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# Randomized Search Strategies for Wideband Signal Acquisition

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**Abstract**—In this paper, we investigate performance of acquisition receivers employing randomized search strategies. In particular, we derive the expressions of the mean acquisition times (MATs) for the uniform randomized search strategy and the non-return-to-self uniform randomized search strategy (NRS). These two search strategies are robust against variation in the maximum dispersion of the channel, making them attractive from the standpoint of a receiver design. We also quantify the MATs of receivers that employ the uniform randomized search strategy and the multi-dwell detectors with optimal thresholds. The MATs are plotted for various search strategies over a range of signal-to-noise ratios, using a channel model that is suitable for wideband transmission. The numerical examples confirm the benefit of the uniform randomized search strategy and the NRS over the conventional serial search strategy.

**Index Terms**—Acquisition, synchronization, spread-spectrum, dense multipath channel, multi-dwell detector, wideband transmission.

## I. INTRODUCTION

Signal acquisition is a task of digital receivers and needs to be performed before data decoding. Signal acquisition serves to coarsely align the phase of the received signal and the phase of the locally generated reference (LGR). Signal acquisition needs to be achieved as fast as possible, especially for applications that involve reliable and time-sensitive transmission. In many cases, signal acquisition is performed without help of acquisition-aided preambles, making signal acquisition a challenging task.

Signal acquisition becomes more challenging when the transmission bandwidth is large. Large transmission bandwidth results in a large number of phases that the receiver needs to examine [1], and hence a slow signal acquisition. This challenge motivates an investigation of techniques that improve performance of wideband signal acquisition.

In general, performance of signal acquisition is commonly measured by a statistic of the acquisition time, a random variable representing the amount of time to achieve (correct) signal acquisition.<sup>1</sup> Examples of common statistics include the expectation and the cumulative distribution function (CDF).

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<sup>1</sup>Randomness in the acquisition time is due to noise, fading, and possibly a randomized decision rule. The use of appropriate statistics to characterize synchronization performance is also common in frame synchronization [2].

The expectation measures the average duration to achieve signal acquisition, also known as the mean acquisition time (MAT) [3]–[7]. The CDF measures the probability that signal acquisition is achieved within a given duration [3]. The expectation is more commonly used than the CDF due to tractability. With this reason, the MAT will be our performance measure.

Techniques to improve the MAT may be classified into a serial search technique [8], a hybrid search technique [9], and a fully-parallel search technique [10]. A serial search technique examines a single signal phase at a given time. The hybrid and fully-parallel search techniques examine many signal phases simultaneously, resulting in a lower MAT at the expense of more hardware complexity than serial search techniques. Serial search techniques will be a focus in this paper, since they require minimum hardware and are common techniques for signal acquisition.

Among different serial search techniques, a promising approach to improve the MAT is to examine signal phases (or cells) according to a specific order, a technique known as a search strategy. Examples of search strategies include the conventional serial search (CSS) [1], [5], the fixed-step serial search with a step size  $N_J$  (FSS- $N_J$ ) [1], the bit-reversal search [5], and the uniform randomized search [11], [12]. The CSS is the most commonly-used search strategy, giving the most natural order  $\{1, 2, 3, \dots\}$  of cells to be examined. The FSS- $N_J$  allows a jump size of  $N_J$  cells after an examination of each cell. The bit reversal search examines the cells in the order that approximately mimics the binary search in computer science. The uniform randomized search examines a cell randomly so that every cell is equally likely to be examined. The uniform randomized search is less well-known than the CSS, FSS- $N_J$ , or the bit-reversal, making the uniform randomized search a natural subject for exploration.

