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Superanalysis of the Interference Effect on Adaptive Antenna Systems

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Abstract—We analyze the performance of optimum combining in a general cochannel interference environment with thermal noise. Specifically, we consider multiple unequal-power interferers, each is spatially correlated across receiving antennas. We develop a new mathematical methodology to analyze the average symbol error probability (SEP) of optimum combining diversity systems in Rayleigh fading. The analysis resorts to the so-called Berezin’s *supermathematics* that treats both commuting and Grassmann anticommuting variables on an equal footing. This superanalysis framework enables us to derive the exact SEP expression for an arbitrary number of interferers with spatial correlation and possibly different power levels. Our results therefore encompass all the previous analytical results, based on the theory of multivariate statistics relating to complex Wishart matrices, for equal-power and/or spatially-uncorrelated interferers. Connecting the powerful supermathematical framework to the analysis of wireless diversity systems with optimum combining, we quantify the interference effects in terms of the degree of power unbalance and the amount of spatial correlation.

I. INTRODUCTION

In the presence of cochannel interference, adaptive antenna arrays with linear diversity combining have long been of interest to maximize the signal-to-interference-plus-noise ratio (SINR) [1]–[9]. The seminal work of [1], [2] laid the early foundations for optimum combining or minimum mean-square error (MMSE) diversity combining to improve the error performance and capacity of wireless communication systems by suppressing multiple interfering signals as well as by mitigating multipath fading of the desired signal. Subsequent to these early work, a large wave of publications has been spawned on the analysis of the average or outage error performance of optimum combining in detail including: i) the exact analysis [3]–[7]; ii) the interference-limited analysis whereby the thermal noise was ignored [8]; and iii) bounds and/or approximations [9]. Most of analyses were based on the joint eigenvalue distribution of complex Wishart matrices or the distributional theory of complex random matrices, for example: 1) equal-power spatially-uncorrelated interferers [3], [4], [8], [9]; 2) unequal-power spatially-uncorrelated interferers [5], [6]; and 3) equal-power spatially-correlated interferers [7].¹ However, there are no reported results for a more general

cochannel interference environment taking into account an arbitrary number of interferers with both unequal power levels and spatial correlation. The main difficulty in the analysis arises due to the absence of a computationally tractable form of the joint eigenvalue distribution of complex random matrices relevant to the SINR in this general setting.

In this paper, we analyze the performance of optimum combining in a general cochannel interference environment with thermal noise. More specifically, we consider multiple unequal-power (UP) interferers, each is spatially correlated (SC) across receiving antennas. This general setting is referred to as a “UP-SC” interference environment. We develop a new mathematical methodology to analyze the average symbol error probability (SEP) of optimum combining diversity systems in such UP-SC interference environments. The key to the success of our approach relies on employing the Berezin’s *supermathematics* [10]–[12]. The supermathematics treats the mathematical analysis and algebra of functions of both commuting and anticommuting variables on the same footing. Using this superanalysis methodology, we lay the foundation for deriving the exact SEP expression in the UP-SC interference environments. Therefore, our results provide a unifying framework for analyzing diversity systems with optimum combining, covering all the previous exact SEP results (e.g., [3]–[6]) as special cases, and thereby we quantify the interference effects on the diversity performance (e.g., the average SEP) in terms of the degree of power unbalance and the amount of spatial correlation. Throughout the paper, we shall use the same notation as in [13] and also omit the detailed derivations due to a space constraint.

