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ABSTRACT

This paper explores the application of contingent claims analysis (CCA) to two “hot” issues in life-cycle finance: (1) investing for retirement and (2) deciding when, if ever, to switch careers. Participants in individual retirement accounts do not have the time or the knowledge to make their own investment decisions. Today they are defaulted into life-cycle mutual funds that pass all risk directly through to the participant. We use CCA to demonstrate how financial firms can design and produce guaranteed contingent benefit contracts that improve participant welfare at no additional cost to the system. In exploring the career-choice issue in the second part of the paper, we use CCA in a somewhat different way. The decision to switch careers is analogous to deciding when to exercise an American-style option to swap one asset for another. By applying the methods used to analyze the option-exercise decision to the career-switching problem, we gain some new insights beyond those derived from the traditional dynamic programming approaches.

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This paper explores the application of contingent claims analysis (CCA) to two important issues in life-cycle finance: (1) investing for retirement and (2) deciding when, if ever, to switch careers. Contingent claims analysis is a methodology that grew out of the option pricing theory of Fischer Black, Robert C. Merton, and Myron Scholes.¹ They derived the option pricing model by showing that there is a self-financing dynamic trading strategy that replicates the payoffs from a call option. In the absence of transaction costs, the law of one price and the force of arbitrage imply that the cost of the initial replicating portfolio is the price of the option. That same approach applies to any derivative security or contingent contract based on traded assets. For every dynamic trading strategy there exists an equivalent contingent contract.²

In reality most investors face substantial transactions costs and cannot trade even approximately continuously as done in the theoretical models. But in a modern, well-developed financial system, the lowest-cost transactors may have marginal trading costs *close to* zero, and can trade *almost* continuously. Thus, the lowest-cost producers of contingent contracts can approximate reasonably well the dynamic trading strategy, and in a competitive environment their cost of replicating the contingent contract is approximately the theoretical price.

In the first part of this paper, we make use of this idea to address the problem faced by consumers saving and investing for retirement. Today's retirement saving plans consist of mutual funds that pass all investment risk through to plan participants. They are being replaced by semi-customized contingent contracts designed and guaranteed by financial intermediaries. From contingent-contract-equivalence it follows that a low-transaction-cost financial

¹ Black and Scholes (1973) and Merton (1973).

²This contingent-contract equivalence to dynamic portfolio strategies can be derived by running the option-pricing derivation "in reverse." This type of procedure is developed in Merton (1989, pp. 250–254, 1992, pp. 450–464).

intermediary can sell to high-transaction-cost customers fully hedged (“immunized”) contracts that have the contingent payoffs associated with an optimized portfolio strategy. The intermediary pursues the dynamic trading strategy at its lower transaction costs and provides the specified contractual benefits to its customers.

In part II of this paper we apply CCA to the issue of career choices. The ability to change careers over one’s working life is isomorphic with an option to exchange one asset for another. Having this option significantly affects the value of one’s human capital, the largest asset that most individuals possess over much of their adult lives. Choosing the optimal strategy involves selecting an initial career and then switching later in life, if the conditions warrant it. Moreover, the switching option affects the consumer’s optimal portfolio mix of safe and risky assets. CCA provides a methodology for studying and evaluating all of these effects.

For the sake of clarity of exposition, throughout this paper we illustrate the main ideas using simple binomial models. To implement the kind of financial engineering required in the real world, however, much more mathematically complex models are required.

I. Retirement Saving Plans and Contingent Contracts

One need not have a crystal ball to envision a future in which financial firms produce personal retirement benefit contracts. The process is already underway. Several major financial firms have launched products of this sort, and others have announced plans to do so.

We envision a 3-stage process for designing and producing such contracts. It is illustrated in Figure 1. In stage 1 the firm determines its production cost schedule (i.e., the cost of state-contingent consumption) taking as inputs the prices of stocks, bonds, and other securities traded in markets. In other words, it computes implicit prices for elementary time-state securities, known in the economics literature as Arrow–Debreu prices.

