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TAX INCIDENCE IN A TWO GOOD MODEL

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1. Introduction

Consideration of the full range of effects of taxation is naturally complicated. An effect of this complication has been the use of simplified models for consideration of specific taxes.[1] Frequently models have assumed three or four commodities and further simplifications about the forms of demand and supply. The simplifications have been in assumptions like inelastic supply of factors, factor supplies depending only on their own price, commodities demands that are independent of the distribution of income, and an absence of pure profits. The model in this paper supplements this literature by a further reduction in the number of commodities, but without any of these other special assumptions. To focus on the impact on distribution, the model further assumes only two consumers and no government expenditures. One further simplification is a failure to analyze the

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[1]For a survey of the incidence literature, see Mieszkowski [1969].

determinants of the shares of pure profits which go to different individuals.

Two types of tax changes will be examined. One is a lump sum redistribution between the two individuals. The second is an increase in the excise tax as it applies to just one consumer, with the proceeds returned to the same consumer as a lump sum transfer. Other differential tax changes can be analyzed as a weighted sum of these two types of transfers. To help identify the various elements that affect incidence, there will be analysis of three different versions of the model--the case of fixed producer prices, the general case of this model, and the case of a worker with no profit income and a capitalist with no labor income.

2. Equilibrium Conditions

Let us denote by q, p, and t the consumer price, producer price and specific tax on the nonnumeraire good. The good will be referred to as the taxed commodity (even when t happens to be zero). Then we have the definitional relation among prices and the tax

$$q = p + t \qquad (1)$$

Both producer and consumer price of the numeraire good are taken to be one. The demands for the taxed good by the two persons will be denoted x^A and x^B. Demands are a function of the consumer price and lump sum incomes of the two individuals respectively. For each individual, lump sum income is the sum of the lump sum subsidy from the government, S^A, S^B, and the share of profits in the economy $\theta^A\pi$, $\theta^B\pi$. In addition to aggregate profits[1] being a function of the producer price, $\pi(p)$, the profit shares, in general, are functions of the producer price also.[2] Since we take the economy to be closed, all profits go to the two consumers

$$\theta^A(p) + \theta^B(p) = 1 \tag{2}$$

Denoting aggregate supply of the taxed good by $y(p)$ we can write market clearance as

$$x^A(p+t, S^A+\theta^A(p)\pi(p)) + x^B(p+t, S^B+\theta^B(p)\pi(p)) = y(p) \tag{3}$$

Price and income derivatives of demand will be denoted $\frac{\partial x^i}{\partial q}$ and $\frac{\partial x^i}{\partial I}$. To complete the model we shall use the government budget constraint rather than market clearance in numeraire good:

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[1]Profits equal the value of net supply of the taxed good at the producer price plus the value of the net supply of the numeraire good.

[2]Profit shares will vary with the producer price when there are many firms with different production functions or factors in inelastic supply which are owned in different proportions by the two consumers.

$$S^A + S^B = t(x^A(p+t, S^A+\theta^A(p)\pi(p)) + x^B(p+t, S^B+\theta^B(p)\pi(p))) \quad (4)$$

These two equations, (3) and (4), will be used to relate the equilibrium price and the transfer to B to the transfer to A, for a given tax rate. Given price and tax changes, the changes in utilities can be calculated by differentiation of the indirect utility functions

$$U^i = v^i(p + t, S^i + \theta^i(p)\pi(p)) \qquad i = A,B \qquad (5)$$

In addition to examining a change in lump sum transfers, we will examine the effects of a change in the tax rate.

3. Fixed Producer Prices[1]

With a technology which is locally linear, tax changes that are not too large will not change producer prices. Before consideration of the effects of the induced price changes, we can use this model to examine the effects of existing excise taxes on the effects of lump sum redistribution. With a linear technology, supply will vary to accommodate to demand and we can consider solely the government budget constraint. To finance a unit increase in the lump

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[1]Replacing scalars by vectors where appropriate, the equations of this section are valid with more than two commodities.

sum subsidy to A, the lump sum subsidy to B must decline by the amount[1]

$$\frac{dS^B}{dS^A} = - \frac{1 - t \dfrac{\partial x^A}{\partial I}}{1 - t \dfrac{\partial x^B}{\partial I}} \tag{6}$$

Thus, if the income derivatives of demand for the taxed good are different for the two consumers, this derivative will be different from -1. A transfer to a consumer with a higher marginal propensity to pay taxes out of income costs relatively less. This is not merely a financial consideration, for the same consideration arises in the change in utilities as income is transferred between consumers. With the tax rate held constant we have[2]

$$\frac{dU^B}{dU^A} = - \left(\frac{\dfrac{\partial v^B}{\partial I}}{\dfrac{\partial v^A}{\partial I}} \right) \left(\frac{1 - t \dfrac{\partial x^A}{\partial I}}{1 - t \dfrac{\partial x^B}{\partial I}} \right) \tag{7}$$

From the consumers's budget constraint, we know that one minus the marginal propensity to pay taxes out of income is

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[1]Equation (6) is derived by implicit differentiation of (4).