II. PRELIMINARIES AND SYSTEM MODEL

A. Definitions

Definition 1 (k -Determinant): For a positive integer m and $\mathcal{A} \subseteq \{1, 2, \dots, m\}$, let $\mathcal{C}_{m,k}^{(\mathcal{A})}$ be the set of all k -subsets of $\{1, 2, \dots, m\} \setminus \mathcal{A}$:

$$\mathcal{C}_{m,k}^{(\mathcal{A})} \triangleq \{\mathcal{S} : \mathcal{S} \subseteq \{1, 2, \dots, m\} \setminus \mathcal{A}, |\mathcal{S}| = k\}. \quad (1)$$

For an $m \times m$ matrix $\mathbf{Q}$, the k -determinant of $\mathbf{Q}$ associated with the subset $\mathcal{A}$, denoted by $\det_{[k:\mathcal{A}]}(\mathbf{Q})$, is defined as the k th elementary symmetric function of the eigenvalue subset

¹Throughout the paper, we use for brevity the term “(un)equal-power spatially-(un)correlated interferers” to denote (un)equal-power interferers, each is spatially (un)correlated across receiving antennas.

of $\mathbf{Q}$:

$$\det_{[k:\mathcal{A}]}(\mathbf{Q}) \triangleq \sum_{\mathcal{B} \in \mathcal{C}_{m,k}^{(\mathcal{A})}} \prod_{i \in \mathcal{B}} \lambda_i(\mathbf{Q}) \quad (2)$$

with the convention $\det_{[0:\mathcal{A}]}(\mathbf{Q}) = 1$.

Definition 2 (Complex Gaussian Matrix): An $m \times n$ complex random matrix $\mathbf{X}$ is said to have a (p, q) -rank complex Gaussian distribution if its entries are jointly complex Gaussian and the covariance of $\text{vec}(\mathbf{X})$ has a Kronecker structure $\Psi^T \otimes \Sigma$, where Σ and Ψ are $m \times m$ and $n \times n$ positive semidefinite matrices of rank $p \leq m$ and $q \leq n$, respectively.

We use the notation $\mathbf{X} \sim \tilde{\mathcal{N}}_{m,n}^{p,q}(\mathbf{M}, \Sigma, \Psi)$ to denote $\mathbf{X}$ is the (p, q) -rank complex Gaussian matrix with $\mathbb{E}\{\mathbf{X}\} = \mathbf{M}$. If Σ and Ψ are positive definite (i.e., $p = m$ and $q = n$), we simply denote $\mathbf{X} \sim \tilde{\mathcal{N}}_{m,n}(\mathbf{M}, \Sigma, \Psi)$ as considered in [13].

Definition 3 (Detquotient Mean): Given $\xi > 0$, the *detquotient mean* of the complex Gaussian matrix $\mathbf{X} \sim \tilde{\mathcal{N}}_{m,n}^{p,q}(\mathbf{0}, \Sigma, \Psi)$, denoted by $\text{DQM}(\Sigma, \Psi; \xi)$, is defined as

$$\text{DQM}(\Sigma, \Psi; \xi) \triangleq \mathbb{E}_{\mathbf{X}} \left\{ \frac{\det(\mathbf{I}_m + \mathbf{X}\mathbf{X}^\dagger)}{\det(\mathbf{I}_m + \xi\mathbf{X}\mathbf{X}^\dagger)} \right\}. \quad (3)$$

B. System Model

We consider a wireless communication system with N_A -antenna reception in the presence of N_I cochannel interferers with thermal noise in the UP-SC interference environment, where the fading rate is assumed to be much slower than the symbol rate. After matched filtering and sampling at the symbol rate, the output signal vector is given by

$$\mathbf{r} = \sqrt{P_0} \alpha_0 s_0 + \sum_{k=1}^{N_I} \sqrt{P_k} \alpha_k s_k + \mathbf{n} \quad (4)$$

where P_0 and P_k are the average received powers; $\alpha_0 \sim \tilde{\mathcal{N}}_{N_A,1}(\mathbf{0}, \mathbf{I}_{N_A}, 1)$ and $\alpha_k \sim \tilde{\mathcal{N}}_{N_A,1}^{\kappa_1,1}(\mathbf{0}, \Phi, 1)$ are the channel gains; s_0 and s_k are the desired and interfering signals with $\mathbb{E}\{|s_0|^2\} = \mathbb{E}\{|s_k|^2\} = 1$; and $\mathbf{n} \sim \tilde{\mathcal{N}}_{N_A,1}(\mathbf{0}, N_0 \mathbf{I}_{N_A}, 1)$ is the additive white Gaussian noise (AWGN) vector, each element has the one-sided power spectral density N_0 .² Note that the noise $\mathbf{n}$ and channel vectors α_i , $i = 0, 1, \dots, N_I$, are mutually independent, and the covariance matrix Φ of rank κ_1 captures the spatial correlation structure of each interfering signal.

