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Aggregate Demand Management In Search Equilibrium

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1. Introduction

To introduce the basic idea of macroeconomics, textbooks generally start with a very simple model. (See, e.g., Dornbusch and Fischer.) Prices are assumed to be constant. There is no money and no capital market. For equilibrium, income adjusts so that consumers and producers can carry out the plans they have made, conditional on income. This paper presents a very simple general equilibrium model of search. There is no capital market and no money. While prices are free to change, the equilibrium price does not vary with economic conditions. Production and consumption strategies determine the equilibrium level of production and income.

With individuals optimizing, the steady state rational expectations equilibrium is inefficient. The inefficiency comes from the fact that a greater level of production improves the trading opportunities of others. In a frictionless market model, this externality would be pecuniary and would have no efficiency implications. In a model with trading frictions this externality matters. In addition to the property that welfare is improved by a small move from equilibrium in the direction of greater economic activity, the model has multiple equilibria, giving a second reason for potential welfare improvement from government intervention.[1]

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[1]The model closest to this in structure is that of M. Hellwig who shows that his search model converges to a Walrasian model as the rate of arrival of trade opportunities rises without limit.

Section 2 to 7 present a continuous time search equilibrium model. In Section 8, the basic mechanism of the trading externality is presented in a static model.

2. Basic Model

We use a highly artificial model of the production and trade processes to highlight the workings of a general equilibrium search model. All individuals are assumed to be alike. Instantaneous utility satisfies

$$U = y - c \tag{1}$$

where y is the consumption of output and c is the cost of production (disutility of labor). The utility function is chosen to be linear as part of the simplification that leads to the conclusion that trade bargains will not vary across pairs who are trading. In addition, the absence of risk aversion permits us to ignore the absence of implicit or explicit wage insurance. Lifetime utility is the present discounted value of instantanous utility

$$V = \int_0^{\infty} e^{-rt} U(t) dt \, . \tag{2}$$

Individuals are assumed to maximize the expected value of lifetime utility.

Production opportunities are a Poisson process. With arrival rate a, each individual learns of production opportunities. Each opportunity has y units of output and costs c ($c \geq \underline{c} > 0$) units to produce. We assume that y is the same for all projects but that c varies across projects with distribution G. Each opportunity is a random draw from G, with costs known before the decision on undertaking the project. There are two further restrictions. Individuals cannot consume the products of their own investment, but trade their own output for that produced by others. Individuals cannot undertake a production project if they have unsold produced output on hand. This extreme assumption on the costs of inventory holding is also part of the simplification of the determination of trade bargains. The fact that all trades involve individuals with y units to sell implies that all units are swapped on a one-for-one basis, and promptly consumed. Thus individuals have 0 or y units for sale. The former are looking for production opportunities and are referred to as unemployed. The latter are trying to sell their output and are referred to as employed.

The trading process is such that for each individual the arrival of potential trading partners is a Poisson process with arrival rate b. For any meeting, there is a probability p that the potential trading partner has a unit to sell, and a probability 1-p that the potential partner does not have a unit to sell; i.e., is unemployed. It is assumed that there is no credit market, so those with nothing to sell are unable to buy. The probability that a potential partner is in the market is a function of the fraction of the population unemployed, u, with p(u) decreasing in u. For example, with undirected search for trading partners p(u) would equal 1-u. The economy is assumed to be sufficiently large so that the expected value of potential production and trade opportunities is realized. The unemployment rate rises from each completed transaction, as a previously employed person becomes eligible to undertake a production opportunity and falls whenever a production opportunity is undertaken. Assuming that all production opportunities with costs below c* are undertaken, we have

$$\dot{u} = bp(u)(1-u) - auG(c^*). \tag{3}$$

That is each of the 1-u employed (per capita) faces the probability bp of having a successful trade meeting and being freed to seek a new opportunity. Each of the u unemployed (per capita) has the flow probability a of learning of an opportunity and accepts the fraction G(c*) of opportunities. In a steady state, we have the equilibrium rate of unemployment by setting $\dot{u}$ equal to zero. From (3) we see that the steady state unemployment rate falls with c*:

$$\left.\frac{du}{dc^*}\right|_{\dot{u}=0} = \frac{auG'(c^*)}{b(1-u)p'(u) - bp(u) - aG(c^*)} < 0 . \tag{4}$$

We turn next to the determination of c*.

