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A REPEATED-TRANSIENT, PARALLEL TYPE THYRATRON

INVERTER WITH SELF EXCITATION

by

CHARL DANIEL CILLIE

B. Sc., University of Stellenbosch, South Africa

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Professor in Charge of Research

Chairman of Departemental Committee on Graduate Students

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### ABSTRACT

The basis of the high voltage direct current transmission system will be static rectifiers and inverters, the basic part of which will be some form of grid controlled vapor tube. In this report the development of a simple type of single phase inverter is described and as far as is possible an analysis of the circuit made. Its characteristics, possibilities and limitations are discussed. The problem is of interest because it represents in their simplest form the problems that will be encountered in the design of the apparatus for the load end of the high voltage d.-c. transmission line.

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#### I. THE THYRATRON TUBE

(1)

Mechanically, the Thyratron tube is nothing more or less than a three-electrode vacuum tube in which a small amount of inert gas or vapor has, been placed. The name also applies to tubes having a mercury pool as cathode, in fact, in its original form it was an ordinary pool type rectifier tube with grids surrounding the anodes.

> Physically, the Thyratron combines the low-voltage drop and consequent high efficiency of the mercury pool rectifier tube with some of the control characteristics of the three-electrode hard vacuum tube. With direct current applied to the anode the grid, however, has "one-way" control only. When made positive, it can start current flow from anode to cathode, but once the conduction process has started no amount of negative grid potential will stop current flow. This is because the whole space surrounding the electrodes is ionized. However, with alternating current applied between anode and cathode the grid has an opportunity to regain control during the negative half-cycle. In this manner, the average plate current can be controlled by a phase displacement between grid and anode voltages.

> > The high efficiency of the Thyratron is due to the

-1-

small and practically constant (i.e. independent of anode current) voltage drop between anode and cathode, which results from the fact that the negative space charge at the cathode is neutralized by positive mercury ions.

2

In the tubes used in this investigation the emission is obtained from an enclosed tungsten filament, having a high heat capacity. From figures 1, 2, and 3, some idea can be had of the external appearance and physical dimensions of the tubes used.

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SINGLE-PHASE, SELF-EXCITED THYRATRON INVERTER, UTILIZING TYPE FG-41 TUBES. E331.9

9-25-30

## II. BRIEF HISTORY OF THE STATIC INVERTER.

## Definition.

The name "inverter" implies any apparatus for the conversion of d.-c. power into a.-c. power. Lately, however, it has been applied to static circuits which are essentially the same as rectifier circuits and containing transformers, condensers, etc., and thermionic tubes, the function of which is the inverse of that of rectifier circuits; viz., to convert direct current into alternating current.

a. The Hard Vacuum Tube Inverter.

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In 1925 D. C. Prince described the principle of operation of a hard-vacuum tube inverter (See Ref. 2(a)). The function of this circuit was the conversion of direct-current into three phase alternating current. Power was "pumped back" into an a.-c. system and the grids of the tubes were controlled and the frequency determined by the latter.

The successful performance of this circuit was perhaps in no small measure due to the positive "two-way" control of the hard-vacuum tubes used. The grids could not only start the current through the tubes but could also shut it off.

Due to the limitations inherent in the use of hard-vacuum tubes in power circuits, their use in inverter circuits has not developed. With the advent of the grid controlled mercury arc tube the inverter idea gathered momentum and although not yet extensively applied, holds considerable promise for the future.

# b. The Direct Current Transformer.

An inverter circuit, utilizing thyratron tubes, was described by D. C. Prince in 1928\*(See Reference 2(b) ). Mercury pool type grid=controlled tubes were used in a single phase circuit. Since the inverted d.-c. power was again rectified at a different d.-c. voltage, the whole assemblage of apparatus was called a "direct current transformer."

In the same article the ideal single phase inverter circuit was also described. This was essentially the same as the circuit utilizing hard-vacuum tubes with the latter replaced by mercury pool type tubes. The voltage wave form was supplied by a synchronous load.

In the "d.-c. transformer" the a.-c. wave-form was
supplied by an oscillatory combination of a condenser, connected between anodes, and a low exciting reactance trans\* Alexanderson Patent 1,800,002 filed 7/15/23 and operated
prior to this date.

former, which combination acted as a flywheel. The cathodes of the two tubes were connected together and the line reactors of both circuits were placed on the same core. Thus a back e.m.f. was provided for the inverter and the balance between input and output resulted in very little energy storage action.

The circuit of this investigation corresponds to that of the inverter part of the d.-c. transformer, with, however, no special provision made for a "flywheel effect " to maintain approximately sine waves of voltage. All voltages and currents are transients. Every half-cycle a new transient is started. Hence, the name "repeated transient" type inverter.

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#### III. THE EXPERIMANTAL INVESTIGATION

### The specific purpose.

With the economies possible in the replacement of rotating machinery by static circuits for supplying high voltage direct current realized, this investigation was concerned with the possibilities of accomplishing the opposite;  $\dot{\mathbf{e}}$ . e., the inversion of high direct current voltages to obtain power in the form of low a.-c. voltages. More specifically, the requirements of the circuit were:

- That it should be self-starting and self-excited,
   both as regards grid and filament power.
- (2) That power supplied at 3000 volts d.-c. should be converted into single phase 110 volts a:c.
- (3) That it should have a convenient regulation characteristic on a variable pure resistance load up to 5 kw. maximum.
- (4) That it should have a frequency of 60 cycles.
- (5) That the cost should be low. This involved the use of standard apparatus.

The output wave-shape was not required to be approximately sinusoidal, but as will be seen the circuit had to all intents and purposes to perform the duties of a motor-generator set.