[2]Since the tradeoff in utilities is the ratio of utility derivatives with respect to S^A, (7) can be obtained by differentiation of (5).

equal to the resource cost of satisfying consumer demand as a consequence of income transfer. Thus the tradeoff in utilities as income is transferred in a lump sum fashion reflects the resource costs of increasing utility. When there are excise taxes present, differences in the costs of increasing utilities can be present, even when the two consumers have equal marginal utilities of income.

4. Income Transfers in the Absence of Excise Taxes[1]

Before considering the general case, it is useful to isolate the effects of the induced price change by considering lump sum redistribution when there is no excise tax. The magnitude of the induced price change will depend on responsiveness of supply to price, $\frac{\partial y}{\partial p}$, and the sum of direct and indirect price effects on aggregate demand. For notational convenience, it is useful to have an expression for the total demand derivative with respect to price. Using the fact that the price derivative of profits equals the level of net supply, we can write the demand derivative as

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[1]The problem analyzed in this section is equivalent to the classical transfer problem. For an exposition see Mundell (1968).

$$\Delta \equiv \frac{d(x^A + x^B)}{dp} = \frac{\partial x^A}{\partial q} + \frac{\partial x^A}{\partial I}(\theta^A y + \pi\theta'^A) + \frac{\partial x^B}{\partial q} + \frac{\partial x^B}{\partial I}(\theta^B y + \pi\theta'^B) \tag{8}$$

Using the Slutsky equation, with s^i as the compensated own price derivatives of the taxed good, and the fact that profit shares add to one, we can rewrite the demand derivative as

$$\begin{aligned} \Delta &= s^A + s^B + \frac{\partial x^A}{\partial I}(\theta^A y - x^A) \\ &\quad + \frac{\partial x^B}{\partial I}(\theta^B y - x^B) + \pi\theta'^A\left(\frac{\partial x^A}{\partial I} - \frac{\partial x^B}{\partial I}\right) \\ &= s^A + s^B + \left(\frac{\partial x^A}{\partial I} - \frac{\partial x^B}{\partial I}\right)(\theta^A y - x^A + \pi\theta'^A) \end{aligned} \tag{9}$$

We can identify two separate elements of the effect of a price increase on aggregate demand, lump sum transfers held constant. One is the effect on compensated demand. This is necessarily negative. The other works through the effect of real incomes on demand. Since consumers own firms, they are on both sides of the market as far as income impacts are concerned. However, they may represent different percentages of the two sides of the market. This will result in a redistribution of real income. In addition, the division of aggregate profits between owners can change as the relative price changes. These effects result in a change in aggregate demand if income derivatives differ. Although it need not always hold, we will assume that net excess demand is decreasing in price,[1]

$$\Delta - \frac{\partial y}{\partial p} < 0 \tag{10}$$

Without this assumption, peculiar results are possible, as we will see in the analysis of a special case below in Section 8.

Since there are now no excise taxes, producer prices do not enter the government budget constraint and the lump sum transfers tradeoff one-for-one

$$\frac{dS^B}{dS^A} = -1. \tag{11}$$

Turning to the market clearance equation we can calculate the change in producer price by implicit differentiation

$$\frac{dp}{dS^A} = - \frac{\frac{\partial x^A}{\partial I} - \frac{\partial x^B}{\partial I}}{\Delta - \frac{\partial y}{\partial p}} \tag{12}$$

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[1]When taxes are introduced, we will continue to assume that excess demand is decreasing in price including the indirect effects on tax revenue.

The change in price necessary to clear the market depends on the demand change induced by redistribution and the price responsiveness of excess demand. We can consider the numerator as giving the impact on demand of redistribution and the denominator as giving the price response to offset the impact effect.

Since prices may change, the effect of a lump sum transfer on utility must consider both direct and indirect effects. Differentiating the indirect utility function and using (9), (12), and Roy's identity $\left(\frac{\partial v}{\partial p} = -x\,\frac{\partial v}{\partial I}\right)$ we have

$$\frac{dU^A}{dS^A} = \frac{\partial v^A}{\partial I}\left[1 + \frac{dp}{dS^A}(-x^A+\theta^A y+\pi\theta'^A)\right]$$

$$= 1 - \frac{\left(\frac{\partial x^A}{\partial I} - \frac{\partial x^B}{\partial I}\right)(\theta^A y - x^A + \pi\theta'^A)}{\Delta - \frac{\partial y}{\partial p}} \qquad (13)$$

$$= \frac{\partial v^A}{\partial I}\left(\frac{s^A + s^B - \frac{\partial y}{\partial p}}{\Delta - \frac{\partial y}{\partial p}}\right)$$

Thus the total effect of the transfer on utility will differ from its impact effect $\left(\frac{\partial v^A}{\partial I}\right)$ insofar as income effects matter for aggregate demand responsiveness.