3. Individual Choice

As modeled, the only decisions to be made are whether to undertake particular production possibilities. Assuming a steady state equilibrium, we can describe this decision as a simple dynamic programming problem. Let us denote the expected present discounted value of lifetime utility for employed and unemployed by W_e and W_u. Then, the utility discount rate times each of these values equals the expected value of the flow of instantaneous utility plus the expected capital gain from a change in status

$$rW_e = bp[y - W_e + W_u]$$
$$rW_u = a \int_0^{c^*} [W_e - W_u - c]dG(c) \quad . \tag{5}$$

With probability bp, an employed person has a trade opportunity giving rise to instantaneous consumption y and a change in status to unemployed. Each unemployed person accepting a production opportunity has an instantaneous utility $-c$ and a change in status to employed.

An unemployed person accepts any opportunity that raises expected utility. Thus we have the criterion

$$c^* = W_e - W_u = \frac{bpy + a \int_0^{c^*} cdG}{r + bp + aG(c^*)} \quad . \tag{6}$$

The level of aggregate demand affects investment decisions since the probability of a sale decreases with the unemployment rate. Differentiating (6) we have

$$\frac{dc^*}{du} = \frac{(y-c^*)\ bp'}{r + bp + aG} < 0,$$

$$\frac{d^2c^*}{du^2} = \frac{(y-c^*)bp'' - 2bp'\frac{dc^*}{du} - aG'(\frac{dc^*}{du})^2}{r + bp + aG} . \qquad (7)$$

To see that $\frac{dc^*}{du}$ is negative, we note that (with positive interest) no one would undertake a project with less output than input ($y > c^*$) and $p' < 0$. With $p'' < 0$, $\frac{d^2c^*}{du^2}$ is also negative. Armed with (3) and (6) we can describe steady state equilibrium.

4. Steady State Equilibrium

A steady state is marked by optimal production decisions (6) and a constant rate of unemployment (3). In each of these equations u and c* are negatively related, allowing the possibility of multiple steady state equilibria. When trade opportunities are better (lower u) individuals are more likely to undertake production (higher c*). A higher rate of production, in turn, improves trade opportunities. In Figure 1 we have drawn (3) and (6) showing multiple equilibria.