Let $\mathbf{z} = \sum_{k=1}^{N_I} \sqrt{P_k} \alpha_k s_k + \mathbf{n}$ be the total interference-plus-noise term and $\text{INR}_k = P_k/N_0$ be the average interference-to-noise power ratio (INR) per branch corresponding to the k th interferer. Conditioned on the α_i 's, the interference-plus-noise covariance matrix can be written as

$$\mathbf{R} = N_0 (\mathbf{I}_{N_A} + \mathbf{H}_1 \mathbf{H}_1^\dagger) \quad (5)$$

²The subscripts 0 and $k \in \{1, 2, \dots, N_I\}$ stand for the desired user and k th interferer, respectively.

where

$$\mathbf{H}_1 = [\sqrt{\text{INR}_1} \alpha_1 \quad \dots \quad \sqrt{\text{INR}_{N_I}} \alpha_{N_I}] \sim \tilde{\mathcal{N}}_{N_A, N_I}^{\kappa_1, \kappa_2}(\mathbf{0}, \Phi, \Lambda)$$

with the INR profile matrix $\Lambda = \text{diag}(\text{INR}_1, \dots, \text{INR}_{N_I})$.³ Note that we can in general consider that any $N_I \times N_I$ matrix $\hat{\Lambda}$ such that $\lambda_k(\hat{\Lambda}) = \text{INR}_k$ is equivalent to the INR profile Λ . If some of interferers have a negligible power level, the INR profile Λ is effectively rank deficient. Hence, the rank κ_2 of Λ reflects the effective number of nonnegligible interferers. With the weighting vector $\mathbf{w}_{\text{oc}} = c \mathbf{R}^{-1} \alpha_0$ (where c is an arbitrary constant), the maximum SINR at the optimum combiner output $\mathbf{w}_{\text{oc}}^\dagger \mathbf{r}$ is [1]

$$\gamma = \text{SNR} \alpha_0^\dagger (\mathbf{I}_{N_A} + \mathbf{H}_1 \mathbf{H}_1^\dagger)^{-1} \alpha_0 \quad (6)$$

where $\text{SNR} = P_0/N_0$ denotes the average SNR per branch.

III. SYMBOL ERROR PROBABILITY

For any arbitrary two-dimensional signaling constellation with polygonal decision boundaries, the average SEP for coherent detection can be written as [14]

$$P_e = \sum_{k=1}^M p_k \sum_{\ell \in \mathcal{B}_k} \frac{1}{2\pi} \int_0^{\phi_{k,\ell}} \mathcal{M}_\gamma \left(\frac{w_{k,\ell}}{4 \sin^2(\theta + \psi_{k,\ell})} \right) d\theta \quad (7)$$

where M is the constellation size; $\{p_k\}_{k=1}^M$ are the *a priori* symbol probabilities; $\phi_{k,\ell}$, $w_{k,\ell}$, $\psi_{k,\ell}$, and $\mathcal{B}_k$ are the quantities depending on the geometry of the constellation (see [14, Fig. 10]); and $\mathcal{M}_\gamma(s) \triangleq \mathbb{E}\{e^{-s\gamma}\}$ is the moment generating function (MGF) of γ . Using (6), we get

$$\mathcal{M}_\gamma(s) = (1 + s \text{SNR})^{-N_A} \text{DQM} \left(\Phi, \Lambda; \frac{1}{1 + s \text{SNR}} \right). \quad (8)$$

A. Detquotient Mean DQM($\Sigma, \Psi; \xi$)

Using the Berezin's supermathematics, we first present the analytical solution for $\text{DQM}(\Sigma, \Psi; \xi)$, which enables us to derive the exact SEP expression for optimum combining in the UP-SC interference environments.