## Method of Attack.

An investigation of this type at the present stage of development resolves itself into three parts:

- Determination of the tubes to be used. This involves the investigation of characteristics and secondary power requirements for filament and grid excitation.
- (2) Development of the circuit and the investigation of its operation, by studying its wave-shapes.
- (3) The development of modifications for the sake of meeting special requirements.

## Determination of the most suitable tube.

The types of Thyratrons manufactured at the present time fall into two general classes according to the purpose for which they are to be used. The essential difference between the two classes is that those types used as "Controlled Rectifiers" need a minimum of 1000 micro-seconds for de-ionization while those used in "Inverter" circuits require approximately 100 micro-seconds.

The types differ as far as constructional details affecting filament supply voltage and tube voltage and current rating are concerned.

# Table I

# Technical Information on Thyratrons Used

Code No.	F. G. 67	F. G. 41*		
Туре	"Inverter"	"Inverter"		
Cathode				
Volts	5.0	5.0		
Amps.	4.5	12.0		
Type	Indirect Heat	Indirect Heat		
Heating time	5 min.	5 min.		
Anode Peak Volts (Max.)	1,000	15,000		
Anode Current (Max)	Same of 12			
Instantaneous	15	75		
Average	2.5	12.5		
Max. Tube Voltage Drop	24	24		
Min. Tube Voltage Drop	10	10		
Temp. Limits ( °C)	20-50	10-50		
Deionization time (micro-seconds)	100	100		

\* This tube was the one actually used. The information contained in Table II applies to the shielded grid type that superseded it.

# THYRATRON FG-41

# TECHNICAL INFORMATION .

Designed as	Inverter Tube
Number of Electrodes	3
Cathode Volts Amperes Type Typical Heating Time(min.)	5.0 12.5 Indirectly heated 5
Maximum Peak Inverse Voltage	15000
Maximum Peak Forward Voltage	15000
Maximum Instantaneous Plate Current(Amps	s.) 75
Maximum Average Plate Current(Amps.)	12.5
Maximum Time of Averaging Plate Current	(sec.) 30
Maximum Instantaneous Grid Current(Amps.	.) 5.0
Maximum Average Grid Current(Amps.)	1.0
Maximum Tube Voltage Drop	24
Minimum Tube Voltage Drop	10
Approximate Starting Characteristic Plate Voltage Grid Voltage 1000 +4.0 15000 -5.0 Approx. 3000 0	9
Overall Dimensions Maximum Length Maximum Diameter	18"(approx.) 5 1/16"
Base Type Numbers	3601 and 1902
Temperature Limits - Ambient(°C.)	10° - 50°
Type of Cooling	Air
Deionization Time(micro-seconds)	100(approx.)

December 17, 1930.

H-4359290

Of the tubes available at the time of this investigation the F. G. 67 type seemed to fill the bill in all requirements except in as far as anode voltage rating was concerned. It had been found very successful in low voltage inverter circuits and its characteristics were well-known. Since in a previous investigation it had been found that this tube would stand up under anode voltages of 1500 volts; i. e., 150% of the rated value, it was decided to find its limitation. Another factor that affected this decision was that the filament heating power of this tube was small and its cost a fraction of that of the next and considerably larger type.

The circuit shown by figure 4 was set up and these tubes tried out. With the grids separately excited from an a.-c. source it is a comparatively easy matter to make this circuit function, provided that the grid voltage is high enough for cleaning up the positive ions in the limited time available with a rectangular wave-shape. The limiting voltage on the tubes was found to be 2400 volts d.-c. At this voltage they failed after operation for half an hour.

The symptoms of breakdown were small sparks on the upper side of the anode and a "corona effect" on the glass where the anode lead was brought out with consequent breaking



FIG. 4. Separately Excited Parallel Inverter Circuit in which Preliminary Tests on the Tubes were Made. The Grids are contro lled from a Separate a-c. source. of the glass bulb. As a result of these tests it was concluded that the spacing of the elements of the tube was inadequate for their use at high voltages.

# Experimentation with Special "Bent-Arm Tube.

There was at this time available some specially constructed tubes, the rating of which were the same as that of the F. G. 41 type. Mechanically, they differed to a very great extent from the latter. Two anodes were placed in the same bulb in bent arms, similar to those in pool type tubes used for high voltages. The cathode had twice the capacity of that of the F. G. 41. In effect the tube was two of the latter combined into one, with the addition of a"holding anode" for maintaining ionization.

With separate excitation on the grids this tube functioned very well, however, not when the holding anode circuit was opened. The latter seemed essential, and the explanation lies in the fact that the tube elements (cathode and anodes) were a comparatively long distance apart, so that the anode voltage (which also is the applied d.-c. voltage) employed was not adequate.

## First self-excited Grid Circuit.

The problems of self-starting and grid circuit design



Fig. 5. First Scheme Used to Obtain Self Excitation of the Grids. For the Waveshapes of this Circuit see Figs. 6 & Z. ( H-4359129 )

8,



Fig. 6. Waveshapes of the circuit of Fig. 5. Load 1.70 kw. Trace A: Grid Current Trace B: Grid Condenser Current Trace C: Timing Wave # 60 Cycles. (C. D. 193080)



Fig. 7. Waveshapes of the Circuit of Fig. 5.Load 1.70 kw. Trace A: Timing Wave = 60 Cycles Trace B: Grid Condenser Current Trace C: Output Voltage ( CD 193081 )

are considered later in greater detail. Many of the facts in connection with these were not realized when self-excitation was first tried with the bent arm tube. For the sake of completeness the first grid circuit found to do the trick is included. The circuit is shown by figure 5 and its waveshapes by figures 6 and 7. It never was seriously considered for this inverter on account of its inherent disadvantages, viz.:

- (1) The grid excitation was not derived in such a way as to be unaffected by the amount of load current.
- (2) Selfstarting could not be obtained. It was found necessary to reduce the d.-c. voltage to less than 1500 and then to touch the grids off with a separate source of a.-c. There is a possibility that this characteristic was more due to the characteristic of the tube used than to any feature of the circuit.
- (3) As a matter of experimental observation, the frequency lacked all control and seemed proportional to load current. Very wide variations with load were observed.

