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Concurrent calorimetric and interferometric studies of steady-state natural convection from miniaturized horizontal single plate-fin systems and plate-fin arrays

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Abstract Concurrent calorimetric and interferometric studies have been conducted to investigate the effect that reduction of the base-plate dimensions has on the steady-state performance of the rate of natural convection heat transfer from miniaturized horizontal single plate-fin systems and plate-fin arrays. The effect was studied through comparison of the present results with those of earlier relevant calorimetric, interferometric, or numerical studies. Results shown that a reduction of the base-plate area by 74 percent increased natural convection coefficient by 1.5 times to $26.0 \text{ W m}^{-2} \text{ K}^{-1}$ for single fin systems and by 1.8 times to $18 \text{ W m}^{-2} \text{ K}^{-1}$ for fin arrays in the range of the base-plate temperature excess of $20\text{--}50^\circ\text{C}$. A simple correlation for the Nusselt number of miniaturized horizontal plate-fin arrays is proposed in the range of Rayleigh number divided by the number of fins to the 2.7 power from 2×10 to 5×10^5 .

List of symbols

A	Heat transfer surface area, m^2
Gr	Grashof number
h	Local natural convection heat transfer coefficient, $\text{W m}^{-2} \text{ K}^{-1}$, defined by (3)
$\bar{h}$	Average natural convection heat transfer coefficient, $\text{W m}^{-2} \text{ K}^{-1}$, defined by (1)

$\bar{h}^*$	Average dissipation coefficient under dominant natural convection conditions, i.e. average natural convection/radiation heat transfer dissipation coefficient, $\text{Wm}^{-2}\text{K}^{-1}$, calculated by setting $Q_r = 0$ in (1)
H	Fin height, mm, see Table 1
I	Measured dc current of main heater, A
k	Thermal conductivity, $\text{Wm}^{-1}\text{K}^{-1}$
l	Half of the fin length, mm
L	Fin length, mm, see Table 1
m	Fringe shift
n	Number of fins in an array, see Table 1
Nu	Average Nusselt number
Pr	Prandtl number
q	Rate of natural convection heat transfer of one side of a plate fin, W
Q	Rate of heat transfer from a single plate-fin system or a plate-fin array, W

Error! Bookmark not defined. Ra Rayleigh number

S	Fin spacing, mm, i.e. the inter-fin separation distance, see Table 1
t	Fin thickness, mm, see Table 1
T	Absolute temperature, K
$\bar{T}$	Average absolute temperature, K
V	Measured dc voltage across main heater, V
w	Uncertainty interval
W	Array base-plate width, mm, see Table 1
y	Vertical distance along the fin wall measured from the fin base, mm

Greek letters

ε	Total emissivity
η	Efficiency
θ	Temperature excess, i.e. the amount by which the local temperature

exceeds that at the ambient air, K

$\bar{\theta}$ Average temperature excess, K

Subscripts

a Air

b Base plate

c Surface of one side of a central or inner plate-fin

cal Calorimetric method

f Plate-fin of a single-fin system or all plate-fins of an array

fm Fin material

H Non-dimensional parameter evaluated with the fin height as the characteristic length

ie Inner surface of an end plate-fin

int Intermediate base area between the maximum and the minimum

l Non-dimensional parameter evaluated with half of the fin length as the prime characteristic length

ℓ (=1, 2, 3, or 4), integer indicating fringe shift location from the fin base

L Non-dimensional parameter evaluated with the fin length as the characteristic length

max Maximum

mid Middle

min Minimum

o Overall

oe Outer surface of an end plate-fin

p Prime surface, i.e. exposed portion of the base-plate upper surface

pf Surface of one side of a plate-fin

r Radiation

S Non-dimensional parameter evaluated with the fin spacing as the characteristic length

t	Total
w	Evaluated at the plate-fin wall

1 Introduction

Most experimental work on natural convection heat transfer from horizontal fin arrays and fin systems applied the calorimetric technique [1-15], two [16, 17] used the Mach-Zender interferometric technique, and three [18-20] applied the differential interferometric technique. Application of earlier calorimetric or interferometric techniques mostly dealt with large fin arrays, except [14, 15], which studied miniaturized horizontal plate-fin arrays. The literature survey showed that there are also two numerical studies without assumption of boundary layer flow adjacent to the fin [21, 22]. The first was on a rectangular fin on a partially heated horizontal base and the other on a short horizontal rectangular fin array. Only four studies were found that dealt with single fin systems [17, 19, 21, 23]. Reference [17] applied a Mach-Zender interferometric technique, [19] applied a differential interferometric technique, [21] a numerical method, and [23] a concurrent calorimetric and interferometric technique.

In references [4-10] the effects of varying fin array geometric variables on the heat dissipation performance (under dominant natural convective conditions) were investigated with a focus on thermal management in electronic cooling, thus limited the fin array base-plate temperature excess, i.e. the amount by which the array base temperature exceeds that at the ambient, to 60 °C, the maximum set by electronic constraints. References [4-9] dealt with large horizontal fin arrays and [10] investigated the natural convection/radiation heat transfer from pin-fin arrays for three different orientations, including an assembly of vertical fins and a horizontal upfacing baseplate.

Wei et al. [13] studied experimentally inflow effects on natural air cooling of array-optimized large plate fins with the base vertically oriented. Four constricted convection configurations were considered i.e. natural air cooling of the plate fins in an infinite cooling air, shrouded by a vertical copper side plate, shrouded by a lower horizontal insulated plate, and enclosed by both lower and side plates. Reference [13] concluded that constriction of natural convective flows results in different effects on the air inflow and

heat transfer in cooling fins and the effects are closely dependent on the array and shrouded configuration. In a similar work Harahap and Setio [12] analyzed the sensitivity of natural convection heat transfer results on the experimental system and method applied by comparing the approach adopted by Starner-Harahap-McManus [1,2] and that of Leung-Probert-Shilston [4-9]. Reference [12] found that the geometry of the cover and structure used to support horizontally-based, vertically finned arrays has an effect on how far an experimental set up could be treated as an almost zero bottom heat loss system or not. A non zero bottom heat loss system would have a heating effect on the air inflow and also the heat transfer in horizontally-based cooling fins. Based on this result, the calorimetric experimental system and method of the present work and that of our earlier work [15] consciously adopted the approach applied by Leung et al. [4-9] to allow comparability of results.