The expression for the change in the utility of B is similar to that for A, implying that the tradeoff in utilities does not depend on properties of demand derivatives

$$\frac{dU^B}{dU^A} = - \frac{\frac{\partial v^B}{\partial I}}{\frac{\partial v^A}{\partial I}} \tag{14}$$

However, the speed with which one moves along this frontier by redistributing income does depend on induced price changes and so the effects of income redistribution on aggregate demand.[1]

5. Income Redistribution

We have now considered lump sum redistribution in two special cases--first where prices don't change and second where excise tax revenues don't change (being zero). We will now turn to the general case, simultaneously differentiating the market clearance equation and the government budget constraint. We will first differentiate with the derivatives of the price and the tax on B appearing in each equation, and then solve for the two derivatives explicitly.

$$\frac{\partial x^A}{\partial I} + \frac{\partial x^B}{\partial I}\frac{dS^B}{dS^A} = -\left(\Delta - \frac{\partial y}{\partial p}\right)\frac{dp}{dS^A} \tag{15}$$

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[1]If there were a third consumer present in the economy, analysis of a transfer between the first two consumers would be altered since $(x^A-\theta^A y-\pi\theta'^A)$ would not necessarily equal $(-x^B+\theta^B y+\pi\theta'^B)$.

$$\left(1 - t\,\frac{\partial x^A}{\partial I}\right) + \left(1 - t\,\frac{\partial x^B}{\partial I}\right)\frac{dS^B}{dS^A} = t\Delta\,\frac{dp}{dS^A} \tag{16}$$

or, solving these two equations,

$$\frac{dp}{dS^A} = \frac{\dfrac{\partial x^B}{\partial I} - \dfrac{\partial x^A}{\partial I}}{\Delta - \left(1 - t\,\dfrac{\partial x^B}{\partial I}\right)\dfrac{\partial y}{\partial p}} \tag{17}$$

$$\begin{aligned}\frac{dS^B}{dS^A} &= -\,\frac{\Delta - \left(1 - t\,\dfrac{\partial x^A}{\partial I}\right)\dfrac{\partial y}{\partial p}}{\Delta - \left(1 - t\,\dfrac{\partial x^B}{\partial I}\right)\dfrac{\partial y}{\partial p}} \\ &= -1 + \frac{t\left(\dfrac{\partial x^B}{\partial I} - \dfrac{\partial x^A}{\partial I}\right)\dfrac{\partial y}{\partial p}}{\Delta - \left(1 - t\,\dfrac{\partial x^B}{\partial I}\right)\dfrac{\partial y}{\partial p}}\end{aligned} \tag{18}$$

Examining these equations, there are several points which can be made. If we have a locally linear technology, $\frac{\partial y}{\partial p}$ is infinite, and these equations do reduce to those analyzed above in Section 3. At the other extreme, if we have a pure exchange economy, $\frac{\partial y}{\partial p}$ is zero. Then the transfers must be one-for-one since excise tax revenue cannot change. This point would be clearer if we had written tax revenue as ty rather than $t(x^A + x^B)$. We can consider the price derivative as being the response to the impact of the initial transfer on demand, recognizing that in addition to the effect of price on excess demand described above, $\Delta - \frac{\partial y}{\partial p}$, there is a further feedback effect of prices on tax revenue, $t \frac{\partial y}{\partial p}$, and so on the lump sum income of B and thus on his demand. Also, the change in prices affects government revenue and thus the level of transfer from B necessary to finance a unit transfer to A. When income derivatives of demand are equal, the transfer affects neither net demand nor net tax revenue implying a one-for-one tradeoff. Where this impact is not zero the impact effect changes prices, and thus tax revenue, by the amount $t \frac{\partial y}{\partial p} \frac{dp}{dS^A}$.

As above we can differentiate the indirect utility functions to calculate utility derivatives which reflect both of the elements described in the previous two sections.

$$\frac{dU^A}{dS^A} = \frac{\partial v^A}{\partial I} \frac{s^A + s^B - \left(1 - t \frac{\partial x^B}{\partial I}\right) \frac{\partial y}{\partial p}}{\Delta - \left(1 - t \frac{\partial x^B}{\partial I}\right) \frac{\partial y}{\partial p}} \tag{19}$$

$$\frac{dU^B}{dS^A} = - \frac{\partial v^B}{\partial I} \frac{s^A + s^B - \left(1 - t \frac{\partial x^A}{\partial I}\right) \frac{\partial y}{\partial p}}{\Delta - \left(1 - t \frac{\partial x^B}{\partial I}\right) \frac{\partial y}{\partial p}} \tag{20}$$