## Second Selfexcited Grid Circuit.

With continued experimentation the grid circuit shown

by figure 8 was evolved. It had none of the disadvantages of its predecessor. Frequency regulation was almost flat above 1.5 kw. load and starting was readily obtained when the line switch was closed on full d-.c. voltage. The voltage and current waveshapes are shown by figures 9 to 17.

### The Performance of F. G. 41 Tubes.

The circuit that resulted from continuous experimentation is shown by figure 8. Its waveshapes are shown by figures 9 to 17. Its performancewas considered highly satisfactory. The use of a standard tube, however, was an essential requirement and although the rating of the F. G. 41 was far in excess of requirements, it seemed to be the only one available.

Satisfactory operation with these tubes was not obtained until they had been modified to the extent of applying a conducting coat of lampblack-aluminum paint to the bulb and connecting this to the grid lead. This procedure insured good grid control at all times and prevented any charge on the glass from creating a secondary, and uncontrolled, grid effect.



D.C. Supply Voltage # 3,000. Tube Filaments Supplied from 125 V. A.C. through Transformer at 5 Volts.

Fig.5.

C.D.C. 9/30/30. H-4359149.



Fig. 9.	Waveshape	s of the Circuit	of	Fig	• 8	at	4.80	kw.
	Load and	75 Cycles.						
	Trace A:	Output Voltage						
	Trace B:	Input Current						
	Trace C:	Grid Voltage	(	CD .	1918	353	)	



Fig.19. Waveshapes of the Circuit of Fig. 8 at 3.70 kw. Trace A: Anode to Cathode Voltage on one Tube Trace B: Output Voltage Trace C: Grid Voltage (CD 191853)



Fig. 12. Waveshapes of the Circuit of Fig. 8 at .85 kw. Load. Tracë A: Input Current Trace B: Grid Current Trace C: Grid Voltage ( CD 193088 )



ig. 14. Waveshapes of the Circuit of Fig. 8, but with very small Inductance in the d-c. Line. 4.8 kw. Load and Frequency of 73 Cycles. Trace A: Output Voltage Trace B: Voltage from Transformer Mid-tap to Ground. ( CD 191854 )



Fig. 16.

Waveshapes of the Circuit of Fig. 8, switching from .27 kw. Load to 4.8 kw. Trace A: Grid Voltage Trace B: D-c. Line Current Trace C: Output Voltage (CD 193089)



Fig. 17. Waveshapes of the Circuit of Fig. 8 at 5.0 kw. Load, and with the Oscillatory Grid Circuit of Fig. 25. Trace A: Anode to Cathode Voltage on one Tube. Trace B: D-c. Line Current Trace C: Grid Voltage (CD 191983)

# Physical Explanation of the Operation of the Inverter.

The type of inverter with which this investigation is concerned does not represent the ideal case, which has been described by D. C. Prince. Mechanically, its operation can best be visualized by considering the action of a common duplex boiler feed pump. There is no flywheel; i. e., no stored energy. The steam acts directly on the piston, which sets the rod in motion, and, through the latter the water in the water cylinder. At the end of the stroke of one rod the latter acts on the steam valves of the other rod, which starts its journey in the opposite direction, to trip the valves of the first rod and start it on its way once more. The action is discontinuous for each part of the pump during alternate half-cycles of operation.

Electrically, the Anyratrons perform the same function as a synchronous switch, externally driven. When the d.-c. line switch is closed, one tube or switch starts conducting. Current flows through both halves of the main transformer, the "power component" going through the tube, and the other going into the condenser to charge it up to the d.-c. line voltage. As soon as current starts flowing the voltage applied to the grid of the **fron**-starting tube is made

negative with respect to its cathode and it is prevented from starting.

At the end of the first half-cycle the condenser in the grid circuit becomes charged and the grid voltage of the tube that was non-conducting approaches zero or becomes more positive until the critical voltage is reached. At this point the second tube becomes conducting, the current from the one previously conducting is transferred to the condenser, with an accompanying negative voltage "impulse" on its anode as shown by figures 10, 11 and 17. At the same instant, the secondary voltage reverses and its grid becomes negative so that by the time the anode voltage passes through zero the de-ionization is complete. The transfer of current from one tube to the other takes place in about 1000 micro-seconds.

Due to the fact that a cathode ray oscillograph was not available the exact de-ionization time was not measured. From figure 11, it can be only guessed. It is very near the rated value of that of the tubes. The successful operation of this circuit is in no small measure due to the fact that the tubes could be de-ionized in about 100 microseconds.

### The Starting Operation.

It has been said that the inverter starts simply

by closing the d-c. line wwitch, with nospecial provisions made that one tube only should start conducting. When both tubes start conducting at the instant the line switch is closed serious damage will result, especially in high voltage circuits such as the one under consideration. It is for this reason that in addition to a circuit breaker, a current limiting resistor and a fuse were employed as shown by figure 8.

An "average" grid voltage-striking voltage characteristic of the F. G. 41 type tube is shown by figure 23. For individual tubes there are in many cases considerable departures from this curve below 2500 volts. The critical grid potential for 3000 volts for practically all tubes used was 9 to 11 volts negative. From the symmetry of the circuit it will be seen that if the two tubes do not have identical characteristics or at least the same critical grid woltage for 3,000 volts, one would conduct for a slightly longer period of time than the other. The inevitable result would be saturation of the main transformer, a condition tha t wouldmmake operation cease in a few cycles.