Figure 1

In stage 2, the firm designs “standard-model” contracts, by maximizing expected utility for “representative” consumers in various categories (e.g., single versus married) under a set of circumstances that are faced by most of them, such as the prevailing tax code and the mandatory social insurance system. As is routinely done today with insurance policies, these standard-model contracts can be modified by adding optional features to suit the special circumstances of different types of consumers. After the contract specifications are finalized, the firm commences stage 3, the process of dynamic replication.

To clarify the design and production process for such personal pension contracts, consider the simple binomial model illustrated in Figure 2. There is a single consumption good, and prices and interest rates are measured in units of that good. There are three periods separated

by three points in time: $t = 0, 1, 2$. There is a single risky asset – a stock, and in each period it can either go up by u or down by d . Each has equal probability of .5. In addition there is a safe asset that pays a fixed real rate of interest, r .

Figure 2

Each node in Figure 2 is a different time-state of the economy. Arrow-Debreu securities are not traded in the market, but the intermediary can synthesize them through dynamic trading in the stock and risk-free bonds and figure out the production costs for each one. Figure 3 shows the dynamic trading strategy required to produce an AD security that pays 1 consumption unit in time-state uu and 0 in all other time-states.

The initial portfolio at $t = 0$, consists of 1.31 shares of stock, each costing 1 consumption unit, and a short position in risk free bonds equal in value to 1.16 consumption units. The net

cost of this portfolio is .15 consumption units, and by the law of one price, this is the price of a *uu* AD security.

Rebalancing of the portfolio occurs at $t = 1$. If the stock index goes up, the portfolio has a value of .39 consumption units, and the long position in stock is increased to 3.33 consumption units and the short bond position is increased to 2.94. No new infusion of cash is required. If the stock index goes down, the portfolio is liquidated, and it yields exactly enough to pay off the principal and interest on the bonds. At $t = 2$, the payoff will be 1 consumption unit in the *uu* state, and zero in all other states.

Figure 3

Once the financial firm has determined the replication costs of the AD securities, it is ready to structure standard-model life-cycle contracts for different categories of customers. For example, let's say it decides to offer a standard-model contract designed for people with logarithmic utility and a rate of time preference equal to zero. Its production cost is 100 units of consumption. It finds the optimal contingent benefits in each time-state by maximizing expected utility for an individual whose total wealth – human plus financial -- is 100 units of consumption.

The use of AD prices enables the firm to solve for the optimal consumption plan using a static optimization framework rather than having to employ the backward induction methods of dynamic programming.³ Formally, the problem is to maximize expected utility from consumption, subject to the budget constraint:

$$\begin{aligned}
& W_0 + Y_0 + \pi^u Y_1^u + \pi^d Y_1^d \\
& \geq C_0 + \pi^u C_1^u + \pi^d C_1^d + \pi^{uu} C_2^{uu} + \pi^{ud} C_2^{ud} + \pi^{du} C_2^{du} + \pi^{dd} C_2^{dd}
\end{aligned}$$

The solution is as shown in the second column of Table 1. Figure 4 describes the investment strategy followed by the financial firm needed to replicate that particular consumption contract.

Table 1. Benefit Structure for Standard-Model Contingent Consumption Contracts

Contingent Consumption	Log Utility	Safe Contract	Log Utility with Habit Formation in Old Age
$C0$	33.33	33.33	33.33
Cu	42.50	34	40.14
Cd	28.33	34	26.76
Cuu	54.18	34.68	54.18
Cud	36.12	34.68	40.14
Cdu	36.12	34.68	36.12
Cdd	24.08	34.68	26.76

Figure 4 shows that the firm does not have to actually create AD securities. Rather, it pursues the same dynamic strategy the client would have pursued to maximize her expected utility had she solved the problem on her own using the methods of dynamic programming. This

³ As shown by John Cox and Chi-Fu Huang (1989), this static formulation is often much easier to solve.

makes it clear that the AD prices computed in stage 1 serve as “shadow” prices in the design and production process rather than transaction prices.

Figure 4.