Of course, these equations reduce to those of the previous two sections if we have fixed prices ($\frac{\partial y}{\partial p}$ infinite) or fixed excise tax revenue, ($\frac{\partial y}{\partial p}$ zero). From these equations we can determine the utility tradeoff as the government redistributes from B to A

$$\frac{dU^B}{dU^A} = - \left(\frac{\frac{\partial v^B}{\partial I}}{\frac{\partial v^A}{\partial I}}\right)\left(\frac{s^A + s^B - \left(1 - t\,\frac{\partial x^A}{\partial I}\right)\frac{\partial y}{\partial p}}{s^A + s^B - \left(1 - t\,\frac{\partial x^B}{\partial I}\right)\frac{\partial y}{\partial p}}\right) \tag{21}$$

Thus the difference in marginal propensities to pay taxes ut of income is a necessary part of the divergence of the utility tradeoff from the ratio of marginal utilities of income. The effect of the difference, however, depends on the relative magnitudes of supply and compensated demand derivatives.

6. Changing Excise Taxes with Fixed Producer Prices

When changing the excise tax, lump sum taxes must generally be changed to continue to balance the government's budget. There are various combinations of excise and lump sum changes which might be considered. The particular one to be considered is that of increasing the excise tax, just for A, returning the proceeds as a lump sum back to A. This variant was chosen for two reasons. One is to facilitate comparison with the analysis of deadweight burden in the one consumer economy. The second is to permit the following method of analysis of a tax-transfer scheme. Consider separately the tax changes, assuming that the revenue for each person is returned to the same person. Then consider a pattern of lump sum transfers to take the revenue from the taxpayers, giving it to the actual recipients. The incidence of the tax-transfer scheme is the sum of these two effects. No doubt there are other ways of dividing up the total effects of a tax transfer scheme which also assist its interpretation.

We now need to distinguish the excise taxes levied on A and B--t^A and t^B. We shall consider a change in t^A, t^B and S^B held constant. The evaluation of this derivative will be done at the point where t^A and t^B are equal. To begin we are considering the case of fixed producer prices. Thus we need only consider the government's budget constraint. By assumption, S^B and U^B are unaffected by the tax increase. (This will not be true in an economy with varying prices.) From the public budget constraint and the indirect utility function we have

$$\frac{dS^A}{dt^A} = \frac{x^A + t\dfrac{\partial x^A}{\partial q}}{1 - t\dfrac{\partial x^A}{\partial I}} = x^A + \frac{ts^A}{1 - t\dfrac{\partial x^A}{\partial I}} \qquad (22)$$

$$\frac{dU^A}{dt^A} = \left(\frac{\partial v^A}{\partial I}\right)\frac{ts^A}{1 - t\dfrac{\partial x^A}{\partial I}} \qquad (23)$$

Naturally, in this economy with fixed producer prices, the presence of an additional consumer in the economy results in no change in the analysis of deadweight burden.

It is a straightforward calculation to check the general principle that the effect on utility of an increase of the excise tax on A with the proceeds going to B can be written as a combination of the above derivatives, $\frac{dU^i}{dt^A} - \frac{dU^i}{dS^A}\frac{dS^A}{dt^A}$, adding the effects of returning the revenue to A to the effects of a lump sum transfer from A.

7. Changing Excise Tax

As in the previous section, we will increase the excise tax on A, returning the revenue as a lump sum to A. Solving the simultaneous differentiation of the two equilibrium conditions we have

$$\frac{dp}{dt^A} = \frac{-s^A}{\Delta - \left(1 - t\,\frac{\partial x^A}{\partial I}\right)\frac{\partial y}{\partial p}} \tag{24}$$

$$\begin{aligned}\frac{dS^A}{dt^A} &= \frac{x^A\Delta - \left(x^A + t\,\frac{\partial x^A}{\partial q}\right)\frac{\partial y}{\partial p}}{\Delta - \left(1 - t\,\frac{\partial x^A}{\partial I}\right)\frac{\partial y}{\partial p}} \\ &= x^A - \frac{ts^A\,\frac{\partial y}{\partial p}}{\Delta - \left(1 - t\,\frac{\partial x^A}{\partial I}\right)\frac{\partial y}{\partial p}}\end{aligned} \tag{25}$$

As above, the price derivative is expressed as an impact effect on demand of excise tax increase and return of the revenue (ignoring the decline in tax revenue as a result of the demand decrease) adjusted for the full effects of price on excess demand including the effect of the change in tax revenue on the demand of A, the recipient of this revenue. When the producer price is fixed, (25) reduces to (22), the derivative calculated above. Alternatively, if supply is completely inelastic, then there can be no change in the equilibrium quantity demanded and the change in tax equals the quantity subjected to the higher excise tax. As above, the change in transfer differs from the impact change (x^A) by the product of the tax rate, the derivative of supply to price and the change in price, that is, the difference equals the price change induced change in tax revenue.