For satisfactory steady state operation then, the two tubes should have the same critical voltage. However.

for satisfactory starting they should be dissimilar; i. e., one should start conducting slightly before the other, because the instant before starting both have zero grid potential. Tests showed that some pairs of tubes would not start, while others would start but would cease operating in a very short length of time. The majority of the tubes tested, however, did "cooperate" quite satisfactorily.

### Regulation Characteristics and their Variation.

The voltage regulation, efficiency and frequency characteristics with varying load of the circuit of figure 8 are shown by figure 19. The efficiency curve does not include the 120 watts needed for cathode heating. It has the same shape as that of a high voltage rectifier circuit that employs mercury vapor tubes. The losses, in order of importance, are: transformer losses, resistance losses in the d.-c. line, grid power, and tube arc drop. Compared with that of a rotating machine the efficiency is remarkable.

The rise in frequency with decreasing load is due to the change in out-put voltage wave-shape from that of a rectangle at full load to a sinusoid at no load.



The cause of the rise and drop in frequency at loads between 1 kw. and zero has not been ascertained but it is not of very great importance as due to the rise in output voltabe the inverter should always be connected to a minimum load of, say, 500 watts.

The load-output voltage curve reaches a critical region, between 500 watts and 1 kw., beyond which a decreased load results in a somewhat excessive voltage rise. An explanation of this is not far to seek. Although the anode condenser is small, it stores energy during each half-cycle, that must be dissipated during the time transfer of the current from one tube to the other takes place. The transformer losses are small and neglecting them the rate of energy dissipation is equal to the square of the output voltage divided by the load resistance. At load resistances less than the critical, therefore, the output voltage must rise because a point has been reached where the rate of energy storage is a minimum and remains substantially at this value up to the open circuit condition.

An analogous condition would arise in electrical machinery if it were not for the fact that a back emf. exists. To make the actual analogy between inverter and rotating machine complete we can rectify the inverter output at light loads and connect the rectifier output back to the d.-c. line. Obviously, however, this would not be economical. If the need for suppressing the light load output voltage does arise, a minimum load left on the inverter will prevent the inconvenience.

Figure 22 shows the fariation of regulation characteristic with circuit constants. The most convenient regulation curve, it will be seen, is obtained with maximum line inductance and a minimum capacitance between anodes. The lower the capacitance and the higher the line inductance the lower will the rate of energy storage and dissipation be and hence the lower the output voltage at light loads. If it were not for losses in the circuit the output foltage on open circuit would be infinite, somewhat analogous to the high speed of a d.-c. motor when its field circuit is opened.

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#### IV. ANALYSIS OF THE INVERTER CIRCUIT.

#### Assumptions.

The main difficulty in the analysis of circuits containing vacuum tubes (or vapor tubes for that matter) lies in the fact that these devices are variable circuit parameters and a convenient method for handling such analyses, without simplifying assumptions, does not yet exist. In analyzing the circuit under consideration the action of the two tubes can be assumed to be that of a synchronous switch. That is, imagine that some mechanism external to the circuit periodically opens and closes the two switches A and B of figure 18. When A is closed B is opened and vice versa. The current through, say A, the instant B is closed is then assumed to be abruptly transferred to the condenser.

Furthermore, the circuit parameters will be assumed linear. The circuit is symmetrical so that one set of differential equations set up in terms of the three loop currents will completely specify its behavior.

### The Symbols Used (See figure 18)

Е	=	Direct Voltage
L		dc. line inductance
R	=	dc. line resistance
г」=	r 5 =	Self-inductance of one-half
		the transformer primary
<sup>L</sup> 12 <sup>=</sup>	r 51_	Mutual Inductance between the
		two primary windings of the main
		transformer.
R <sub>1</sub> =	R <sub>2</sub> =	Resistance of one-half of the
		transformer primary winding.
<sup>L</sup> 3	=	Self-inductance of transformer
		secondary winding.
<sup>L</sup> 13 <sup>=</sup>	<sup>L</sup> 23 <sup>=</sup>	Mutual inductance between one-
		half the transformer primary
		winding and the secondary winding.
il, i	2 and i 3	= Licop currents for the respective
		loops.

**R**<sub>3</sub> = Combined resistance of load and transformer secondary winding

35

Let



Fig. 18. The Circuit for which the Differential Equations were set up and Solved. The Currents are plotted in Figures 20 and 21. The Differential Equations of the Circuit

#### For the first loop:

#### For the second loop:

 $\frac{R_{1}i_{2}+L_{1}di_{2}+R_{2}i_{2}+L_{2}di_{2}}{dt^{2}} + \frac{2L_{12}di_{2}}{dt^{2}} + \frac{1}{6} \int_{2}^{1} 2^{dt}$   $- \frac{L_{1}di_{1}}{dt^{1}} - \frac{R_{1}i_{1}}{L_{12}di_{1}} - \frac{L_{23}di_{13}}{dt^{2}} - \frac{L_{13}di_{3}}{dt^{3}} = 0 \quad \dots \quad (2)$ 

#### For the third loop:

 $R_{3i_3} + L_{3di_3} + L_{13di_1} - L_{23di_2} - L_{13di_2} = 0$  .....(3)

Replacing the derivative operator by p and noting that  $L_1 = L_2$ ,  $L_{13} = L_{23}$ , and  $R_1 = R_2$ , these three equations can be put into determinantal form as follows:

$$D(p) = \begin{pmatrix} L_{p} + R + L_{1}p + R_{1} \end{pmatrix} - (L_{1}p + L_{12}p + R_{1}) + L_{13}p \\ -(L_{1}p + L_{12}p + R_{1}) + (2R_{1} + 2L_{1}p + 2L_{12}p + \frac{1}{6p}) - 2L_{13}p \\ -L_{13}p - 2L_{13}p + (R_{3} + L_{3}p) \end{pmatrix} = 0 \dots (4)$$

For the sake of not making the determinantal equation too cumbersome the assumption will be made that the linkage reactance of the transformer is negligible. The effects and justification of this will be considered later.