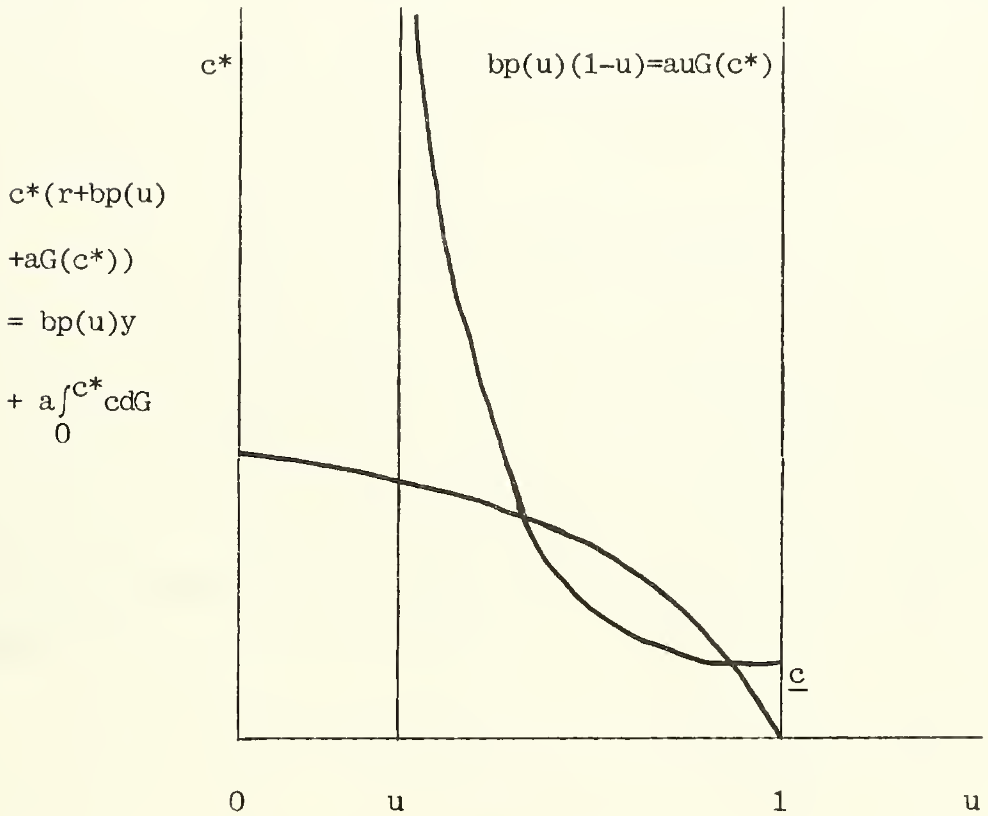

Figure 1

Steady state unemployment rates are bounded below by the unemployment level reached if all production opportunities are accepted (G = 1). The rate equals one for c* below c, the lower bound of possible production costs. The production cost cutoff goes to zero as p(u) goes to zero, as it will as u goes to one. As u goes to zero and p(u) goes to one, the production choice cutoff has a finite upper bound.

If agents expect the current unemployment rate to be permanent then the economy is always on the optimal steady state production decision curve, (6), and the equilibria in Figure 1 with the lowest unemployment rate and with a rate of one are stable. Since G does not necessarily have nice properties, there can be more equilibria than shown.

5. Long Run Stimulation Policy

To explore aggregate demand policy, we will assume that the government has sufficient policy tools to control production decisions. Below we will consider a production cost subsidy to induce private decisions at the optimal steady state. In this section we will examine a small permanent change in c* away from a steady state equilibrium with no intervention. In the next section we will examine the optimal path for c*(t) from an arbitrary initial position. In a steady state equilibrium, we have a flow of utility per capita satisfying

$$Q(t) = bp(u)(1-u)y - au \int_0^{c^*} c\,dG, \qquad (8)$$

where bp(u)(1-u) is the rate of sales, with consumption of y per sale, and auG is the rate of production, with an average cost of $\int_0^{c^*} c\,dG/G$ per project undertaken. For social welfare we are interested in the present discounted value of Q

$$W = \int_0^\infty e^{-rt} Q(t)dt. \tag{9}$$

Starting at a steady state equilibrium ($\dot{u}=0$), the change in W from a permanent change in c* satisfies (for a derivation of (10) see Diamond (1980))

$$r\frac{\partial W}{\partial c^*} = -auc^*G'(c^*) - (by((1-u)p' - p) - a\int_0^{c^*} cdG) \frac{auG'(c^*)}{r + bp - b(1-u)p' + aG(c^*)} . \tag{10}$$

The first term represents the increase in production costs at the steady state unemployment rate while the second represents the change in both output and investment along the unemployment rate trajectory induced by the change in investment rule. At an equilibrium without intervention (where (6) holds) we can write this as

$$r\frac{\partial W}{\partial c^*} = -auc^*G' + \frac{(-(1-u)byp' + c^*(r + bp + aG))auG'}{r + bp - b(1-u)p' + aG}$$

$$= \frac{-auG'(1-u)bp'}{r + bp - b(1-u)p' + aG} (y-c^*) > 0. \tag{11}$$

Thus, without intervention, there is too little activity in the economy.