Lemma 1: Let $\mathbf{Q} \in \mathbb{C}^{m \times m}$ be positive semidefinite. Then, we have the following inverse Fourier transforms for determinantal functions of $\mathbf{Q}$:

i) For a positive constant ζ ,

$$\begin{aligned} & \mathcal{F}_{(\omega)}^{-1} \left\{ \det(\mathbf{I}_m + j\omega \zeta \mathbf{Q})^{-1} \right\}(t) \\ &= \sum_{i=1}^{\varrho(\mathbf{Q})} \sum_{j=1}^{\tau_i(\mathbf{Q})} \frac{\mathcal{X}_{i,j}(\mathbf{Q}) t^{j-1}}{\Gamma(j) \zeta^j \lambda_{(i)}^j(\mathbf{Q})} \exp \left(-\frac{t}{\zeta \lambda_{(i)}(\mathbf{Q})} \right) u(t) \end{aligned}$$

where $j = \sqrt{-1}$, $\Gamma(z)$ is the Euler's gamma function, $u(z)$ is the Heaviside step function, and $\mathcal{X}_{i,j}(\mathbf{Q})$ is the (i, j) th characteristic coefficient of $\mathbf{Q}$ [13, Definition 4].⁴

³Since the fading rate is much slower than the symbol rate, the interference-plus-noise vector $\mathbf{z}$ conditioned on the interference channel vectors $\alpha_1, \alpha_2, \dots, \alpha_{N_I}$ is $\sim \tilde{\mathcal{N}}_{N_A,1}(\mathbf{0}, \mathbf{R}, 1)$.

⁴We shall denote the inverse Fourier transform of $X(j\omega)$ by

$$\mathcal{F}_{(\omega)}^{-1} \{X(j\omega)\}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega.$$

ii) For an arbitrary constant ζ ,

$$\mathcal{F}_{(\omega)}^{-1}\{\det(\mathbf{I}_m + j\omega\zeta\mathbf{Q})\}(t) = \sum_{k=0}^m \zeta^k \det_{[k:\emptyset]}(\mathbf{Q}) \delta^{(k)}(t)$$

where $\delta^{(k)}(z)$ denotes the k th derivative of the Dirac's delta function $\delta(z)$.

Theorem 1: Let $\mathbf{\Sigma} \in \mathbb{C}^{m \times m}$ and $\mathbf{\Psi} \in \mathbb{C}^{n \times n}$ be positive semidefinite. Then, for $\xi > 0$, we have

$$\begin{aligned} \text{DQM}(\mathbf{\Sigma}, \mathbf{\Psi}; \xi) &= \mathcal{G}(\mathbf{\Sigma}, \mathbf{\Psi}) \mathcal{J}(\mathbf{\Sigma}, \mathbf{\Psi}; \xi) \\ &- \xi \sum_{k=1}^m \sum_{\ell=1}^n \left\{ \lambda_k^2(\mathbf{\Sigma}) \lambda_\ell^2(\mathbf{\Psi}) \mathcal{H}_{k,\ell}(\mathbf{\Sigma}, \mathbf{\Psi}) \right. \\ &\quad \times \mathcal{J}\left(\begin{bmatrix} \mathbf{\Sigma} & 0 \\ 0 & \lambda_k(\mathbf{\Sigma}) \end{bmatrix}, \begin{bmatrix} \mathbf{\Psi} & 0 \\ 0 & \lambda_\ell(\mathbf{\Psi}) \end{bmatrix}; \xi\right) \Big\} \end{aligned} \quad (9)$$