Noting that under the above assumption  $L_{12} = L_{1}$ , the determinantal equation will be as follows:

$$2(R_{1}LL_{3} + R_{3}LL_{1} + 2R_{3}L_{1}^{2}) p^{3}$$

$$+ (2R_{1}R_{3}L + LL_{3} + 2R_{1}R_{3}L_{1} + R_{1}^{2}L_{3} + 2RR_{1}L_{3} + 4RR_{3}L_{1}) p^{2}$$

$$+ (LR_{3} + L_{1}R_{3} + R_{1}^{2}R_{3} + R_{1}L_{3} + 2RR_{1}R_{3} + \frac{RL_{3}}{C}) p$$

$$+ \frac{R_{1}R_{3}}{C} + \frac{RR_{3}}{C} = 0 \dots (5)$$

For any given set of circuit constants the three roots of this equation can be found. They will have the usual form:

(1) Non-oscillatory case:

$$\begin{array}{c} p_1 = -a \\ p_2 = -b \\ p_3 = -c \end{array} \right) \quad \text{all real}$$

The solution for any current in the circuit will be:

(2) Critically damped case:

$$p_1 = -d$$

$$p_2 = p_3 = -f$$
both real

The general form of solution for any current will be:

$$i = I_{ss} + K_4 e^{-dt} + K_5 e^{-ft} + K_6 t e^{-ft}$$
(3) Oscillatory case:

$$p_{1} = -r \quad (real)$$

$$p_{2} = -u + jw$$

$$p_{3} = -u - jw$$
(Conjugate complex)

and the current will be of the form:

$$i = I_{ss} + K_7 e^{-rt} + K_8 e^{-ut} sin (wt + \Theta)$$

where  $I_{SS}$  in all three cases is the value of the current

at t = CO , or the steady state value, and  $K_{1}$  to Kgare constants.

For any set of circuit constants the values of the K's in equations (6), (7) and (8) can be found if the initial conditions are known.

In order to calculate the currents two solutions must be made; viz., one for the starting half-cycle and the other for the "steady-state" or all subsequent halfcycles; i. e., assuming that the steady state is reached in one half-cycle, as the oscillograms indicate.

#### The Starting Half-cycle.

When tube A starts, at t = 0, we know that i is zero and all the applied d.-c. voltage appears across the line inductance so that  $\left(\frac{di}{dt}\right)$  will be  $\frac{E}{L}$ . For finding the third condition; viz., the value of  $\left(\frac{d^2i}{dt^2}\right)$  we must go back to the fifferential equations. Multiphying equation (1) by 2 and adding (2) we have

 $2Lpi_1 + 2Ri_1 + R_1i_1 + \frac{1}{Cp}i_2 = E$  .....(9)

Differentiating (2) with respect to t, d we have

$$2L \frac{d^{2}i}{dt^{2}} + (2R + R_{1})\frac{di}{dt} + \frac{i}{c^{2}} = 0 \dots (10)$$
  
so that  $\frac{d2i}{dt^{2}} = -\left[ (2R + R)\frac{di}{dt^{2}} + \frac{i}{c^{2}} \right] \dots (11)$   
$$2L$$

Already having the value of  $\left(\frac{di}{dt}^2\right)_0$  and noting that

 $i_2 = 0$ , when t = 0,  $\frac{d^2 i_1}{dt^2}$  can be found.

For finding  $\frac{di_2}{dt}$  we know that the rise in line current at t = 0 divides equally between the two parts of the transformer primary so that  $\frac{di_2}{dt} = \frac{1di_1}{2}$ . The

third condition for the solution of i 2 is gathered from the fact that the charge on the condenser initially is zero and hence

In the integration of (12) a constant will appear which is simply the ultimate charge on the condenser. Neglecting resistance drops this will be CE, and, being in opposition to the direct supply voltage it will be positive in sign so that

$$\int \mathbf{i}_2 d\mathbf{t} = -CE$$

To sum up:

When 
$$t = 0$$
,  $i_1 = 0$   

$$\frac{di_1}{dt} = \frac{\mathbf{E}}{\mathbf{L}}$$

$$d^2 \mathbf{i}_1 = -\frac{\left[(2\mathbf{R} + \mathbf{R}_1)\mathbf{E}\right]}{2\mathbf{L}}$$

$$i_2 = 0$$

$$\frac{di_2}{dt} = \frac{\mathbf{E}}{2\mathbf{L}}$$

$$\int \mathbf{i}_2 dt = -\mathbf{C}\mathbf{E}$$

$$\mathbf{I}_{SS} (i_1) = \frac{\mathbf{E}}{\mathbf{R} + \mathbf{R}_1}$$

For the circuit constants shown in Figure 18 the two sets of simultaneous equations are:

$$K_1 + K_2 \sin \Theta = -20.25$$
 .....(14)

-2.06 K<sub>1</sub> - 552 K<sub>2</sub>sin  $\Theta$  + 55.0 K<sub>2</sub> cos $\Theta$  = 300...(15)

 $4.24K_1 + 301,500 K_2 \sin \theta - 60,600 K_2 \cos \theta = -3720 \dots$  (16)

 $K_{1} + K_{2}\sin \theta = 0 \qquad ....(17)$   $-2.06 K_{1} - 552K_{2}\sin \theta + 55.0 K_{2}\cos \theta = 150 \qquad ....(18)$   $-K_{1} - K_{2} (-552 \sin \theta -55.0 \cos \theta) = .00009 \qquad ....(19)$   $\frac{K_{2}}{552^{2}} + 55.0^{2}$ 

Solving the above we have:

 $i_{1} = 20.25 - 19.33e^{-2.06t} + .59e^{-552t} \sin(55t + 11.5^{\circ}) \dots (20)$  $i_{2} = 2.73e^{-552t} \sin 55t \dots (21)$ 

These are plotted in figures 20, and 21, where the output frequency is arbitrarily taken as 62.5 cycles/sec. so that one-half cycle takes .008 seconds.