The uniform randomized search is attractive from the standpoint of a receiver design. This search strategy spreads the correct signal phases uniformly in the search sequence, a property that typically results in a small MAT. The uniform randomized search is robust against the variation in the number of multipath clusters and the maximum dispersion of the channel. Despite these attractions, the MAT of the uniform randomized search is unknown except for a few cases.<sup>2</sup>

In this paper, we investigate performance of the uniform randomized search and its natural variation, the non-return-to-self uniform randomized search (NRS). The NRS has the same

<sup>2</sup>These few cases are (1) single-dwell detectors [11] and (2) multi-dwell detectors for channels with the uniform (or flat) power dispersion profile (PDP) [12].

benefits as the uniform randomized search's and has never been explored anywhere. The main contributions of the paper are:

- the expressions for the MATs of the uniform randomized search and the NRS in an arbitrary operating environment;
- a performance comparison between the uniform randomized search and the NRS in a fading channel;
- the expression for the MATs of the uniform randomized search in the asymptotic signal-to-noise ratio (SNR) regimes for multi-dwell detectors with optimal thresholds; and
- a performance comparison among various search strategies, including the CSS, the uniform randomized search, and the NRS, in a wideband channel.

These results provide fundamental understanding and guide a design of rapid acquisition receivers employing randomized search strategies.

This paper is organized as follows. Section II describes the system model. Section III derives the MATs of the uniform randomized search and the NRS. Section IV derives the asymptotic MAT of the uniform randomized search with optimal thresholds. Section V presents numerical examples. Section VI concludes the paper and summarizes important findings.

## II. SYSTEM MODEL

During signal acquisition, the acquisition receiver aims to align a received signal phase and the LGR phase to be within a small time difference,  $T_{\text{res}}$ . Signal phases belong to a known interval, referred to as the time uncertainty, which at most ranges from 0 to the period of the spreading sequence. Time difference  $T_{\text{res}}$  partitions the time uncertainty into subintervals or cells of length  $T_{\text{res}}$ . Signal acquisition aims to find an *in-phase cell* or a time interval that contains a received signal phase.<sup>3</sup>

Partitioning of the time uncertainty results in two key parameters:  $N_{\text{unc}}$  and  $N_{\text{hit}}$ . Parameter  $N_{\text{unc}}$  denotes the number of cells in total and can range from a small number to several thousands, depending on the transmission bandwidth. Parameters  $N_{\text{hit}}$  denotes the number of in-phase cells and can range from tens to hundreds, depending on the maximum dispersion of the channel. In a wideband channel,  $N_{\text{unc}}$  and  $N_{\text{hit}}$  satisfy  $1 \ll N_{\text{hit}} \ll N_{\text{unc}}$ . Parameters  $N_{\text{unc}}$  and  $N_{\text{hit}}$  affect the search strategy and the MAT.

The uniform randomized search and the NRS can be characterized with the help of parameter  $N_{\text{unc}}$ . The uniform randomized search produces a sequence  $\{U_1, U_2, U_3, \dots\}$  of cells, where  $U_j$ 's are independent and uniformly distributed over  $\{1, 2, \dots, N_{\text{unc}}\}$ . The NRS produces a sequence  $\{N_1, N_2, N_3, \dots\}$ , where  $N_j$ 's are independent,  $N_1$  is uniformly distributed over  $\{1, 2, \dots, N_{\text{unc}}\}$ , and  $N_k$  is uniformly

distributed over  $\{1, 2, \dots, N_{\text{unc}}\} \setminus \{N_{k-1}\}$  for  $k \geq 2$ . The sequence from the uniform randomized search or the NRS provides the order of cells for the acquisition receiver to test.<sup>4</sup>

To test a given cell in the sequence, the acquisition receiver employs a commonly-used multi-dwell detector. The multi-dwell detector consists of  $N_{\text{dwell}} \geq 1$  decision stages, followed by a final verification stage (see [8, Fig. 1]). The decision stages and the verification stage perform independent statistical tests to validate whether or not a given cell is an in-phase cell. A statistical test for decision stage  $i$  lasts for  $t_i$  time-units, known as a dwell time. A statistical test for the final verification stage lasts for  $t$  time-units, known as the penalty time. All statistical tests are proceeded in serial, starting from decision stage 1. If the cell undergoing a test passes *all*  $N_{\text{dwell}}$  decision stages, the receiver enters the final verification stage. Otherwise, the receiver discards the current cell and tests the next cell in the sequence.

Multi-dwell detectors admit the following two characteristics. First, the final verification stage is reliable and for the purpose of analysis has a negligible probability of error. This characteristic is a typical consideration [4] and can be achieved by a design of the final verification stage with large  $t$ . Second, the multi-dwell detectors are operated in stationary and slowly faded channels. This characteristic is also a typical consideration [4], [13] and results in a constant probability of error at each decision stage. With these two characteristics, multi-dwell detectors will eventually detect an in-phase cell and has the MAT that does not change over time.