6. Short Run Stabilization Policy

Continuing with the assumption that the government can control production decisions, we can examine the optimal policy for an arbitrary initial position. That is, the optimal stabilization policy satisfies

$$\max_{c^*(t)} \int_0^\infty e^{-rt} Q(t)\,dt$$

$$\text{where } Q(t) = bp(u(t))(1-u(t))y - au(t)\int_0^{c^*(t)} c\,dG \tag{12}$$

$$\dot{u}(t) = bp(u(t))(1-u(t)) - au(t)G(c^*(t))$$

$$u(0) = u_0.$$

The Euler equation is

$$\dot{c}^*(t) = rc^* + (y-c^*)b(p'(1-u)-p) + a\int_0^{c^*}(c^* - c)\,dG. \tag{13}$$

The phase diagram is shown in Figure 2 under the assumption that the state with lowest unemployment is the optimum.

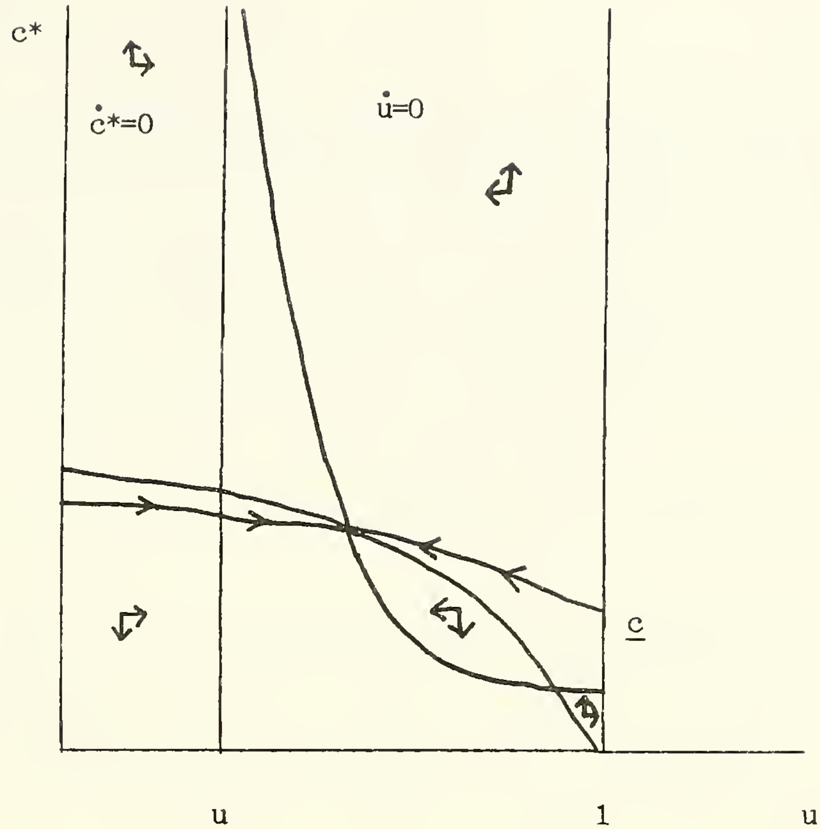

Figure 2

Comparing the equation for $\dot{c}^*=0$, (13), and the private choice of c^* in a steady state, (6), the former is always above the latter as a function of u.

7. Subsidizing Production

The asymptotically optimal steady state is described by setting c^* (in (13)) equal to zero (or alternatively by setting $\frac{\partial W}{\partial c^*}$ (in (10)) equal to zero). By subsidizing the cost of producton, individuals can be induced to select this cutoff cost. In this section we derive the equation for this subsidy. We assume that the subsidy is financed by a lumpsum tax (payable in labor) that falls on the employed and unemployed equally.