Calculation of the Currents during the Second Half-Cycle.

When the second half-cycle starts conditions are different from those of the first half-cycle. The instant tube B starts the condenser voltage adds to that of the d.-c. supply and full line voltage again appears across the line inductance (see figure 13).

 $i_1$ , however, does not start from zero. Its magnitude is 1.2 amperes; i.e., the current that is flowing through tube A when tube B starts to conduct at t = .008 seconds for the first half cycle and t = 0for the second half cycle.

For the third condition; i. e., the value of  $\frac{di}{dt}$ , we find the initial conditions contradictory and not in accordance with the fundamental differential equations. These were set up on the basis of one tube only being conducting. Hence, with the condenser fully charged, either aiding or bucking the direct voltage,  $i_2$  must be nero at t = 0. Conversely, if  $i_2$  has some value the condenser cannot possibly be fully charged.

The third initial condition for finding  $i_1$  then, based on the initial value of  $i_2$  (see equation (11) ) is useless. Similarly, the initial conditions for finding  $i_2$  are useless. The key to the situation is of course the fact that all the current through tube A at t = .008seconds is not abruptly transferred to the condenser. Some of the current is transferred at once but until some time before the positive ion cleanup is complete there is a current flow in tube A. At the instant de-ionization is complete and the voltage across the condenser passes through zero, the condenser only carries current. For purposes of analysis then, we cannot assume the action of the tubes to correspond to that of a synchronous switch and what happens during the de-ionization period cannot be calculated on this basis.

## Calculation of d.-c. Line Current during the Second Half Cycle.

The fact that the initial conditions for finding i<sub>1</sub> are contradictory does not preclude its calculation. i<sub>1</sub> can be calculated accurately. The oscillograms show that after the starting half-cycle all others are alike, so that the initial conditions, as far as i<sub>1</sub> is concerned, for the second half-cycle must be the same as for the third. Since at the end of the starting half-cycle, only

one component of  $i_1$ ; viz.,  $(20.25 - 19.33e^{-2.06t})$  still has a value we conclude that this component of i is the same for <u>all</u> half-cycles. Knowing il and  $\frac{dil}{dt}$  at t = 0, the other component of the form  $Ke^{-552t}sin(55t+2an)$ be found. For the circuit under consideration we have:

At t = 0

Solution of (22) and (23) gives

 $i_1 = 8.26 e^{-552t}sin(55t + 1.9^{\circ})$  .....(24) (as a component of  $i_1$ )

and the complete form of il hence is:

 $20.25 - 19.33 e^{-2.06t} + 8.26e^{-552t} sin (55t + 0)....(25)$ 

This is plotted in figures 20 and 21.

#### Calculation of other currents during Second Half-Cycle.

In order to form a picture of the current com-

ponents in the circuit during the second half-cycle two calculations were made. They are shown by figures 20 and 21.

In figure 21, the assumption was made that  $i_2$  (the condenser current) starts from zero. It will of course have the same form as it had during the first half-cycle with twice the amplitude since the condenser is initially charged, aiding the direct voltage. It was further assumed that the current in tube A dies down rapidly and becomes zero at the instant the charge on the condenser is zero.

In figure 20, the assumption was made that the condenser at the end of the first half cycle takes over all the current that has been flowing in tube A, that is, that the current through tube A stops abruptly and that it has the form  $K \in \frac{-552t}{\sin}(55t+\theta)$  and for our two initial conditions we have that  $i_{\perp} = 1.20$  amperes and  $\int i_{\perp} dt = -CE$ .

We hence solve

 $-552 \text{ K sin}\Theta + 55.0 \text{ K cos }\Theta = -55.3 \dots (27)$ 

and find  $i_2 = -11.2 e^{-552t} sin (55t - 6.2^{\circ}) \dots (28)$ 

We find that  $i_{2}$  decreases sharply, comes to zero at .002 seconds and becomes negative, as shown by the dotted line (figure 20). The negative portion of course does not actually exist to the extent shown, because it means that during the rest of the second half-cycle the condenser discharges and charges again, which we know is not the case. The physical interpretation of the negative loop of  $i_{2}$  seems to be that the current was brought in initially from some outside source (in this case tube A), and, after the condenser is charged at t = .002 seconds, it will tend to discharge again. Since tube A has stopped conducting some time prior to t = .002 seconds,  $i_{2}$  must be discontinuous at this point.

#### Results of Analysis.

It has already been shown that the current abruptly through the outgoing tube cannot cease/with the condenser capacity assumed. At the beginning of the second halfcycle it has some value and it takes a short time to come to zero. Since in the circuit under consideration no oscillograms were taken of the components calculated, there



Circuit of Fig. 18.

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are no means of checking the calculations. The actual case will be somewhere between the components shown by figure 20 and those shown by figure 21 with a tendency more towards those of the  $\frac{former}{latter}$ . Since this is the extreme case; i. e., the shortest possible de-ionization period available, the analysis provides a means of checking up on probable tube performance.

That the actual oscillograms of the line current merely resemble the calculated current is no doubt due to the non-linearity of the circuit parameters and is to be expected in work of this nature.

#### The Effect of Leakage Reactance in the Transformer.

The transformer used was an ordinary distribution transformer with small leakage reactance. If leakage reactance is taken into consideration in the analysis it would merely result in the addition of small high frequency currents corresponding to the oscillating frequency of the leakage reactance and the condenser capacity. These currents and their induced voltages are noticeable in all oscillograms, where the lines are thick. Figures 13 and 14, however, show the high frequency voltage across the transformer primary very clearly. The amplitude of the high frequency voltage is small and no harm is done. If an ordinary small leakage reactance transformer is used, leakage reactance can be neglected both as far as as calculations and as far/its effects on operation are concerned.