The dwell times, the penalty time, and the probabilities of error at decision stages can be represented succinctly by polynomials, known as path-gain polynomials [3]. Path-gain polynomials can be used to obtain the MATs for many search strategies [1], [3], [5]. As an example, the path gains of a double-dwell ( $N_{\text{dwell}} = 2$ ) detector are given by [3], [4]

$$\begin{aligned} H_{Dj}(z) &= p_{D1}^{(j)} p_{D2}^{(j)} z^{t_1+t_2}, \\ H_{Mj}(z) &= \left[1 - p_{D1}^{(j)}\right] z^{t_1} + p_{D1}^{(j)} \cdot \left[1 - p_{D2}^{(j)}\right] \cdot z^{t_1+t_2}, \text{ and} \\ H_0(z) &= [1 - p_{F1}] z^{t_1} + p_{F1} \cdot [1 - p_{F2}] \cdot z^{t_1+t_2} \\ &\quad + p_{F1} p_{F2} z^{t_1+t_2+t}. \end{aligned}$$

Here  $p_{Di}^{(j)}$  is the probability that decision stage  $i$  detects the  $j^{\text{th}}$  in-phase cell,  $p_{Fi}$  is the probability of false alarm in decision stage  $i$  ( $i = 1, 2$  and  $1 \leq j \leq N_{\text{hit}}$ ). In the next section, we will derive the expressions of the MATs for the uniform randomized search and the NRS.

## III. PERFORMANCE ANALYSIS

To obtain the MAT of a randomized search strategy, we reduce the path-gain polynomials into the equivalent, simple forms. The reduction serves to simplify the derivation of the MAT and proceeds as follows.

Each path-gain polynomial  $H(z)$  will be reduced to  $h(z) \triangleq H(1)z^{H'(1)/H(1)}$ . The reduction yields the MAT-equivalent

<sup>3</sup>In some applications such as Rake reception design, the objective of signal acquisition is to find several in-phase cells. In that case, signal acquisition usually starts by finding one in-phase cell and then progressing to find the remaining ones.

<sup>4</sup>The exclusion of cell  $N_{k-1}$  in the NRS prevents the same cell to be tested twice in a row.

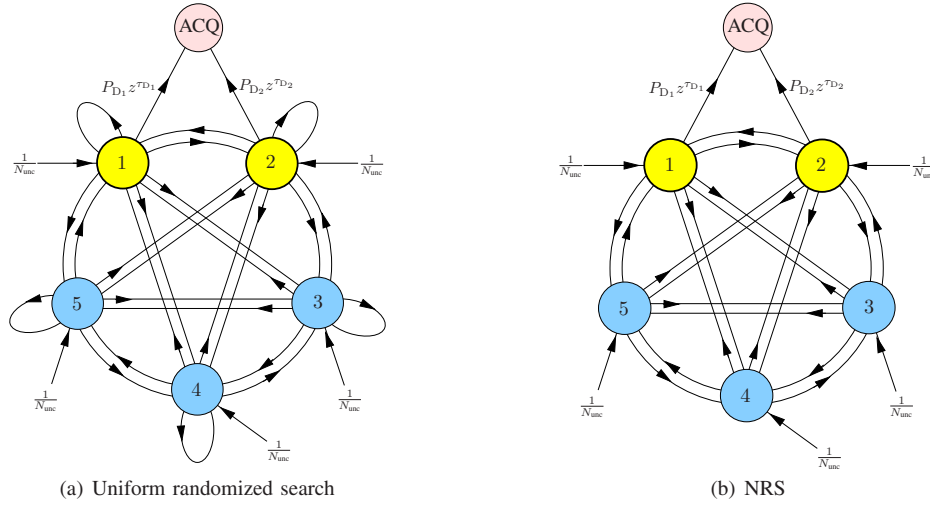


Fig. 1. Circular flow diagrams for (a) the uniform randomized search ( $\alpha = 1/N_{\text{unc}}$ ) and (b) the NRS ( $\alpha = 1/(N_{\text{unc}} - 1)$ ) are illustrated for a simple case of  $N_{\text{unc}} = 5$  and  $N_{\text{hit}} = 2$ . Unlabeled, outgoing edges from node  $j \in \{1, 2\}$  has the path-gains of  $\alpha P_{M_j} z^{\tau_{M_j}}$ . Outgoing edges from nodes 3–5 have the path-gains of  $\alpha z^{\tau_P}$ .