The impact of the tax-transfer on B depends on how he is affected by the price increase. It depends, that is, on his relative position on the two sides of this market and the effect on his profit share.

$$\frac{dU^B}{dt^A} = - \frac{\partial v^B}{\partial I}\left(x^B - \theta^B y - \pi\theta'^B\right)\frac{dp}{dt^A} \tag{26}$$

The change in utility of A combines the direct effects on his utility (like those appearing in the one consumer economy) with those that flow from his relative position in the market and changing profit share

$$\frac{dU^A}{dt^A} = \left(\frac{\partial v^A}{\partial I}\right)\frac{-ts^A \frac{\partial y}{\partial p} + s^A\left(x^A - \theta^A y - \pi\theta'^A\right)}{\Delta - \left(1 - t\,\frac{\partial x^A}{\partial I}\right)\frac{\partial y}{\partial p}} \tag{27}$$

To see the relationship of this to deadweight burden in the one consumer economy we can examine the sum of utilities, assuming equal marginal utilities of income, for the redistributions net out once we add across all consumers.

$$\frac{d(U^A + U^B)}{dt^A} = \frac{\partial v^A}{\partial I} \frac{-ts^A \frac{\partial y}{\partial p}}{\Delta - \left(1 - t\frac{\partial x^A}{\partial I}\right)\frac{\partial y}{\partial p}} = \frac{\partial v^A}{\partial I} t \frac{\partial y}{\partial p} \frac{dp}{dt^A} \tag{28}$$

Thus, as in the one consumer case, the marginal deadweight loss equals the loss in tax revenue (ty) as a consequence of the response of equilibrium to the tax increase.

8. An Example

For a particularly simple example, we can consider the economy with a single worker, A (who has no profit income) and a single capitalist, B (who does no work). We shall follow the same sequence of steps as above, using the natural notation of w for wage, L for labor and f(L) for production. For the compensated supply derivative of labor relative to the wage we will use s. For convenience we will consider the percentage wage tax τ. Diagrammatic analysis will be presented for the simpler cases.

In the absence of an excise tax the equilibrium in the economy can be described by the single equation that the wage equals the marginal product of labor

$$w = f'(L(w, S^A)) \tag{29}$$

If there is no lump sum transfer to the worker, $S^A = 0$, then equilibrium will appear as in Figure 1.

[Insert Figure 1 here]

If we now transfer income to the worker, the change in the wage satisfies

$$\frac{dw}{dS^A} = \frac{f'' \frac{\partial L}{\partial I}}{1 - f'' \frac{\partial L}{\partial w}} \tag{30}$$

There are three different cases which need to be considered separately. Before considering these cases let us examine one of the borderline cases because of its simplicity. If labor supply is unresponsive to income then the wage does not change and we have the situation shown in Figure 2. A

[Insert Figure 2 here]

parallel upward shift in the worker's budget line (corresponding to the lump sum transfer he receives) leaves the equilibrium position of the firm unchanged. Since the supply of labor of the capitalist is necessarily unchanged, we have equal income derivatives of labor supply on both sides of the transfer.

If the labor supply is increasing with income, the denominator of (30) is necessarily positive and the equilibrium wage falls. When we shift the worker's budget upward, keeping it parallel, he now wants to work more than previously. This lowers the equilibrium wage, represented by a rotation of his budget line. The new equilibrium is found when an indifference curve is tangent to a budget line which parallels the production function's tangent at the given supply of labor, as shown, for example, in Figure 3.

[Insert Figure 3 here]

When labor supply is decreasing with income, the denominator of (30) may be positive or negative. If it is positive, the equilibrium is as shown in Figure 4. The decreased

[Insert Figure 4 here]

desire to work as a result of the income transfer raises the equilibrium wage. If the denominator is negative we have a supply curve of labor which is sufficiently backward bending to cross the demand for labor in an "unstable" way. The income transfer which reduces labor supply at any wage then results in an increase in the level of work in equilibrium. Such a situation is shown in Figure 5. As can be

[Insert Figure 5 here]

seen from the diagram, worker utility is now lowered as a consequence of a transfer to the worker. To see this we can calculate the derivative of worker utility with respect to the transfer

$$\frac{dU^A}{dS^A} = \left(\frac{\partial v^A}{\partial I}\right)\frac{1 - f''s}{1 - f''\frac{\partial L}{\partial w}} \tag{31}$$

Since the numerator is positive, the sign of the utility change is the same as the sign of the denominator. The utility of the capitalist moves in the opposite direction to that of the worker, since all equilibria without excise taxes are Pareto optimal.