Harahap and McManus [2] studied and used recorded schlieren-shadowgraph flow patterns as a model for carrying out a similarity analysis on the governing equations to find parameters for generalizing calorimetrically measured average natural convection coefficients of large horizontal fin arrays. Mannan [3] presented sketches of observed cigarette smoke patterns of the flow field associated with a comprehensive investigation of the effect of varying the fin spacing, length, height, and the base temperature excess on the natural convection performance of large horizontal fin arrays. Based on a statistical analysis of his extensive data, Mannan [3] concluded that the effect of fin height is not significant, except for arrays with small fin spacing. Mobedi and Yüncü [22] conducted a three dimensional numerical study to reveal the mechanism of natural convective heat transfer fluid flow and heat transfer from short horizontal rectangular fin arrays and reported good agreement between numerical results and experimental data of [2, 3]. Flow pattern studies of [2, 3, 22] found that the fin height to length ratio H/L is an important parameter for horizontal fin arrays. At a fixed fin spacing S , the larger the value of $H/L \geq 0.25$ the closer the flow field evolves toward the single chimney flow pattern with the consequent result of higher natural convection heat transfer performance.

Mobedi et al. [21] presented results of a numerical investigation to reveal the mechanism of two dimensional steady state laminar air flow and heat transfer from a

vertical rectangular fin attached to a partially heated horizontal base. The flow pattern of the problem considered was a two dimensional transverse chimney flow similar to the flow pattern on one side of the vertical symmetrical axis sketched by [19] for the horizontal single plate fin system and in [17] for the single longitudinal fin positioned pointing upwards from a horizontal cylinder visualized using a Mach-Zender interferometer. Reference [21] concluded that half of the base plate width to fin height ratio $W/2H$ is an important geometric parameter and decreasing the value of this parameter has the effect of increasing the average natural convection coefficient.

Leung and Probert [9] calorimetrically measured steady state rates of heat dissipation under dominant natural-convective conditions of short-protrusion rectangular, relatively small fin arrays with the base vertically and horizontally oriented. For the horizontal aspect, [9] found an optimal fin spacing S of 9.5 ± 0.5 mm which is almost temperature invariant and also insensitive to change of fin height H from 10 to 17 mm.

Jones and Smith [16] employed a Mach-Zender interferometer to analyze the effect of varying the fin height, fin spacing and the temperature excess on maximizing the natural convection heat transferred from large horizontal fin arrays, while undergoing a cooling down process under quasi-steady conditions. The fins were assumed isothermal. Inner or central fins were treated the same as end fins in calculating the overall natural convection coefficient. On the basis of data correlation, Jones and Smith [16] concluded that fin spacing is the prime geometric variable. The effects of varying the fin length could not be reflected by this correlation because only one fin length was applied in their experiments as was generally the case with earlier interferometric studies [16, 17, 18, 19, 20].

Sobhan et al. [18, 19] applied the differential interferometric technique to investigate unsteady and steady natural convection heat transfer from horizontal single fin systems and fin arrays. Tall plate-fins $H = 70$ mm outside the range of short fin height employed by [9] were tested.

Rao and Venkateshan [20] extended the method applied by [18, 19] to study the coupling between natural convection and radiation in the open channels of a fin array by including the radiation heat transfer term. They found that for special cases where the reduction in convection heat loss due to the radiation-convection interaction was more or

less compensated by the radiation itself, it was possible to compare their data with the predictions of the correlations due to Sobhan et al. [19], and that of Jones and Smith [16]. Data of [20] showed fair agreement with the prediction of the correlation equation proposed by Sobhan et al. [19] but showed discrepancy at lower Rayleigh numbers with the prediction of the correlation equation proposed by Jones and Smith [16].

Heindel et al. [14] studied the enhancement of natural convection heat transfer from an array of highly-finned, discrete heat sources mounted to one wall of a cavity filled with a dielectric liquid, and an opposite wall of highly-finned copper cold plate maintained at 15 °C. The cavity was oriented with the wall mounted with the discrete heat sources both vertically and horizontally. Dense miniaturized parallel fin arrays with the base area 12.7 x 12.7 mm were employed to cool an array of 3 x 3 heat sources. The fin thickness and spacing were 0.20 mm and 0.51 mm, respectively. The horizontal cavity orientation was found to perform better than the vertical orientation.

Harahap et al. [15] applied the calorimetric technique to study the effect of reducing the base-plate area on the natural air cooling heat dissipation performance of miniaturized horizontal fin arrays with an experimental approach adopted from that applied by Leung et al. [4-9]. Reduction of the fin array base plate area from a maximum square of 49 x 49 mm ($A_{b,max} = 24.51 \times 10^{-4} \text{ m}^2$) to a minimum square of 25 x 25 mm ($A_{b,min} = 6.25 \times 10^{-4} \text{ m}^2$) with various intermediate area rectangles in between those extremes, was found to have the effect of increasing the values of the average dissipation coefficients under dominant natural convection conditions. One of two sets of fin arrays tested had an 11 mm spacing, i.e., in the near the optimal range for maximum heat transfer performance suggested by [9], a fin height of 13.5 mm, i.e. within the range of short fin protrusions suggested by [9] and a fin thickness of 1 mm. It was also observed that the average heat dissipation coefficient values increased with reduction of the array base-plate area over the range of the fin length alignment parameter or the base-plate width to length ratio W/L from 0.51 to 1.96. Alignment of the fin length parallel to the shorter side of a rectangular base-plate ($W/L > 1.0$) consistently resulted in higher average dissipation coefficients than if the length of the fins were aligned parallel to the longer side ($W/L < 1.0$) of a rectangle base-plate of the same area..

In the present study, the calorimetric and interferometric methods were concurrently applied to gain further insight into the effects of reduction of the base plate reported in [15] on the natural convection performance of a set of three single plate-fin systems and a set of four plate-fin arrays with spacing near the optimal value established in [9].

2 Experimental set up and procedure

The small integral single plate-fin systems and plate-fin arrays tested were made from Alcoa aluminum alloy 7050 ($k_{fm} = 157.49 \text{ W/m.K}$ and $\epsilon = 0.11$ [25]) and manufactured by wire cutting to N7 tolerances of ± 0.1 , ± 0.2 , and $\pm 0.3 \text{ mm}$ for array geometric dimensions in the ranges from 0 to 6, 6 to 30, 30 to 120 mm, respectively (see Table 1). Miniaturized horizontal plate fin arrays investigated as test objects in the present work and symbols used for geometric dimensions are shown in Fig. 1. Table 1 shows the test object codes, values of geometric dimensions, base-plate width to length ratio, base plate area, and number of fins for three single plate-fin systems and four plate-fin arrays tested in the present work. Except for the code identifying the test object, all other symbols used for arrays in Table 1 and Fig. 1 are consistent with those given in Table 1 and Fig. 3 of [15] and are explained in the List of Symbols of the present paper. In comparing Table 1 of the present work and that of [15], it is noted that thicker fins ($t = 1.5 \text{ mm}$) were used in the present work compared to those tested in [15] ($t = 1.0 \text{ mm}$) thus reducing fin spacing S to either 10.25 or 10.37 mm. These values were still within the range of the near optimal spacing suggested by [9].