path-gain polynomials in the forms  $P_{D_j} z^{\tau_{D_j}}$ ,  $P_{M_j} z^{\tau_{M_j}}$ , and  $z^{\tau_P}$  [1]. For example, path-gain polynomials for a double-dwell detector are reduced to have

$$\begin{aligned} P_{D_j} &\triangleq H_{D_j}(1) = p_{D1}^{(j)} p_{D2}^{(j)}, & P_{M_j} &\triangleq H_{M_j}(1) = 1 - p_{D1}^{(j)} p_{D2}^{(j)}, \\ \tau_{D_j} &\triangleq H'_{D_j}(1)/H_{D_j}(1) = t_1 + t_2, \\ \tau_{M_j} &\triangleq H'_{M_j}(1)/H_{M_j}(1) \\ &= \frac{[1 - p_{D1}^{(j)}] t_1 + p_{D1}^{(j)} \cdot [1 - p_{D2}^{(j)}] (t_1 + t_2)}{1 - p_{D1}^{(j)} p_{D2}^{(j)}}, \text{ and} \\ \tau_P &\triangleq H'_0(1)/H_0(1) = [1 - p_{F1}] t_1 + p_{F1} [1 - p_{F2}] (t_1 + t_2) \\ &\quad + p_{F1} p_{F2} (t_1 + t_2 + t). \end{aligned}$$

The reduction takes into account the probabilities of detection and false alarm through parameters  $\{P_{D_j}, P_{M_j}, \tau_{D_j}, \tau_{M_j}, \tau_P\}$ .

#### A. Uniform Randomized Search

Let  $T_j$  denote the conditional MAT of the uniform randomized search, conditioned on the event that the acquisition receiver tests cell  $j$  as the first cell,  $1 \leq j \leq N_{\text{unc}}$ . By symmetry of the uniform randomized search, we assume without loss of generality that cells  $j = 1, 2, \dots, N_{\text{hit}}$  are the in-phase cells. By symmetry of the non-in-phase cells,  $T_j$ 's for the non-in-phase cells  $j$  are identical, giving the equality  $T_{N_{\text{hit}}+1} = T_{N_{\text{hit}}+2} = \dots = T_{N_{\text{unc}}} \triangleq T$  for some  $T$ .

The MAT for the uniform randomized search is given by

$$T_{\text{rnd}} = \frac{N_{\text{hit}}}{N_{\text{unc}}} \sum_{j=1}^{N_{\text{hit}}} T_j + \frac{N_{\text{unc}} - N_{\text{hit}}}{N_{\text{hit}}} T, \quad (1)$$

where  $T_j$ 's and  $T$  satisfy

$$\begin{aligned} T_j &= P_{D_j} \tau_{D_j} + P_{M_j} \tau_{M_j} + \frac{P_{M_j}}{N_{\text{unc}}} \sum_{i=1}^{N_{\text{hit}}} T_i \\ &\quad + \frac{N_{\text{unc}} - N_{\text{hit}}}{N_{\text{unc}}} P_{M_j} T, \quad j = 1, 2, 3, \dots, N_{\text{hit}}, \end{aligned} \quad (2)$$

$$T = \tau_P + \frac{1}{N_{\text{unc}}} \sum_{i=1}^{N_{\text{hit}}} T_i + \frac{N_{\text{unc}} - N_{\text{hit}}}{N_{\text{unc}}} T. \quad (3)$$

Summing (2) over  $j$  and using the relationship in (3), we solve for  $\sum_{i=1}^{N_{\text{hit}}} T_i$  and  $T$  in terms of  $\{N_{\text{hit}}, N_{\text{unc}}, P_{D_j}, P_{M_j}, \tau_{D_j}, \tau_{M_j}, \tau_P\}$  and obtain the MAT of the uniform randomized search

$$T_{\text{rnd}} = \frac{\left( \sum_{j=1}^{N_{\text{hit}}} P_{D_j} \tau_{D_j} + P_{M_j} \tau_{M_j} \right) + (N_{\text{unc}} - N_{\text{hit}}) \tau_P}{N_{\text{hit}} - \sum_{j=1}^{N_{\text{hit}}} P_{M_j}}. \quad (4)$$

The expression is valid for any shape of the PDP, any number of multipath clusters, and any number of decision stages.