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#### V. DESIGN OF THE MAIN CIRCUIT.

#### The Inverter not an Oscillator.

Although the inverter supplies alternating current, it is not comparable to an oscillator. The name Repeated Transient Inverter implies that sine waves do not exist and hence the analysis, that has barely been started in this report, is a new game. It is not played according to the rules applying to oscillators. In the design of an oscillator the first step is to see that the circulating energy is large enough, the required minimum ratio of volt-amperes to watts being 4 • The mechanical problem of determining the correct flywheel size for an engine is analogous. After this has been assured the calculations of the proper voltages and proper phase relations of grid and anode voltages must be undertaken.

An inverter can be made to oscillate wither by designing the circuit on an oscillatory basis or by supplying a synchronous load. The latter method is by far the more practical and economical and represents the ideal case.

The design of the circuit under consideration

can be started directly by the calculation of proper voltages, phase angles do not exist, and the grid circuit can be considered as a separate problem.

#### Voltage and Current Relations.

The relation between the d.-c. supply voltage and the a.-c. output voltage determines the ratio of transformation of the transformer, which is simply  $\frac{2E_{3.c.}}{E_{a.c}}$ Provisions must be made for a mid tap in the primary side. Since each half of the primary carries all the load current for one-half the time its rating must be for

## secondary amps.(full load)

For the inverter, then, similar to the case of the full wave rectifier the transformer must be of special design. The current rating of its primary must be 2 times that of the secondary.

#### VOLtage Regulation.

From the standpoint of most convenient voltage

regulation, which involves the suppression of the output voltage at light load the ratio of line inductance to condenser capacity must be a maximum. On the other hand, most successful operation demands an adequate period for de-ionization of the tubes. Maximum de-ionization time is attained with sine waves of voltage which means a large ratio of capacitance to transformer inductance. For an inverter of the type considered here the transformer has no less than ordinary inductance and the regulation characteristic desired must be the most convenient so that the line reactor size is determined by economical considercharacteristics ations and the capacitance a function of the tube; i. e., the condenser must be just large enough to provide sufficient de-ionization time. More cannot be said as regards these elements in the design of the circuit since the calculations of Chapter IV lack experimental check. There is no doubt but that with adequately checked analyses along these lines a firm basis for the design of the circuit can be established.

#### Grid Voltage.

With the regulation characteristic the most con-

venient, the de-ionization time is a minimum. This positive ion cleanup process is accomplished not only by means of a negative voltage on the anode but also by a negative voltage on the grids. Hence the grid voltage must be made the maximum allowable without jeopardizing the life span of the tubes. However, here again there is a limitation; viz., the fact that the voltage rise at light load, especially with an oscillatory grid cirmay cuit,/be excessive. Hence the maximum grid voltage should be based on the voltage at the minimum load.

# Inductive Load and the Effects of Filters in Lines with the Load.

An inverter of this type cannot handle inductive loads unless the condenser capacity is increased to compensate for the load inductance. The tubes cannot supply lagging current. The same applies to the insertion of a filter in series with the load, except that in this case the condenser must be increased only sufficiently to take care of the harmonics. The effects of a filter in series with the load can be analyzed as in Chapter IV with the additional circuit constants added to the differential equations.

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#### VI. GRID CIRCUIT DESIGN.

#### Introduction: Action Explained by Means of Analysis.

In the "ideal" inverter circuit, where sine waves af voltage are maintained on the output side the excitation of the grids does not represent a serious problem. The grids are supplied by the load through a transformer that has a condenser connected in series with its primary to cause the necessary phase shift of grid voltage with respect to that of the anode. This case is analogous to that of a steam engine, with a flywheel and a governor. The valves move in synchronism with the piston and may be advanced or retarded in phase.

The valves in our duplex boiler feed pump are not permanently connected to the piston. They are merely respectively tripped over at the end of each stroke of "the other" piston; the valves of piston A are tripped by rod B and vice versa. The analogy between the inverter and the feed pump, however, holds only in as far as the "main" circuit is concerned. In order to include the grid circuit we must modify our pump. Let the valves be controlled by a "gadget" that is "charged up" (a spring

is compressed or a pendulum is released, to start swinging) when one of the pistons starts its motion. Also imagine that by the time this piston reaches the end end of the stroke the compressed spring is released by a timing device or the pendulum reaches a point in its swing where it acts on the valves of the other piston to start it on its way. The analogy is crude, but the point we are trying to make is that in our modified pump, as in the inverter, the valves are not positively tripped at the end of the stroke. They are acted upon or "set" when one piston starts on its stroke, to trip automatically at the end of the stroke. Each piston rod at the start of its motion "sets" the valves of the other.

When the inverter starts, the grid of the starting tube is made positive and that of the non-starting tube negative. Due to the charging up of the blocking condenser in the rid circuit both waves die down, the first becomes more negative and the second more positive. As soon as the grid voltage of the incoming tube reaches the critical grid potential, the tube starts. There is thus no <u>positive action</u> that starts the tubes and no oscillation in the main circuit that determines the fre-

quency. The latter simply depends on the constants of the grid circuit.

#### The Ideal Grid Voltage Wave-shape.

It has already been pointed out that in inverter experimentation separate excitation from an a.-c. source is frequently employed to excite the grids. In other words, the inverter is "driven". When the grid is made positive with respect to the cathode the tube conducts, while it is prevented from starting to conduct, when negative. At the beginning of the negative half-cycle of the grid voltage wave, conduction is stopped by transferring the tube current through a condenser and making the anode, for a brief space of time, negative with respect to the cathode.