Table 1

Fig. 1

2.1 Calorimetric aspect

Figure 2 gives a general idea of the zero conduction loss experimental set-up and system of the calorimetric aspect of the present investigation. A detailed description of this set up and the corresponding experimental procedure has been reported in Reference [15] and will not be repeated here. However, instead of calculating the average heat transfer dissipation coefficient as in [15], the present work dealt with determination of the average natural convection coefficient by applying the following equation

Fig.2

$$\bar{h}_{cal} = \frac{I.V - Q_r}{\bar{\theta}_b . A_t} \quad (1)$$

where $I.V$ is the steady state direct-current power input to the main heater, Q_r is the rate of radiation heat transfer from the test object, $\bar{\theta}_b$ is the average array base-plate temperature excess, and A_t is the total heat transfer area,

$$A_t = W.L + 2.n.H.L + 2.n.H.t \quad (n = 1 \text{ for a single fin system}) \quad (2)$$

The rate of radiation loss correction Q_r was calculated and applied by employing an Interactive Heat Transfer Program [24] assuming gray diffuse and isothermal surfaces in L - and U -shaped enclosures formed as appropriate by the plate-fin and exposed portion of the base plate, and a fictitious black surface representing the surrounding ambient, respectively, for the single plate-fin system and the plate-fin array.

2.2 Interferometric aspect

The differential interferometer applied in the present investigation was set up similar to the one used by Carr [26], taking advantage of recent developments in laser, computer, and digital camera technologies. Details of the working principle and the set up of components of the differential interferometer applied in the present investigation have been described in [23]. Figure 3 shows the arrangement of the components of the differential interferometer. A general idea of the set up when in use can be obtained by following the paths of rays originating from the helium neon laser going through various components until finally reaching the screen.

Fig. 3

The application of the differential interferometer in the present work differed from that of Sobhan et al. [18, 19] and Rao et al. [20]. References [18, 19, 20] analyzed interferograms by an iteration procedure based on the one dimensional rectangular fin governing differential equation, which was numerically integrated using measured fin base and tip temperatures as boundary conditions. In the present case, the temperature variation along the height of the plate-fin was measured by four fine Teflon coated type T thermocouples (0.13 mm in diameter) inserted into four tunnels (1 mm in diameter and $\frac{1}{4}$ of the fin-length deep) drilled horizontally and parallel to the fin length either from the

front or the back end of the 1.5 mm thick fin. The four locations of the thermocouple beads inside the tunnels were 4 mm apart along the plate-fin height, i.e. at heights 0, 4, 8, and 12 mm from the base. The two lowest thermocouples were inserted from the front end (Fig. 4(a)) and the other two from the back-end (Fig. 4(b)), and the wires were carefully led downwards into the wooden box support so as not to block or interfere with interferometric rays passing along the length of the surface of the plate-fins. Figures 4(a) and 4(b) also show that a distinction was made between a central and an end plate-fin by separately measuring the temperature variation along the height of these two types of fins. The temperature variation measured in this fashion, under the one-dimensional assumption, was considered as the plate-fin wall or surface temperature variation, and taken as applicable to both surfaces of a plate-fin. Furthermore, the temperature variation of the central fin was assumed to apply also for all inner fins. The observation and recording of temperatures along the height of the plate-fin were made using an Omega-Engineering 2.00 program in conjunction with a data logger with 8 simultaneous input ports. This temperature measurement system was part of the interferometric aspect of the concurrent method and thus separate from the temperature measurement system of the calorimetric aspect, which consisted of a Labtech data acquisition program with a data acquisition of Advantech possessing 16 input ports for simultaneous measurements as described in detail in [15, 23]. Figure 3 shows the lay-out of two computers associated with the two temperature measurement systems.

Fig. 4

At steady state, as established calorimetrically, measurements of temperatures by eight thermocouple beads installed in the central and end plate-fins were recorded using the Omega Engineering data logger and program 2.00 in computer 2 of Fig 3. Measured temperatures were then fitted by an appropriate polynomial equation using a Microsoft Excel program to represent the plate-fin wall temperature variation along the height of the central and end plate-fins.

The differential interferometer was then switched on and adjusted to give an optimal fringe pattern image, which was recorded with a digital camera. A recorded fringe pattern image was further processed using a Microsoft Visio program to determine the

fringe shift at the wall of the plate-fin. The local natural convection heat transfer coefficient at the location of the determined fringe shift was calculated from [23, 26]

$$h(y_\ell) = 0.031 \left[\frac{k_{aw} T_w^2(y_\ell)}{L \theta_w(y_\ell)} \right] m(y_\ell) \quad (3)$$

Here, the local plate-fin wall temperature T_w , the local fin-plate wall temperature excess θ_w , and the fringe shift m were all evaluated at height y_ℓ , i.e. the fringe shift location measured from the fin base, k_{aw} is the air thermal conductivity evaluated at T_w , and L is the fin length. The numerical factor in Eq. (3) was characteristic of the differential interferometer and experimental settings used in this investigation. In all tests conducted, only four fringes were associated with each height of a plate-fin, thus $\ell = 1, 2, 3$, or 4.

The average rate of natural convection heat transfer from the surface of one side of a plate-fin of a single fin system or of an array was numerically calculated from

$$q = (L + t) \sum_{\ell=1}^4 h(y_\ell) \theta_w(y_\ell) \Delta Y_\ell \quad (4)$$

with

$$(\Delta Y)_0 = 0, \quad (5)$$

$$(\Delta Y)_1 = y_1 + \frac{(y_2 - y_1)}{2}, \quad (6)$$

$$(\Delta Y)_\ell = [y_\ell - (\Delta Y)_{\ell-1} - (\Delta Y)_{\ell-2}] + \frac{(y_{\ell+1} - y_\ell)}{2} \quad (\ell = 2, \text{ or } 3), \quad (7)$$

and

$$(\Delta Y)_4 = (H + \frac{t}{2} - y_4) + \frac{(y_4 - y_3)}{2}. \quad (8)$$

Equations (4) and (8) included convection loss from the front and back edges, and the top edge of a plate-fin by using $L + t$ and $H + t/2$, instead of only L and H , respectively. The average natural convection coefficient of the surface of one side of a plate-fin was calculated from

$$\bar{h}_{pf} = \frac{q}{A_{pf} \cdot \bar{\theta}_w} \quad (9)$$

where $A_{pf} = (L + t) \cdot (H + t/2)$ is the corrected surface area of one side of a plate-fin and $\bar{\theta}_w$ is the average plate-fin wall temperature excess.