When  $N_{\text{dwell}} = 1$ , the expression (4) is consistent with the known results [11, eq. (22)] for a receiver with one decision stage. When the PDP is uniform, the expression is also consistent with the known results [12, eq. (2)] for the uniform PDP.

#### B. Non-Return-To-Self Uniform Randomized Search

The MAT of the NRS can be obtained by a similar approach. Let  $S_j$  denote the conditional MAT of the NRS, conditioned on the event that the acquisition receiver tests cell  $j$  as the first cell. By symmetry of the NRS and the symmetry of the non-in-phase cells, we assume without loss of generality that (a) cells  $j = 1, 2, \dots, N_{\text{hit}}$  are the in-phase cells and (b)  $S_{N_{\text{hit}}+1} = S_{N_{\text{hit}}+2} = \dots = S_{N_{\text{unc}}} \triangleq S$  for some  $S$ . We express the MAT of the NRS as a weighted sum of  $S_j$ 's and  $S$ , write down the linear equations for  $S_j$ 's and  $S$ , and solve for  $\sum_{i=1}^{N_{\text{hit}}} S_i$  and  $S$ . Then, we obtain the MAT of the NRS, shown at the bottom of the next page, where  $\beta_j \triangleq P_{D_j} \tau_{D_j} + P_{M_j} \tau_{M_j}$ .

We can compare the MATs of the uniform randomized search and the NRS for a typical UWB environment. In particular, when  $1/N_{\text{unc}} \simeq 0$ , we can verify that

$$T_{\text{nrs}} \simeq T_{\text{rnd}}.$$

TABLE I  
APPROXIMATIONS OF THE ASYMPTOTIC MATS FOR DIFFERENT SEARCH  
STRATEGIES IN THE CHANNEL WITH ONE CLUSTER OF MULTIPATH

search strategy	low SNR	high SNR
CSS	$\frac{N_{\text{unc}}}{N_{\text{hit}}} t$	$\frac{N_{\text{unc}}}{2} t_1$
FSS- $N_{\text{hit}}$	$\frac{N_{\text{unc}}}{2N_{\text{hit}}} t$	$\frac{N_{\text{unc}}}{2N_{\text{hit}}} t_1$
uniform randomized search	$\frac{N_{\text{unc}}}{N_{\text{hit}}} t$	$\frac{N_{\text{unc}}}{N_{\text{hit}}} t_1$

That is, the uniform randomized search and the NRS perform comparably well in a wideband channel.

#### IV. PERFORMANCE UNDER THE OPTIMAL THRESHOLDS

For the multi-dwell detector, each statistical test is usually a threshold test. The thresholds affect probabilities of detection and false alarm, which in turn, affect the MAT of a randomized search strategy. In this section, we derive the expressions for the MATs when the thresholds are selected optimally.

We will obtain the MAT of the uniform randomized search in the low and high SNR regimes, which constitute the extreme operating environments. The thresholds will be selected to minimize the MAT. We will not consider the NRS here, since the NRS and the uniform randomized search perform comparably well in wideband channels.

To derive the asymptotic MATs under the optimal thresholds, we assume that the relationship between the probabilities of detection and false alarm satisfies assumptions (A1)–(A2) of [8, Sec. II].<sup>5</sup> Let  $T_{\text{rnd/low}}$  and  $T_{\text{rnd/high}}$  denote the MATs of the uniform randomized search in the low and high SNR regimes, respectively, under the optimal thresholds. Taking the limits of (4) as the SNR approaches infinity (for high SNR) or zero (for low SNR) and minimizing the limits over the thresholds give

$$T_{\text{rnd/low}} = \frac{N_{\text{unc}}}{N_{\text{hit}}} \sum_{i=1}^{N_{\text{dwell}}} t_i + \left( \frac{N_{\text{unc}}}{N_{\text{hit}}} - 1 \right) t$$

$$T_{\text{rnd/high}} = \left( \frac{N_{\text{unc}}}{N_{\text{hit}}} - 1 \right) t_1 + \sum_{i=1}^{N_{\text{dwell}}} t_i.$$

The minimizations can be obtained in closed form. The derivation is omitted for brevity.