With a subsidy of s per project completed, the individually optimal cutoff rule becomes

$$c^*-s = W_e-W_u = \frac{bpy + a\int_0^{c^*}(c-s)dG}{r + bp + aG(c^*)} \;. \qquad (14)$$

The asymptotically optimal level satisfies

$$c^* = \frac{bpy - bp'(1-u)y + a\int_0^{c^*}cdG}{r + bp - bp'(1-u) + aG(c^*)} \;.$$

Equating the expressions for c* and solving we have

$$s = \frac{-b(1-u)p'(ry + a\int_0^{c^*}(y-c)dG)}{(r+bp)(r+bp-b(1-u)p' + aG)} \;. \qquad (15)$$

This subsidy level is positive as can be seen from (13) which implies $y > c^*$ when $\dot{c}^*$ equals zero.[1]

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[1]L. Weiss suggested calculating the effect of unemployment compensation, financed by a tax output. Such a policy can be fitted into the model by giving each unemployed person a probability of receiving an output bundle just equal to the after tax output level of a project. Such a policy moves in the wrong direction since the incentive to production of having more potential trading partners is smaller than the disincentives coming from the sum of output taxation and unemployment subsidization.

8. Static Model

The dynamic model used above seems useful for understanding both the workings of the externality and the design of policy. Given that model to motivate the equilibrium trade possibilities, one can describe the externality more simply in terms of a static model. Let us consider an aggregate production function

$$y = f(c) \tag{16}$$

with $f' > 0$, $f'' < 0$. Let $\pi(y)$ be the probability of making a sale as a function of the aggregate output level. Unsold output is assumed to be wasted so that welfare satisfies

$$U = y\pi(y) - c. \tag{17}$$

If individuals view π as a parameter, equilibrium occurs at a level of production satisfying

$$\pi(f(c))f'(c) = 1. \tag{18}$$

For efficiency, the aggregate relationship between sales probability and production level must be recognized, giving an optimality condition

$$(\pi + y\pi')f'(c) = 1. \tag{19}$$

By subsidizing the cost of production (financed by lump sum taxation) the decentralized economy can be induced to produce at a point which satisfies the social optimality condition.

9. Summary and Conclusions

It is common in theoretical economics to use a tropical island metaphor to describe the workings of a model. The island described here has many individuals, not one. When unemployed they stroll along the beaches examining palm trees. Some trees have cocoanuts. All bunches have the same number of nuts, but differ in their heights above the ground. Having spotted a bunch the individual decides whether to climb the tree. There is a taboo against eating nuts one has picked oneself. Having climbed a tree, the worker goes searching for a trade - nuts for nuts - which will result in consumption. This represents, artificially, the realistic aspect of the small extent of consumption of one's own production in modern economies. The ease in finding a trading partner depends on the number of potential partners available. Thus the equilibrium level of production is not efficient if everyone correctly predicts the difficulty of successful trading. Of course, overoptimism can result in the efficient production level. There is no mechanism to ensure that individual by individual, or on average, forecasts of time to completed trade are correct. Errors would be particularly likely in a nonsteady state path.

There are several properties of this type of macro search model which seem particularly attractive. Even without lags in the ability of the government to affect private decisions, the government does not have the power to move instantaneously to a full employment position. Recognizing the costs of starting a production process there is an optimal rate of convergence to the optimally full employment steady state, reflecting the higher real costs of moving too quickly. Knowledge of private forecasts would be essential to the optimal design of tools to alter private decisions but are not necessary for recognizing a situation calling for intervention (except to the extent that the bases of private forecasts might improve the government forecast).

The model presented here is very special. One cannot draw policy conclusions directly from such a model. There are two purposes for its construction. One is to form a basis for further generalization and study. The second is to provide an example to contrast with models which assume, unrealistically, the existence of a frictionless, instantaneous trade coordination mechanism. While the construction of realistic models of trade frictions (and wage rigidities) is needed for good policy analysis, the existence of this simple model should indicate the inappropriateness of basing policy on a model with perfect markets.

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