Equations (4) to (9) were applied to the central and end fins of an array under the following assumptions. Measured data obtained from one side of a central plate-fin was assumed to apply to both sides of that plate-fin, and applied also to all inner fins, if an array had more than 3 fins. End plate-fins, however, had to be treated differently and separately from inner plate-fins, because the average rate of natural convection heat transfer from the surfaces of an end plate-fin would be different than that from the surfaces of an inner plate-fin. As measurements of the local temperature variation and fringe shifts were carried out on one end fin only, it was further assumed that they apply to both end plate-fins of an array. Under these assumptions, the rate of natural convection heat transfer from all plate-fins of an array was then calculated from

$$Q_f = (n - 2)2q_c + 2q_{ie} + 2q_{oe} \quad (10)$$

where q_c , q_{ie} , and q_{oe} are similar to q in Eq. (4) but evaluated using their respective local natural convection coefficient $h(y_\ell)$ and local plate-fin wall temperature excess $\theta_w(y_\ell)$ values of one surface of the central plate-fin, inner and outer surface of the end plate-fin, respectively, and n is the number of fins in an array. It is to be noted that $Q_f = 2q$ for the single plate-fin system.

2.3 Setting-up the concurrent calorimetric-interferometric system

The calorimetric experimental set up as described in [15, 23] and shown in Fig. 2 was integrated into the concurrent system by placing the wooden supporting box, mounted with any one of the test objects listed in Table 1 and its corresponding heater assembly inside the box, at the center of the differential interferometer table top, i.e. the test section of the interferometer (Fig.3). The power supply cables of the main and guard heaters as well as the thermocouple wires of the calorimetric set up were passed through a hole at the center of the table top to the underside before directed to respective connections and instruments

(A, B, C, D, and E in Fig. 3). Also to be noted in Fig. 3 is that the Omega Data Logger (F) and computer 2 (G) were associated with measurement of the local temperature variation along the plate-fin height of the interferometric method. Finally, a black wooden box shield with windows as sketched in Fig. 3 enclosed the wooden supporting box and the test object assembly from the general surroundings of the test room to minimize the effects of disturbances created by movements of air on the natural convection from the test object being tested.

The concurrent system allowed the combination of measured data of both aspects to give what is referred to in this paper as concurrent results. The rate of natural convection heat transfer from all plate-fins of an array Q_f (see Eq. (10)), which was obtained interferometrically, when applied in an energy balance, together with the radiation loss corrected main heater power input of the calorimetric method, gives the rate of natural convection heat transfer from the exposed portion of the base plate or the primary surface,

$$Q_p = (I.V) - Q_r - Q_f \quad (11)$$

The overall fin array natural convection efficiency was then calculated from

$$\eta_o = \frac{Q_p + Q_f}{I.V} \times 100\% \quad (12)$$

where $I.V$ is the heating dc-power input to the main heater [15].

3 Uncertainty estimation

The second-power method [27] was applied to determine the uncertainty interval of measured average natural convection coefficients. Analysis of results showed that the maximum uncertainty for the set of average natural convection coefficients for both the single plate-fin system and plate-fin array occurred when the electrical heating input is lowest.

Variables affecting the uncertainty interval of calorimetrically measured average natural convection coefficients were I , V , W , L , H , t , and $\bar{\theta}_b$. (see Eqs. (1) and (2)). The uncertainty intervals of I and V were estimated to be the same as the accuracies of respective measuring instruments, respectively, ± 0.0005 A and ± 0.005 V. The tolerances

of manufacture of each single fin system and fin array (see Table 1) were used as estimates for the uncertainty intervals of W , L , H , and t . For all single fin systems (except one, with $H = 20 \pm 0.2$ mm) and all fin arrays, H and t were kept constant, thus the uncertainty intervals were $H = 13.5 \pm 0.2$ mm and $t = 1.5 \pm 0.1$ mm. The uncertainty interval of the average base-plate temperature excess $\bar{\theta}_b$ was assumed to be ± 0.5 K

Using these variable uncertainty assumptions the largest maximum uncertainty of calorimetrically measured average natural convection coefficients $w_{\bar{h}_{cal}}^- / \sqrt{\bar{h}_{cal}}$ of the single fin system was 2.39% for $L25W25H13.5A_{b,min}$, and 2.01% for the fin array $L25W25n3A_{b,min}$.

For measurement of the local natural convection coefficient of the interferometric aspect the uncertainty variables were y_ℓ , $T_w(y_\ell)$, $\theta_w(y_\ell)$, and $m(y_\ell)$ [see Eq. (3)] with the following respective uncertainty intervals ± 0.01 mm, ± 0.1 K, ± 0.1 K, and ± 0.01 mm. The calculated uncertainty interval of the local natural convection coefficient $w_{h(y_\ell)}$ was used in determining the uncertainty interval of the average rate of natural convection heat transfer from the surface of one side of a plate-fin w_q together with the other uncertainties of variables affecting the measurement of q , namely, w_{L+t} , $w_{\theta_w(y_\ell)}$, and $w_{(\Delta Y)_\ell}$ using the following values ± 0.02 or ± 0.03 (depending on the dimensional value of $L+t$), ± 0.1 K, and ± 0.01 mm, respectively. Finally, the calculated uncertainty interval of the rate of natural convection heat transfer from the surface of one side of a plate-fin w_q together with uncertainty intervals of area of a plate-fin $w_{A_{pf}}$ and the average plate-fin temperature excess $w_{\bar{\theta}_{pf}}^-$ were applied to calculate the uncertainty interval of the average natural convection coefficient of the surface of one side of a plate-fin $w_{\bar{h}_{pf}}^-$.

The uncertainty intervals of interferometrically measured average plate-fin natural convection coefficients $w_{\bar{h}_{pf}}^- / \sqrt{\bar{h}_{pf}}$ were found to be in the range between 2.50 to 3.25 % and one highest value exceeded 3.25%, i.e. at 3.84% for the inner surface of the end plate-fin of array $L25W25n3A_{b,min}$.

