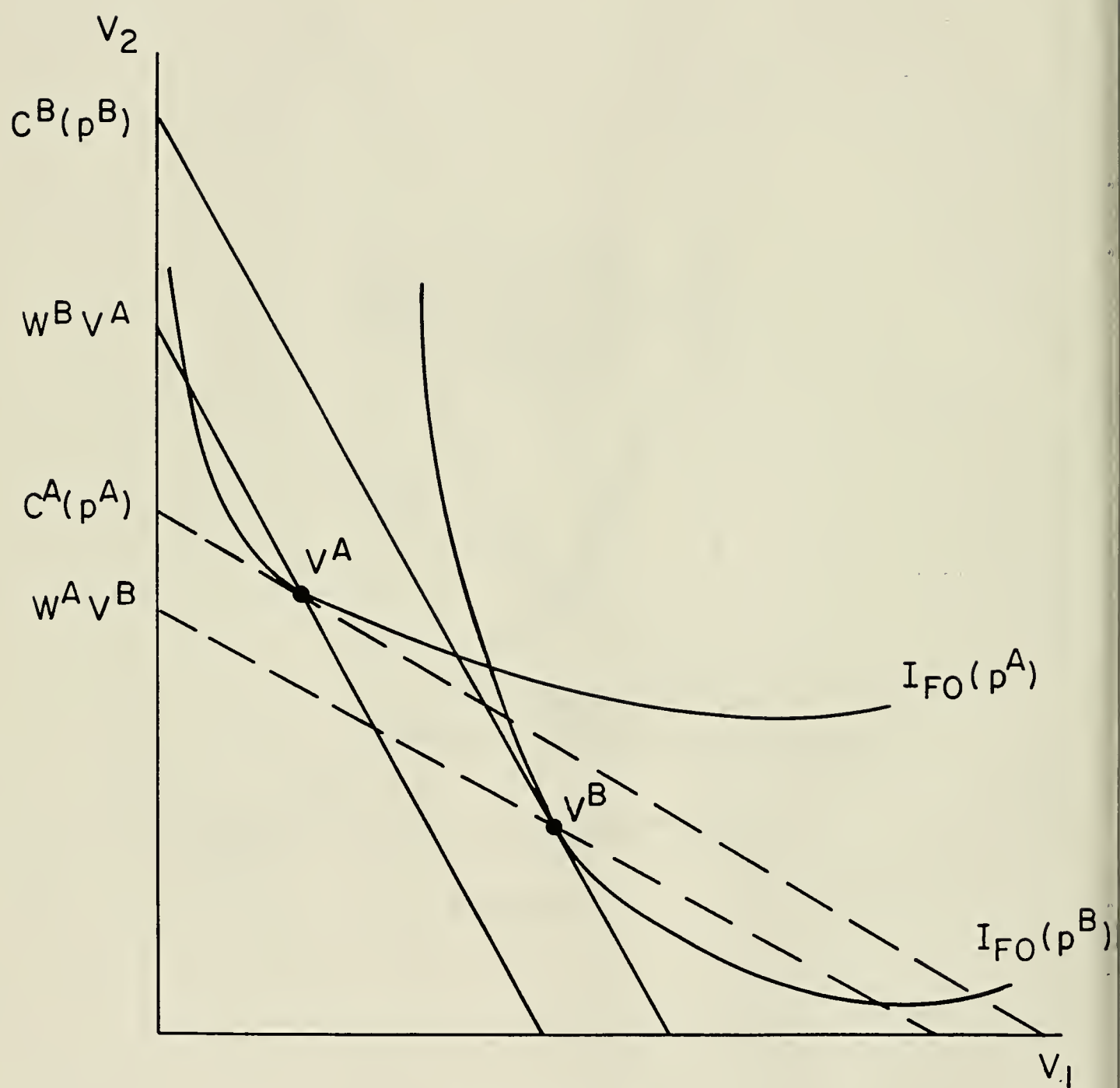


Figure 7.1

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