With a fixed tax on wage income, the equilibrium equation becomes

$$w = f'(L(w(1-\tau), S^A)) \tag{32}$$

A fixed transfer is being provided to workers. Thus any increase in wage tax revenue when the transfer is increased goes to the capitalist. The change in wage satisfies

$$\frac{dw}{dS^A} = \frac{f''\frac{\partial L}{\partial I}}{1 - f''\frac{\partial L}{\partial w}(1 - \tau)} \tag{33}$$

This gives rise to utility changes

$$\frac{dU^A}{dS^A} = \left(\frac{\partial v^A}{\partial I}\right)\frac{1 - f''(1 - \tau)s}{1 - f''\frac{\partial L}{\partial w}(1 - \tau)} \tag{34}$$

$$\frac{dU^B}{dS^A} = -\left(\frac{\partial v^B}{\partial I}\right)\frac{1 - \tau w \frac{\partial L}{\partial I} - f''(1 - \tau)s}{1 - f'' \frac{\partial L}{\partial w}(1 - \tau)} \tag{35}$$

Recognizing that the marginal propensities to pay tax out of income of the capitalist and worker are 0 and $\tau w \frac{\partial L}{\partial I}$ respectively, this can be seen to have the same form as the general case.

Turning now to a change in the ad valorem tax, with the revenue returning to the worker, we have the equilibrium equations

$$w = f'(L(w(1-\tau), S^A) \tag{36}$$

$$S^A = \tau wL(w(1-\tau), S^A) \tag{37}$$

Upon differentiation we have the change in the wage as

$$\frac{dw}{d\tau} = \frac{-f''ws}{1 - \tau w \frac{\partial L}{\partial I} - f''\left[\tau L \frac{\partial L}{\partial I} + (1-\tau)\frac{\partial L}{\partial w}\right]} \tag{38}$$

The change in the utility of the capitalist just depends on the change in the wage

$$\frac{dU^B}{d\tau} = - \frac{\partial v^B}{\partial I} L \frac{dw}{d\tau} \tag{39}$$

The change in lump sum income and utility of the worker satisfy

$$\frac{dS^A}{d\tau} = w \frac{L - \tau w \frac{\partial L}{\partial w} - f''L \frac{\partial L}{\partial w}}{1 - \tau w \frac{\partial L}{\partial I} - f''\left[\tau L \frac{\partial L}{\partial I} + (1-\tau)\frac{\partial L}{\partial w}\right]} \tag{40}$$

$$\frac{dU^A}{d\tau} = w \frac{\partial v^A}{\partial I} \frac{-\tau ws - sLf''}{1 - \tau w \frac{\partial L}{\partial I} - f''\left[\tau L \frac{\partial L}{\partial I} + (1-\tau)\frac{\partial L}{\partial w}\right]} \tag{41}$$

Since the worker gains in transfers what the capitalist loses, $L \frac{dw}{d\tau}$, the worker's utility will rise if the marginal deadweight cost of the tax is less than the financial gain from the increased wage.

References

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E E   marks the equilibrium without transfers