4 Results and discussion

4.1 Calorimetric aspect

The variation of the steady-state natural convection heat transfer rate per unit base-plate area $(I.V - Q_r)/W.L$ and the steady-state average natural convection coefficient $\bar{h}_{cal}$ with the average base-plate temperature excess $\bar{\theta}_b$ for three single plate-fin systems and four plate-fin arrays having different values of the base plate area A_b , and the total heat transfer area A_t are presented in Fig. 5. The natural convection heat transfer rate per unit base-plate area can be seen to increase with the average base-plate temperature excess and with reduction of the base-plate area. It can also be seen that for the same square base-plate area ($W/L = 1$) the steady-state natural convection heat transfer rate per unit base-plate area of the single plate-fin system is less than for the plate-fin array, because the latter configuration has a higher number of fins, thus a higher total heat transfer area.

Fig. 5

In Fig. 5 values of the average natural convection coefficients can be seen to increase with the average base-plate temperature excess and with reduction of the base-plate area. The average natural convection coefficient data for the set of single plate-fin systems can be seen to have shifted upwards above those for the set of plate-fin arrays with the same base area A_b .

For plate-fin arrays with the near optimal fin spacing tested in the present work, Fig. 5 shows that reduction of the base plate area increases the values of the average natural convection coefficients over the range $30 < \bar{\theta}_b < 60$ K, similar to the corresponding results for the average dissipation coefficients in [15] which is already discussed in the Introduction. An increase of the average natural convection coefficient of 31.0 % on the average in the range $30 < \bar{\theta}_b < 60$ K can be seen in Fig. 5 as the value of $H/L = 0.27$ of the array with $A_{b,max}$ was increased to $H/L = 0.54$ for the array with $A_{b,int1}$, i.e. by approximately halving the fin length, under constant $H/S = 1.30$, and $n = 5$. Similarly, as the value of $H/L = 0.27$ for the array with $A_{b,int2}$ is increased to $H/L = 0.54$ for the array with $A_{b,min}$, again by

approximately halving the fin length while keeping $H/S = 1.31$, and $n = 3$ constant, resulted in an increase of the average natural convection coefficient by 35.0 % on the average in the range $30 < \bar{\theta}_b < 60$ K. These results are in agreement with the conclusions of [2, 3, 22] on the effect of increasing the parameter H/L .

However, for the present intermediate cases of arrays with the same rectangular base-plate area, Fig. 5 shows that values of the natural convection coefficients for array with $W/L = 1.96$ are only slightly higher than those for the array with $W/L = 0.51$ over the range $30 < \bar{\theta}_b < 60$ K. This W/L effect was more pronounced for the case of the dissipation coefficients dealt with in [15]. The increase of $H/L = 0.27$ for array $L49W25n3A_{b,int2}$ with $H/S = 1.31$, and $n = 3$ to $H/L = 0.54$ for array $L25W49n5A_{b,int1}$ with $H/S = 1.30$, and $n = 5$ did not show a significant increase of the values of the average natural convection coefficient. It is noted that this increase in the value of H/L involved different n values. Thus, the importance of the W/L parameter has been overshadowed by the importance of the number of fins n .

Figure 5 shows that reduction of the square-shaped base area of the single plate-fin system from $A_{b,max}$ to $A_{b,min}$ by 74 percent, while keeping the plate-fin height constant at $H = 13.5$ mm, increases the value of the average natural convection coefficient by a factor of about one and a half in the range $25 \text{ K} < \bar{\theta}_b < 50$ K. However, for the reduction of the base area from $A_{b,max}$ to $A_{b,mid}$ by 55 percent and at the same time increasing the plate-fin height by 48 percent (from 13.5 to 20.0 mm) shows no increasing effect on the average values of natural convection coefficients of the single fin system with $A_{b,mid}$ relative to those of the single fin system with $A_{b,max}$.

The effect that decreasing of the half width to fin height ratio $W/2H$ has on increasing the values of the natural convection coefficient as concluded by [21], albeit based on a simplified two dimensional mathematical model, could still be expected to apply for the case of the single plate-fin system with finite length. In Fig. 5 it can be seen that as the value of $W/2H = 1.81$ for the single plate-fin system with $A_{b,max}$ decreases to $W/2H = 0.92$ for the single plate-fin system with $A_{b,min}$, under constant $H = 13.5$ mm, the values of the average natural convection coefficient of the latter increase relative to those of the former.

However, reduction of the value of $W/2H = 1.81$ for the single fin system with $A_{b,max}$ to $W/2H = 0.82$ for the single fin system with $A_{b,mid}$ by reducing half of the base width $W/2$ from 24.5 mm to 16.5 mm and at the same time increasing the fin height H from 13.5 mm to 20 mm, shows no increasing effect on the values of the average natural convection coefficient of the single plate-fin system with $A_{b,mid}$ relative to those of the single plate-fin system with $A_{b,max}$ as can be observed in Fig.5. It is noted that the value of $H = 20$ mm of the single plate-fin system with $A_{b,mid}$ is already above the maximum value of $H = 17$ mm of the range of short upward protrusion plate-fins suggested by [9], thus may have been sensitive to the variation of the fin height. This case will be examined further in conjunction with interferometric measurement results.

The conclusion that can be drawn from the above discussion is that increasing the H/L ratio by reducing L under conditions of constant H , S , and n , i.e. reducing the base plate area, has the effect of increasing the average natural convection coefficient of a fin array. This is not the case, however, if the number of fins n of the array are different. For the single fin system ($n = 1$) decreasing the value of the $W/2H$ ratio by decreasing half of the width of the base plate, i.e. reducing the base plate area, under condition of constant H , has the effect of increasing the average natural convection coefficient. These conclusions suggest that the number of fins n is an important geometric parameter for both test object configurations studied in the present work.

4.2 Interferometric aspect

Figure 6(a) shows a typical measured temperature drop profile from plate-fin base to tip for the single plate-fin system with short plate-fin upward protrusion of $H = 13.5$ mm fitted by a second order polynomial. The drop is small of the order of 1K. The corresponding increase of the local natural convection coefficient from base to tip along the plate-fin height is shown in Fig. 6(b). This increase possesses the features of a second order polynomial similar to the results obtained by Sobhan et al. [19] up to a height of about 15 mm from the base of their tall fins ($H = 70$ mm). Beyond that height, the results in [19] show a reduction and then followed by an increase of the rate of development of the local natural convection coefficient with height until a maximum value is reached at a height of

Fig. 6

20 mm from the base. A similar feature is shown by the single fin system with $H = 20$ mm of the present study as shown in Fig. 6(b), which corresponds to a typical measured temperature drop profile from plate-fin base to tip shown in Fig. 6(a) fitted with a fourth order polynomial. These results show that the temperature variation along the height of single plate-fin system is sensitive to the fin height and has a varying effect on the local natural convection heat transfer coefficient from the fin-plate.