If the critical grid potentials of both tubes have exactly the same value each tube will conduct for exactly the same length of time, provided the a.-c. voltage wave applied is symmetrical with respect to the time axis. If, however, the critical grid potentials are not the same (this has been stressed before) and the grid voltage does not have a high rate of change at the time



it passes through the critical value (getting more positive) one tube will conduct for a longer period than the other with the result that the main transformer will become saturated and operation will cease. In other words, with unequal conducting periods, the symmetry of the circuit will be upset.

The requirement for the grid voltage wave then is that, when it reverses, it must have a high rate of change. Grid characteristics sometimes differ and to eliminate any effects they may have on inverter stability, the ideal voltage wave should be rectangular (or square). Reversal would take place in zero time and no matter how far apart the critical grid potentials are, provided they do not exceed the amplitude of the applied rectangular wave, the control will be as good as can be desired.

#### Non-oscillatory and Oscillatory Grid Circuits.

The most convenient source of grid power (especially in high voltage applications) obviously is the output side of the main transformer. It is alterna-



ting and rectangular in shape. However, as far as the grids are concerned it must be regarded as a direct voltage, the polarity of which is reversed every halfcycle by the action of the grids on the main circuit.

We have now reached the point where we must design the "gadget" for our modified boiler feed pump. The simplest circuit possible is indicated by figure 24(a), which is simply a dondenser and resistance in series. The drop across the latter is impressed on the grids through other, and much larger, resistances. The wave will be the familiar exponentially decreasing transient and the grid voltages will be one half-cycle out of phase with respect to the resistance mid-point; i. e., when one grid is positive the other is negative to the same extent.

As the condenser charges up the current through the resistance approaches zero and the grid voltage also approaches zero until the critical grid potential of the tube next to strike is reached. When it strikes the polarity of our d.-c. supply (inverter output) reverses and the process repeats.

In the light of what has been said before in regard to the possible difference in critical grid potentials of the two tubes figures 24(b) and (c) are selfexplanatory. They are developments of 24 (a) and show how a difference in critical grid potentials with an oscillatory circuit will tend to decrease instability resulting from a longer period of conduction in one tube than in the other.

#### Remarks in Connection with Grid Circuit Design.

In most of the experimental work in connection with inverter under consideration a critically damped circuit was used. In other words inductance played a minor part. The step-up transformer was necessary to obtain an adequate voltage. For obtaining an oscillatory wave, the capacitance was increased and the inductance decreased. The latter was accomplished simply by the addition of a reactor in parallel with the grid transformer primary. For purposes of calculation it is merely necessary to tune the capacitance and the inductance in series to the desired

frequency and to make the damping small. The oscillatory wave used is shown by figure 17 and the circuit constants indicated in figure 25.

The advent of shielded grid thyratrons may eliminate the necessity of using an oscillatory grid circuit for this type of inverter. At the time this investigation was carried on shielded grid tubes were not available, and a conducting coat of paint was used on the bulb of the tubes. With a shielded grid the tube characteristics should be much more uniform, for then the charge on the glass is prevented from having any influence on the action of the tube.

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Fig. 25. Oscillatory Grid Circuit Employed with that of Fig. 8. For the Waveshapes of this Circuit see Fig.17. (H-4359151)

#### VII. DISCUSSION OF RESULTS AND CONCLUSIONS.

#### Discussion of Results:

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In the foregoing the development of a static circuit, utilizing thyratrons, has been described and the way for analysis of the circuit pointed out.

The success of the circuit is due to one important factor; viz., the fact that the latest type hot cathode thyratrons have a very short de-ionization period, much shorter than the former mercury pool type tubes.

The analysis shows some of the fifficulties involved in analyzing circuits with vacuum tubes, difficulties that will have to be overcome with the advent of the high voltage direct current transmission line. Inverters of the type described will no doubt some day form an important adjunct to the much-talked-of new development.

As a result of experimentation a specific circuit is described and its performance characteristics

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given, while as a result of analysis the probable waveshapes of currents in the circuit are calculated. Here we say "probable", because the experimental data were not adequate to provide a complete check.

The inverter serves the same purpose as a motorgenerator set over which it, however, has a number of advantages in the form of greater efficiency and much lower first cost. These two factors are enhanced the higher the direct voltage goes.

### Conclusions:

(1) With present-day hot cathode tubes static inversion of direct current is a feasible and economical

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proposition, even with no alternating current available to drive the grids and the circuit not designed on an oscillatory basis.

(2) The mathematical analysis of power circuits containing vapor tubes cannot be accomplished successfully unless assisted by full physical pictures of actual conditions, but with the physics firmly in hand a rigorous analysis is possible.

## Suggestions for Further Research.

The problem attacked in this investigation is an almost ideal one for the integraph. But before it is attacked it will be necessary to obtain more oscillograms of the currents in the main part of the circuit during the commutation period in order to ascertain physically the true state of affairs.

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#### REFERENCES

## (1) Thyratron Tubes and Their Characteristics.

- (a) "Hot Cathode Thyratrons", by A. W. Hull, Gen. Elec. Rev., 32, 213 and 390 (1929).
- (b) "Characteristics of Mercury Arcs" by W. B. Nottingham, Journal of Frank. Inst., 211, 271 (1932)

# (2) <u>Inverters</u>

- (a) "The Inverter" by D. C. Prince, Gen. Elec. Rev., 28, 676 (1925)
- (b) "The Direct-Current Transformer" by D. C. Prince, Gen. Elec. Rev. 31, 347 (1928).
- (3) Analysis of Circuits Containing Mercury Vapor Rectifiers.

"The analysis of Polyphase Rectifier Circuits" by Daniel Silverman, M. I. T. Doctor's Thesis, 1930.