E'E'   marks the equilibrium with transfers

Figure 1

Competitive Equilibrium - No Transfers

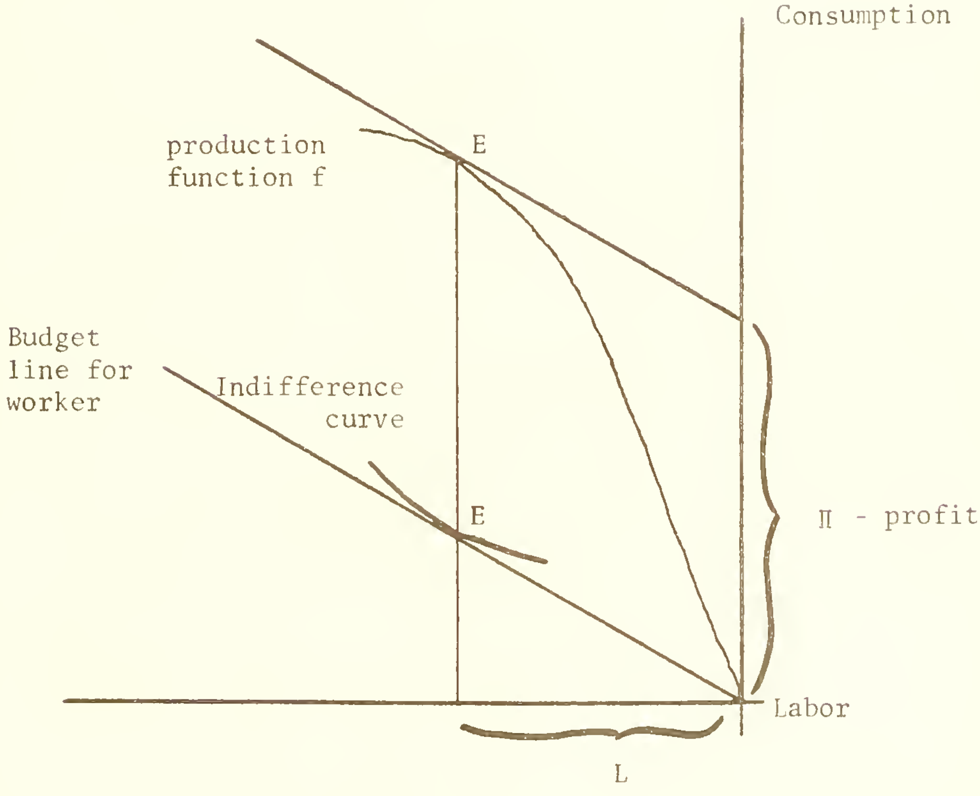

Figure 2

Lump Sum Transfer: $\frac{\partial L}{\partial I} = 0$

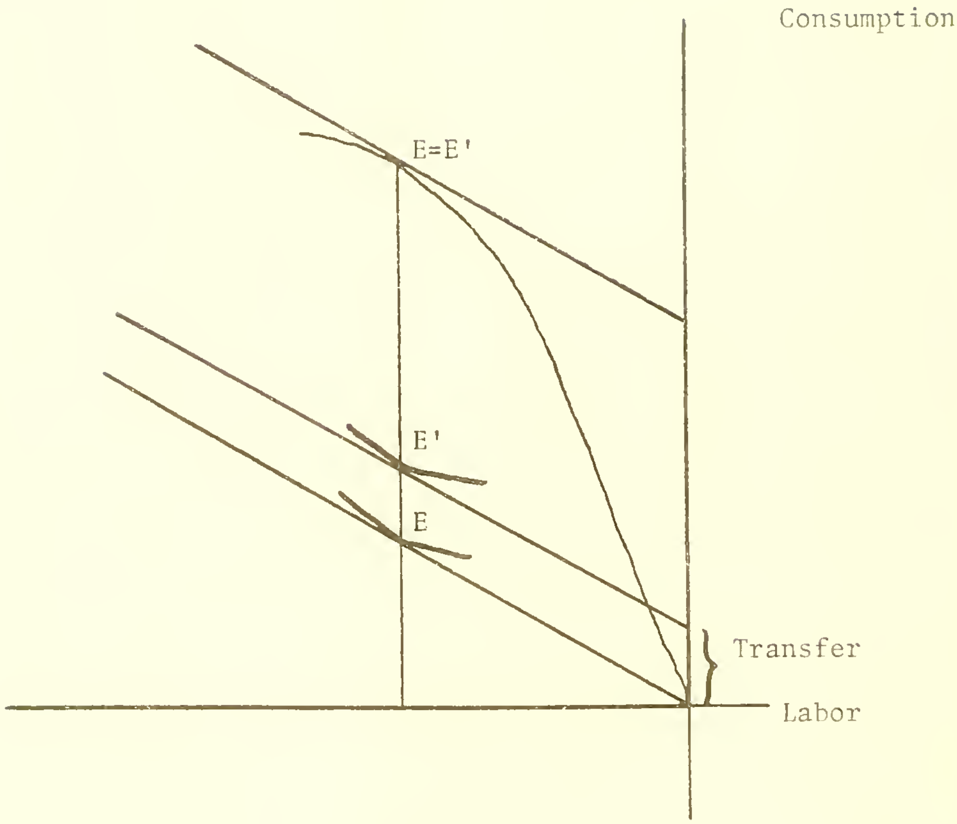

Figure 3

Lump Sum Transfer: $\frac{\partial L}{\partial I} > 0$