Figure 6(a) also shows typical variation of measured local temperatures along the height of a central and an end plate-fin of an array with short plate-fin upward protrusion of $H = 13.5$ mm, also fitted by a second order polynomial. The local temperatures along the height of the central plate-fin are higher than those at corresponding heights from base of the end plate-fin. The temperature drop profile of the central plate-fin is almost linear, and that of the end plate-fin is similar to that of the plate-fin of a single plate-fin system with $H = 13.5$ mm. The corresponding increases of the values of the local natural convection coefficient at four locations along plate-fin height are shown in Fig. 6(b). The typical development of the local natural convection coefficient values along the plate-fin height is highest for the outer surface of the end plate-fin, followed by that of the central plate-fin, and the lowest is that for the inner surface of the end plate-fin. The variation of the values of the local heat transfer coefficient shown in Fig. 6(b) are similar to those obtained by Sobhan et al [19] for arrays at the lower portion of their tall fins ($H = 70$ mm) until a height of about 15 mm from the base.

In Fig. 7 the variation of the average fin-plate natural convection coefficient $\bar{h}_{pf}$ with the average plate-fin temperature excess $\bar{\theta}_w$ is shown as the base-plate area of the single plate-fin system was reduced. Reduction from $A_{b,max}$ to $A_{b,min}$ shows the expected effect of increased values of the average plate-fin natural convection coefficients over the range $25 \text{ K} < \bar{\theta}_w < 50 \text{ K}$. Reduction of the base plate area from $A_{b,max}$ to $A_{b,mid}$ and at the same time increasing the plate-fin height H from 13.5 to 20 mm, however, shows a decrease of the values of the plate-fin average natural convection coefficients to values lower than those of $A_{b,max}$ for the higher values of $\bar{\theta}_w > 40 \text{ K}$, with the consequent lowering of the performance of the rate of natural convection heat transfer from the plate-fin. This characteristic,

Fig. 7

however, is not exhibited by the calorimetric average natural convection coefficient of single fin system $L33W33H20A_{b,mid}$, although no net increase of these overall average values above those of $L49W49H13.5A_{b,max}$ was noticeable in Fig. 5.

Figure 7 also shows the typical variation of the values of the average plate-fin natural convection coefficient $\bar{h}_{pf}$ with the average plate-fin temperature excess $\bar{\theta}_w$ for the central, inner and outer end plate-fin surfaces. Typical for the arrays tested, the values of the average plate-fin natural convection coefficient of the outer surface of the end plate-fin are highest, followed by those of the central (inner) plate-fin, and the lowest are for the inner surface of the end plate-fin. The order of this variation is consistent with those for respective values of the local natural convection coefficient along the fin height of the corresponding plate-fin surfaces as shown in Fig. 6(b).

Finally, the variation of the values of the average plate-fin natural convection coefficient with the average plate-fin temperature excess of the central plate-fin, inner surface of the end plate-fin, and the outer surface of the end plate-fin, among the four fin arrays tested, as the base plate areas were reduced, are compared in Fig. 8. The effect that reduction of the base-plate area has in increasing the values of the average plate-fin natural convection coefficients over the range $30 \text{ K} < \bar{\theta}_w < 60 \text{ K}$ can be seen in Fig. 8, for the surface of the central plate-fin and the outer surface of the end plate-fin. This effect is similar to that which was found for the average natural convection coefficients of the calorimetric aspect, i.e. the array with $A_{b,min}$ has the highest values of the average plate-fin natural convection coefficients followed by those of the intermediate arrays with $W/L = 1.96$ and $W/L = 0.51$, and the lowest are those for the array with $A_{b,max}$ over the range $30 \text{ K} < \bar{\theta}_w < 60 \text{ K}$. However, this order does not apply for cluster of data of the inner surface of the end plate-fin. Instead, increase of values of the average plate-fin natural convection coefficient as the base-plate area was reduced applies only if arrays have the same number of fins, namely, reduction from $A_{b,max}$ to $A_{b,int,1}$ for $n = 5$ and from $A_{b,int,2}$ to $A_{b,min}$ for $n = 3$. The effect of reduction of the base-plate area related to the array number of fins n can be seen to be valid also for the surface of the central plate-fin and the outer surface of the end

Fig. 8

plate-fin., which suggests that the number of fins n is an important geometric parameter as was observed in the conclusion of discussion of results of the calorimetric aspect.

4.3 Concurrent method

Table 2 shows the breakdown of the heating power input, $I.V.$, into percentage of natural convection loss from the plate fin of a single fin system and all plate-fins of a fin array Q_f , the prime surface Q_p , and loss due to radiation Q_r . The values of the heating power input shown in the table are those at mid range of the power input applied during tests of each test object. Values of corresponding overall efficiencies of the test objects calculated according to Eq. (12) are included in the last column of this table.

Table 2

5 Correlations analysis

5.1 Single plate-fin systems

For the horizontal large single fin system, Sobhan et al. [19] used the fin height as the characteristic length in generalizing their data and proposed the following correlation

$$Nu_H = 2.36 \times 10^{-4} Ra_H^{0.808} \left[1 - \exp\left(-1.253 \times 10^6 / Ra_H\right) \right]^{0.362} \quad (13)$$

Figure 9 shows comparison of the present data for miniaturized single plate-fin systems generalized following Sobhan et al. [19], i.e. by defining the Nusselt and Rayleigh numbers as $Nu_H = \bar{h}_{pf} H / k$ and $Ra_H = g\beta\bar{\theta}_w H^3 / \nu\alpha$, respectively. Property values of air were evaluated at the average film temperature. It is seen that present data fall above the prediction of the Eq. (13). It is to be recalled that only one fin length was applied in the interferometric studies of [19] on single fin systems with tall fins.

Fig. 9

Following [2, 15, 23] in using the fin length as the characteristic length in the Nusselt and Rayleigh numbers and applying the parameter W/H (proportional to $W/2H$) as suggested by [21], the calorimetric data for the miniaturized single plate-fin system was successfully correlated as is also shown in the upper part of Fig.9, giving

$$Nu_L = (10^{-5} Ra_L + 18.322) \left(\frac{W}{H} \right)^{0.25} \quad (14)$$

In this correlation, Nusselt numbers of the single plate-fin system were calculated from the average natural convection coefficients as $Nu_L = \bar{h}_{cal} L / k$, and Rayleigh numbers used corresponding values of the average base-plate temperature excesses, thus $Ra_L = g\beta\bar{\theta}_b L^3 / \nu\alpha$. The property values of air were evaluated at the average film temperature.