Under a typical operating condition, we have  $N_{\text{hit}} \ll N_{\text{unc}}$  and  $t_i \ll t$  for each  $i$ . These relations imply that the

<sup>5</sup>The assumptions require that the probabilities of detection approach 1 in the high SNR regime and approach the probability of false alarm in the low SNR regime. For UWB, an important scenario in which (A1)–(A2) are satisfied is a non-coherent detector in a Nakagami- $m$  fading channel [14, Sec. V-A].

asymptotic MATs of the uniform randomized search can be approximated by the expressions in the last row of Table I. For comparison purposes, the table also contains the approximations of the asymptotic MATs for the CSS and FSS- $N_{\text{hit}}$  [8] under the same conditions on  $N_{\text{hit}}$ ,  $N_{\text{unc}}$ ,  $t_i$ 's, and  $t$ .<sup>6</sup>

From the table, we can gain insights into performance of the uniform randomized search. The uniform randomized search is approximately  $N_{\text{hit}}/2$  times faster than the CSS in the high SNR regime. It is approximately 2 times slower than the FSS- $N_{\text{hit}}$  in the low and high SNR regimes. Although slower than the FSS- $N_{\text{hit}}$ , the uniform randomized search does not require knowledge of the maximum dispersion of the channel ( $N_{\text{hit}}$ ) and is invariant to the variation in the number of multipath clusters. With this benefit, the uniform randomized can be attractive from the standpoint of a receiver design.

#### V. NUMERICAL EXAMPLES

To demonstrate performance of the randomized search strategies, we consider a transmitter that employs a direct sequence with the spreading gain  $N_{\text{SG}} = 65535$  and the chip duration  $T_C = 1$  ns. The signal is propagated through a Nakagami- $m$  fading channel, which is modeled as a tapped delay line with  $N_{\text{hit}} = 101$  taps and a tap spacing of one chip. The  $m$ -parameter for the fading is given by  $m_k = 3.5 - \frac{2}{73}(k-1)$ , and the PDP is exponential decaying. The channel model is a simplification of the IEEE 802.15.4a standard [15], [16] for UWB.

The receiver uses a non-coherent detection with two decision stages, commonly referred to as the search mode and the verification mode. The architecture for the search mode is similar to that in [12], except that here the receiver employs a bank of  $N_C$  correlators. The correlation duration is set to  $t_1 \triangleq MT_C = 100$  ns. The verification mode employs a commonly-used majority test [17] with  $N_T = 4$  independent subtests. Each subtest is similar to the test in search mode except that (a) the correlation duration is increased to  $2MT_C$  (and hence  $t_2 = N_T \cdot 2M$ ) and (b) the threshold is set differently for detection reliability. If  $K = 2$  or more subtests conclude that the cell undergoing a test is a correct cell, the receiver initiates the final verification, which lasts for  $t = 10^5$  ns. Parameters for multi-dwell detector are consistent with existing literature [4].

Probabilities of detection and false alarm can be derived in closed-form expressions. A main step in the derivation utilizes the distribution of the sum of independent gamma random variables [18]. The derivation is omitted for brevity.

<sup>6</sup>In a channel with one cluster of multipath (Table I), the CSS and FSS- $N_{\text{hit}}$  are the slowest and the fastest deterministic search strategies, respectively, in the asymptotic SNR regimes (see [8] for a precise statement). With this reason, the CSS and FSS- $N_{\text{hit}}$  are suitable baselines for comparison.