When multiple sources of risk are considered – for example, when interest rate risk is introduced – the binomial model is no longer adequate. Continuous-time modeling and more advanced optimization methods are required.⁴

II. The Option to Switch Careers

For most people, income from work is the major source of funding for lifetime consumption. The present value of an individual’s lifetime potential earnings from work is called human capital. Zvi Bodie, Robert C. Merton, and William Samuelson (1992), modeled human capital as equivalent to a portfolio of risky stocks and risk-free bonds. In recent work, Doriana Ruffino

⁴See Jerome Detemple, Rene Garcia, and Marcel Rindisbacher (2003), and Jonathan Treussard (2008).

and Jonathan Treussard (2007) and Treussard (2007) have modeled the ability to switch careers as an American-style spread option.⁵

In their models, a career is characterized by a stochastic process for labor income with a certain mean and standard deviation. People will choose to switch when an alternative career exceeds the value of their existing career by more than some threshold amount. Ruffino and Treussard show that if people can switch careers it increases the value of their human capital and reduces its risk. Let's look at the option to switch careers in a simple binomial model illustrated in Figure 5.

There are two careers, one safe and the other risky. Labor income in the safe career grows at a constant rate. Labor income for the risky career is perfectly correlated with a traded stock index, so its risk is explicitly priced in the market and can be hedged. The structure of stock and bond returns is the same as in Figure 1, and thus, the AD prices derived in Part I are used to value the income flows in Part II.

The individual must decide in which occupation to start, knowing that she may switch to the alternative career at $t=1$ or at $t=2$. We assume that switching can occur only once – thus, if the option to switch is exercised at $t=1$, the individual cannot switch back to her original career at $t=2$.

Let us first consider the present value of each career, ignoring the option to switch. In Figure 5 the last two entries in each vector represent the present value of either staying in the safe or risky career until retirement. From Figure 5 we see that the safe career has the higher present value -- 300 consumption units versus 294 for the risky career.

⁵ Unlike European options that can be exercised only at their maturity dates, American options can be exercised any time up to (and including) maturity.

Figure 5

When we take account of the option to switch careers either at $t=1$ or $t=2$, we see that there are two nested choices: *whether* to exercise the option and, if so, *when* to exercise it. At each node, the individual has the choice between switching careers and remaining in her current occupation for one more period.

Option $Val_t = \text{Max}[\text{Alt Career}_t - \text{Initial Career}_t - \text{Cost}, \pi^u \times \text{Option Val}_{t+1}^u + \pi^d \times \text{Option Val}_{t+1}^d]$, where π^u and π^d are Arrow-Debreu prices.

Now consider the option to change careers when the switching cost is zero. At $t=1$, the individual has the choice between switching careers and remaining in her current occupation. Table 2 below reports option values, optimal exercise policies, and the resulting value of human capital depending on whether the individual starts out in the safe or risky career.

Table 2

Start in Safe Career					Start in Risky Career			
State	Value of Switching Option	Switch?	Labor Income	Value of Human Capital	Value of Switching Option	Switch?	Labor Income	Value of Human Capital
<i>o</i>	6.99	-	100	<u>306.87</u>	13.39	-	100	<u>307.11</u>
<i>u</i>	18.00	Yes	112.14	221.92	0.61	No	112.14	222.53
<i>d</i>	0	No	101.98	203.92	22.23	Yes	101.98	203.92
<i>uu</i>	21.76	Yes	125.76	125.76	0	No	125.76	125.76
<i>ud</i>	0	No	102.96	102.96	1.03	Yes	104	104
<i>du</i>	0	No	104	104	1.03	Yes	104	104
<i>dd</i>	0	No	104	104	19.70	Yes	104	104

The optimal strategy is to start in the risky career, and then switch to the safe career as soon as labor income goes down. Thus the individual will stay in the risky career for all three periods unless labor income goes down either at $t = 1$ or $t = 2$. The PV of human capital under the optimal strategy is 307.11 consumption units.

If, however, there were a switching cost of 10 consumption units, the optimal strategy would be to start in the safe career, and switch if labor income in the risky career goes up at $t=1$. The present value of the optimal strategy in this case is 302.99 versus a present value of 300.95 for the alternative strategy of starting in the risky career.

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