5.2 Plate-fin arrays

In Fig. 10 data of miniaturized plate-fin arrays of the present study is compared to the predictions of the fin spacing based correlations for large fin arrays proposed by Jones and Smith [16]

Fig. 10

$$Nu_S = 6.7 \times 10^{-4} Ra_S \left[1 - \exp\left\{ - (7460 / Ra_S)^{0.44} \right\} \right]^{1.7} \quad (15)$$

and by Sobhan et al. [19]

$$Nu_S = 2.201 \times 10^{-2} (k_a / k_{fm})^{-0.233} Ra_S^{0.377} \quad (16)$$

Present data generalized using correlation parameters proposed by [16, 19], i.e.

$Nu_S = \bar{h}_{cal} S / k$ and $Ra_S = g\beta\bar{\theta}_b S^3 / \nu\alpha$, show discrepancy with the prediction of Eq.

(15) and show poor agreement with the prediction of Eq. (16). The same result has also been observed by Rao et al. [20] using their data.

As can also be seen in Fig 10 that the calorimetric data of the present study was found to be well predicted by the correlation proposed in [15] for the rate of heat dissipation under dominant natural convection conditions from miniaturized horizontal plate-fin arrays

$$Nu_l = 0.203 \left[Gr_l Pr(nS/H) \right]^{0.393} (S/l)^{0.470} (H/l)^{0.87} (W/L)^{-0.620} \quad (17)$$

Here $l = L/2$ was used as the characteristic geometric parameter in defining Nusselt and

Grashof numbers as $Nu_l = \bar{h}_{cal}^* l / k$ and $Gr_l = g\beta\bar{\theta}_b l^3 / \nu^2$. In Eq. (17), the fin length alignment parameter W/L as well as the H/l ratio, which is proportional to the H/L ratio, can be seen to have significant influence on the Nusselt number. It should also be noted that the number of fins n appears in the correlation. Equation (17), however, involves too many parameters thus could be impractical for application in thermal design.

At the end of Sections 4.1 and 4.2 it was concluded that the number of fins n of an array is an important determinative geometric parameter. The single plate-fin system can be viewed as a limiting case of an array with the number of fins n equal to one. Following up on this conclusion, the data of the present work was combined with those of our previous work [15], and generalized using the fin length L and the number of fins n as the characteristic geometric parameters. The result of this generalization is also shown in upper part of Fig. 10, giving a simple correlation equation

$$Nu_L = 2.921(Ra_L/n^{2.70})^{0.202} \quad (18)$$

Here, the number fins n are odd integers in the range from 1 to 13. Nusselt and Rayleigh numbers were calculated, respectively, using the average dissipation coefficients under dominant natural convection conditions, $\bar{h}^*$, and the average base-plate temperature excess, $\bar{\theta}_b$. Properties of air were evaluated at the film temperature. This correlation equation is expected to apply well for horizontally-based short upward protusion plate-fin arrays in the range of height H from 10 to 17 mm and for values of spacing S near the optimal value of 9 ± 0.5 mm. The simplicity of this equation makes it attractive for applications in thermal design. Moreover, in electronic thermal management one is more interested in achievable high natural convection/radiation dissipation coefficients [4-10, 15] rather than through what mode the heat is transferred for the cooling purpose.

6 Conclusions

Concurrent calorimetric and interferometric measurements of steady state natural convection heat transfer have been performed to determine the effect of miniaturizing the base plate on the overall and plate-fin average natural convection coefficients of single plate fin systems and fin arrays.

The calorimetric aspect of this work extended the type of test object investigated in our earlier work [15], by adding single plate-fin systems to plate-fin arrays for testing. It was found that increasing the H/L ratio under conditions of constant H , S , and n , i.e. reducing the base-plate area through reducing L , has the effect of increasing the average natural convection coefficient of fin arrays. For single fin systems, decreasing the $W/2H$

ratio by decreasing half of the width of the base plate under constant H , has the effect of increasing the average natural convection coefficient. A single plate-fin system ($n=1$) having the same base-plate area as a plate-fin array ($n>1$) was found to have higher average natural convection coefficient values over the range of the average base-plate temperature excess tested in the present work.

Typical measured temperature drop profile and its corresponding increase of interferometrically measured local natural convection coefficient from the plate-fin base to the tip of a single plate-fin system, and of a central (inner) and an end plate-fin of an array with short plate-fin upward protrusion tested in this work exhibited the features of a second order polynomial. This feature, thus also the plate-fin rate of natural convection performance, was found to be sensitive to increase of fin height. For an array, typical development of the natural convection coefficient values along the plate-fin height is highest for the outer surface of the end plate-fin, followed by that of the central plate-fin and the lowest is that for the inner surface of the end plate fin. Interferometrically measured average plate-fin natural convection coefficients increase with the average plate-fin temperature excess and with reduction of the base-plate area by increasing the H/L ratio under conditions of constant H , S , and n for the plate-fin arrays, and by decreasing the $W/2H$ ratio by decreasing half of the base plate width under constant H for the single plate-fin system. This result is similar to the effect that reducing the base-plate area has on the calorimetrically measured average natural convection coefficients. Both the calorimetric and the interferometric techniques led to a same conclusion that the importance of the W/L ratio as a generalizing parameter established earlier in [15] has been overshadowed by the importance of the number of fins, n .

Performance of calorimetric and interfermetric measurements concurrently has enabled quantitative delineation of the rate of natural convection heat transfer from the fin part of an array from that of the primary surface.

Correlations for the Nusselt number obtained by earlier investigators for large single fin systems [19] and fin arrays [16, 19] were found unable to predict corresponding generalized natural convection data for miniaturized horizontally single plate-fin systems and plate-fin arrays of the present work. The correlation proposed in [15] for miniaturized

horizontally based fin arrays was found to predict data of plate-fin arrays of the present work very well.

The conclusions of discussions on calorimetric and interferometric results of this study suggest that the fin length L and the number of fins n are prime geometric variables for generalization. Following on this suggestion a simple correlation equation for arrays with short upward protrusion fins is proposed under conditions of the present study, and those of [15].