with

$$\begin{aligned} \mathcal{G}(\mathbf{\Sigma}, \mathbf{\Psi}) &= \sum_{k=0}^{\min\{m,n\}} k! \det_{[k:\emptyset]}(\mathbf{\Sigma}) \det_{[k:\emptyset]}(\mathbf{\Psi}) \\ \mathcal{H}_{p,q}(\mathbf{\Sigma}, \mathbf{\Psi}) &= \sum_{k=0}^{\min\{m,n\}-1} k! \det_{[k:\{p\}]}(\mathbf{\Sigma}) \det_{[k:\{q\}]}(\mathbf{\Psi}) \\ \mathcal{J}(\mathbf{\Sigma}, \mathbf{\Psi}; \xi) &= \sum_{i=1}^{\varrho(\mathbf{\Sigma})} \sum_{j=1}^{\tau_i(\mathbf{\Sigma})} \sum_{p=1}^{\varrho(\mathbf{\Psi})} \sum_{q=1}^{\tau_p(\mathbf{\Psi})} \left\{ \mathcal{X}_{i,j}(\mathbf{\Sigma}) \mathcal{X}_{p,q}(\mathbf{\Psi}) \right. \\ &\quad \times {}_2F_0\left(j, q; -\xi \lambda_{\langle i \rangle}(\mathbf{\Sigma}) \lambda_{\langle p \rangle}(\mathbf{\Psi})\right) \Big\} \end{aligned}$$

where ${}_pF_q(\cdot)$ is the generalized hypergeometric function.

Corollary 1: For $\mathbf{\Sigma} = \sigma \mathbf{I}_m$ and $\mathbf{\Psi} = \rho \mathbf{I}_n$, we have

$$\begin{aligned} \text{DQM}(\sigma \mathbf{I}_m, \rho \mathbf{I}_n; \xi) &= T_{m,n}^{(\sigma, \rho)} {}_2F_0(m, n; -\xi \sigma \rho) \\ &- \xi \sigma^2 \rho^2 mn T_{m-1, n-1}^{(\sigma, \rho)} {}_2F_0(m+1, n+1; -\xi \sigma \rho) \end{aligned} \quad (10)$$

where

$$T_{m,n}^{(\sigma, \rho)} = \sum_{k=0}^{\min\{m,n\}} k! (\sigma \rho)^k \binom{m}{k} \binom{n}{k}. \quad (11)$$

Corollary 2: For $\mathbf{\Sigma} = \sigma \mathbf{1}_m$ and $\mathbf{\Psi} = \rho \mathbf{1}_n$, we have

$$\begin{aligned} \text{DQM}(\sigma \mathbf{1}_m, \rho \mathbf{1}_n; \xi) &= (1 + \sigma \rho mn) {}_2F_0(1, 1; -\xi \sigma \rho mn) \\ &- \xi (\sigma \rho mn)^2 {}_2F_0(2, 2; -\xi \sigma \rho mn). \end{aligned} \quad (12)$$

B. General Result

Using (7), (8), and Theorem 1, we obtain the exact SEP expression for optimum combining in the UP-SC interference environment as (13), shown at the bottom of the page, where $\mathcal{P}_{k,\ell}(\mathbf{A}, \mathbf{B})$ for $m \times m$ and $n \times n$ positive-semidefinite matrices $\mathbf{A}$ and $\mathbf{B}$ is given by (14) with

$$\begin{aligned} \mathcal{V}_{i,j}^{\phi, \psi}(\nu, a, b) &\triangleq \frac{1}{2\pi} \int_0^\phi \left(1 + \frac{a}{4 \sin^2(\theta + \psi)}\right)^{-\nu} \\ &\times {}_2F_0\left(i, j; -\frac{b}{1 + \frac{a}{4 \sin^2(\theta + \psi)}}\right) d\theta \end{aligned} \quad (15)$$

for $i, j, \nu \in \mathbb{N}$ and $a, b > 0$. Note that the expression (13) is valid for arbitrary spatial correlation $\mathbf{\Phi}$ and INR profile matrix $\mathbf{\Lambda}$. Together with systematic implementations of the characteristic coefficients and k -determinant provide a simple and unifying way to calculate the exact SEP.