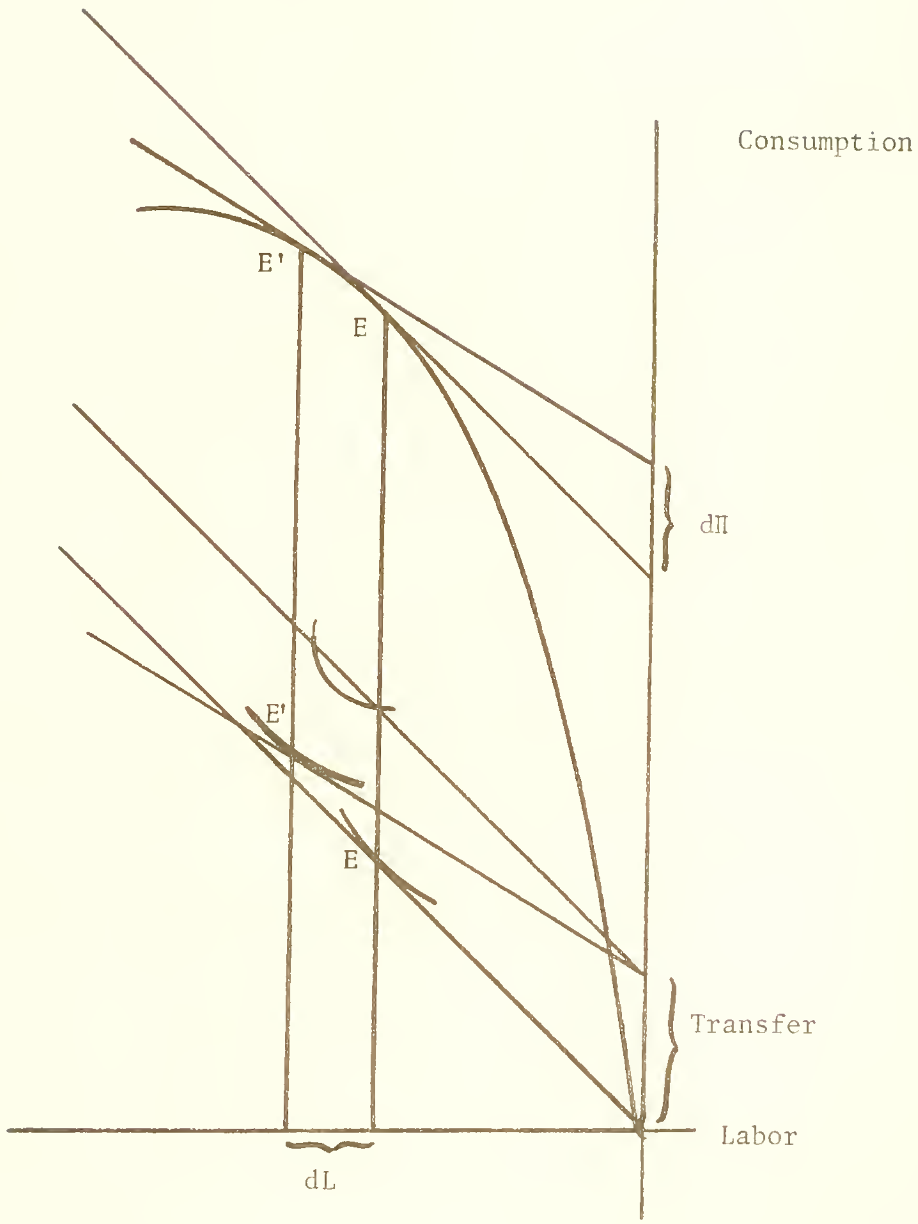

Figure 4

Lump Sum Transfer: $\frac{\partial L}{\partial I} < 0$ and $1 - f''\frac{\partial L}{\partial w} > 0$

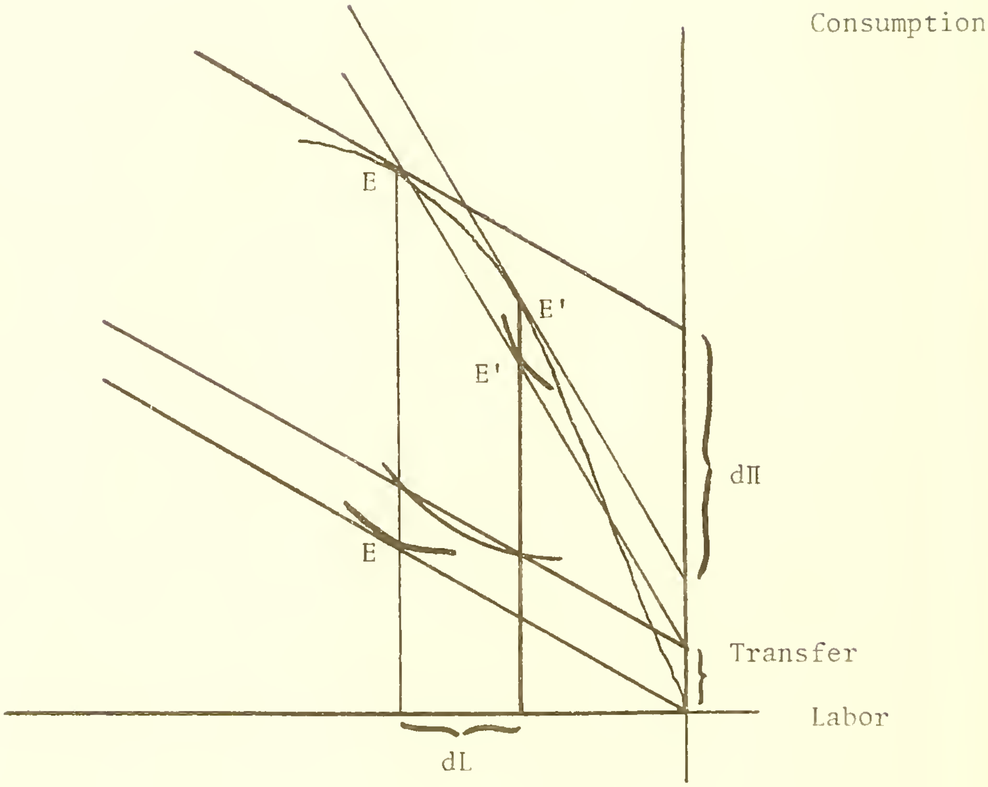

Figure 5

Lump Sum Transfer: $\frac{\partial L}{\partial I} < 0$ and $1 - f''\frac{\partial L}{\partial w} < 0$