$$T_{\text{nrs}} = \left[ \frac{\sum_{j=1}^{N_{\text{hit}}} \frac{(N_{\text{unc}}-1)\beta_j + (N_{\text{unc}}-1)(N_{\text{unc}}-N_{\text{hit}})\tau_P P_{M_j}/N_{\text{hit}}}{N_{\text{unc}}-1+P_{M_j}}}{N_{\text{hit}} - N_{\text{unc}} \sum_{j=1}^{N_{\text{hit}}} \frac{P_{M_j}}{N_{\text{unc}}-1+P_{M_j}}} \right] + \frac{(N_{\text{unc}} - N_{\text{hit}})(N_{\text{unc}} - 1)\tau_P}{N_{\text{unc}}N_{\text{hit}}} \quad (5)$$



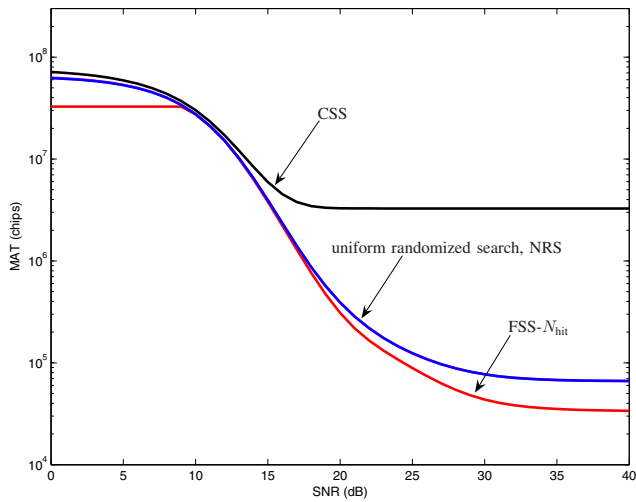


Fig. 2. Intelligent search strategies (the FSS- $N_{\text{hit}}$ , uniform randomized search, NRS) are faster than the CSS ( $N_C = 4$  correlators).

We now plot the MATs as functions of the SNR for various search strategies (see Fig. 2). Here, the thresholds in the search and verification modes are selected optimally to minimize the MATs. The FSS- $N_{\text{hit}}$ , uniform randomized search, and NRS outperform the commonly-used CSS. MATs of the uniform randomized search and the NRS are virtually indistinguishable, a finding that is consistent with Section III-B. The uniform randomized search and NRS are slower than the FSS- $N_{\text{hit}}$ , but they are more robust against the variation in the operating environment.

The acquisition receiver satisfies assumptions (A1)–(A2) (see [14, Sec. V-A] for the proof). Hence, the asymptotic results from Section IV are valid for our numerical examples.<sup>7</sup>

From the figure, the uniform randomized search and the NRS are approximately 50 times faster than the CSS in the high SNR regime and approximately 1.2 times faster than the CSS in the low SNR regime, the findings that are consistent with Table I. The MAT of the uniform randomized search is  $6.29 \times 10^7$  chips at 0 dB and  $6.62 \times 10^4$  chips at 40 dB. These numbers are also consistent with the asymptotes of  $T_{\text{rnd/low}} = 6.55 \times 10^7$  chips and  $T_{\text{rnd/high}} = 6.58 \times 10^4$  chips.

## VI. CONCLUSION

In this paper, we derive the expressions of the MATs for the uniform randomized search and the NRS. The expressions are valid for any shape of the PDP and any number of multipath clusters. The uniform randomized search and the NRS perform equally well under a typical wideband environment. Performance of the uniform randomized search and the NRS is robust against the variation in the operating

environment, making these two search strategies attractive from the standpoint of a receiver design.

We quantify the performance of the uniform randomized search in the asymptotic SNR regimes when the thresholds of a multi-dwell detector are selected optimally to minimize the MAT. We compare the asymptotic MATs of the uniform randomized search to those of the CSS and the FSS- $N_{\text{hit}}$ . Furthermore, we plot the MATs of the randomized search strategies over a range of SNRs. The asymptotic MATs from the plots are consistent with the analysis. The results quantify the benefits of the uniform randomized search and the NRS.

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<sup>7</sup>The MAT of the FSS- $N_{\text{hit}}$  are relatively constant for SNRs approximately below 10 dB. This pattern arises from the fact that the FSS- $N_{\text{hit}}$  practically achieves the low-SNR limit over that SNR range. A similar remark can also be made from the figure for the CSS with SNRs above 20 dB. The SNR (such as 10 dB or 20 dB) at which a search strategy practically reaches its MAT limit is not yet well-understood.