C. Case Study

As two extreme interference environments, we now particularize the average SEP in (13) for the best and worst interference environments when the total INR is fixed to

$$\text{INR}_{\text{tot}} \triangleq \sum_{k=1}^{N_I} \text{INR}_k = N_I \bar{\gamma}_I. \quad (16)$$

1) Worst interference case: When the desired signal is disturbed by equal-power spatially-uncorrelated interfering signals, the optimum combining receiver suffers the most severe interference. For this worst case, i.e., $\mathbf{\Phi} = \mathbf{I}_{N_A}$ and $\mathbf{\Lambda} = \bar{\gamma}_I \mathbf{I}_{N_I}$, we can reduce the average SEP (13) using Corollary 1 to

$$\begin{aligned} P_e^{(\text{worst})} &= \sum_{k=1}^M p_k \sum_{\ell \in \mathcal{B}_k} \left\{ T_{N_A, N_I}^{(1, \text{INR})} \mathcal{V}_{N_A, N_I}^{\phi_{k,\ell}, \psi_{k,\ell}}(N_A, w_{k,\ell} \text{SNR}, \bar{\gamma}_I) \right. \\ &\quad \left. - N_A N_I \bar{\gamma}_I^2 T_{N_A-1, N_I-1}^{(1, \text{INR})} \mathcal{V}_{N_A+1, N_I+1}^{\phi_{k,\ell}, \psi_{k,\ell}}(N_A+1, w_{k,\ell} \text{SNR}, \bar{\gamma}_I) \right\} \end{aligned}$$

which is simple to evaluate.

$$P_e = \sum_{k=1}^M p_k \sum_{\ell \in \mathcal{B}_k} \left\{ \mathcal{G}(\mathbf{\Phi}, \mathbf{\Lambda}) \mathcal{P}_{k,\ell}(\mathbf{\Phi}, \mathbf{\Lambda}) - \sum_{r=1}^{N_A} \sum_{t=1}^{N_I} \lambda_r^2(\mathbf{\Phi}) \text{INR}_t^2 \mathcal{H}_{r,t}(\mathbf{\Phi}, \mathbf{\Lambda}) \mathcal{P}_{k,\ell} \left(\begin{bmatrix} \mathbf{\Phi} & 0 \\ 0 & \lambda_r(\mathbf{\Phi}) \end{bmatrix}, \begin{bmatrix} \mathbf{\Lambda} & 0 \\ 0 & \text{INR}_t \end{bmatrix} \right) \right\} \quad (13)$$

$$\mathcal{P}_{k,\ell}(\mathbf{A}, \mathbf{B}) = \sum_{i=1}^{\varrho(\mathbf{A})} \sum_{j=1}^{\tau_i(\mathbf{A})} \sum_{p=1}^{\varrho(\mathbf{B})} \sum_{q=1}^{\tau_p(\mathbf{B})} \mathcal{X}_{i,j}(\mathbf{A}) \mathcal{X}_{p,q}(\mathbf{B}) \mathcal{V}_{j,q}^{\phi_{k,\ell}, \psi_{k,\ell}} \left(m, w_{k,\ell} \text{SNR}, \lambda_{\langle i \rangle}(\mathbf{A}) \lambda_{\langle p \rangle}(\mathbf{B}) \right) \quad (14)$$

2) *Best interference case*: Let $\mathbf{\Lambda} = \bar{\gamma}_I \mathbf{1}_{N_I}$ for the circumstance in the presence of a dominant strong-power interferer, which is equivalent to the case of INR profile $\{\text{INR}_k\}_{k=1}^{N_I} = \{N_I \bar{\gamma}_I, 0, \dots, 0\}$. In this degenerate interference-power situation together with spatially full correlation $\mathbf{\Phi} = \mathbf{1}_{N_A}$, we can reduce the average SEP (13) using Corollary 2 to

$$P_e^{(\text{best})} = \sum_{k=1}^M p_k \sum_{\ell \in \mathcal{B}_k} \left\{ (1 + N_A N_I \bar{\gamma}_I) \times \mathcal{V}_{1,1}^{\phi_{k,\ell}, \psi_{k,\ell}}(N_A, w_{k,\ell} \text{SNR}, N_A N_I \bar{\gamma}_I) - (N_A N_I \bar{\gamma}_I)^2 \mathcal{V}_{2,2}^{\phi_{k,\ell}, \psi_{k,\ell}}(N_A + 1, w_{k,\ell} \text{SNR}, N_A N_I \bar{\gamma}_I) \right\}.$$