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Fig.1 Miniaturized horizontally-based plate-fin arrays tested in the present work: geometric parameters are defined in the list of symbols, see also Table 1

Fig. 2 Schematic of the calorimetric experimental system and method of the present investigation. SPFS – single plate-fin system, OB – outer box, MHUP – main heater upper plate, MHLP – main heater lower plate, C – computer, TP – thermocouple port, T – thermocouple wires, DCPS – DC power supply, GH – guard heater, AM – ammeter, VM – voltmeter, GHLP – guard heater lower plate, GHUP – guard heater upper plate, RI – rock wool insulation, SWB – support wooden box, AS – asbestos slab, MH – main heater

Fig. 3 Lay-out of instrumentation and the test object in the concurrent calorimetric and interferometric set up. L HeNe - Helium Neon Laser, SF - Spatial Filter, K - Condenser, PW1 - Wollaston Prism 1, PW2 - Wollaston Prism 2, PW3 - Wollaston Prism 3, CC1 - Spherical Mirror 1, CC2 - Spherical Mirror 2, L1 - Convex Lens 1, L2 - Convex Lens 2, P - Analyzer, A - Computer 1 for *Labtech* Program, B - DC Power Supply Regulator, C - True RMS Multimeter (Fluke 87 III), D - True RMS Multimeter (Fluke 79 III), E - *Advantech* Data Acquisition Card, F - *Omega* Data Logger, G - Computer 2 for *Omega Engineering* program

(a)

(b)

Fig.4 (a) Installation of thermocouples, at the base and at a height of 4 mm from the base, in tunnels 12.0 mm deep, drilled from the front end of the 1.5 mm thick end and central plate-fins for array $L49W49H13.5 A_{b,max}$. (b) Installation of thermocouples, at a height of 8 mm and 12 mm from the base, in tunnels 12.0 mm deep, drilled from the back end of the 1.5 mm thick central and end plate-fins for array $L49W49H13.5n5 A_{b,max}$

Fig. 5 Variation of the rate of steady state natural convection per unit base-plate area, $(I.V - Q_r)/(W.L)$, and the average natural convection coefficient, $\bar{h}_{cal}$, with the average base-plate temperature excess, $\bar{\theta}_b$, as the base-plate areas of the single plate-fin system and the plate-fin array were miniaturized from $A_{b,max}$ to $A_{b,min}$

Fig. 6 (a) Typical measured temperature variations along the height of a plate-fin of the single fin system with $H = 13.5$ mm and 20 mm, and along the height of a central and an end plate-fin of the fin array with $H = 13.5$ mm Dashed curve is a second order polynomial fit of the measured temperatures for fins with $H = 13.5$ mm, and it is a fourth order polynomial fit for the fin with $H = 20$ mm ; (b) Typical variation of the values of the local natural convection coefficient along the height of the plate-fin for a plate-fin of the single fin system with $H = 13.5$ mm and 20 mm, and along the height of a central plate-fin, an end plate-fin inner surface, and end plate-fin outer surface of the fin array with $H = 13.5$ mm, each of them corresponding to its temperature variation shown in (a)

Fig. 7 Variation of the average plate-fin natural convection coefficient with the average plate-fin temperature excess as the base plate areas of the single fin system were miniaturized from $A_{b,max}$ to $A_{b,min}$. Note that $A_{b,mid}$ shows an anomaly due sensitivity of the base to tip variation of the local temperature on the fin height as discussed in connection with Figs. 6(a) and 6(b). The lower cluster of black data points pertain to plate-fin arrays showing a typical variation of the values of the average plate-fin natural convection coefficient with the average plate-fin temperature excess for the surface of the central (inner) plate-fin, the inner surface of the end plate-fin, and the outer surface of the end plate-fin

Fig. 8 Comparison of the average plate-fin natural convection coefficient variation with the average plate-fin temperature excess of the surface of the central plate-fin, the inner surface and the outer surface of the end plate-fin, as the base-plate area was reduced from $A_{b,max}$ to $A_{b,min}$

Fig. 9 Comparison of Nusselt numbers results for the miniaturized horizontal single plate-fin systems of this study with the prediction of the correlation for large horizontal single fin systems due to Sobhan et al. [19], Eq. (13). The upper part shows results of generalization of present average natural convection coefficient data of the single plate-fin system with parameters proposed in this study, i.e. employing the fin length as the characteristic dimension and the parameter W/H

Fig. 10 Comparison of Nusselt number results for the horizontal miniaturized the plate-fin arrays of the present study with predictions of the correlations proposed by Jones and Smith [16] Eq. (15) and Sobhan et al. [19], Eq. (16), both for large fin arrays, and that by Harahap et al. [15], Eq. (17) for horizontal miniaturized the plate-fin arrays. Generalization of present data of the single plate-fin systems, the plate-fin arrays, and data of [15] for fin arrays with $S = 3 \text{ mm}$ and $S = 11 \text{ mm}$, showing dependence of the Nusselt number on the Rayleigh number and the number of fins n

Table 1 Summary of test object codes and geometric parameters of the single plate-fin systems and plate-fin arrays tested

Test Object Code	H (mm)	t (mm)	S (mm)	W (mm)	L (mm)	W/L	$A_b \times 10^4$ (m ²)	n
<i>Single Fin System</i>								
$L49W49H13.5 A_{b,max}$	13.5 ± 0.2	1.5 ± 0.1	-	49 ± 0.3	49	1.00	24.01	1
$L33W33H20 A_{b,mid}$	20	1.5	-	33 ± 0.3	33	1.00	10.89	1
$L25W25H13.5 A_{b,min}$	13.5	1.5	-	25 ± 0.2	25	1.00	6.25	1
<i>Fin Array</i>								
$L49W49 n5 A_{b,max}$	13.5	1.5	10.37 ± 0.2	49	49	1.00	24.01	5
$L49W25 n3 A_{b,int2}$	13.5	1.5	10.25	25	49	0.51	12.25	3
$L25 W49 n5 A_{b,int1}$	13.5	1.5	10.37	49	25	1.96	12.25	5
$L25 W25 n3 A_{b,min}$	13.5	1.5	10.25	25	25	1.00	6.25	3

The emissivity of the fin surfaces is 0.11 at 24 °C [25]

Table 2 Breakdown of the mid-test-run total heating power input into percentage loss of natural convection from the plate-fins, prime surface, and loss due to radiation

No.	Test Object Code	$I\dot{V}$ [W]	Q_f [%]	Q_p [%]	Q_r [%]	η_o [%]
<i>Single Fin System</i>						
1	<i>L49W49H1 3.5 $A_{b,max}$</i>	2.59	13.51	82.24	4.25	95.75
2	<i>L33W33H20 $A_{b,mid}$</i>	1.82	20.33	75.27	4.40	95.60
3	<i>L25W25H1 3.5 $A_{b,min}$</i>	1.37	19.71	77.37	2.92	97.08
<i>Fin Array</i>						
4	<i>L49W49n5 $A_{b,max}$</i>	5.30	40.30	53.83	5.85	94.15
5	<i>L49W25n3 $A_{b,int2}$</i>	3.06	38.40	56.70	4.90	95.10
6	<i>L25W49n5 $A_{b,int1}$</i>	3.37	43.03	52.52	4.45	95.55
7	<i>L25W25n3 $A_{b,min}$</i>	2.32	33.10	63.28	3.62	97.08