D. Achievable Diversity

Theorem 2: Let $\text{SIR}_k = P_0/P_k$, $k = 1, 2, \dots, N_I$, be the average signal-to-interference power ratio (SIR) for the k th interferer and

$$d_{\text{oc}} \triangleq \lim_{\substack{\text{SNR} \rightarrow \infty \\ \text{SIR}_k \rightarrow c_k}} \frac{-\log P_e}{\log \text{SNR}} \quad (17)$$

be the diversity order achieved by optimum combining where c_k 's are finite positive constants. Then, the diversity order is

$$d_{\text{oc}} = N_A - \min\{\kappa_1, \kappa_2\}. \quad (18)$$

Theorem 2 reveals that the *rank-deficient* interference correlation or INR profile decreases the effective number of interferers and hence only $\min\{\kappa_1, \kappa_2\}$ receiving antennas are required to suppress the interfering signals. Therefore, regardless of the number of interferers and their power profile, if each interfering signal is fully correlated at the receiver, one of receiving antennas can be exploited to suppress these degenerate interfering signals and the other antennas can realize the diversity order of $N_A - 1$.

IV. NUMERICAL RESULTS

Fig. 1 shows the analytical and simulated average SEP results for 8-PSK versus SNR when $N_I = 3$ and N_A varies from 2 to 5. In this example, the spatial correlation for interfering signals follows the exponential correlation $\mathbf{\Phi} = [0.5^{|i-j|}]$ and the SIR's for three interferers are set to $\{\text{SIR}_k\}_{k=1}^{N_I} = \{0 \text{ dB}, 5 \text{ dB}, 10 \text{ dB}\}$. We can see that the analysis agrees exactly with the simulation results and observe that the achievable diversity order (i.e., the high-SNR slope of the SEP curve) is equal to $d_{\text{oc}} = N_A - \min\{N_A, N_I\}$, as stated in Theorem 2, since $\mathbf{\Phi}$ and $\mathbf{\Lambda}$ are full rank. Hence, the high-SNR slope vanishes for overloaded cases ($N_A \leq N_I$) in which all antennas are used up to suppress the interfering signals.

The effect of interference correlation on the SEP performance can be ascertained by referring to Fig. 2 where the analytical SEP of 8-PSK at SNR = 25 dB is depicted as a function of the correlation coefficient ρ for the exponential interference correlation $\mathbf{\Phi} = [\rho^{|i-j|}]$. In this figure, N_I , N_A , and $\{\text{SIR}_k\}_{k=1}^{N_I}$ are the same as in Fig. 1. We can observe

Fig. 1. Average SEP of 8-PSK versus SNR: $N_I = 3$, $\mathbf{\Phi} = [0.5^{|i-j|}]$, $\{\text{SIR}_k\}_{k=1}^{N_I} = \{0 \text{ dB}, 5 \text{ dB}, 10 \text{ dB}\}$, and $N_A = 2, 3, 4, 5$.

that the SEP decreases with the correlation coefficient ρ , i.e., the amount of correlation. This SEP decrease is especially pronounced, due to strong interference correlation, at the value of ρ close to one. This reveals that the SEP as a functional of the eigenvalues of the interference correlation matrix $\mathbf{\Phi}$ is *monotonically decreasing in a sense of Schur (MDS)*, namely, the more spatially-correlated interference results in the less severe interfering effect, and hence less significantly degrades the SEP performance.⁵

In what follows, we consider an *intra-class* INR profile matrix for interfering signals:

$$\mathbf{\Lambda}_{\text{intra}}(\rho) = \bar{\gamma}_I \begin{bmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{bmatrix}_{N_I \times N_I}, \quad \rho \in [0, 1]$$

which is equivalent to the INR profile

$$\begin{aligned} \{\text{INR}_k\}_{k=1}^{N_I} &= \left\{ \lambda_k(\mathbf{\Lambda}_{\text{intra}}(\rho)) \right\}_{k=1}^{N_I} \\ &= \{(1 - \rho + N_I \rho) \bar{\gamma}_I, (1 - \rho) \bar{\gamma}_I, \dots, (1 - \rho) \bar{\gamma}_I\}. \end{aligned}$$

Note that the intra-class INR profile can be interpreted as a situation consisting of one strong interferer and $N_I - 1$ relatively low equal-power interferers with the aggregate INR equal to $\text{INR}_{\text{tot}} = N_I \bar{\gamma}_I$.

Fig. 3 demonstrates the effect of interference-power profile on the SEP performance, where the analytical SEP for 8-PSK versus SNR is depicted for the intra-class INR profile $\mathbf{\Lambda}_{\text{intra}}(\rho)$. In this example, $N_A = 4$, $N_I = 3$, $\mathbf{\Phi} = [0.5^{|i-j|}]$, $\bar{\gamma}_I = \text{SNR}$, and $\rho = 0$ (equal-power interferers), 0.3, 0.5, 0.7, 0.8, 0.9,

⁵See, e.g., [13, Appendix I] and references therein for the basic concept of Schur monotonicity.

Fig. 2. Average SEP of 8-PSK as a function of the correlation coefficient ρ at $\text{SNR} = 25$ dB: $N_I = 3$, $\Phi = [\rho^{|i-j|}]$, $\text{SIR} \{\text{SIR}_k\}_{k=1}^{N_I} = \{0 \text{ dB}, 5 \text{ dB}, 10 \text{ dB}\}$, and $N_A = 2, 3, 4, 5$.

Fig. 3. Average SEP of 8-PSK versus SNR: $N_A = 4$, $N_I = 3$, $\Phi = [0.5^{|i-j|}]$, $\Lambda = \Lambda_{\text{intra}}(\rho)$, $\bar{\gamma}_I = \text{SNR}$, and $\rho = 0$ (equal-power interferers), $0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 1$ (dominant strong interferer).

0.95, 1 (dominant strong interferer). We can observe that the SEP decreases with ρ , i.e., the degree of power unbalance among interferers. In particular, the achievable diversity order is equal to $d_{\text{oc}} = 3$ (i.e., $d_{\text{oc}} = 4 - \min\{3, 1\}$) for $\rho = 1$ (dominant strong interferer) due to the rank deficiency of Λ , while achieving the diversity order equal to $d_{\text{oc}} = 1$ (i.e., $d_{\text{oc}} = 4 - \min\{3, 3\}$) for other values of ρ . This reveals that the SEP as a functional of the INR profile $\{\text{INR}_k\}_{k=1}^{N_I}$ (or equivalently, the eigenvalues of the INR profile matrix Λ) is again MDS, namely, the more unbalanced (or less spread out)

interference-power profile results in the less severe interfering effect, providing a better SEP performance.

V. CONCLUSIONS

We derived the exact average SEP for optimum combining in a flat Rayleigh-fading environment in the presence of multiple cochannel interferers and thermal noise, taking into account unequal power levels among interfering signals, each is spatially correlated across receiving antennas. Using the Berezin's supermathematics, we established a unifying framework to calculate the exact SEP for such general interference environments. On account of the generality of interference modeling, our results are substantial enough to encompass all the previous exact SEP analysis for equal-power and/or spatially-uncorrelated interferers as special cases, which hence consolidate the effects of interference power unbalance and spatial correlation on the SEP of diversity systems.

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